Chapter 10

Polar Coordinates; Vectors

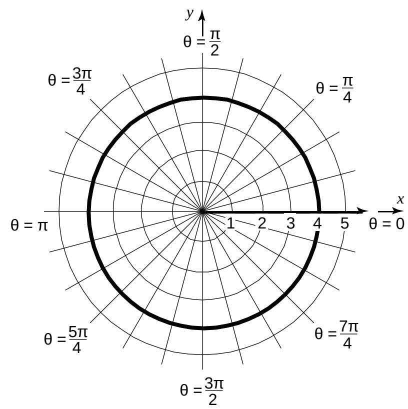
10.2 Polar Equations and Graphs

1. $r = 4$

The equation is of the form $r = a,\ a > 0$. It is a circle, center at the pole and radius 4.

Transform to rectangular form:

$$r = 4$$
$$r^2 = 16$$
$$x^2 + y^2 = 16$$

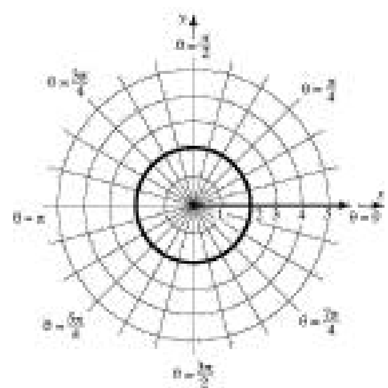


2. $r = 2$

The equation is of the form $r = a,\ a > 0$. It is a circle, center at the pole and radius 2.

Transform to rectangular form:

$$r = 2$$
$$r^2 = 4$$
$$x^2 + y^2 = 4$$

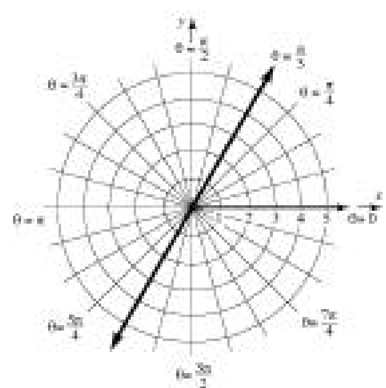


3. $\theta = \dfrac{\pi}{3}$

The equation is of the form $\theta = \alpha$. It is a line, passing through the pole at an angle of $\dfrac{\pi}{3}$.

Transform to rectangular form:

$$\theta = \frac{\pi}{3}$$
$$\tan\theta = \tan\left(\frac{\pi}{3}\right)$$
$$\frac{y}{x} = \sqrt{3} \rightarrow y = \sqrt{3}x$$

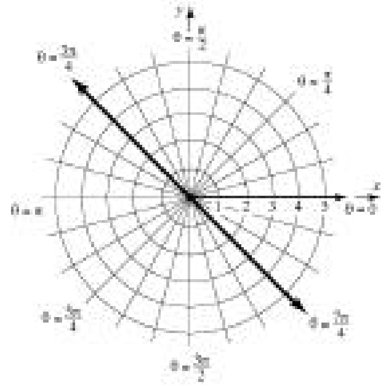


4. $\theta = -\frac{\pi}{4}$

The equation is of the form $\theta = \alpha$. It is a line, passing through the pole at an angle of $-\frac{\pi}{4}$.

Transform to rectangular form:

$$\theta = -\frac{\pi}{4}$$
$$\tan\theta = \tan\left(-\frac{\pi}{4}\right)$$
$$\frac{y}{x} = -1$$
$$y = -x$$

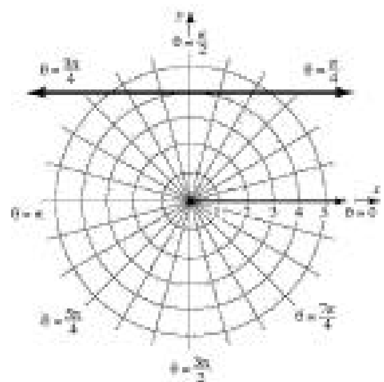


5. $r\sin\theta = 4$

The equation is of the form $r\sin\theta = b$. It is a horizontal line, 4 units above the pole.

Transform to rectangular form:

$$r\sin\theta = 4$$
$$y = 4$$

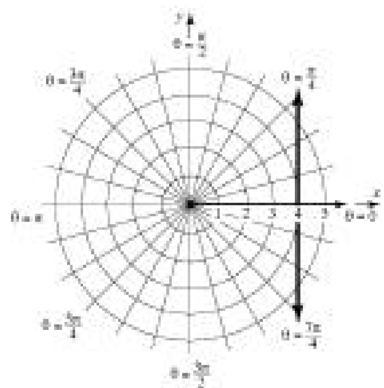


6. $r\cos\theta = 4$

The equation is of the form $r\cos\theta = a$. It is a vertical line, 4 units to the right of the pole.

Transform to rectangular form:

$$r\cos\theta = 4$$
$$x = 4$$

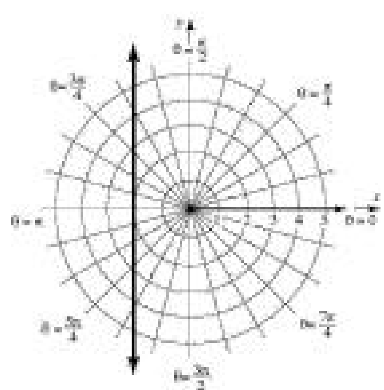


7. $r\cos\theta = -2$

The equation is of the form $r\cos\theta = a$. It is a vertical line, 2 units to the left of the pole.

Transform to rectangular form:

$$r\cos\theta = -2$$
$$x = -2$$

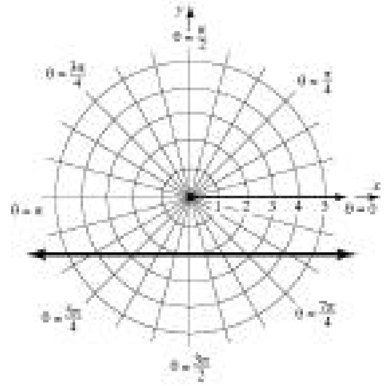


8. $r\sin\theta = -2$

The equation is of the form $r\sin\theta = b$. It is a horizontal line, 2 units below the pole.

Transform to rectangular form:

$$r\sin\theta = -2$$
$$y = -2$$

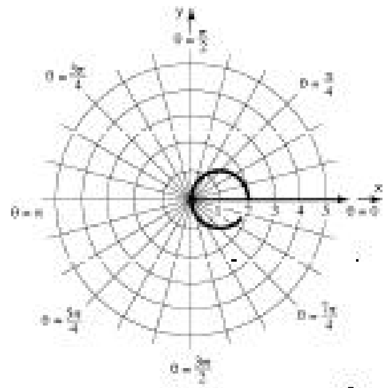


9. $r = 2\cos\theta$

The equation is of the form $r = 2a\cos\theta,\ a > 0$. It is a circle, passing through the pole, and center on the polar axis.

Transform to rectangular form:

$$r = 2\cos\theta$$
$$r^2 = 2r\cos\theta$$
$$x^2 + y^2 = 2x$$
$$x^2 - 2x + y^2 = 0$$
$$(x-1)^2 + y^2 = 1$$

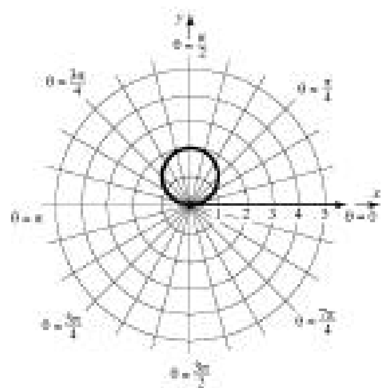


10. $r = 2\sin\theta$

The equation is of the form $r = 2a\sin\theta,\ a > 0$. It is a circle, passing through the pole, and center on the line $\theta = \dfrac{\pi}{2}$.

