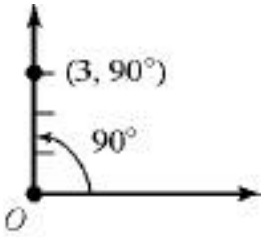
# Chapter 10

# Polar Coordinates; Vectors

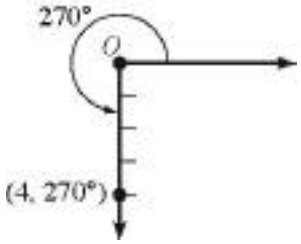
## 10.1 Polar Coordinates

1. A
2. B
3. C
4. C
5. B
6. D
7. A
8. D

9. $(3, 90^\circ)$

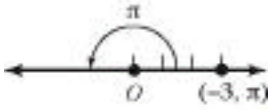



10. $(4, 270^\circ)$

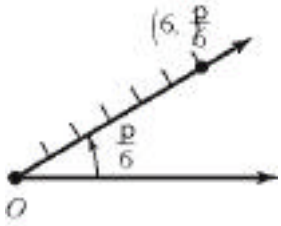



11. $(-2, 0)$

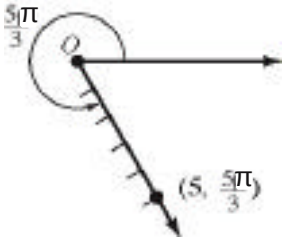

12. $(-3, \pi)$



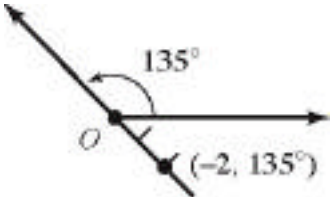

13. $\left(6, \frac{\pi}{6}\right)$



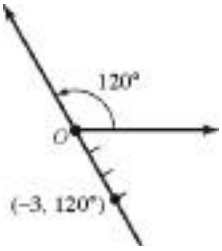

14. $\left(5, \frac{5\pi}{3}\right)$



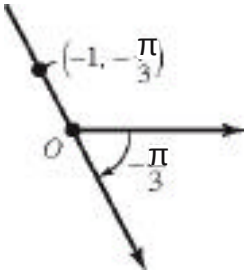

15. $(-2, 135^\circ)$



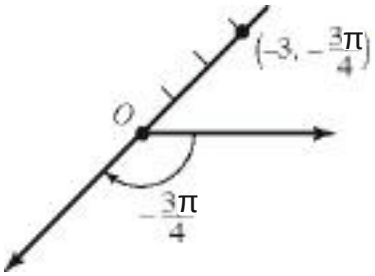

16. $(-3, 120^\circ)$



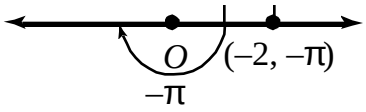
17. $\left(-1, -\frac{\pi}{3}\right)$



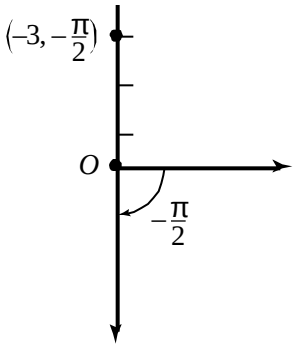

18. $\left(-3, -\frac{3\pi}{4}\right)$



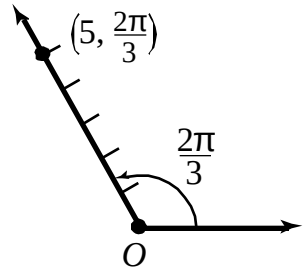

19. $(-2, -\pi)$




20. $\left(-3, -\frac{\pi}{2}\right)$




21. $\left(5, \frac{2\pi}{3}\right)$

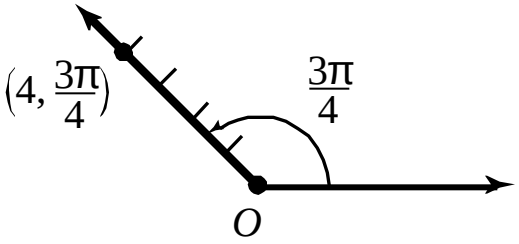



(a) $r > 0$, $-2\pi \le \theta < 0$ $\left(5, -\frac{4\pi}{3}\right)$

(b) $r < 0$, $0 \le \theta < 2\pi$ $\left(-5, \frac{5\pi}{3}\right)$

(c) $r > 0$, $2\pi \le \theta < 4\pi$ $\left(5, \frac{8\pi}{3}\right)$

22. $\left(4, \frac{3\pi}{4}\right)$

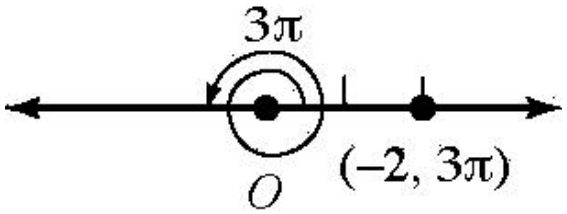



