# Chapter 8

# Analytic Trigonometry

## 8.8 Trigonometric Equations (II)

1. $2\cos^2\theta + \cos\theta = 0 \to \cos\theta(2\cos\theta + 1) = 0$

$\cos\theta = 0 \quad \to \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

or $2\cos\theta + 1 = 0$

$2\cos\theta = -1 \to \cos\theta = -\frac{1}{2} \to \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

2. $\sin^2\theta - 1 = 0 \to (\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta + 1 = 0 \to \sin\theta = -1 \to \theta = \frac{3\pi}{2}$

or $\sin\theta - 1 = 0 \to \sin\theta = 1 \to \theta = \frac{\pi}{2}$

3. $2\sin^2\theta - \sin\theta - 1 = 0 \to (2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta + 1 = 0 \to 2\sin\theta = -1 \to \sin\theta = -\frac{1}{2} \to \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

or $\sin\theta - 1 = 0 \to \sin\theta = 1 \to \theta = \frac{\pi}{2}$

4. $2\cos^2\theta + \cos\theta - 1 = 0 \to (\cos\theta + 1)(2\cos\theta - 1) = 0$

$\cos\theta + 1 = 0 \to \cos\theta = -1 \to \theta = \pi$

or $2\cos\theta - 1 = 0 \to 2\cos\theta = 1 \to \cos\theta = \frac{1}{2} \to \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

5. $(\tan\theta - 1)(\sec\theta - 1) = 0$

$\tan\theta - 1 = 0 \to \tan\theta = 1 \to \theta = \frac{\pi}{4}, \frac{5\pi}{4}$

or $\sec\theta - 1 = 0 \to \sec\theta = 1 \to \theta = 0$

6. $(\cot\theta + 1)\left(\csc\theta - \frac{1}{2}\right) = 0$

$\cot\theta + 1 = 0 \to \cot\theta = -1 \to \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

or $\csc\theta - \frac{1}{2} = 0 \to \csc\theta = \frac{1}{2}$, which is impossible

the solutions are $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

7. $\sin\theta - \cos^2\theta = 1 + \cos\theta$

8. $\cos^2\theta - \sin^2\theta + \sin\theta = 0$

$$\left(1-\sin^2\theta\right) - \sin^2\theta + \sin\theta = 0 \rightarrow 1 - 2\sin^2\theta + \sin\theta = 0$$

$$2\sin^2\theta - \sin\theta - 1 = 0 \rightarrow \left(2\sin\theta + 1\right)\left(\sin\theta - 1\right) = 0$$

$$2\sin\theta + 1 = 0 \rightarrow \sin\theta = -\frac{1}{2} \rightarrow \theta = \frac{\pi}{6}, \frac{7\pi}{6}$$

$$\text{or } \sin\theta - 1 = 0 \rightarrow \sin\theta = 1 \rightarrow \theta = \frac{\pi}{2}$$

9. $\sin^2\theta = 6\left(\cos\theta + 1\right)$

$$1 - \cos^2\theta = 6\left(\cos\theta + 1\right) \rightarrow 1 - \cos^2\theta = 6\cos\theta + 6$$

$$\cos^2\theta + 6\cos\theta + 5 = 0 \rightarrow \left(\cos\theta + 5\right)\left(\cos\theta + 1\right) = 0$$

$$\cos\theta + 5 = 0 \rightarrow \cos\theta = -5, \text{ which is impossible}$$

$$\text{or } \cos\theta + 1 \rightarrow \cos\theta = -1 \rightarrow \theta = \pi$$

the solution is $\theta = \pi$.

10. $2\sin^2\theta = 3\left(1 - \cos\theta\right)$

$$2\left(1-\cos^2\theta\right) = 3\left(1 - \cos\theta\right) \rightarrow 2 - 2\cos^2\theta = 3 - 3\cos\theta$$

$$2\cos^2\theta - 3\cos\theta + 1 = 0 \rightarrow \left(2\cos\theta - 1\right)\left(\cos\theta - 1\right) = 0$$

$$2\cos\theta - 1 = 0 \rightarrow \cos\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\text{or } \cos\theta - 1 = 0 \rightarrow \cos\theta = 1 \rightarrow \theta = 0$$

11. $\cos\left(2\theta\right) + 6\sin^2\theta = 4$

$$1 - 2\sin^2\theta + 6\sin^2\theta = 4 \rightarrow 4\sin^2\theta = 3 \rightarrow \sin^2\theta = \frac{3}{4} \rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

12. $\cos\left(2\theta\right) = 2 - 2\sin^2\theta$

$$1 - 2\sin^2\theta = 2 - 2\sin^2\theta \rightarrow 0 = 1, \text{ which is impossible}$$

therefore the equation has no real solution.