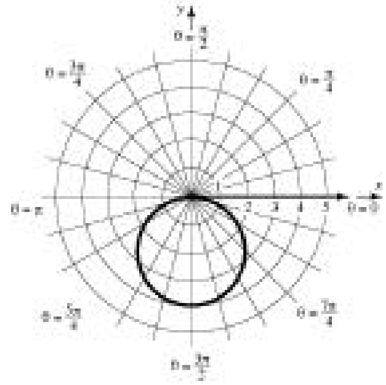
Transform to rectangular form:

$$r = 2\sin\theta$$
$$r^2 = 2r\sin\theta$$
$$x^2 + y^2 = 2y$$
$$x^2 + y^2 - 2y = 0$$
$$x^2 + (y-1)^2 = 1$$



11. $r = -4\sin\theta$

The equation is of the form $r = 2a\sin\theta,\ a > 0$. It is a circle, passing through the pole, and center on the line $\theta = \dfrac{\pi}{2}$.

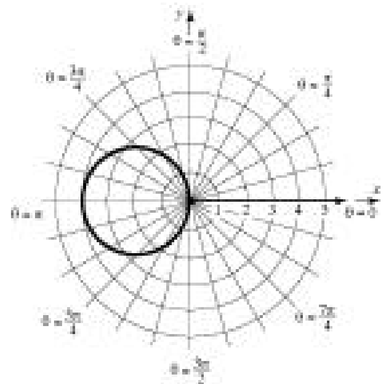


Transform to rectangular form:

$$\begin{aligned} r &= -4\sin\theta \\ r^2 &= -4r\sin\theta \\ x^2 + y^2 &= -4y \\ x^2 + y^2 + 4y &= 0 \\ x^2 + (y+2)^2 &= 4 \end{aligned}$$

12. $r = -4\cos\theta$

The equation is of the form $r = 2a\cos\theta,\ a > 0$. It is a circle, passing through the pole, and center on the polar axis.



Transform to rectangular form:

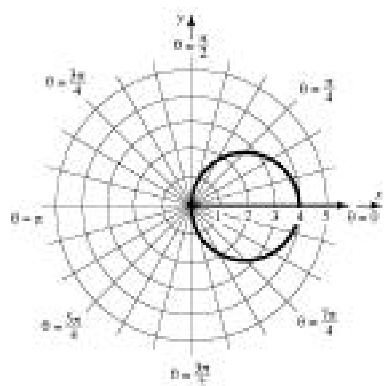
$$\begin{aligned} r &= -4\cos\theta \\ r^2 &= -4r\cos\theta \\ x^2 + y^2 &= -4x \\ x^2 + 4x + y^2 &= 0 \\ (x+2)^2 + y^2 &= 4 \end{aligned}$$

13. $r\sec\theta = 4$

Transform to rectangular form:

$$\begin{aligned} r\sec\theta &= 4 \\ r\,\frac{1}{\cos\theta} &= 4 \\ r &= 4\cos\theta \\ r^2 &= 4r\cos\theta \\ x^2 + y^2 &= 4x \\ x^2 - 4x + y^2 &= 0 \\ (x-2)^2 + y^2 &= 4 \end{aligned}$$

The equation is a circle, passing through the pole, center on the polar axis and radius 2.

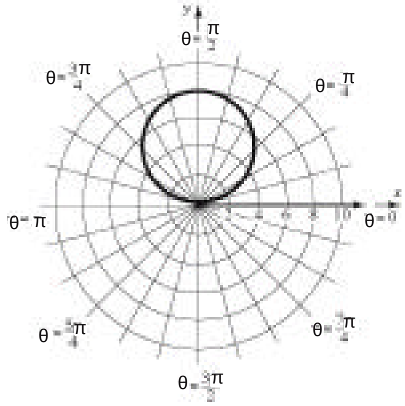


14. $r\csc\theta = 8$

Transform to rectangular form:

$$r\csc\theta = 8$$
$$r\left(\frac{1}{\sin\theta}\right) = 8$$
$$r = 8\sin\theta$$
$$r^2 = 8r\sin\theta$$
$$x^2 + y^2 = 8y$$
$$x^2 + y^2 - 8y = 0$$
$$x^2 + (y-4)^2 = 16$$

The equation is a circle, passing through the pole, center on the line $\theta = \frac{\pi}{2}$ and radius 4.

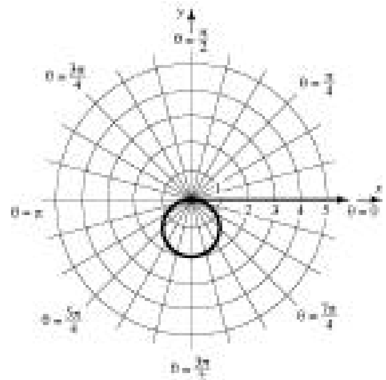


15. $r\csc\theta = -2$

Transform to rectangular form:

$$r\csc\theta = -2$$
$$r\left(\frac{1}{\sin\theta}\right) = -2$$
$$r = -2\sin\theta$$
$$r^2 = -2r\sin\theta$$
$$x^2 + y^2 = -2y$$
$$x^2 + y^2 + 2y = 0$$
$$x^2 + (y+1)^2 = 1$$

The equation is a circle, passing through the pole, center on the line $\theta = \frac{\pi}{2}$ and radius 1.

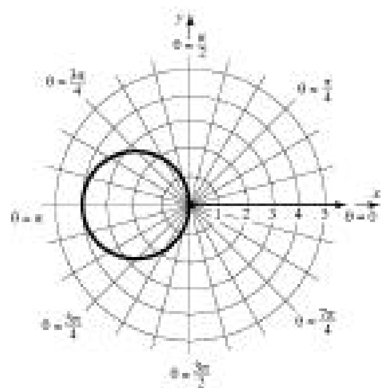


16. $r\sec\theta = -4$

Transform to rectangular form:

$$r\sec\theta = -4$$
$$r\left(\frac{1}{\cos\theta}\right) = -4$$
$$r = -4\cos\theta$$
$$r^2 = -4r\cos\theta$$
$$x^2 + y^2 = -4x$$
$$x^2 + 4x + y^2 = 0$$
$$(x+2)^2 + y^2 = 4$$

The equation is a circle, passing through the pole, center on the polar axis and radius 2.



