Handling Loops in Bounded Model Checking of C Programs via *k*-Induction

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Joint work with
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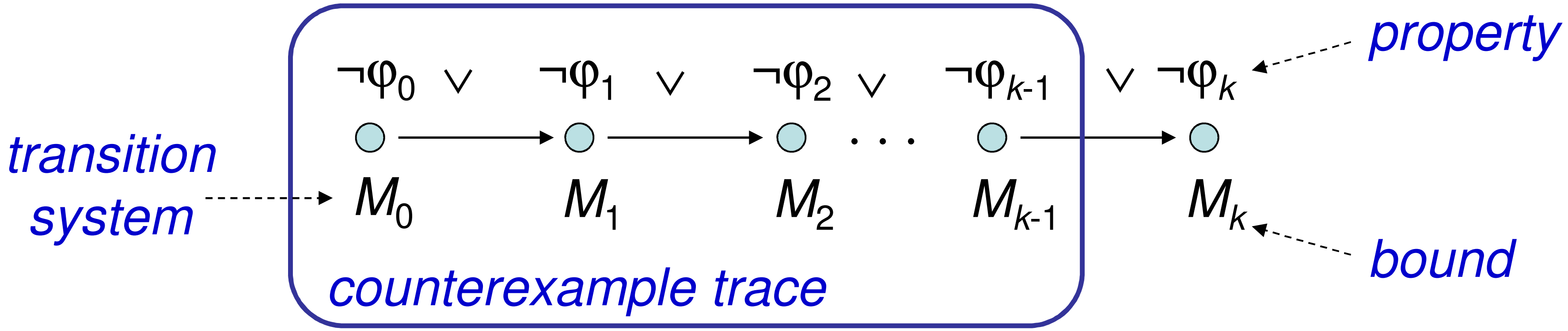
Bounded Model Checking (BMC)

basic Idea: check negation of given property up to given depth



- transition system M unrolled k times
  - for programs: loops, arrays, …
- translated into verification condition ψ such that

  ψ satisfiable iff φ has counterexample of max. depth k

- has been applied successfully to verify (embedded) software

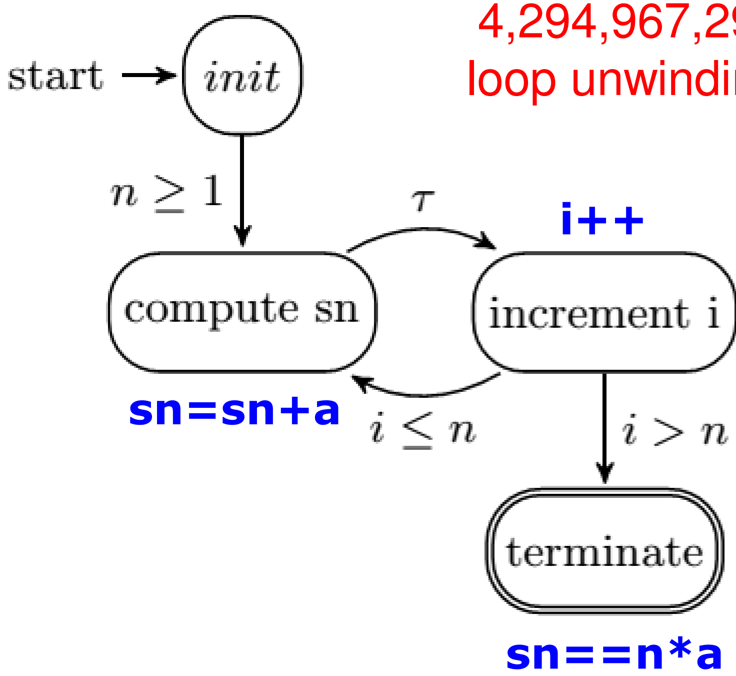
Difficulties in proving the correctness of programs with loops in BMC

- BMC techniques can falsify properties up to a given depth *k*
  - they can prove correctness only if an upper bound of *k* is known (**unwinding assertion**)
    - » BMC tools typically fail to verify programs that contain bounded and unbounded loops

$$S_n = \sum_{i=1}^{n} a = na, n \geq 1$$

the loop will be unfolded 2^{n-1} times (in the worst case, 2^{32-1} times on 32 bits integer)