13.
$$\cos\theta = \sin\theta$$
$$\frac{\sin\theta}{\cos\theta} = 1 \rightarrow \tan\theta = 1$$
$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

14.
$$\cos\theta + \sin\theta = 0$$
$$\sin\theta = -\cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = -1 \rightarrow \tan\theta = -1$$
$$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$$

15.
$$\tan\theta = 2\sin\theta$$
$$\frac{\sin\theta}{\cos\theta} = 2\sin\theta$$
$$\sin\theta = 2\sin\theta\cos\theta$$
$$0 = 2\sin\theta\cos\theta - \sin\theta$$
$$0 = \sin\theta(2\cos\theta - 1)$$
$$2\cos\theta - 1 = 0 \rightarrow \cos\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$
$$\text{or } \sin\theta = 0 \rightarrow \theta = 0, \pi$$

16.
$$\sin(2\theta) = \cos\theta$$
$$2\sin\theta\cos\theta = \cos\theta$$
$$2\sin\theta\cos\theta - \cos\theta = 0$$
$$(\cos\theta)(2\sin\theta - 1) = 0$$
$$\cos\theta = 0 \rightarrow \cos\theta = 0 \rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$
$$\text{or } 2\sin\theta = 1 \rightarrow \sin\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6}. \frac{5\pi}{6}$$

17.
$$\sin\theta = \csc\theta$$
$$\sin\theta = \frac{1}{\sin\theta}$$
$$\sin^2\theta = 1 \rightarrow \sin\theta = \pm 1$$
$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

18.
$$\tan\theta = \cot\theta$$
$$\tan\theta = \frac{1}{\tan\theta}$$
$$\tan^2\theta = 1 \rightarrow \tan\theta = \pm 1$$
$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

19.
$$\cos(2\theta) = \cos\theta$$
$$2\cos^2\theta - 1 = \cos\theta \rightarrow 2\cos^2\theta - \cos\theta - 1 = 0$$
$$(2\cos\theta + 1)(\cos\theta - 1) = 0$$
$$2\cos\theta + 1 = 0 \rightarrow \cos\theta = -\frac{1}{2} \rightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$
$$\text{or } \cos\theta - 1 = 0 \rightarrow \cos\theta = 1 \rightarrow \theta = 0$$

20.
$$\sin(2\theta)\sin\theta = \cos\theta$$
$$2\sin\theta\cos\theta\sin\theta = \cos\theta \rightarrow 2\sin^2\theta\cos\theta - \cos\theta = 0$$
$$(2\sin^2\theta - 1)\cos\theta = 0$$
$$2\sin^2\theta - 1 = 0 \rightarrow 2\sin^2\theta = 1 \rightarrow \sin^2\theta = \frac{1}{2} \rightarrow \sin\theta = \pm\frac{\sqrt{2}}{2}$$
$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$
$$\text{or } \cos\theta = 0 \rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

21. $\sin(2\theta)+\sin(4\theta)=0$

$\sin(2\theta)+2\sin(2\theta)\cos(2\theta)=0 \rightarrow \sin(2\theta)\left(1+2\cos(2\theta)\right)=0$

$$1+2\cos(2\theta)=0 \rightarrow \cos(2\theta)=-\frac{1}{2} \rightarrow 2\theta=\frac{2\pi}{3}+2k\pi \rightarrow \theta=\frac{\pi}{3}+k\pi$$

$$2\theta=\frac{4\pi}{3}+2k\pi \rightarrow \theta=\frac{2\pi}{3}+k\pi$$

$$\text{or } \sin(2\theta)=0 \rightarrow 2\theta=0+2k\pi \rightarrow \theta=k\pi$$

$$2\theta=\pi+2k\pi \rightarrow \theta=\frac{\pi}{2}+k\pi$$

$\theta=0,\ \frac{\pi}{3},\frac{\pi}{2},\frac{2\pi}{3},\pi,\frac{4\pi}{3},\frac{3\pi}{2},\frac{5\pi}{3}$

22. $\cos(2\theta)+\cos(4\theta)=0$

$2\cos(3\theta)\cos(-\theta)=0 \rightarrow 2\cos(3\theta)\cos\theta=0$

$$\cos(3\theta)=0 \rightarrow 3\theta=\frac{\pi}{2}+2k\pi \rightarrow \theta=\frac{\pi}{6}+\frac{2k\pi}{3}$$

$$3\theta=\frac{3\pi}{2}+2k\pi \rightarrow \theta=\frac{\pi}{2}+\frac{2k\pi}{3}$$

$$\text{or } \cos\theta=0 \rightarrow \theta=\frac{\pi}{2}+2k\pi$$

$$\theta=\frac{3\pi}{2}+2k\pi$$

$\theta=\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6},\frac{3\pi}{2},\frac{7\pi}{6},\frac{11\pi}{6}$