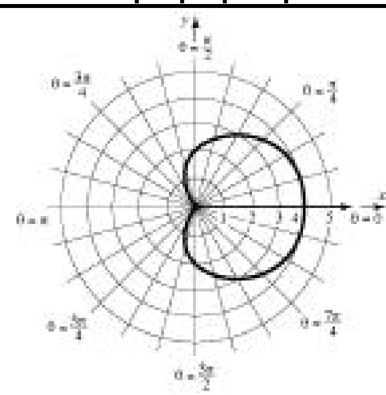
17. E	18. A	19. F	20. B
21. H	22. G	23. D	24. C

25. $r = 2 + 2\cos\theta$ The graph will be a cardioid. Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 2 + 2\cos(-\theta) = 2 + 2\cos\theta$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 2 + 2\cos(\pi - \theta) = 2 + 2(\cos(\pi)\cos\theta + \sin(\pi)\sin\theta)$$
$$= 2 + 2(-\cos\theta + 0) = 2 - 2\cos\theta$$

The test fails.
The pole: Replace r by $-r$. $-r = 2 + 2\cos\theta$. The test fails.
Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 2 + 2\cos\theta$	4	$2 + \sqrt{3} \approx 3.7$	3	2	1	$2 - \sqrt{3} \approx 0.3$	0

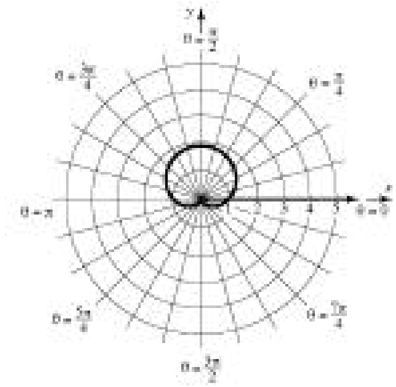


26. $r = 1 + \sin\theta$ The graph will be a cardioid. Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 1 + \sin(-\theta) = 1 - \sin\theta$.
The test fails.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 1 + \sin(\pi - \theta) = 1 + (\sin(\pi)\cos\theta - \cos(\pi)\sin\theta)$$
$$= 1 + (0 + \sin\theta) = 1 + \sin\theta$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.
The pole: Replace r by $-r$. $-r = 1 + \sin\theta$. The test fails.
Due to symmetry to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 1 + \sin\theta$	0	$1 - \frac{\sqrt{3}}{2} \approx 0.1$	$\frac{1}{2}$	1	$\frac{3}{2}$	$1 + \frac{\sqrt{3}}{2} \approx 1.9$	2

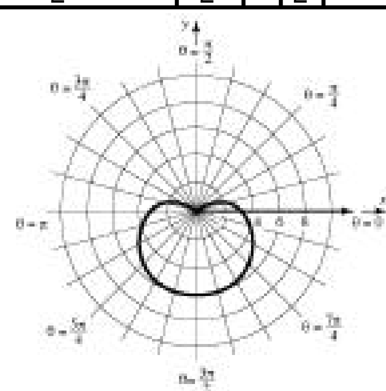


27. $r = 3 - 3\sin\theta$ The graph will be a cardioid. Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 3 - 3\sin(-\theta) = 3 + 3\sin\theta$.
The test fails.
The line $\theta = \dfrac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 3 - 3\sin(\pi - \theta) = 3 - 3(\sin(\pi)\cos\theta - \cos(\pi)\sin\theta)$$
$$= 3 - 3(0 + \sin\theta) = 3 - 3\sin\theta$$

The graph is symmetric with respect to the line $\theta = \dfrac{\pi}{2}$.
The pole: Replace r by $-r$. $-r = 3 - 3\sin\theta$. The test fails.
Due to symmetry to the line $\theta = \dfrac{\pi}{2}$, assign values to θ from $-\dfrac{\pi}{2}$ to $\dfrac{\pi}{2}$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 3 - 3\sin\theta$	6	$3 + \frac{3\sqrt{3}}{2} \approx 5.6$	$\frac{9}{2}$	3	$\frac{3}{2}$	$3 - \frac{3\sqrt{3}}{2} \approx 0.4$	0



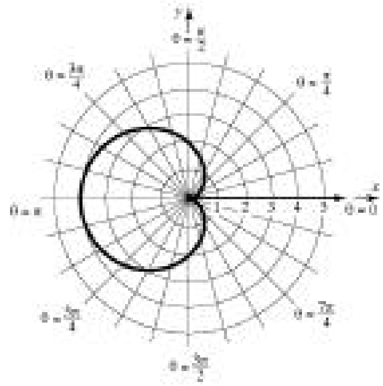
28. $r = 2 - 2\cos\theta$ The graph will be a cardioid. Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 2 - 2\cos(-\theta) = 2 - 2\cos\theta$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \dfrac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 2 - 2\cos(\pi - \theta) = 2 - 2(\cos(\pi)\cos\theta + \sin(\pi)\sin\theta)$$
$$= 2 - 2(-\cos\theta + 0) = 2 + 2\cos\theta$$

The test fails.

The pole: Replace r by $-r$. $-r = 2 - 2\cos\theta$. The test fails.
Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 2 - 2\cos\theta$	0	$2-\sqrt{3} \approx 0.3$	1	2	3	$2+\sqrt{3} \approx 3.7$	4

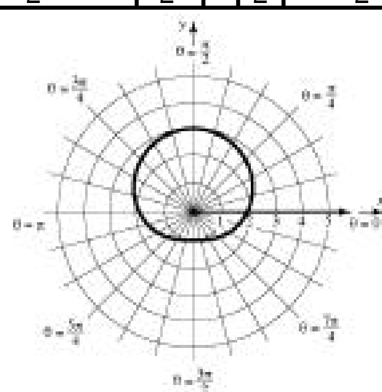


29. $r = 2 + \sin\theta$ The graph will be a limacon without an inner loop.
Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 2 + \sin(-\theta) = 2 - \sin\theta$.
The test fails.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 2 + \sin(\pi - \theta) = 2 + (\sin(\pi)\cos\theta - \cos(\pi)\sin\theta)$$
$$= 2 + (0 + \sin\theta) = 2 + \sin\theta$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.
The pole: Replace r by $-r$. $-r = 2 + \sin\theta$. The test fails.
Due to symmetry to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 2 + \sin\theta$	1	$2 - \frac{\sqrt{3}}{2} \approx 1.1$	$\frac{3}{2}$	2	$\frac{5}{2}$	$2 + \frac{\sqrt{3}}{2} \approx 2.9$	3

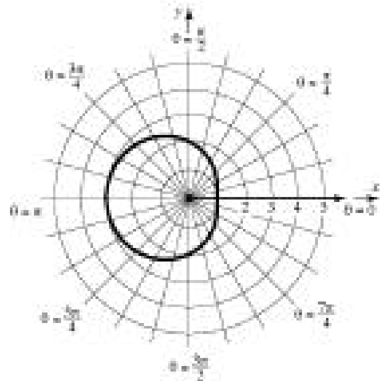


30. $r = 2 - \cos\theta$ The graph will be a limacon without an inner loop.
Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 2 - \cos(-\theta) = 2 - \cos\theta$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 2 - \cos(\pi - \theta) = 2 - (\cos(\pi)\cos\theta + \sin(\pi)\sin\theta)$$
$$= 2 - (-\cos\theta + 0) = 2 + \cos\theta$$

The test fails.
The pole: Replace r by $-r$. $-r = 2 - \cos\theta$. The test fails.
Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 2 - \cos\theta$	1	$2 - \frac{\sqrt{3}}{2} \approx 1.1$	$\frac{3}{2}$	2	$\frac{5}{2}$	$2 + \frac{\sqrt{3}}{2} \approx 2.9$	3

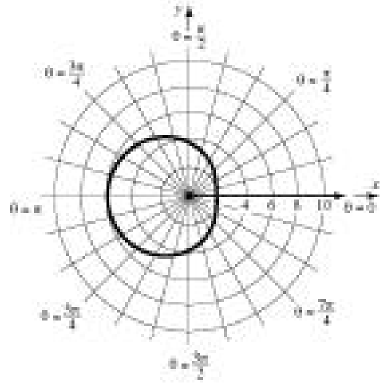


31. $r = 4 - 2\cos\theta$ The graph will be a limacon without an inner loop.
Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 4 - 2\cos(-\theta) = 4 - 2\cos\theta$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 4 - 2\cos(\pi - \theta) = 4 - 2(\cos(\pi)\cos\theta + \sin(\pi)\sin\theta)$$
$$= 4 - 2(-\cos\theta + 0) = 4 + 2\cos\theta$$