(a) $r > 0$, $-2\pi \le \theta < 0$ $\left(4, -\frac{5\pi}{4}\right)$

(b) $r < 0$, $0 \le \theta < 2\pi$ $\left(-4, \frac{7\pi}{4}\right)$

(c) $r > 0$, $2\pi \le \theta < 4\pi$ $\left(4, \frac{11\pi}{4}\right)$

23. $(-2, 3\pi)$

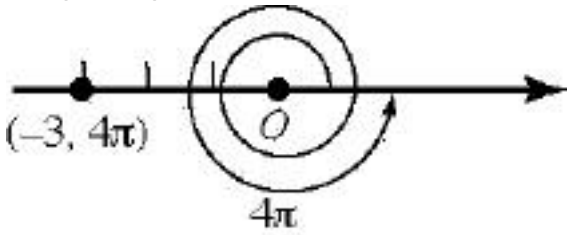



(a) $r > 0,\ -2\pi \le \theta < 0$ $(2, -2\pi)$

(b) $r < 0,\ 0 \le \theta < 2\pi$ $(-2, \pi)$

(c) $r > 0,\ 2\pi \le \theta < 4\pi$ $(2, 2\pi)$

24. $(-3, 4\pi)$

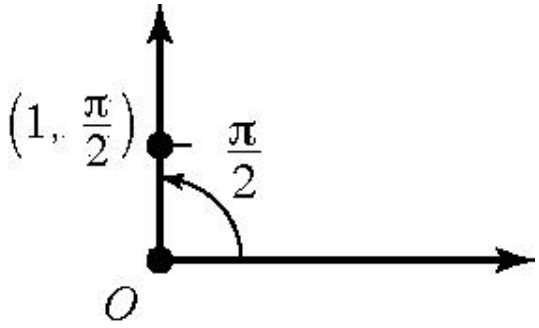



(a) $r > 0,\ -2\pi \le \theta < 0$ $(3, -\pi)$

(b) $r < 0,\ 0 \le \theta < 2\pi$ $(-3, 0)$

(c) $r > 0,\ 2\pi \le \theta < 4\pi$ $(3, 3\pi)$

25. $\left(1, \frac{\pi}{2}\right)$

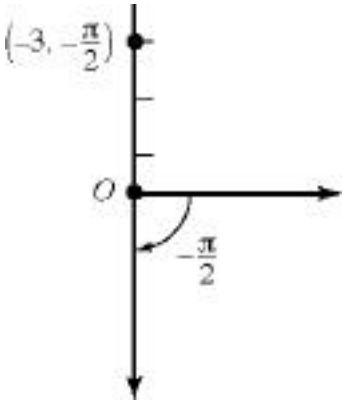



(a) $r > 0,\ -2\pi \le \theta < 0$ $\left(1, -\frac{3\pi}{2}\right)$

(b) $r < 0,\ 0 \le \theta < 2\pi$ $\left(-1, \frac{3\pi}{2}\right)$

(c) $r > 0,\ 2\pi \le \theta < 4\pi$ $\left(1, \frac{5\pi}{2}\right)$

26. $(2, \pi)$

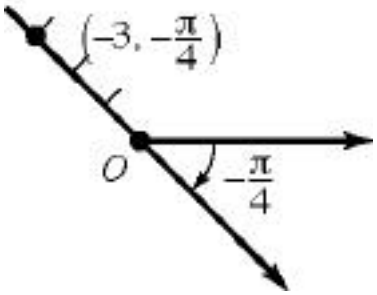



(a) $r > 0,\ -2\pi \le \theta < 0$ $(2, -\pi)$

(b) $r < 0,\ 0 \le \theta < 2\pi$ $(-2, 0)$

(c) $r > 0,\ 2\pi \le \theta < 4\pi$ $(2, 3\pi)$

27. $\left(-3, -\frac{\pi}{4}\right)$

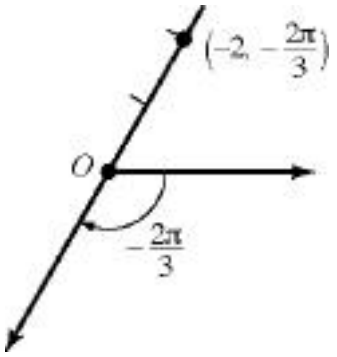



(a) $r > 0,\ -2\pi \le \theta < 0$ $\left(3, -\frac{5\pi}{4}\right)$

(b) $r < 0,\ 0 \le \theta < 2\pi$ $\left(-3, \frac{7\pi}{4}\right)$

(c) $r > 0,\ 2\pi \le \theta < 4\pi$ $\left(3, \frac{11\pi}{4}\right)$