23. $\cos(4\theta)-\cos(6\theta)=0$

$\cos\left(5\theta-\theta\right)-\cos\left(5\theta+\theta\right)=0 \rightarrow -2\sin(5\theta)\sin(-\theta)=0$

$$2\sin\left(5\theta\right)\sin\theta=0$$

$$\sin(5\theta)=0 \rightarrow 5\theta=0+2k\pi \rightarrow \theta=\frac{2k\pi}{5}$$

$$5\theta=\pi+2k\pi \rightarrow \theta=\frac{\pi}{5}+\frac{2k\pi}{5}$$

$$\text{or } \sin\theta=0 \rightarrow \theta=0+2k\pi$$

$$\theta=\pi+2k\pi$$

$\theta=0,\ \frac{\pi}{5},\frac{2\pi}{5},\frac{3\pi}{5},\frac{4\pi}{5},\pi,\frac{6\pi}{5},\frac{7\pi}{5},\frac{8\pi}{5},\frac{9\pi}{5}$

24. $\sin(4\theta)-\sin(6\theta)=0$

$$2\sin(-\theta)\cos(5\theta)=0 \to -2\sin\theta\cos(5\theta)=0$$

$$\cos(5\theta)=0 \to 5\theta=\frac{\pi}{2}+2k\pi \to \theta=\frac{\pi}{10}+\frac{2k\pi}{5}$$

$$5\theta=\frac{3\pi}{2}+2k\pi \quad \to \quad \theta=\frac{3\pi}{10}+\frac{2k\pi}{5}$$

$$\text{or } \sin\theta=0 \to \theta=0+2k\pi$$

$$\theta=0+2k\pi$$

$$\theta=0,\frac{\pi}{10},\frac{3\pi}{10},\frac{\pi}{2},\frac{7\pi}{10},\frac{9\pi}{10},\pi,\frac{11\pi}{10},\frac{13\pi}{10},\frac{3\pi}{2},\frac{17\pi}{10},\frac{19\pi}{10}$$

25. $$1+\sin\theta=2\cos^2\theta$$

$$1+\sin\theta=2(1-\sin^2\theta) \to 1+\sin\theta=2-2\sin^2\theta$$

$$2\sin^2\theta+\sin\theta-1=0 \to (2\sin\theta-1)(\sin\theta+1)=0$$

$$2\sin\theta-1=0 \to \sin\theta=\frac{1}{2} \to \theta=\frac{\pi}{6},\frac{5\pi}{6}$$

$$\text{or } \sin\theta+1=0 \to \sin\theta=-1 \to \theta=\frac{3\pi}{2}$$

26. $$\sin^2\theta=2\cos\theta+2$$

$$1-\cos^2\theta=2\cos\theta+2$$

$$\cos^2\theta+2\cos\theta+1=0 \to (\cos\theta+1)^2=0 \to \cos\theta+1=0$$

$$\cos\theta=-1 \to \theta=\pi$$

27. $$\tan^2\theta=\frac{3}{2}\sec\theta$$

$$\sec^2\theta-1=\frac{3}{2}\sec\theta \to 2\sec^2\theta-2=3\sec\theta$$

$$2\sec^2\theta-3\sec\theta-2=0 \to (2\sec\theta+1)(\sec\theta-2)=0$$

$2\sec\theta+1=0 \to \sec\theta=-\frac{1}{2}$, which is impossible

$$\text{or } \sec\theta-2=0 \to \sec\theta=2 \to \theta=\frac{\pi}{3},\frac{5\pi}{3}$$

the solutions are $\theta=\frac{\pi}{3},\frac{5\pi}{3}$

28. $$\csc^2\theta=\cot\theta+1$$

$1 + \cot^2\theta = \cot\theta + 1 \rightarrow \cot^2\theta - \cot\theta = 0$

$\cot\theta(\cot\theta - 1) = 0$

$\cot\theta = 0 \rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

or $\cot\theta = 1 \rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$

29. $3 - \sin\theta = \cos(2\theta)$

$3 - \sin\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta - \sin\theta + 2 = 0$

This is a quadratic equation in $\sin\theta$. The discriminant is $b^2 - 4ac = 1 - 16 = -15 < 0$. The equation has no real solutions.