The test fails.
The pole: Replace r by $-r$. $-r = 4 - 2\cos\theta$. The test fails.
Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 4 - 2\cos\theta$	2	$4 - \sqrt{3} \approx 2.3$	3	4	5	$4 + \sqrt{3} \approx 5.7$	6

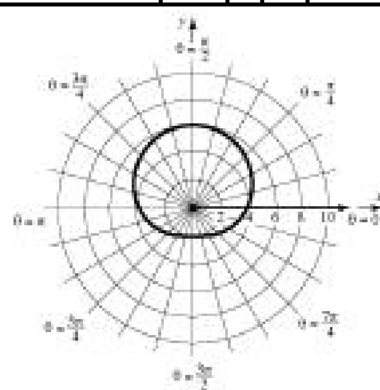


32. $r = 4 + 2\sin\theta$ The graph will be a limacon without an inner loop.
Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 4 + 2\sin(-\theta) = 4 - 2\sin\theta$.
The test fails.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 4 + 2\sin(\pi - \theta) = 4 + 2(\sin(\pi)\cos\theta - \cos(\pi)\sin\theta)$$
$$= 4 + 2(0 + \sin\theta) = 4 + 2\sin\theta$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.
The pole: Replace r by $-r$. $-r = 4 + 2\sin\theta$. The test fails.
Due to symmetry to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 4 + 2\sin\theta$	2	$4 - \sqrt{3} \approx 2.3$	3	4	5	$4 + \sqrt{3} \approx 5.7$	6



33. $r = 1 + 2\sin\theta$ The graph will be a limacon with an inner loop.
Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 1 + 2\sin(-\theta) = 1 - 2\sin\theta$.
The test fails.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

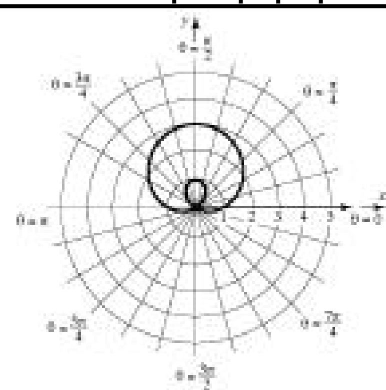
$$r = 1 + 2\sin(\pi - \theta) = 1 + 2(\sin(\pi)\cos\theta - \cos(\pi)\sin\theta)$$
$$= 1 + 2(0 + \sin\theta) = 1 + 2\sin\theta$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

The pole: Replace r by $-r$. $-r = 1 + 2\sin\theta$. The test fails.

Due to symmetry to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 1 + 2\sin\theta$	-1	$1-\sqrt{3} \approx -0.7$	0	1	2	$1+\sqrt{3} \approx 2.7$	3



34. $r = 1 - 2\sin\theta$ The graph will be a limacon with an inner loop.

Check for symmetry:

Polar axis: Replace θ by $-\theta$. The result is $r = 1 - 2\sin(-\theta) = 1 + 2\sin\theta$.
The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

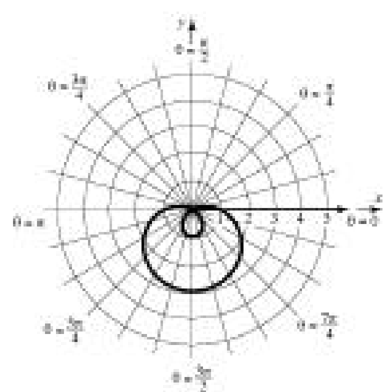
$$r = 1 - 2\sin(\pi - \theta) = 1 - 2(\sin(\pi)\cos\theta - \cos(\pi)\sin\theta)$$
$$= 1 - 2(0 + \sin\theta) = 1 - 2\sin\theta$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

The pole: Replace r by $-r$. $-r = 1 - 2\sin\theta$. The test fails.

Due to symmetry to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 1 - 2\sin\theta$	3	$1+\sqrt{3} \approx 2.7$	2	1	0	$1-\sqrt{3} \approx -0.7$	-1

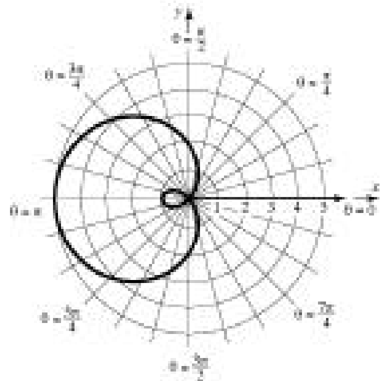


35. $r = 2 - 3\cos\theta$ The graph will be a limacon with an inner loop.
Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 2 - 3\cos(-\theta) = 2 - 3\cos\theta$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 2 - 3\cos(\pi - \theta) = 2 - 3(\cos(\pi)\cos\theta + \sin(\pi)\sin\theta)$$
$$= 2 - 3(-\cos\theta + 0) = 2 + 3\cos\theta$$

The test fails.
The pole: Replace r by $-r$. $-r = 2 - 3\cos\theta$. The test fails.
Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 2 - 3\cos\theta$	-1	$2 - \frac{3\sqrt{3}}{2} \approx -0.6$	$\frac{1}{2}$	2	$\frac{7}{2}$	$2 + \frac{3\sqrt{3}}{2} \approx 4.6$	5

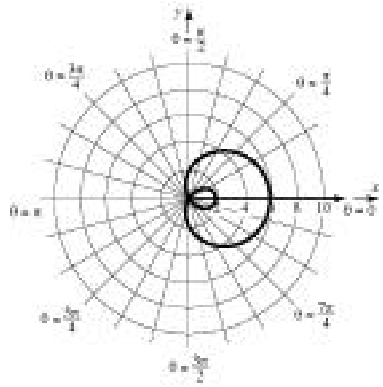


36. $r = 2 + 4\cos\theta$ The graph will be a limacon with an inner loop.
Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 2 + 4\cos(-\theta) = 2 + 4\cos\theta$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 2 + 4\cos(\pi - \theta) = 2 + 4(\cos(\pi)\cos\theta + \sin(\pi)\sin\theta)$$
$$= 2 + 4(-\cos\theta + 0) = 2 - 4\cos\theta$$

The test fails.
The pole: Replace r by $-r$. $-r = 2 + 4\cos\theta$. The test fails.
Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 2 + 4\cos\theta$	6	$2 + 2\sqrt{3} \approx 5.5$	4	2	0	$2 - 2\sqrt{3} \approx -1.5$	-2



37. $r = 3\cos(2\theta)$ The graph will be a rose with four petals. Check for symmetry:

Polar axis: Replace θ by $-\theta$. $r = 3\cos(2(-\theta)) = 3\cos(-2\theta) = 3\cos(2\theta)$.

The graph is symmetric with respect to the polar axis.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

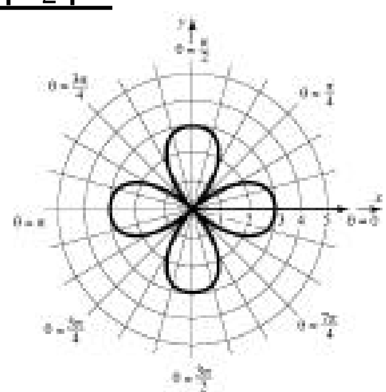
$$r = 3\cos(2(\pi - \theta)) = 3\cos(2\pi - 2\theta)$$
$$= 3(\cos(2\pi)\cos(2\theta) + \sin(2\pi)\sin(2\theta)) = 3(\cos 2\theta + 0) = 3\cos(2\theta)$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

The pole: Since the graph is symmetric to both the polar axis and the line $\theta = \frac{\pi}{2}$, it is also symmetric to the pole.