28. $\left(-2, -\frac{2\pi}{3}\right)$




(a) $r > 0,\ -2\pi \le \theta < 0$ $\left(2, -\frac{5\pi}{3}\right)$

(b) $r < 0,\ 0 \le \theta < 2\pi$ $\left(-2, \frac{4\pi}{3}\right)$

(c) $r > 0,\ 2\pi \le \theta < 4\pi$ $\left(2, \frac{7\pi}{3}\right)$

29. $x = r\cos\theta = 3\cos\left(\frac{\pi}{2}\right) = 3 \cdot 0 = 0$

$y = r\sin\theta = 3\sin\left(\frac{\pi}{2}\right) = 3 \cdot 1 = 3$

The rectangular coordinates of the point $\left(3, \frac{\pi}{2}\right)$ are $(0, 3)$.

30. $x = r\cos\theta = 4\cos\left(\frac{3\pi}{2}\right) = 4 \cdot 0 = 0$

$y = r\sin\theta = 4\sin\left(\frac{3\pi}{2}\right) = 4 \cdot (-1) = -4$

The rectangular coordinates of the point $\left(4, \frac{3\pi}{2}\right)$ are $(0, -4)$.

31. $x = r\cos\theta = -2\cos(0) = -2 \cdot 1 = -2$

$y = r\sin\theta = -2\sin(0) = -2 \cdot 0 = 0$

The rectangular coordinates of the point $(-2, 0)$ are $(-2, 0)$.

32. $x = r\cos\theta = -3\cos(\pi) = -3(-1) = 3$

$y = r\sin\theta = -3\sin(\pi) = -3 \cdot 0 = 0$

The rectangular coordinates of the point $(-3, \pi)$ are $(3, 0)$.

33. $x = r\cos\theta = 6\cos\left(150^\circ\right) = 6\left(-\frac{\sqrt{3}}{2}\right) = -3\sqrt{3}$

$y = r\sin\theta = 6\sin\left(150^\circ\right) = 6\left(\frac{1}{2}\right) = 3$

The rectangular coordinates of the point $\left(6, 150^\circ\right)$ are $\left(-3\sqrt{3},\ 3\right)$.