30. $\cos(2\theta) + 5\cos\theta + 3 = 0$

$2\cos^2\theta - 1 + 5\cos\theta + 3 = 0 \rightarrow 2\cos^2\theta + 5\cos\theta + 2 = 0$

$(2\cos\theta + 1)(\cos\theta + 2) = 0$

$2\cos\theta = -1 \rightarrow \cos\theta = -\frac{1}{2} \rightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

or $\cos\theta = -2$, which is impossible

the solutions are $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

31. $\sec^2\theta + \tan\theta = 0$

$\tan^2\theta + 1 + \tan\theta = 0$

This is a quadratic equation in $\tan\theta$. The discriminant is $b^2 - 4ac = 1 - 4 = -3 < 0$. The equation has no real solutions.

32. $\sec\theta = \tan\theta + \cot\theta$

$$\frac{1}{\cos\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \rightarrow \frac{1}{\cos\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$\frac{1}{\cos\theta} = \frac{1}{\sin\theta\cos\theta} \rightarrow \frac{\sin\theta\cos\theta}{\cos\theta} = 1 \rightarrow \sin\theta = 1 \rightarrow \theta = \frac{\pi}{2}$$

Since $\sec\left(\frac{\pi}{2}\right)$ and $\tan\left(\frac{\pi}{2}\right)$ do not exist, there is no real solution.

33. $\sin\theta - \sqrt{3}\cos\theta = 1$

Divide each side by 2:

$$\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta = \frac{1}{2}$$

Rewrite in the difference of two angles form where

$\cos\phi = \frac{1}{2}$ and $\sin\phi = \frac{\sqrt{3}}{2}$ and $\phi = \frac{\pi}{3}$:

$$\sin\theta\cos\phi - \cos\theta\sin\phi = \frac{1}{2} \rightarrow \sin(\theta - \phi) = \frac{1}{2}$$

$$\theta - \phi = \frac{\pi}{6} \quad \text{or} \quad \theta - \phi = \frac{5\pi}{6}$$

$$\theta - \frac{\pi}{3} = \frac{\pi}{6} \quad \text{or} \quad \theta - \frac{\pi}{3} = \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{2} \quad \text{or} \quad \theta = \frac{7\pi}{6}$$

34. $\sqrt{3}\sin\theta + \cos\theta = 1$

Divide each side by 2:

$$\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta = \frac{1}{2}$$

Rewrite in the sum of two angles form where

$$\cos\phi = \frac{\sqrt{3}}{2} \text{ and } \sin\phi = \frac{1}{2} \text{ and } \phi = \frac{\pi}{6}:$$

$$\sin\theta\cos\phi + \cos\theta\sin\phi = \frac{1}{2} \rightarrow \sin(\theta + \phi) = \frac{1}{2}$$

$$\theta + \phi = \frac{\pi}{6} \quad \text{or} \quad \theta + \phi = \frac{5\pi}{6}$$

$$\theta + \frac{\pi}{6} = \frac{\pi}{6} \quad \text{or} \quad \theta + \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\theta = 0 \quad \text{or} \quad \theta = \frac{2\pi}{3}$$

35. $\tan(2\theta) + 2\sin\theta = 0$

$$\frac{\sin(2\theta)}{\cos(2\theta)} + 2\sin\theta = 0$$

$$\frac{\sin 2\theta + 2\sin\theta\cos 2\theta}{\cos 2\theta} = 0 \rightarrow 2\sin\theta\cos\theta + 2\sin\theta(2\cos^2\theta - 1) = 0$$

$$2\sin\theta\left(\cos\theta + 2\cos^2\theta - 1\right) = 0 \rightarrow 2\sin\theta\left(2\cos^2\theta + \cos\theta - 1\right) = 0$$

$$2\sin\theta(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$$2\cos\theta - 1 = 0 \rightarrow \cos\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\text{or} \quad 2\sin\theta = 0 \rightarrow \sin\theta = 0 \rightarrow \theta = 0, \pi$$

$$\text{or} \quad \cos\theta + 1 = 0 \rightarrow \cos\theta = -1 \rightarrow \theta = \pi$$

the solutions are $\theta = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

36. $$\tan(2\theta) + 2\cos\theta = 0$$

$$\frac{\sin(2\theta)}{\cos(2\theta)} + 2\cos\theta = 0$$

$$\frac{\sin(2\theta) + 2\cos\theta\cos 2\theta}{\cos(2\theta)} = 0 \rightarrow 2\sin\theta\cos\theta + 2\cos\theta(1 - 2\sin^2\theta) = 0$$

$$2\cos\theta\left(\sin\theta + 1 - 2\sin^2\theta\right) = 0 \rightarrow -2\cos\theta\left(2\sin^2\theta - \sin\theta - 1\right) = 0$$

$$-2\cos\theta(2\sin\theta + 1)(\sin\theta - 1) = 0$$

$$2\sin\theta + 1 = 0 \rightarrow \sin\theta = -\frac{1}{2} \rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

or $-2\cos\theta = 0 \rightarrow \cos\theta = 0 \rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

or $\sin\theta - 1 = 0 \rightarrow \sin\theta = 1 \rightarrow \theta = \frac{\pi}{2}$

the solutions are $\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

37. $\sin\theta + \cos\theta = \sqrt{2}$

Divide each side by $\sqrt{2}$: $\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = 1$