Due to symmetry, assign values to θ from 0 to $\frac{\pi}{2}$.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 3\cos 2\theta$	3	$\frac{3}{2}$	0	$-\frac{3}{2}$	-3



38. $r = 2\sin(3\theta)$ The graph will be a rose with three petals. Check for symmetry:

Polar axis: Replace θ by $-\theta$. $r = 2\sin(3(-\theta)) = 2\sin(-3\theta) = -2\sin(3\theta)$.

The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

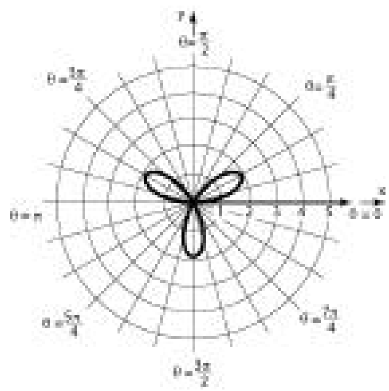
$$r = 2\sin(3(\pi - \theta)) = 2\sin(3\pi - 3\theta)$$
$$= 2(\sin(3\pi)\cos(3\theta) - \cos(3\pi)\sin(3\theta)) = 2(0 + \sin(3\theta)) = 2\sin(3\theta)$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

The pole: Replace r by $-r$. $-r = 2\sin(3\theta)$. The test fails.

Due to symmetry to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 2\sin(3\theta)$	2	0	$-\sqrt{2} \approx -1.4$	-2	0	2	$\sqrt{2} \approx 1.4$	0	-2



39. $r = 4\sin(5\theta)$ The graph will be a rose with five petals. Check for symmetry:

Polar axis: Replace θ by $-\theta$. $r = 4\sin(5(-\theta)) = 4\sin(-5\theta) = -4\sin(5\theta)$.

The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

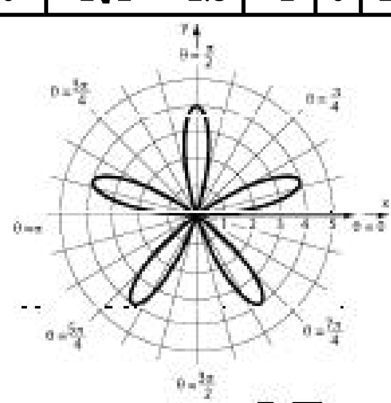
$$r = 4\sin(5(\pi - \theta)) = 4\sin(5\pi - 5\theta)$$
$$= 4(\sin(5\pi)\cos(5\theta) - \cos(5\pi)\sin(5\theta)) = 4(0 + \sin(5\theta)) = 4\sin(5\theta)$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

The pole: Replace r by $-r$. $-r = 4\sin(5\theta)$. The test fails.

Due to symmetry to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 4\sin(5\theta)$	-4	0	$-2\sqrt{2} \approx -2.8$	-2	0	2	$-2\sqrt{2} \approx -2.8$	0	4



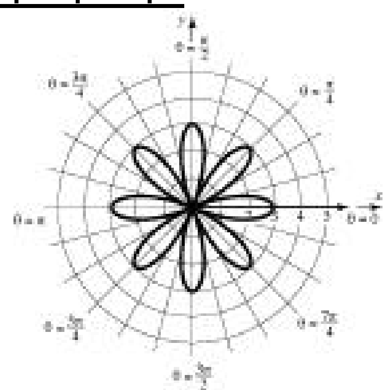
40. $r = 3\cos(4\theta)$ The graph will be a rose with eight petals. Check for symmetry:
Polar axis: Replace θ by $-\theta$. $r = 3\cos(4(-\theta)) = 3\cos(-4\theta) = 3\cos(4\theta)$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 3\cos(4(\pi - \theta)) = 3\cos(4\pi - 4\theta)$$
$$= 3(\cos(4\pi)\cos 4\theta + \sin(4\pi)\sin 4\theta) = 3(\cos 4\theta + 0) = 3\cos(4\theta)$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.
The pole: Since the graph is symmetric to both the polar axis and the line $\theta = \frac{\pi}{2}$, it is also symmetric to the pole.

Due to symmetry, assign values to θ from 0 to $\frac{\pi}{2}$.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 3\cos(4\theta)$	3	$-\frac{3}{2}$	-3	$-\frac{3}{2}$	3



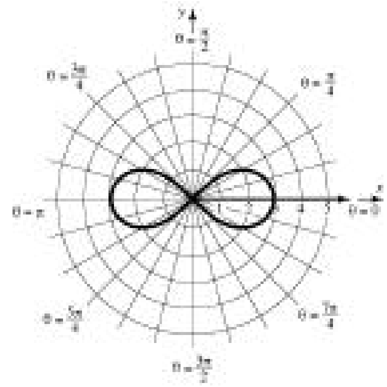
41. $r^2 = 9\cos(2\theta)$ The graph will be a lemniscate. Check for symmetry:
Polar axis: Replace θ by $-\theta$. $r^2 = 9\cos(2(-\theta)) = 9\cos(-2\theta) = 9\cos(2\theta)$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r^2 = 9\cos(2(\pi - \theta)) = 9\cos(2\pi - 2\theta)$$
$$= 9(\cos(2\pi)\cos 2\theta + \sin(2\pi)\sin 2\theta) = 9(\cos 2\theta + 0) = 9\cos(2\theta)$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.
The pole: Since the graph is symmetric to both the polar axis and the line $\theta = \frac{\pi}{2}$, it is also symmetric to the pole.

Due to symmetry, assign values to θ from 0 to $\frac{\pi}{2}$.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = \pm\sqrt{9\cos(2\theta)}$	± 3	$\pm\frac{3\sqrt{2}}{2} \approx \pm 2.1$	0	not defined	not defined

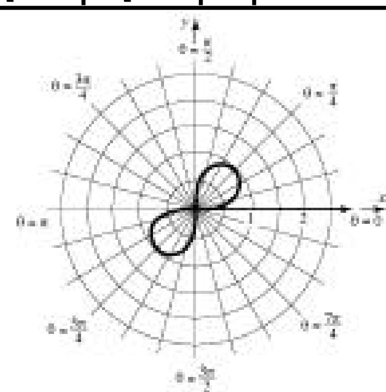


42. $r^2 = \sin(2\theta)$ The graph will be a lemniscate. Check for symmetry:
Polar axis: Replace θ by $-\theta$. $r^2 = \sin(2(-\theta)) = \sin(-2\theta) = -\sin(2\theta)$.
The test fails
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r^2 = \sin(2(\pi - \theta)) = \sin(2\pi - 2\theta)$$
$$= \sin(2\pi)\cos 2\theta - \cos(2\pi)\sin(2\theta) = 0 - \sin(2\theta) = -\sin(2\theta)$$

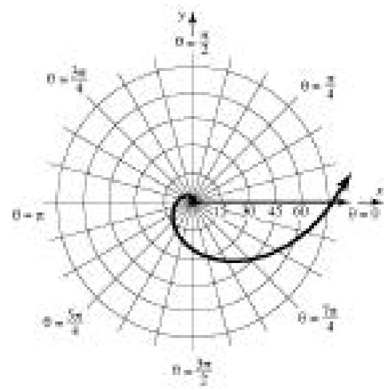
The test fails.
The pole: Replace r by $-r$. $(-r)^2 = \sin(2\theta) \rightarrow r^2 = \sin(2\theta)$.
The graph is symmetric to the pole.
Due to symmetry to the pole, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = \pm\sqrt{\sin(2\theta)}$	0	$\pm\sqrt{\frac{\sqrt{3}}{2}}$	$\pm\sqrt{\frac{\sqrt{3}}{2}}$	0	undefined	undefined	0



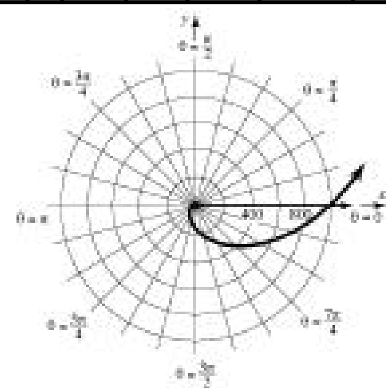
43. $r = 2^\theta$ The graph will be a spiral. Check for symmetry:
Polar axis: Replace θ by $-\theta$. $r = 2^{-\theta}$. The test fails.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$. $r = 2^{\pi-\theta}$. The test fails.
The pole: Replace r by $-r$. $-r = 2^\theta$. The test fails.