34. $x = r\cos\theta = 5\cos\left(300^\circ\right) = 5\left(\frac{1}{2}\right) = \frac{5}{2}$

$y = r\sin\theta = 5\sin\left(300^\circ\right) = 5\left(-\frac{\sqrt{3}}{2}\right) = -\frac{5\sqrt{3}}{2}$

The rectangular coordinates of the point $\left(5,\ 300^\circ\right)$ are $\left(\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)$

35. $x = r\cos\theta = -2\cos\left(\frac{3\pi}{4}\right) = -2\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}$

$y = r\sin\theta = -2\sin\left(\frac{3\pi}{4}\right) = -2\left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

The rectangular coordinates of the point $\left(-2, \frac{3\pi}{4}\right)$ are $\left(\sqrt{2}, -\sqrt{2}\right)$.

36. $x = r\cos\theta = -3\cos\left(\frac{2\pi}{3}\right) = -3\left(-\frac{1}{2}\right) = \frac{3}{2}$

$y = r\sin\theta = -3\sin\left(\frac{2\pi}{3}\right) = -3\left(\frac{\sqrt{3}}{2}\right) = -\frac{3\sqrt{3}}{2}$

The rectangular coordinates of the point $\left(-3, \frac{2\pi}{3}\right)$ are $\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

37. $x = r\cos\theta = -1\cos\left(-\frac{\pi}{3}\right) = -1\left(\frac{1}{2}\right) = -\frac{1}{2}$

$y = r\sin\theta = -1\sin\left(-\frac{\pi}{3}\right) = -1\left(-\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

The rectangular coordinates of the point $\left(-1, -\frac{\pi}{3}\right)$ are $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

38. $x = r\cos\theta = -3\cos\left(-\frac{3\pi}{4}\right) = -3\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\sqrt{2}}{2}$

$y = r\sin\theta = -3\sin\left(-\frac{3\pi}{4}\right) = -3\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\sqrt{2}}{2}$

The rectangular coordinates of the point $\left(-3, -\frac{3\pi}{4}\right)$ are $\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$.

39. $x = r\cos\theta = -2\cos(-180^\circ) = -2(-1) = 2$
$y = r\sin\theta = -2\sin(-180^\circ) = -2(0) = 0$
The rectangular coordinates of the point $(-2, -180^\circ)$ are $(2, 0)$.

40. $x = r\cos\theta = -3\cos(-90^\circ) = -3(0) = 0$
$y = r\sin\theta = -3\sin(-90^\circ) = -3(-1) = 3$
The rectangular coordinates of the point $(-3, -90^\circ)$ are $(0, 3)$.

41. $x = r\cos\theta = 7.5\cos(110^\circ) \approx 7.5(-0.3420) \approx -2.57$
$y = r\sin\theta = 7.5\sin(110^\circ) \approx 7.5(0.9397) \approx 7.05$
The rectangular coordinates of the point $(7.5, 110^\circ)$ are $(-2.57, 7.05)$.

42. $x = r\cos\theta = -3.1\cos(182^\circ) \approx -3.1(-0.9994) \approx 3.10$
$y = r\sin\theta = -3.1\sin(182^\circ) \approx -3.1(-0.0349) \approx 0.11$
The rectangular coordinates of the point $(-3.1, 182^\circ)$ are $(3.10, 0.11)$.

43. $x = r\cos\theta = 6.3\cos(3.8) \approx 6.3(-0.7910) \approx -4.98$
$y = r\sin\theta = 6.3\sin(3.8) \approx 6.3(-0.6119) \approx -3.85$
The rectangular coordinates of the point $(6.3, 3.8)$ are $(-4.98, -3.85)$.

44. $x = r\cos\theta = 8.1\cos(5.2) \approx 8.1(0.4685) \approx 3.79$
$y = r\sin\theta = 8.1\sin(5.2) \approx 8.1(-0.8835) \approx -7.16$
The rectangular coordinates of the point $(8.1, 5.2)$ are $(3.79, -7.16)$.

45. $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 0^2} = \sqrt{9} = 3 \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{0}{3}\right) = \tan^{-1}(0) = 0$
Polar coordinates of the point (3, 0) are (3, 0).