Rewrite in the sum of two angles form where $\cos\phi = \frac{1}{\sqrt{2}}$ and $\sin\phi = \frac{1}{\sqrt{2}}$ and $\phi = \frac{\pi}{4}$:

$$\sin\theta\cos\phi + \cos\theta\sin\phi = 1 \rightarrow \sin(\theta + \phi) = 1$$

$$\theta + \phi = \frac{\pi}{2}$$

$$\theta + \frac{\pi}{4} = \frac{\pi}{2} \rightarrow \theta = \frac{\pi}{4}$$

38. $\sin\theta + \cos\theta = -\sqrt{2}$

Divide each side by $\sqrt{2}$: $\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = -1$

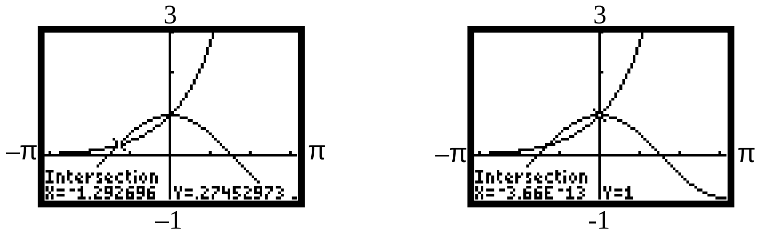
Rewrite in the sum of two angles form where $\cos\phi = \frac{1}{\sqrt{2}}$ and $\sin\phi = \frac{1}{\sqrt{2}}$ and $\phi = \frac{\pi}{4}$:

$$\sin\theta\cos\phi + \cos\theta\sin\phi = -1 \rightarrow \sin(\theta + \phi) = -1$$

$$\theta + \phi = \frac{3\pi}{2}$$

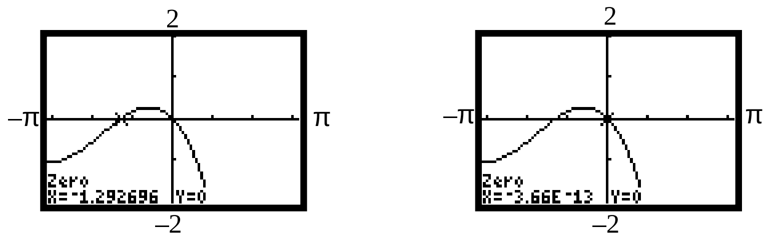
$$\theta + \frac{\pi}{4} = \frac{3\pi}{2} \rightarrow \theta = \frac{5\pi}{4}$$