4,294,967,295 loop unwindings



ESBMC: SMT-based BMC of single- and multi-threaded software

Goal: prove that an invariant is *k*-inductive

SMT-b … n CBMC:

- symbolicall … utes C into SSA, produces QF formulae
- unrolls loops up to a maximum bound *k*
- assertion failure *iff* corresponding formula is satisfiable
  - safety properties (array bounds, pointer dereferences, overflows,...)
  - user-specified properties

Multi-threaded programs:

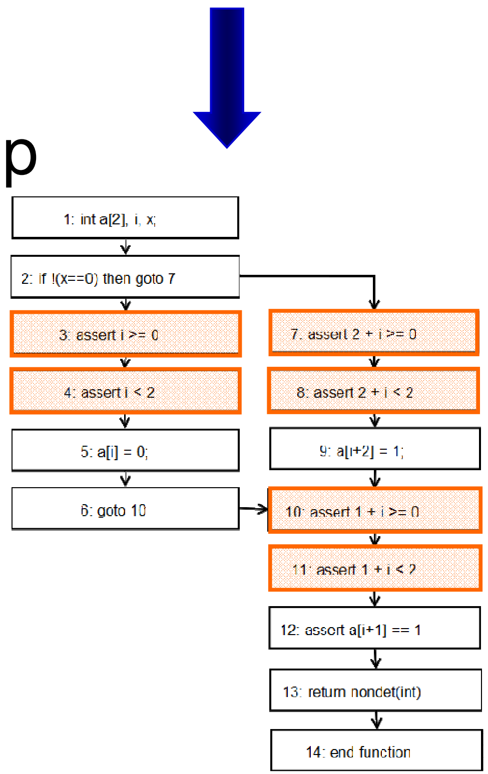
- produces one SSA program for each possible thread interleaving
- interleaves only at “visible” instructions
- optional context bound

Software BMC using ESBMC

- program modelled as state transition system
  - *state*: program counter and program variables
  - derived from control-flow graph
  - checked safety properties give extra nodes
- program unfolded up to given bounds
  - loop iterations
  - context switches
- unfolded program optimized to reduce blow-up
  - constant propagation
  - forward substitutions

  } crucial

```
int main() {
  int a[2], i, x;
  if (x==0)
    a[i]=0;
  else
    a[i+2]=1;
  assert(a[i+1]==1);
}
```



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- front-end converts unrolled and optimized program into SSA

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  assert(a[i+1]==1);
}
```

$g_1 = x_1 == 0$
$a_1 = a_0 \text{ WITH } [i_0{:=}0]$
$a_2 = a_0$
$a_3 = a_2 \text{ WITH } [2+i_0{:=}1]$
$a_4 = g_1 \; ? \; a_1 : a_3$
$t_1 = a_4[1+i_0] == 1$

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- extraction of *constraints C* and *properties P*
  - specific to selected SMT solver, uses theories
- satisfiability check of $C \wedge \neg P$

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int main() {
  int a[2], i, x;
  if (x==0)
    a[i]=0;
  else
    a[i+2]=1;
  assert(a[i+1]==1);
}
```

$$C := \begin{bmatrix} g_1 := (x_1 = 0) \\ \wedge\, a_1 := store(a_0, i_0, 0) \\ \wedge\, a_2 := a_0 \\ \wedge\, a_3 := store(a_2, 2 + i_0, 1) \\ \wedge\, a_4 := ite(g_1, a_1, a_3) \end{bmatrix}$$

$$P := \begin{bmatrix} i_0 \geq 0 \wedge i_0 < 2 \\ \wedge\, 2 + i_0 \geq 0 \wedge 2 + i_0 < 2 \\ \wedge\, 1 + i_0 \geq 0 \wedge 1 + i_0 < 2 \\ \wedge\, select(a_4, i_0 + 1) = 1 \end{bmatrix}$$

Software BMC using ESBMC

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  - *state*: program counter and program variables
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  - checked safety properties give extra nodes
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  if (x==0)
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  else
    a[i+2]=1;
  assert(a[i+1]==1);
```

ESBMC finds real errors in applications, but it is susceptible to producing time-out or memory-out for correct programs

$$C := \left[\begin{array}{l} \ldots = 0) \\ \wedge a_1 := store(a_0, i_0, 0) \\ \wedge a_2 := a_0 \\ \wedge a_3 := store(a_2, 2+i_0, 1) \\ \wedge a_4 := ite(g_1, a_1, a_3) \end{array}\right]$$

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Induction-Based Verification

***k*-induction** checks loop-free programs...

- **base case (*$base_k$*):** find a counter-example with up to k loop unwindings (plain BMC)
- **forward condition (*fwd_k*):** check that P holds in all states reachable within k unwindings
- **inductive step (*$step_k$*):** check that whenever P holds for k unwindings, it also holds after next unwinding
  - havoc state
  - run k iterations
  - assume invariant
  - run final iteration

⇒ iterative deepening if inconclusive

Loop-free Programs (*base*$_k$ and *fwd*$_k$)

- A loop-free program is represented by a **straight-line program** (without loops) using *if*-statements

```
for(B; c; D) { E; }   ⇒   B while(c) { E; D;}
```

```
L1: while(c) {             L1: if(!c) goto L2
      E; D;          ⇒           E; D;
    }                            goto L1
                           L2: ASSUME or ASSERT
```

```
L1: while(cond1) {         L1: if(!cond1) goto L4
      LOOP1 BODY                   LOOP1 BODY
L2:   while(cond2) {  ⇒    L2:     if(!cond2) goto L3
        LOOP2 BODY                   LOOP2 BODY
      }                              goto L2
    }                      L3:       goto L1
                           L4:  ASSUME or ASSERT
```

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for(B; c; D) { E; }   ⇒   B while(c) { E; D;}
```

base$_k$ and _fwd_$_k$ translations can easily be implemented on top of plain BMC

```
L1                                  2
                          L2: ASSUME or ASSERT
```

```
L1: while(cond1) {
      LOOP1 BODY
L2:   while(cond2) {
        LOOP2 BODY
      }
    }
```

⇒

```
L1: if(!cond1) goto L4
      LOOP1 BODY
L2:   if(!cond2) goto L3
        LOOP2 BODY
        goto L2
L3:     goto L1
L4:  ASSUME or ASSERT
```

Loop-free Programs (*step$_k$*)

- In the inductive step, loops are converted into:

```
while(c) { E; }        ⇒        A while(c) { S; E; U; } R;
```

  - **A:** assigns **non-deterministic values** to all loops variables (the state is havocked before the loop)
  - **c:** is the **halt condition** of the loop
  - **S: stores the current state** of the program variables before executing the statements of E
  - **E:** is the actual **code inside the loop**
  - **U: updates all program variables** with local values after executing E

The *k*-induction algorithm

```
k=1
while k<=max_iterations do
    if  base_{P,ϕ,k} then
        return trace s[0..k]
    else
      k=k+1
      if fwd_{P,ϕ,k} then
        return true
      else if step_{P′,ϕ,k} then
        return true
    end if
end
return unknown
```

The *k*-induction algorithm

I : initial condition
T : transition relation of P
σ : termination condition
ϕ : safey property

$I \wedge T \wedge \sigma \Rightarrow \phi$

inserts **unwinding assumption** after each loop

k=1
while *k<=max_iterations* **do**
  if $base_{P,\phi,k}$ **then**
    return *trace s[0..k]*
  else
    k=k+1
    if $fwd_{P,\phi,k}$ **then**
      return *true*
    else if $step_{P',\phi,k}$ **then**
      return *true*
  end if
end
return *unknown*

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      else if step_{P′,ϕ,k} then
        return true
    end if
end
return unknown
```

$I \wedge T \wedge \sigma \Rightarrow \phi$

γ: transition relation of *P′*

$\gamma \wedge \sigma \Rightarrow \phi$

havoc variables that occur in the loop's termination condition

The *k*-induction algorithm

k=1

while *k<=max_iterations* **do**

  if $base_{P,\phi,k}$ **then** ← $I \wedge T \wedge \sigma \Rightarrow \phi$

   return *trace s[0..k]*

  else

   k=k+1

   if $fwd_{P,\phi,k}$ **then** ← $I \wedge T \Rightarrow \sigma \wedge \phi$

    return *true*

   else if $step_{P',\phi,k}$ **then** ← $\gamma \wedge \sigma \Rightarrow \phi$

    return *true*

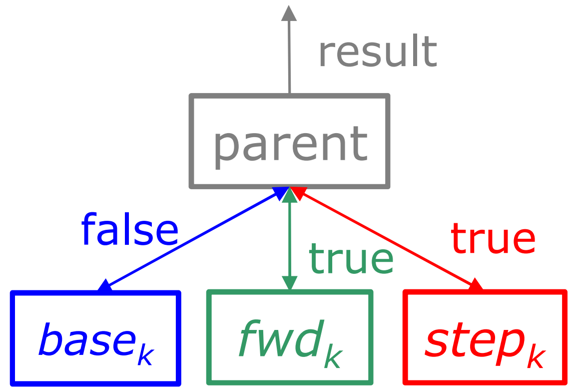
  end if

end

return *unknown*

> unable to **falsify** or **prove** the property

Parallel *k*-Induction Algorithm



- The parallel implementation consists of **four different processes**
  - running in different processing cores
  - splitting each step potentially **divides the work clock-time by a factor of three**
- Parent process initializes three child processes, executes the logic of the *k*-induction algorithm, and shows the verification results
  - **two pipes** are used in each process for the **inter-process communication**
- Once the solution is found, the child process communicates to the parent process, which sends signals to the other two child processes to finalize them

Running example

Prove that $S_n = \sum_{i=1}^{n} a = na$ for $n \geq 1$

```
int main() {
  long long int i=1, sn=0;
  unsigned int n;
  assume (n>=1);
  while(i<=n) {
    sn = sn + a;
    i++;
  }
  assert(sn==n*a);
}
```

Running example: *base case*

Insert an **unwinding assumption** consisting of the termination condition after the loop

– find a counter-example with *k* loop unwindings

```
int main() {
  unsigned int n=nondet_uint();
  long long int i=1, sn=0;
  assume (n>=1);
  if (i<=n) {          ⎫
    sn = sn + a;       ⎬  k-copies
    i++;               ⎪
  }                    ⎭
  ...
  assume(i>n); //unwinding assumption
  assert(sn==n*a);
}
```

Running example: *forward condition*

Insert an **unwinding assertion** consisting of the termination condition after the loop

- check that *P* holds in all states reachable with k unwindings

```
int main() {
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  assume (n>=1);
  if (i<=n) {
    sn = sn + a;        } k-copies
    i++;
  }
  ...
  assert(i>n);  //unwinding assertion
  assert(sn==n*a);
}
```

Running example: *inductive step*

Havoc (only) the variables that occur in the loop's termination and branch conditions

```
unsigned int nondet_uint();
typedef struct state {
    unsigned int i, n, sn;
} statet;
int main() {
    unsigned int i, n=nondet_uint(), sn=0, k;
    assume(n>=1);
    statet cs, sv[n];
    cs.i=nondet_uint();
    cs.sn=nondet_uint();
    cs.n=n;
```

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define the type of the program state

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state vector

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```

define the type of the program state

state vector

explore all possible values implicitly

Running example: *inductive step*

ESBMC is called to verify the assertions where the first arbitrary state is emulated by **nondeterminism**

```
for(i=1; i<=n; i++) {
    sv[i-1]=cs;
    sn = sn + a;
    cs.i=i;
    cs.sn=sn;
    cs.n=n;
    assume(sv[i-1]!=cs);
  }
assume(i>n);
assert(sn ==  n*a);
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capture the state *cs* before the iteration

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capture the state *cs* before the iteration

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capture the state *cs* before the iteration

capture the state cs after the iteration

constraints are included by means of assumptions

insert unwinding assumption

Removing Redundant States

- An assume instruction checks whether the current state is different from the previous one
  - prevent redundant states to be inserted into the state vector

```
assume(sv[i-1]!=cs);
```

- We compare $sv_j[i]$ to cs_j for $0 < j \leq k$ and $0 \leq i \leq k$

$$sv_1[0] \neq cs_1$$

$$sv_1[0] \neq cs_1 \wedge sv_2[1] \neq cs_2$$

...

$$sv_1[0] \neq cs_1 \wedge sv_2[1] \neq cs_2 \wedge \ldots \wedge sv_k[i] \neq cs_k$$