46. $r = \sqrt{x^2 + y^2} = \sqrt{0^2 + 2^2} = \sqrt{4} = 2 \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{0}\right) = \frac{\pi}{2}$
Polar coordinates of the point (0, 2) are $\left(2, \frac{\pi}{2}\right)$.

47. $r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + 0^2} = \sqrt{1} = 1 \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{0}{-1}\right) = \tan^{-1}(0) = 0$

The point lies on the negative x-axis thus $\theta = \pi$.
Polar coordinates of the point (–1, 0) are $(1, \pi)$.

48. $r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-2)^2} = \sqrt{4} = 2 \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-2}{0}\right) = \frac{\pi}{2}$

The point lies on the negative y-axis thus $\theta = -\frac{\pi}{2}$.

Polar coordinates of the point (0, –2) are $\left(2, -\frac{\pi}{2}\right)$.

49. The point (1, –1) lies in quadrant IV.

$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-1}{1}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$

Polar coordinates of the point $(1, -1)$ are $\left(\sqrt{2}, -\frac{\pi}{4}\right)$.

50. The point (–3, 3) lies in quadrant II.

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 3^2} = 3\sqrt{2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{3}{-3}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$

Polar coordinates of the point $(-3, 3)$ are $\left(3\sqrt{2}, \frac{3\pi}{4}\right)$.

51. The point $\left(\sqrt{3}, 1\right)$ lies in quadrant I.

$r = \sqrt{x^2 + y^2} = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{4} = 2 \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

Polar coordinates of the point $\left(\sqrt{3}, 1\right)$ are $\left(2, \frac{\pi}{6}\right)$.

52. The point $\left(-2, -2\sqrt{3}\right)$ lies in quadrant III.

$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + \left(-2\sqrt{3}\right)^2} = \sqrt{16} = 4$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-2\sqrt{3}}{-2}\right) = \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$

Polar coordinates of the point $\left(-2, -2\sqrt{3}\right)$ are $\left(4, \frac{4\pi}{3}\right)$.

53. The point (1.3, –2.1) lies in quadrant IV.

$r = \sqrt{x^2 + y^2} = \sqrt{1.3^2 + (-2.1)^2} = \sqrt{6.1} \approx 2.47$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-2.1}{1.3}\right) \approx \tan^{-1}(-1.6154) \approx -1.02$

Polar coordinates of the point $(1.3, -2.1)$ are $(2.47, -1.02)$.

54. The point (–0.8, –2.1) lies in quadrant III.

$r = \sqrt{x^2 + y^2} = \sqrt{(-0.8)^2 + (-2.1)^2} = \sqrt{5.05} \approx 2.25$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-2.1}{-0.8}\right) \approx \tan^{-1}(2.625) \approx 1.21$

$\theta = \pi + 1.21 \approx 4.35$

Polar coordinates of the point $(-0.8, -2.1)$ are $(2.25, 4.35)$.

55. The point (8.3, 4.2) lies in quadrant I.

$r = \sqrt{x^2 + y^2} = \sqrt{8.3^2 + 4.2^2} = \sqrt{86.53} \approx 9.30$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4.2}{8.3}\right) \approx \tan^{-1}(0.5060) \approx 0.47$

Polar coordinates of the point $(8.3, 4.2)$ are $(9.30, 0.47)$.

56. The point (–2.3, 0.2) lies in quadrant II.

$r = \sqrt{x^2 + y^2} = \sqrt{(-2.3)^2 + 0.2^2} = \sqrt{5.33} \approx 2.31$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{0.2}{-2.3}\right) \approx \tan^{-1}(-0.0870) \approx -0.09$

$\theta = \pi - 0.09 \approx 3.05$

Polar coordinates of the point $(-2.3, 0.2)$ are $(2.31, 3.05)$.