θ	$-\pi$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$r = 2^\theta$	0.1	0.3	0.6	1	1.7	3.0	8.8	26.2	77.9



44. $r = 3^{\theta}$ The graph will be a spiral. Check for symmetry:
Polar axis: Replace θ by $-\theta$. $r = 3^{-\theta}$. The test fails.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$. $r = 3^{\pi-\theta}$. The test fails.
The pole: Replace r by $-r$. $-r = 3^{\theta}$. The test fails.

θ	$-\pi$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$r = 3^{\theta}$	0.03	0.2	0.4	1	2.4	5.6	31.5	177.2	995

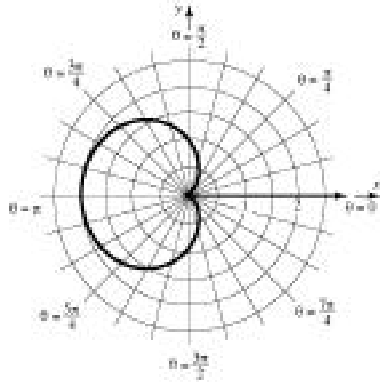


45. $r = 1 - \cos\theta$ The graph will be a cardioid. Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 1 - \cos(-\theta) = 1 - \cos\theta$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 1 - \cos(\pi - \theta) = 1 - (\cos(\pi)\cos\theta + \sin(\pi)\sin\theta)$$
$$= 1 - (-\cos\theta + 0) = 1 + \cos\theta$$ The test fails

The pole: Replace r by $-r$. $-r = 1 - \cos\theta$. The test fails.
Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 1 - \cos\theta$	0	$1 - \frac{\sqrt{3}}{2} \approx 0.1$	$\frac{1}{2}$	1	$\frac{3}{2}$	$1 + \frac{\sqrt{3}}{2} \approx 1.9$	2



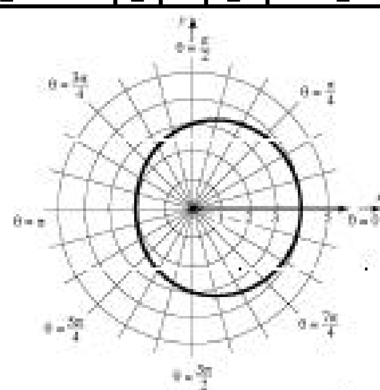
46. $r = 3+\cos\theta$ The graph will be a limacon without an inner loop.
Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 3+\cos(-\theta) = 3+\cos\theta$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 3+\cos(\pi-\theta) = 3+(\cos(\pi)\cos\theta+\sin(\pi)\sin\theta)$$
$$= 3+(-\cos\theta+0) = 3-\cos\theta$$

The test fails.
The pole: Replace r by $-r$. $-r = 3+\cos\theta$. The test fails.

Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 3+\cos\theta$	4	$3+\frac{\sqrt{3}}{2}\approx 3.9$	$\frac{7}{2}$	3	$\frac{5}{2}$	$3-\frac{\sqrt{3}}{2}\approx 2.1$	2



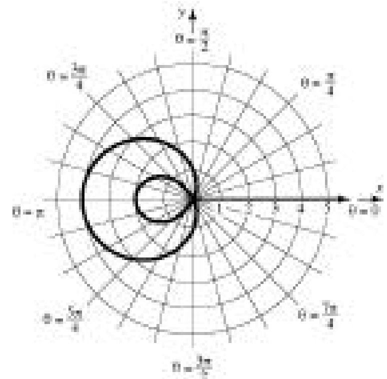
47. $r = 1-3\cos\theta$ The graph will be a limacon with an inner loop.
Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 1-3\cos(-\theta) = 1-3\cos\theta$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 1-3\cos(\pi-\theta) = 1-3(\cos(\pi)\cos\theta+\sin(\pi)\sin\theta)$$
$$= 1-3(-\cos\theta+0) = 1+3\cos\theta$$

The test fails.

The pole: Replace r by $-r$. $-r = 1-3\cos\theta$. The test fails.
Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 1-3\cos\theta$	-2	$1-\frac{3\sqrt{3}}{2}\approx -1.6$	$-\frac{1}{2}$	1	$\frac{5}{2}$	$1+\frac{3\sqrt{3}}{2}\approx 3.6$	4

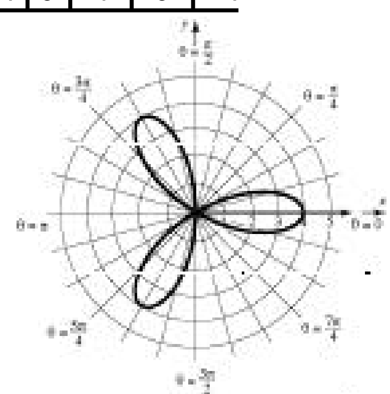


48. $r = 4\cos(3\theta)$ The graph will be a rose with three petals. Check for symmetry:
Polar axis: Replace θ by $-\theta$. $r = 4\cos(3(-\theta)) = 4\cos(-3\theta) = 4\cos(3\theta)$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi-\theta$.

$$r = 4\cos(3(\pi-\theta)) = 4\cos(3\pi-3\theta)$$
$$= 4(\cos(3\pi)\cos 3\theta + \sin(3\pi)\sin 3\theta) = 4(-\cos 3\theta + 0) = -4\cos(3\theta)$$

The test fails.
The pole: Replace r by $-r$. $-r = 4\cos(3\theta)$. The test fails.
Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 4\cos 3\theta$	4	0	-4	0	4	0	-4



49. $r = \dfrac{2}{1-\cos\theta}$ Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = \dfrac{2}{1-\cos(-\theta)} = \dfrac{2}{1-\cos\theta}$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi-\theta$.

$$r = \frac{2}{1-\cos(\pi-\theta)} = \frac{2}{1-(\cos(\pi)\cos\theta + \sin(\pi)\sin\theta)}$$

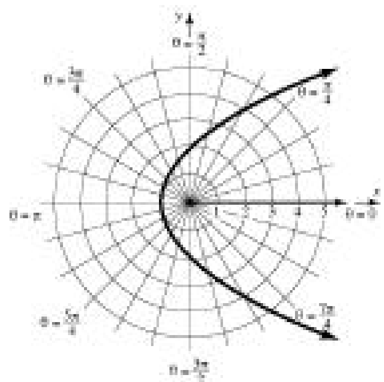
$$= \frac{2}{1-(-\cos\theta+0)} = \frac{2}{1+\cos\theta}$$

The test fails.

The pole: Replace r by $-r$. $-r = \frac{2}{1-\cos\theta}$. The test fails.

Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = \frac{2}{1-\cos\theta}$	undefined	$\frac{2}{\left(1-\frac{\sqrt{3}}{2}\right)} \approx 14.9$	4	2	$\frac{4}{3}$	$\frac{2}{\left(1+\frac{\sqrt{3}}{2}\right)} \approx 1.1$	1



50. $r = \frac{2}{1-2\cos\theta}$ Check for symmetry:

Polar axis: Replace θ by $-\theta$. The result is $r = \frac{2}{1-2\cos(-\theta)} = \frac{2}{1-2\cos\theta}$.

The graph is symmetric with respect to the polar axis.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = \frac{2}{1-2\cos(\pi-\theta)} = \frac{2}{1-2(\cos(\pi)\cos\theta + \sin(\pi)\sin\theta)}$$

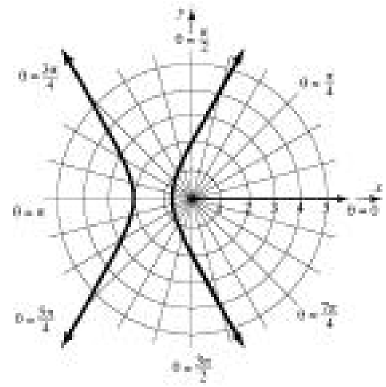
$$= \frac{2}{1-2(-\cos\theta+0)} = \frac{2}{1+2\cos\theta}$$

The test fails.

The pole: Replace r by $-r$. $-r = \frac{2}{1-2\cos\theta}$. The test fails.

Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = \frac{2}{1-2\cos\theta}$	-2	$\frac{2}{1-\sqrt{3}} \approx -2.7$	undefined	2	1	$\frac{2}{1+\sqrt{3}} \approx 0.7$	$\frac{2}{3}$



51. $r = \dfrac{1}{3-2\cos\theta}$ Check for symmetry:

Polar axis: Replace θ by $-\theta$. The result is $r = \dfrac{1}{3-2\cos(-\theta)} = \dfrac{1}{3-2\cos\theta}$.

The graph is symmetric with respect to the polar axis.

The line $\theta = \dfrac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = \frac{1}{3-2\cos(\pi-\theta)} = \frac{1}{3-2(\cos(\pi)\cos\theta + \sin(\pi)\sin\theta)}$$

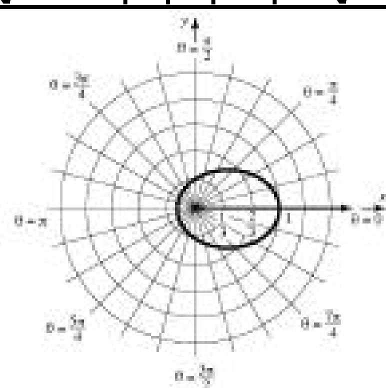
$$= \frac{1}{3-2(-\cos\theta+0)} = \frac{1}{3+2\cos\theta}$$

The test fails.

The pole: Replace r by $-r$. $-r = \dfrac{1}{3-2\cos\theta}$. The test fails.

Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = \frac{1}{3-2\cos\theta}$	1	$\frac{1}{3-\sqrt{3}} \approx 0.8$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3+\sqrt{3}} \approx 0.2$	$\frac{1}{5}$



52. $r = \dfrac{1}{1-\cos\theta}$ Check for symmetry:

Polar axis: Replace θ by $-\theta$. The result is $r = \dfrac{1}{1-\cos(-\theta)} = \dfrac{1}{1-\cos\theta}$.

The graph is symmetric with respect to the polar axis.

The line $\theta = \dfrac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = \frac{1}{1-\cos(\pi-\theta)} = \frac{1}{1-(\cos(\pi)\cos\theta+\sin(\pi)\sin\theta)}$$

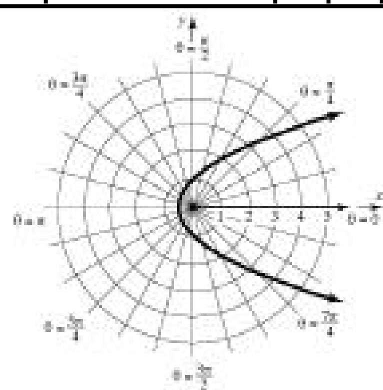
$$= \frac{1}{1-(-\cos\theta+0)} = \frac{1}{1+\cos\theta}$$

The test fails.

The pole: Replace r by $-r$. $-r = \dfrac{1}{1-\cos\theta}$. The test fails.

Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = \frac{2}{1-\cos\theta}$	undefined	$\frac{2}{\left(1-\frac{\sqrt{3}}{2}\right)} \approx 14.9$	4	2	$\frac{4}{3}$	$\frac{2}{\left(1+\frac{\sqrt{3}}{2}\right)} \approx 1.1$	1



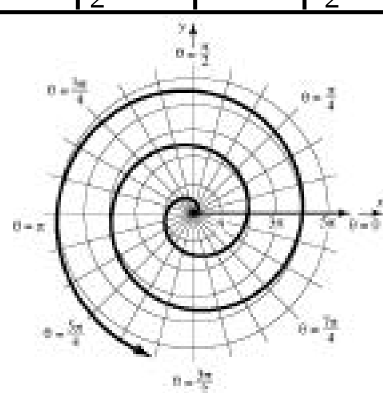
53. $r = \theta,\ \theta \ge 0$ Check for symmetry:

Polar axis: Replace θ by $-\theta$. $r = -\theta$. The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$. $r = \pi - \theta$. The test fails.

The pole: Replace r by $-r$. $-r = \theta$. The test fails.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$r = \theta$	0	$\frac{\pi}{6} \approx 0.5$	$\frac{\pi}{3} \approx 1.0$	$\frac{\pi}{2} \approx 1.6$	$\pi \approx 3.1$	$\frac{3\pi}{2} \approx 4.7$	$2\pi \approx 6.3$



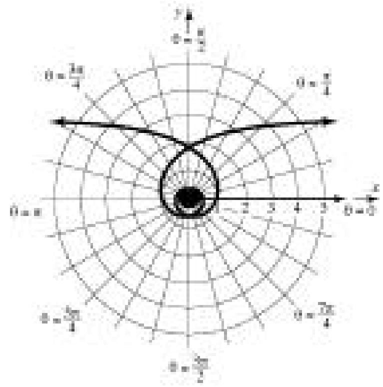
54. $r = \frac{3}{\theta}$ Check for symmetry:

Polar axis: Replace θ by $-\theta$. $r = \frac{3}{-\theta}$. The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$. $r = \frac{3}{\pi - \theta}$. The test fails.

The pole: Replace r by $-r$. $-r = \frac{3}{\theta}$. The test fails.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$r = \frac{3}{\theta}$	undefined	$\frac{18}{\pi} \approx 5.7$	$\frac{9}{\pi} \approx 2.9$	$\frac{6}{\pi} \approx 1.9$	$\frac{3}{\pi} \approx 1.0$	$\frac{2}{\pi} \approx 0.6$	$\frac{3}{2\pi} \approx 0.5$



55. $r = \csc\theta - 2 = \frac{1}{\sin\theta} - 2$, $0 < \theta < \pi$ Check for symmetry:

Polar axis: Replace θ by $-\theta$. $r = \csc(-\theta) - 2 = -\csc\theta - 2$.
The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = \csc(\pi - \theta) - 2 = \frac{1}{\sin(\pi - \theta)} - 2$$

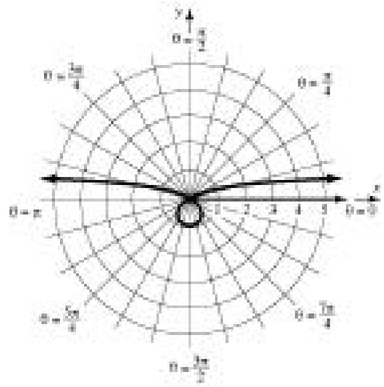
$$= \frac{1}{\sin(\pi)\cos\theta - \cos(\pi)\sin\theta} - 2 = \frac{1}{\sin\theta} - 2 = \csc\theta - 2$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

The pole: Replace r by $-r$. $-r = \csc\theta - 2$. The test fails.