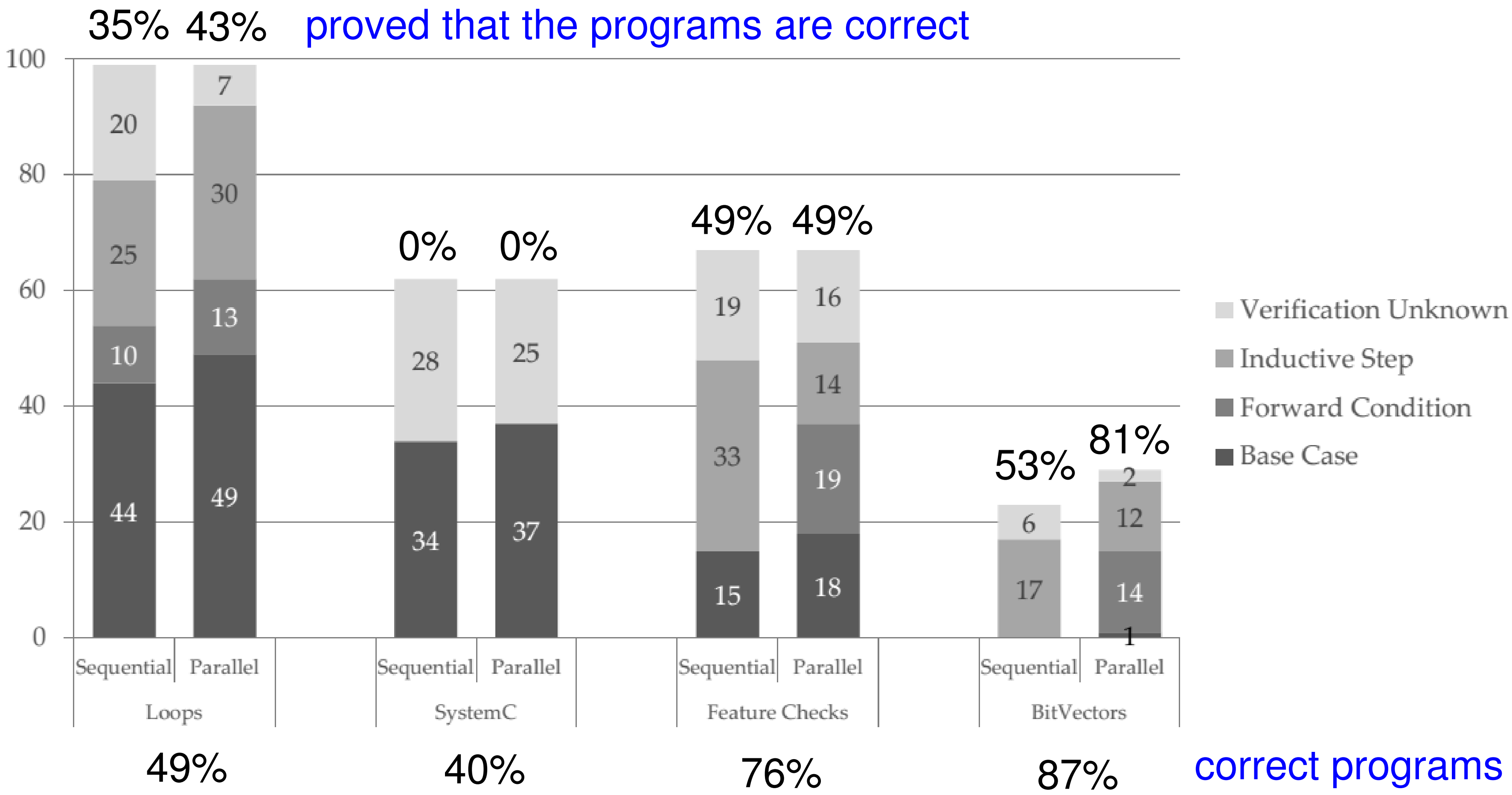
- We could compare $sv_k[i]$ to all cs_k for $i < k$ (since **inequalities are not transitive**)
  - however, the number of constraints can grow very large quickly

Experimental Evaluation

- **Goal:** evaluate the performance of the sequential and parallel implementations using the SV-COMP benchmarks
  - **Loops** (99 programs)
    - » 49% correct and 51% incorrect programs
  - **SystemC** (62 programs)
    - » 40% correct and 60% incorrect programs
  - **FeatureChecks** (67 programs)
    - » 76% correct and 24% incorrect programs
  - ***BitVectors*** (32 programs)
    - » 87% correct and 13% incorrect programs
- Set-up:
  - ESBMC v1.22 together with the SMT solver Z3 v4.0
  - support the logics *QF_AUFBV* and *QF_AUFLIRA*
  - standard desktop PC, time-out 900 seconds

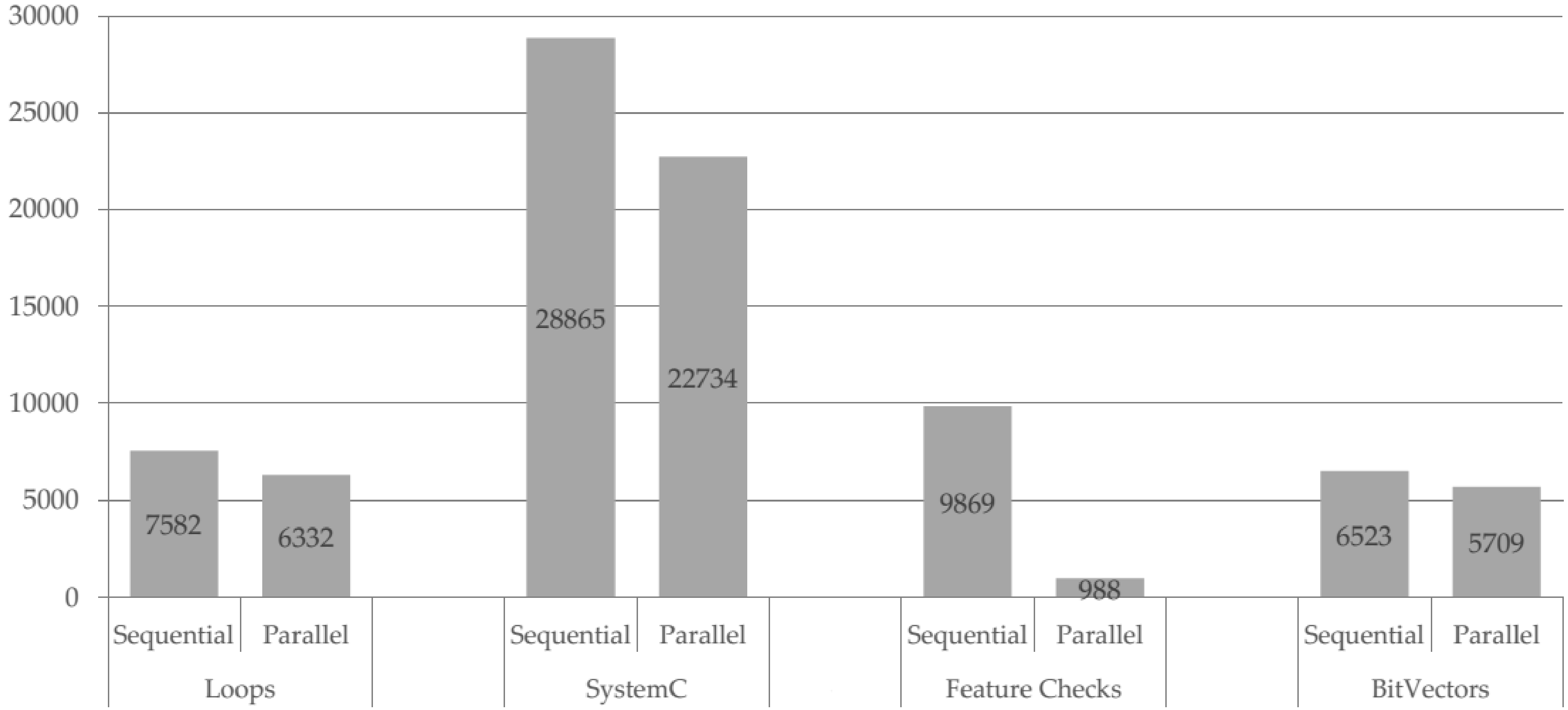
Verification Results for Each Step

- Most of **unknown** results occurred due to nested loops
  - base case produced two **false alarms** due to the memory model adopted by ESBMC