57. $2x^2 + 2y^2 = 3$

$2(x^2 + y^2) = 3$

$2r^2 = 3 \rightarrow r^2 = \frac{3}{2}$

58. $x^2 + y^2 = x$

$r^2 = r\cos\theta$

59. $x^2 = 4y$

$(r\cos\theta)^2 = 4r\sin\theta$

$r^2\cos^2\theta - 4r\sin\theta = 0$

60. $y^2 = 2x$

$(r\sin\theta)^2 = 2r\cos\theta$

$r^2\sin^2\theta - 2r\cos\theta = 0$

61. $2xy = 1$

$2(r\cos\theta)(r\sin\theta) = 1$

$2r^2\sin\theta\cos\theta = 1$

$r^2\sin 2\theta = 1$

62. $4x^2y = 1$

$4(r\cos\theta)^2 r\sin\theta = 1$

$4r^2\cos^2\theta\, r\sin\theta = 1$

$4r^3\cos^2\theta\sin\theta = 1$

63. $x = 4$

$r\cos\theta = 4$

64. $y = -3$

$r\sin\theta = -3$

65. $r = \cos\theta$

$r^2 = r\cos\theta$

$x^2 + y^2 = x$

$x^2 - x + y^2 = 0$

66. $r = \sin\theta + 1$

$r^2 = r\sin\theta + r$

$x^2 + y^2 = y + \sqrt{x^2 + y^2}$

67.
$$r^2 = \cos\theta$$
$$r^3 = r\cos\theta$$
$$\left(x^2 + y^2\right)^{3/2} = x$$
$$\left(x^2 + y^2\right)^{3/2} - x = 0$$

68.
$$r = \sin\theta - \cos\theta$$
$$r^2 = r\sin\theta - r\cos\theta$$
$$x^2 + y^2 = y - x$$
$$x^2 + x + y^2 - y = 0$$

69.
$$r = 2$$
$$\sqrt{x^2 + y^2} = 2$$
$$x^2 + y^2 = 4$$

70.
$$r = 4$$
$$r^2 = 16$$
$$x^2 + y^2 = 16$$

71.
$$r = \frac{4}{1 - \cos\theta}$$
$$r(1 - \cos\theta) = 4$$
$$r - r\cos\theta = 4$$
$$\sqrt{x^2 + y^2} - x = 4$$
$$\sqrt{x^2 + y^2} = x + 4$$
$$x^2 + y^2 = x^2 + 8x + 16$$
$$y^2 = 8(x + 2)$$

72.
$$r = \frac{3}{3 - \cos\theta}$$
$$r(3 - \cos\theta) = 3$$
$$3r - r\cos\theta = 3$$
$$3\sqrt{x^2 + y^2} - x = 3$$
$$3\sqrt{x^2 + y^2} = x + 3$$
$$9(x^2 + y^2) = x^2 + 6x + 9$$
$$9x^2 + 9y^2 = x^2 + 6x + 9$$
$$8x^2 - 6x + 9y^2 - 9 = 0$$

73. Rewrite the polar coordinates in rectangular form:

$$P_1 = (r_1, \theta_1) \quad \rightarrow \quad P_1 = (r_1\cos\theta_1, r_1\sin\theta_1)$$
$$P_2 = (r_2, \theta_2) \quad \rightarrow \quad P_2 = (r_2\cos\theta_2, r_2\sin\theta_2)$$
$$d = \sqrt{(r_2\cos\theta_2 - r_1\cos\theta_1)^2 + (r_2\sin\theta_2 - r_1\sin\theta_1)^2}$$
$$= \sqrt{r_2^2\cos^2\theta_2 - 2r_1r_2\cos\theta_2\cos\theta_1 + r_1^2\cos^2\theta_1 + r_2^2\sin^2\theta_2 - 2r_1r_2\sin\theta_2\sin\theta_1 + r_1^2\sin^2\theta_1}$$
$$= \sqrt{r_2^2(\cos^2\theta_2 + \sin^2\theta_2) + r_1^2(\cos^2\theta_1 + \sin^2\theta_1) - 2r_1r_2(\cos\theta_2\cos\theta_1 + \sin\theta_2\sin\theta_1)}$$
$$= \sqrt{r_2^2 + r_1^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$