39. Use INTERSECT to solve:



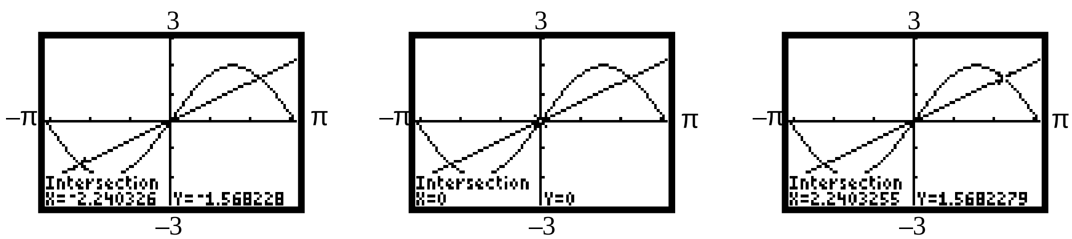

$x \approx -1.29, 0$

40. Use ZERO to solve:



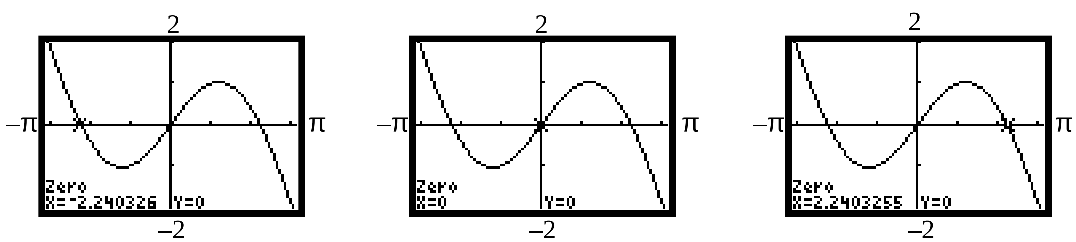

$x \approx -1.29, 0$

41. Use INTERSECT to solve:



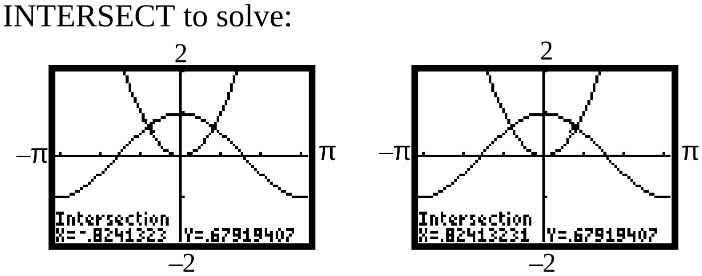

$x \approx -2.24, 0, 2.2$

42. Use ZERO to solve:



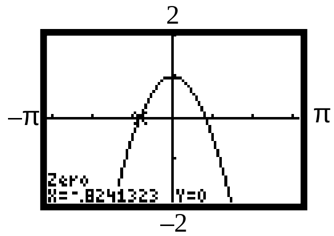

$x \approx -2.24, 0, 2.2$

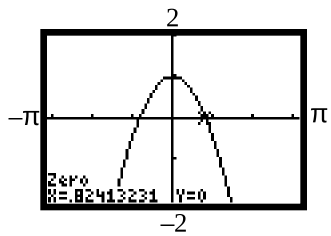
43. Use INTERSECT to solve:




$x \approx -0.82, 0.82$

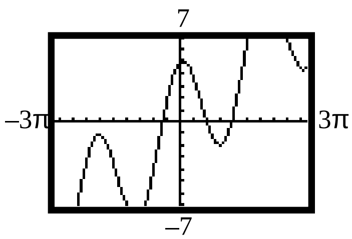
44. Use ZERO to solve:



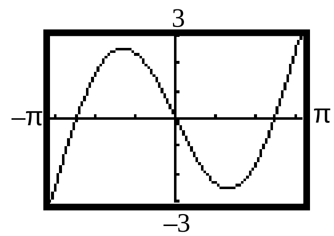

$x \approx -0.82, 0.82$

45. $x + 5\cos x = 0$
Find the intersection of
$y_1 = x + 5\cos x$ and $y_2 = 0$:



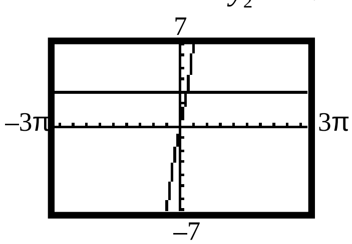

$x \approx -1.31, 1.98, \quad 3.84$

46. $x - 4\sin x = 0$
Find the intersection of
$y_1 = x - 4\sin x$ and $y_2 = 0$:



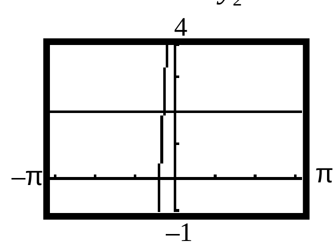

$x \approx -2.47, 0, 2.47$

47. $22x - 17\sin x = 3$
Find the intersection of
$y_1 = 22x - 17\sin x$ and $y_2 = 3$:



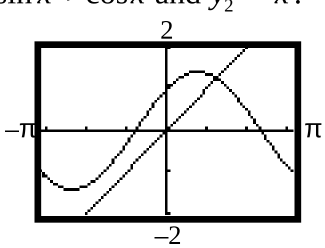

$x \approx 0.52$

48. $19x + 8\cos x = 2$
Find the intersection of
$y_1 = 19x + 8\cos x$ and $y_2 = 2$:



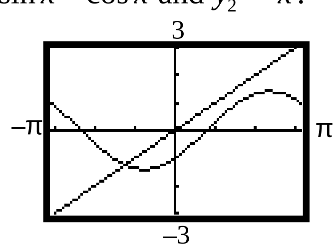

$x \approx -0.30$

49. $\sin x + \cos x = x$
Find the intersection of
$y_1 = \sin x + \cos x$ and $y_2 = x$:



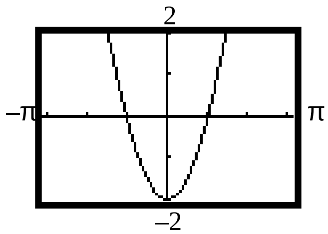

$x \approx 1.26$

50. $\sin x - \cos x = x$
Find the intersection of
$y_1 = \sin x - \cos x$ and $y_2 = x$:



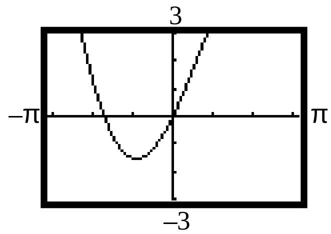

$x \approx -1.26$

51. $x^2 - 2\cos x = 0$
Find the intersection of
$y_1 = x^2 - 2\cos x$ and $y_2 = 0$:



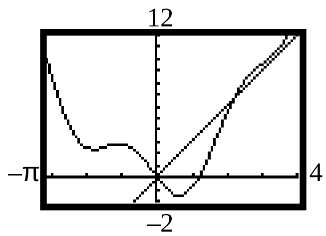

$x \approx -1.02, 1.02$

52. $x^2 + 3\sin x = 0$
Find the intersection of
$y_1 = x^2 + 3\sin x$ and $y_2 = 0$:



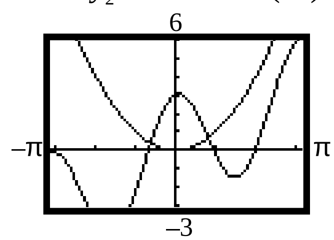

$x \approx -1.72, 0$

53. $x^2 - 2\sin 2x = 3x$
Find the intersection of
$y_1 = x^2 - 2\sin 2x$ and $y_2 = 3x$:



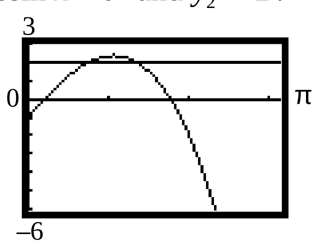

$x \approx 0, 2.15$

54. $x^2 = x + 3\cos(2x)$
Find the intersection of
$y_1 = x^2$ and $y_2 = x + 3\cos(2x)$:



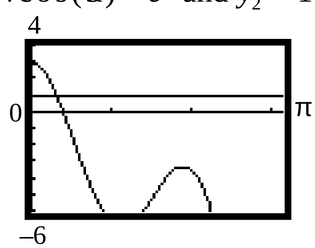

$x \approx -0.62, 0.81$

55. $6\sin x - e^x = 2, \quad x > 0$
Find the intersection of
$y_1 = 6\sin x - e^x$ and $y_2 = 2$:




$x \approx 0.76, 1.35$

56. $4\cos(3x) - e^x = 1, \quad x > 0$
Find the intersection of
$y_1 = 4\cos(3x) - e^x$ and $y_2 = 1$:




$x \approx 0.31$

57. (a) Solve: $\cos(2\theta) + \cos\theta = 0, \quad 0^\circ < \theta < 90^\circ$

$$2\cos^2\theta - 1 + \cos\theta = 0 \rightarrow 2\cos^2\theta + \cos\theta - 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$$2\cos\theta - 1 = 0 \rightarrow \cos\theta = \frac{1}{2} \rightarrow \theta = 60^\circ, 300^\circ$$

$$\text{or } \cos\theta + 1 = 0 \rightarrow \cos\theta = -1 \rightarrow \theta = 180^\circ$$

The solution is $60^\circ$.

(b) Solve: $\cos(2\theta)+\cos\theta=0,\ 0^\circ<\theta<90^\circ$

$$2\cos\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)=0 \rightarrow \cos\left(\frac{3\theta}{2}\right)=0 \text{ or } \cos\left(\frac{\theta}{2}\right)=0$$

$$\frac{3\theta}{2}=90^\circ \rightarrow \theta=60^\circ$$

$$\frac{3\theta}{2}=270^\circ \rightarrow \theta=180^\circ$$

$$\frac{\theta}{2}=90^\circ \rightarrow \theta=180^\circ$$

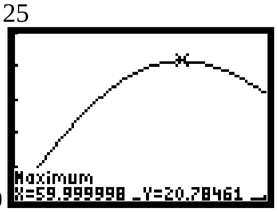
$$\frac{\theta}{2}=270^\circ \rightarrow \theta=540^\circ$$

The solution is 60˚.

(c) $A(60^\circ)=16\sin 60^\circ(\cos 60^\circ+1)=16\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}+1\right)=12\sqrt{3}\text{ in}^2\approx 20.78\text{ in}^2$

(d) Graph and use the MAXIMUM feature:

25




0 90

0

The maximum area is approximately 20.78 in$^2$ when the angle is 60˚.