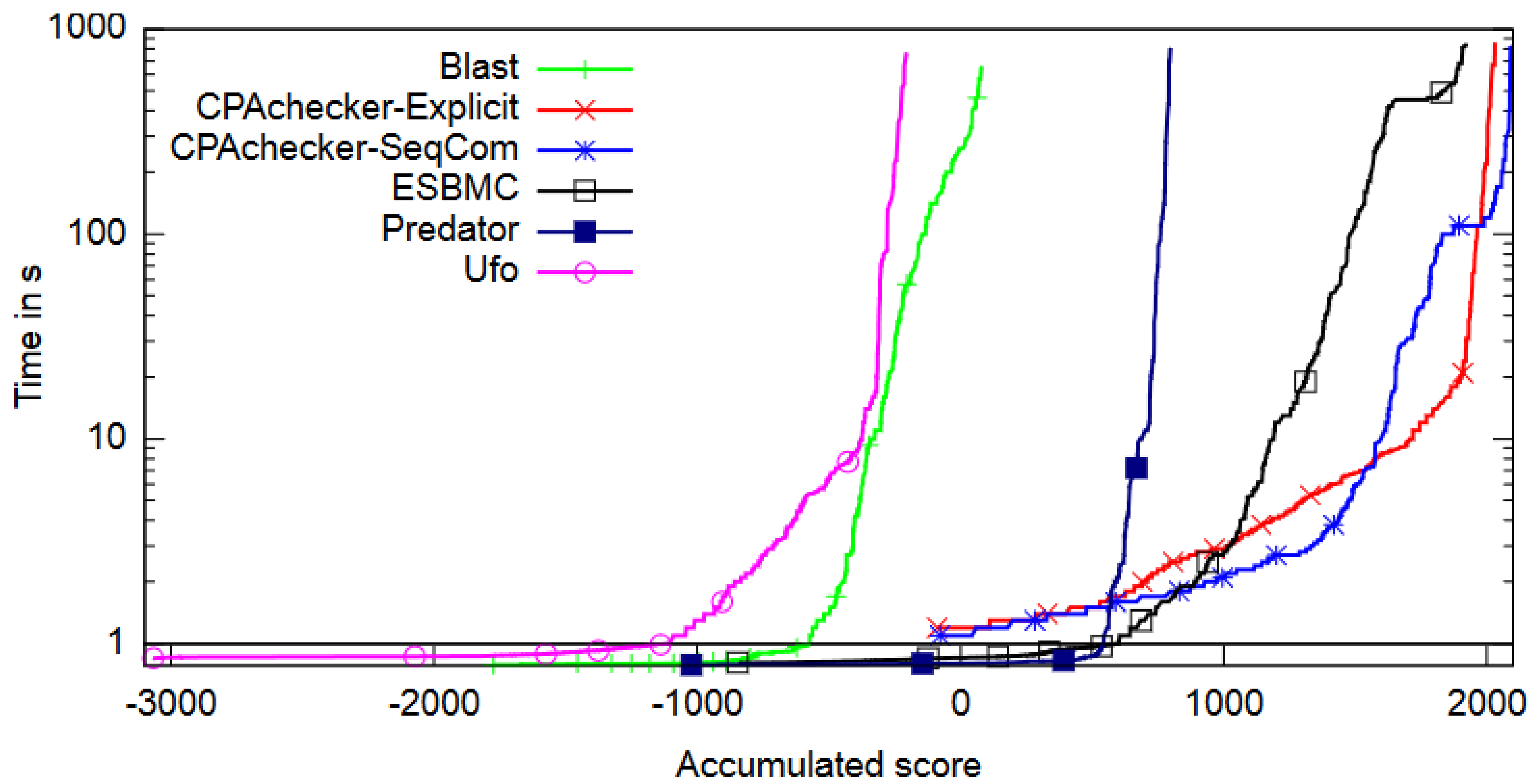
Verification Time per Category

- Sequential *k*-induction verifies 70% of the benchmarks in 52839 seconds, and the parallel *k*-induction verifies 80% in 35763 seconds
  - » **speedup of 32%**



SV-COMP 2013 Results – Overall Ranking

- Sequential *k*-induction participated in the 2nd edition of the SV-COMP
  - **verify by induction** that the safety property holds
    - » If that fails, **search for a bounded reachable state**



Strengths:

- robust *k*-induction algorithm for C programs
  - this marks the first application of the *k*-induction algorithm to a broader range of C programs
- combines plain BMC with *k*-induction
  - *k*-induction by itself is by far not as strong as plain BMC

    $\Rightarrow$ although it produced substantially fewer false results

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    $\Rightarrow$ although it produced substantially fewer false results

Weaknesses:

- scalability (like other BMCs...)
  - loop unrolling
  - interleavings
- investigate whether redundant constraints can be avoided
  - using the results of already completed steps
- refine invariants to strengthen the induction hypothesis