Due to symmetry, assign values to θ from 0 to $\frac{\pi}{2}$.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = \csc\theta - 2$	not defined	0	$\sqrt{2} - 2 \approx -0.6$	$\frac{2\sqrt{3}}{3} - 2 \approx -0.8$	-1



56. $r = \sin\theta\tan\theta$ Check for symmetry:

Polar axis: Replace θ by $-\theta$.

$r = \sin(-\theta)\tan(-\theta) = (-\sin\theta)(-\tan\theta) = \sin\theta\tan\theta$.

The graph is symmetric with respect to the polar axis.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = \sin(\pi - \theta)\tan(\pi - \theta) = \left(\sin(\pi)\cos\theta - \cos(\pi)\sin\theta\right)\left(\frac{\tan(\pi) - \tan\theta}{1 + \tan(\pi)\tan\theta}\right)$$

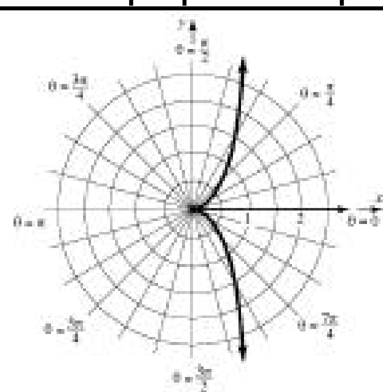
$$= \sin\theta\left(\frac{-\tan\theta}{1}\right) = -\sin\theta\tan\theta$$

The test fails.

The pole: Replace r by $-r$. $-r = \sin\theta\tan\theta$. The test fails.

Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = \sin\theta\tan\theta$	0	$\frac{1}{2}\frac{\sqrt{3}}{3} \approx 0.3$	$\frac{3}{2}$	undefined	$-\frac{3}{2}$	$\frac{1}{2}\left(-\frac{\sqrt{3}}{3}\right) \approx -0.3$	0



57. $r = \tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ Check for symmetry:

Polar axis: Replace θ by $-\theta$. $r = \tan(-\theta) = -\tan\theta$. The test fails.

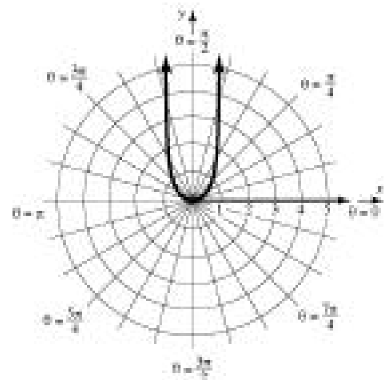
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = \tan(\pi - \theta) = \frac{\tan(\pi) - \tan\theta}{1 + \tan(\pi)\tan\theta} = \frac{-\tan\theta}{1} = -\tan\theta$$

The test fails.

The pole: Replace r by $-r$. $-r = \tan\theta$. The test fails.

θ	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$r = \tan\theta$	$-\sqrt{3} \approx -1.7$	-1	$-\frac{\sqrt{3}}{3} \approx -0.6$	0	$\frac{\sqrt{3}}{3} \approx 0.6$	1	$\sqrt{3} \approx 1.7$



58. $r = \cos\left(\frac{\theta}{2}\right)$ Check for symmetry:

Polar axis: Replace θ by $-\theta$. $r = \cos\left(-\frac{\theta}{2}\right) = \cos\frac{\theta}{2}$.

The graph is symmetric to the polar axis.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

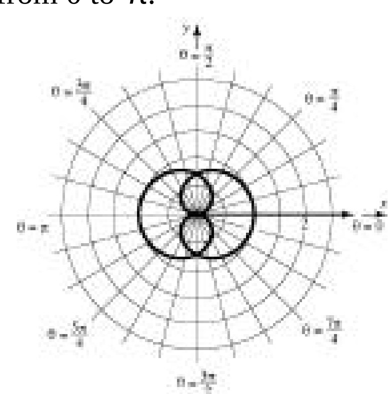
$$r = \cos\left(\frac{\pi - \theta}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \cos\left(\frac{\pi}{2}\right)\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\theta}{2}\right) = \sin\left(\frac{\theta}{2}\right)$$

The test fails.

The pole: Replace r by $-r$. $-r = \cos\frac{\theta}{2}$. The test fails.

Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = \cos\left(\frac{\theta}{2}\right)$	1	0.97	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0.26	0



59. Convert the equation to rectangular form:

$r\sin\theta = a \rightarrow y = a$

The graph of $r\sin\theta = a$ is a horizontal line a units above the pole if $a > 0$, and $|a|$ units below the pole, if $a < 0$.

60. Convert the equation to rectangular form:

$$r\cos\theta = a \rightarrow x = a$$

The graph of $r\cos\theta = a$ is a vertical line a units to the right of the pole if $a > 0$, and $|a|$ units to the left of the pole, if $a < 0$.

61. Convert the equation to rectangular form:

$$r = 2a\sin\theta, a > 0$$

$$r^2 = 2ar\sin\theta$$

$$x^2 + y^2 = 2ay \rightarrow x^2 + y^2 - 2ay = 0 \rightarrow x^2 + (y-a)^2 = a^2$$

Circle: radius a, center at rectangular coordinates $(0, a)$.

62. Convert the equation to rectangular form:

$$r = -2a\sin\theta, a > 0$$

$$r^2 = -2a\ r\sin\theta$$

$$x^2 + y^2 = -2ay \rightarrow x^2 + y^2 + 2ay = 0 \rightarrow x^2 + (y+a)^2 = a^2$$

Circle: radius a, center at rectangular coordinates $(0, -a)$.

63. Convert the equation to rectangular form:

$$r = 2a\cos\theta, a > 0$$

$$r^2 = 2ar\cos\theta$$

$$x^2 + y^2 = 2ax \rightarrow x^2 - 2ax + y^2 = 0 \rightarrow (x-a)^2 + y^2 = a^2$$

Circle: radius a, center at rectangular coordinates $(a, 0)$.

64. Convert the equation to rectangular form:

$$r = -2a\cos\theta, a > 0$$

$$r^2 = -2a\ r\cos\theta$$

$$x^2 + y^2 = -2ax \rightarrow x^2 + 2ax + y^2 = 0 \rightarrow (x+a)^2 + y^2 = a^2$$

Circle: radius a, center at rectangular coordinates $(-a, 0)$.

65. (a) $r^2 = \cos\theta$: $r^2 = \cos(\pi - \theta) \rightarrow r^2 = -\cos\theta$ Test fails.
$(-r)^2 = \cos(-\theta) \rightarrow r^2 = \cos\theta$ New test works.

(b) $r^2 = \sin\theta$: $r^2 = \sin(\pi - \theta) \rightarrow r^2 = \sin\theta$ Test works.
$(-r)^2 = \sin(-\theta) \rightarrow r^2 = -\sin\theta$ New test fails.

66. Symmetry with respect to the pole: In a polar equation, replace θ by $\pi + \theta$. If an equivalent equation results, the graph is symmetric with respect to the pole.

(a) $r^2 = \sin\theta$: $r^2 = \sin(\pi + \theta) \rightarrow r^2 = -\sin\theta$ New test fails.
$(-r)^2 = \sin\theta \rightarrow r^2 = \sin\theta$ Test works.

(b) $r = \cos^2\theta$: $-r = \cos^2\theta$ Test fails.
$r = \cos^2(\pi + \theta) = (-\cos\theta)^2 = \cos^2\theta$ New test works.