58. (a) $\sin(2\theta)+\cos(2\theta)=0$

Divide each side by $\sqrt{2}$:

$$\frac{1}{\sqrt{2}}\sin(2\theta)+\frac{1}{\sqrt{2}}\cos(2\theta)=0$$

Rewrite in the sum of two angles form where $\cos\phi=\frac{1}{\sqrt{2}}$ and $\sin\phi=\frac{1}{\sqrt{2}}$ and $\phi=\frac{\pi}{4}$:

$$\sin(2\theta)\cos\phi+\cos(2\theta)\sin\phi=0 \rightarrow \sin(2\theta+\phi)=0$$

$$2\theta+\phi=0+k\pi$$

$$2\theta+\frac{\pi}{4}=0+k\pi \rightarrow 2\theta=-\frac{\pi}{4}+k\pi$$

$$\theta=-\frac{\pi}{8}+\frac{k\pi}{2}$$

$$\theta=\frac{3\pi}{8} \text{ or } 67.5^\circ$$

(b) $\sin(2\theta)+\cos(2\theta)=0$

$$\sin(2\theta)=-\cos(2\theta) \rightarrow \frac{\sin(2\theta)}{\cos(2\theta)}=-1$$

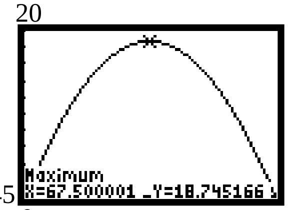
$$\tan(2\theta)=-1 \rightarrow 2\theta=\frac{3\pi}{4} \rightarrow \theta=\frac{3\pi}{8} \text{ or } 67.5^\circ$$

(c) $R = \dfrac{32^2\sqrt{2}}{32}\left(\sin(2\cdot 67.5°) - \cos(2\cdot 67.5°) - 1\right) = 32\sqrt{2}\left(\sin 135° - \cos 135° - 1\right)$

$= 32\sqrt{2}\left(\dfrac{\sqrt{2}}{2} - \dfrac{-\sqrt{2}}{2} - 1\right) = 32\sqrt{2}\left(\sqrt{2} - 1\right) = 64 - 32\sqrt{2}$ feet

(d) Graphing:

20




45 90

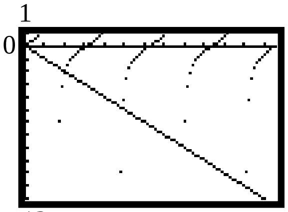
0

The angle that maximizes the distance is 67.5° and the maximum distance is 18.75 feet.

59. Graph:

1

0 4π



–12

The first two positive solutions are 2.03 and 4.91.

60. (a) Let L be the length of the ladder with x and y being the lengths of the two parts in each hallway.

$L = x + y$

$\cos\theta = \dfrac{3}{x} \quad\rightarrow\quad x = \dfrac{3}{\cos\theta} = 3\sec\theta$

$\sin\theta = \dfrac{4}{y} \quad\rightarrow\quad y = \dfrac{4}{\sin\theta} = 4\csc\theta$

$L(\theta) = 3\sec\theta + 4\csc\theta$

(b) $3\sec\theta\tan\theta - 4\csc\theta\cot\theta = 0$

$3\sec\theta\tan\theta = 4\csc\theta\cot\theta$

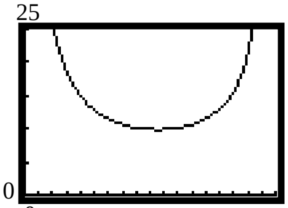
$\dfrac{\sec\theta\tan\theta}{\csc\theta\cot\theta} = \dfrac{4}{3} \rightarrow \tan^3\theta = \dfrac{4}{3} \rightarrow \tan\theta = \sqrt[3]{\dfrac{4}{3}} \approx 1.10064$

$\theta = 47.74°$

(c) $L = 3\sec 47.74° + 4\csc 47.74° \approx 9.87$ feet

(d) Graphing:

25



0 90

0

The graph shows a minimum, not a maximum.

61. (a) $107 = \frac{(34.8)^2 \sin 2\theta}{9.8}$

$\sin(2\theta) = \frac{107(9.8)}{(34.8)^2} = 0.8659$

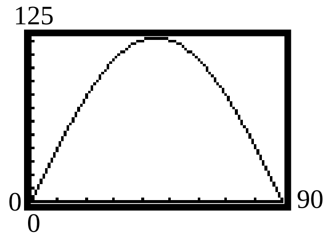
$2\theta = \sin^{-1} 0.8659 = 59.98^\circ$ or $120.02^\circ$

$\theta = 29.99^\circ$ or $60.01^\circ$

(b) Graph and use the MAXIMUM feature:

The maximum distance is 123.58 meters when the angle is 45˚.

(c) Graph:




62. (a) $110 = \frac{40^2 \sin(2\theta)}{9.8}$

$\sin(2\theta) = \frac{110 \ 9.8}{40^2} \approx 0.67375$

$2\theta = 42.357^\circ$ or $137.643^\circ$

$\theta = 21.18^\circ$ or $68.82^\circ$

(b) The maximum distance is approximately 163.3 meter

(c) Graphing: