Chapter 10

Polar Coordinates; Vectors

10.3 The Complex Plane; De Moivre's Theorem

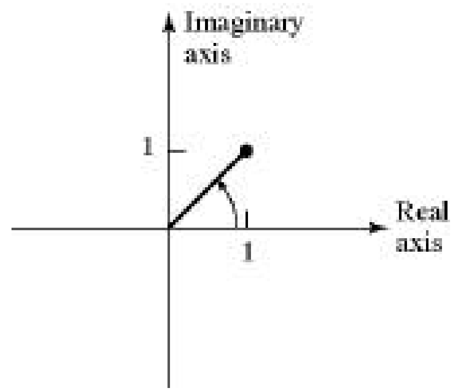
1. $r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

   $\tan\theta = \dfrac{y}{x} = 1 \quad \rightarrow \quad \theta = 45^\circ$

   The polar form of $z = 1 + i$ is

   $z = r(\cos\theta + i\sin\theta)$

   $= \sqrt{2}\left(\cos(45^\circ) + i\sin(45^\circ)\right)$



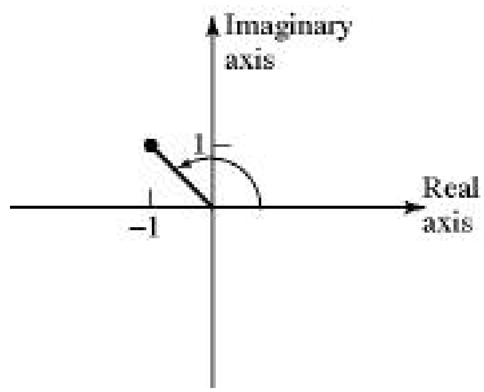
2. $r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

   $\tan\theta = \dfrac{y}{x} = -1 \quad \rightarrow \quad \theta = 135^\circ$

   The polar form of $z = -1 + i$ is

   $z = r(\cos\theta + i\sin\theta)$

   $= \sqrt{2}\left(\cos(135^\circ) + i\sin(135^\circ)\right)$



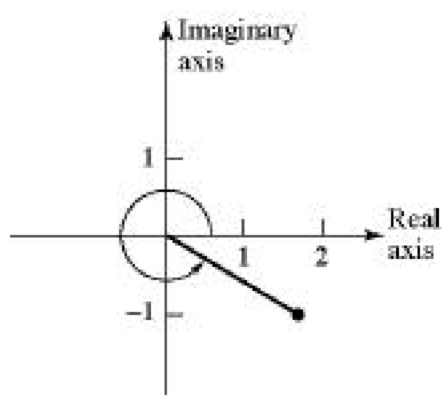
3. $r = \sqrt{x^2 + y^2} = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = \sqrt{4} = 2$

   $\tan\theta = \dfrac{y}{x} = \dfrac{-1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3} \quad \rightarrow \quad \theta = 330^\circ$

   The polar form of $z = \sqrt{3} - i$ is

   $z = r(\cos\theta + i\sin\theta)$

   $= 2\left(\cos(330^\circ) + i\sin(330^\circ)\right)$



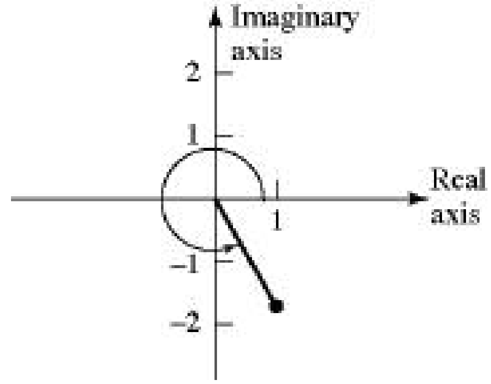
4. $r = \sqrt{x^2 + y^2} = \sqrt{1^2 + \left(-\sqrt{3}\right)^2} = \sqrt{4} = 2$

$\tan\theta = \dfrac{y}{x} = \dfrac{-\sqrt{3}}{1} = -\sqrt{3} \quad \to \quad \theta = 300^c$

The polar form of $z = 1 - \sqrt{3}i$ is

$z = r\left(\cos\theta + i\sin\theta\right)$

$\quad = 2\left(\cos\left(300^\circ\right) + i\sin\left(300^\circ\right)\right)$



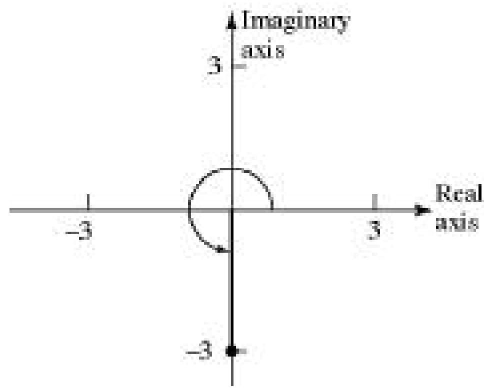
5. $r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-3)^2} = \sqrt{9} = 3$

$\tan\theta = \dfrac{y}{x} = \dfrac{-3}{0} = \text{undefined} \to \theta = 270^\circ$

The polar form of $z = -3i$ is

$z = r\left(\cos\theta + i\sin\theta\right)$

$\quad = 3\left(\cos\left(270^\circ\right) + i\sin\left(270^\circ\right)\right)$



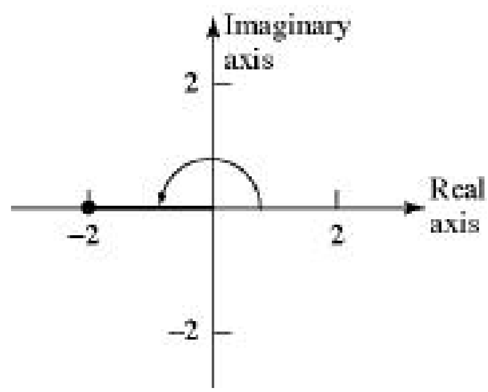
6. $r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + 0^2} = \sqrt{4} = 2$

$\tan\theta = \dfrac{y}{x} = \dfrac{0}{-2} = 0 \quad \to \quad \theta = 180^\circ$

The polar form of $z = -2$ is

$z = r\left(\cos\theta + i\sin\theta\right)$

$\quad = 2\left(\cos\left(180^\circ\right) + i\sin\left(180^\circ\right)\right)$



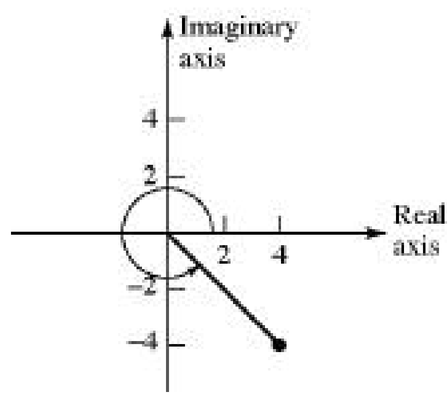
7. $r = \sqrt{x^2 + y^2} = \sqrt{4^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-4}{4} = -1 \quad \to \quad \theta = 315^\circ$

The polar form of $z = 4 - 4i$ is

$z = r\left(\cos\theta + i\sin\theta\right)$

$\quad = 4\sqrt{2}\left(\cos\left(315^\circ\right) + i\sin\left(315^\circ\right)\right)$



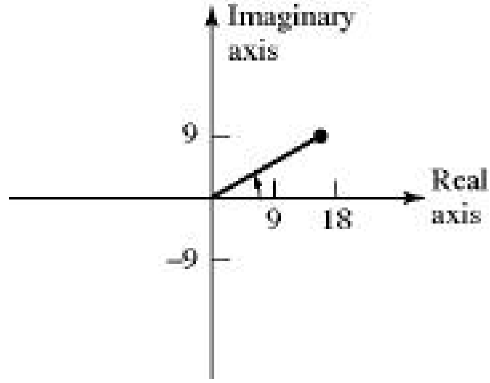
8. $r=\sqrt{x^2+y^2}=\sqrt{\left(9\sqrt{3}\right)^2+9^2}=\sqrt{324}=18$

$\tan\theta=\dfrac{y}{x}=\dfrac{9}{9\sqrt{3}}=\dfrac{\sqrt{3}}{3}\quad\rightarrow\quad\theta=30^\circ$

The polar form of $z=9\sqrt{3}+9i$ is

$z=r(\cos\theta+i\sin\theta)$

$=18\left(\cos(30^\circ)+i\sin(30^\circ)\right)$



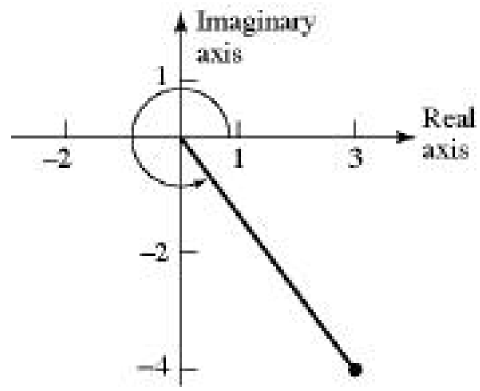
9. $r=\sqrt{x^2+y^2}=\sqrt{3^2+(-4)^2}=\sqrt{25}=5$

$\tan\theta=\dfrac{y}{x}=\dfrac{-4}{3}\quad\rightarrow\quad\theta=306.9$

The polar form of $z=3-4i$ is

$z=r(\cos\theta+i\sin\theta)$

$=5\left(\cos(306.9^\circ)+i\sin(306.9^\circ)\right)$



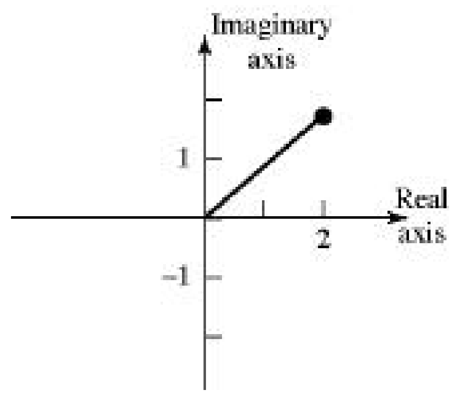
10. $r=\sqrt{x^2+y^2}=\sqrt{2^2+\left(\sqrt{3}\right)^2}=\sqrt{7}$

$\tan\theta=\dfrac{y}{x}=\dfrac{\sqrt{3}}{2}\quad\rightarrow\quad\theta=40.9^\circ$

The polar form of $z=2+\sqrt{3}i$ is

$z=r(\cos\theta+i\sin\theta)$

$=\sqrt{7}\left(\cos(40.9^\circ)+i\sin(40.9^\circ)\right)$



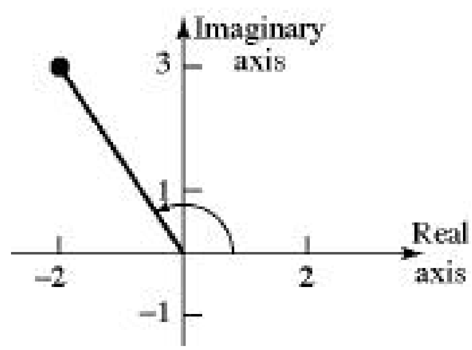
11. $r=\sqrt{x^2+y^2}=\sqrt{(-2)^2+3^2}=\sqrt{13}$

$\tan\theta=\dfrac{y}{x}=\dfrac{3}{-2}=-\dfrac{3}{2}\quad\rightarrow\quad\theta=123.7^\circ$

The polar form of $z=-2+3i$ is

$z=r(\cos\theta+i\sin\theta)$

$=\sqrt{13}\left(\cos(123.7^\circ)+i\sin(123.7^\circ)\right)$



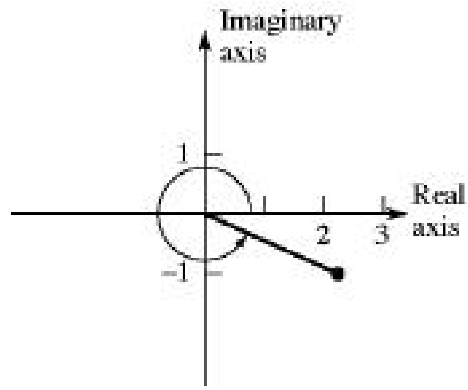
12. $r=\sqrt{x^2+y^2}=\sqrt{\left(\sqrt{5}\right)^2+(-1)^2}=\sqrt{6}$

$\tan\theta=\dfrac{y}{x}=\dfrac{-1}{\sqrt{5}}=-\dfrac{\sqrt{5}}{5} \quad \rightarrow \quad \theta=335.9^{\circ}$

The polar form of $z=\sqrt{5}-i$ is

$z=r(\cos\theta+i\sin\theta)$

$=\sqrt{6}\left(\cos\left(335.9^{\circ}\right)+i\sin\left(335.9^{\circ}\right)\right)$



13. $2\left(\cos\left(120^{\circ}\right)+i\sin\left(120^{\circ}\right)\right)=2\left(-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)=-1+\sqrt{3}\,i$

14. $3\left(\cos\left(210^{\circ}\right)+i\sin\left(210^{\circ}\right)\right)=3\left(-\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i\right)=-\dfrac{3\sqrt{3}}{2}-\dfrac{3}{2}i$

15. $4\left(\cos\left(\dfrac{7\pi}{4}\right)+i\sin\left(\dfrac{7\pi}{4}\right)\right)=4\left(\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i\right)=2\sqrt{2}-2\sqrt{2}\,i$

16. $2\left(\cos\left(\dfrac{5\pi}{6}\right)+i\sin\left(\dfrac{5\pi}{6}\right)\right)=2\left(-\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i\right)=-\sqrt{3}+i$

17. $3\left(\cos\left(\dfrac{3\pi}{2}\right)+i\sin\left(\dfrac{3\pi}{2}\right)\right)=3(0-1i)=-3i$

18. $4\left(\cos\left(\dfrac{\pi}{2}\right)+i\sin\left(\dfrac{\pi}{2}\right)\right)=4(0+1i)=4i$

19. $0.2\left(\cos\left(100^{\circ}\right)+i\sin\left(100^{\circ}\right)\right)\approx 0.2(-0.1736+0.9848i)=-0.0347+0.1970\,i$

20. $0.4\left(\cos\left(200^{\circ}\right)+i\sin\left(200^{\circ}\right)\right)\approx 0.4(-0.9397-0.3420i)=-0.3759-0.1368\,i$

21. $2\left(\cos\left(\dfrac{\pi}{18}\right)+i\sin\left(\dfrac{\pi}{18}\right)\right)\approx 2(0.9848+0.1736i)=1.9696+0.3472i$

22. $3\left(\cos\left(\dfrac{\pi}{10}\right)+i\sin\left(\dfrac{\pi}{10}\right)\right)\approx 3(0.9511+0.3090i)=2.8533+0.9270i$

23. $z\;w=2\left(\cos\left(40^{\circ}\right)+i\sin\left(40^{\circ}\right)\right)\;\;4\left(\cos\left(20^{\circ}\right)+i\sin\left(20^{\circ}\right)\right)$

$=2\;\;4\left(\cos(40^{\circ}+20^{\circ})+i\sin(40^{\circ}+20^{\circ})\right)=8\left(\cos\left(60^{\circ}\right)+i\sin\left(60^{\circ}\right)\right)$

$$\frac{z}{w}=\frac{2\left(\cos\left(40^\circ\right)+i\sin\left(40^\circ\right)\right)}{4\left(\cos\left(20^\circ\right)+i\sin\left(20^\circ\right)\right)}=\frac{2}{4}\left(\cos(40^\circ-20^\circ)+i\sin(40^\circ-20^\circ)\right)$$
$$=\frac{1}{2}\left(\cos\left(20^\circ\right)+i\sin\left(20^\circ\right)\right)$$

24. $$z\ w=\left(\cos\left(120^\circ\right)+i\sin\left(120^\circ\right)\right)\ \left(\cos\left(100^\circ\right)+i\sin\left(100^\circ\right)\right)$$
$$=\left(\cos(120^\circ+100^\circ)+i\sin(120^\circ+100^\circ)\right)=\cos\left(220^\circ\right)+i\sin\left(220^\circ\right)$$
$$\frac{z}{w}=\frac{\left(\cos\left(120^\circ\right)+i\sin\left(120^\circ\right)\right)}{\left(\cos\left(100^\circ\right)+i\sin\left(100^\circ\right)\right)}=\left(\cos(120^\circ-100^\circ)+i\sin(120^\circ-100^\circ)\right)$$
$$=\cos\left(20^\circ\right)+i\sin\left(20^\circ\right)$$

25. $$z\ w=3\left(\cos\left(130^\circ\right)+i\sin\left(130^\circ\right)\right)\ 4\left(\cos\left(270^\circ\right)+i\sin\left(270^\circ\right)\right)$$
$$=3\ 4\left(\cos(130^\circ+270^\circ)+i\sin(130^\circ+270^\circ)\right)=12\left(\cos\left(400^\circ\right)+i\sin\left(400^\circ\right)\right)$$
$$=12\left(\cos(400^\circ-360^\circ)+i\sin(400^\circ-360^\circ)\right)=12\left(\cos\left(40^\circ\right)+i\sin\left(40^\circ\right)\right)$$
$$\frac{z}{w}=\frac{3\left(\cos\left(130^\circ\right)+i\sin\left(130^\circ\right)\right)}{4\left(\cos\left(270^\circ\right)+i\sin\left(270^\circ\right)\right)}=\frac{3}{4}\left(\cos(130^\circ-270^\circ)+i\sin(130^\circ-270^\circ)\right)$$
$$=\frac{3}{4}\left(\cos(-140^\circ)+i\sin(-140^\circ)\right)=\frac{3}{4}\left(\cos\left(220^\circ\right)+i\sin\left(220^\circ\right)\right)$$

26. $$z\ w=2\left(\cos\left(80^\circ\right)+i\sin\left(80^\circ\right)\right)\ 6\left(\cos\left(200^\circ\right)+i\sin\left(200^\circ\right)\right)$$
$$=2\ 6\left(\cos(80^\circ+200^\circ)+i\sin(80^\circ+200^\circ)\right)=12\left(\cos 280^\circ+i\sin 280^\circ\right)$$
$$\frac{z}{w}=\frac{2\left(\cos\left(80^\circ\right)+i\sin\left(80^\circ\right)\right)}{6\left(\cos\left(200^\circ\right)+i\sin\left(200^\circ\right)\right)}=\frac{2}{6}\left(\cos(80^\circ-200^\circ)+i\sin(80^\circ-200^\circ)\right)$$
$$=\frac{1}{3}\left(\cos(-120^\circ)+i\sin(-120^\circ)\right)=\frac{1}{3}\left(\cos\left(240^\circ\right)+i\sin\left(240^\circ\right)\right)$$

27. $$z\ w=2\left(\cos\left(\frac{\pi}{8}\right)+i\sin\left(\frac{\pi}{8}\right)\right)\ 2\left(\cos\left(\frac{\pi}{10}\right)+i\sin\left(\frac{\pi}{10}\right)\right)=2\ 2\left(\cos\left(\frac{\pi}{8}+\frac{\pi}{10}\right)+i\sin\left(\frac{\pi}{8}+\frac{\pi}{10}\right)\right)$$
$$=4\left(\cos\left(\frac{9\pi}{40}\right)+i\sin\left(\frac{9\pi}{40}\right)\right)$$
$$\frac{z}{w}=\frac{2\left(\cos\left(\frac{\pi}{8}\right)+i\sin\left(\frac{\pi}{8}\right)\right)}{2\left(\cos\left(\frac{\pi}{10}\right)+i\sin\left(\frac{\pi}{10}\right)\right)}=\frac{2}{2}\left(\cos\left(\frac{\pi}{8}-\frac{\pi}{10}\right)+i\sin\left(\frac{\pi}{8}-\frac{\pi}{10}\right)\right)=\cos\left(\frac{\pi}{40}\right)+i\sin\left(\frac{\pi}{40}\right)$$

28. $$z\ w=4\left(\cos\left(\frac{3\pi}{8}\right)+i\sin\left(\frac{3\pi}{8}\right)\right)\ 2\left(\cos\left(\frac{9\pi}{16}\right)+i\sin\left(\frac{9\pi}{16}\right)\right)$$
$$=4\ 2\left(\cos\left(\frac{3\pi}{8}+\frac{9\pi}{16}\right)+i\sin\left(\frac{3\pi}{8}+\frac{9\pi}{16}\right)\right)=8\left(\cos\left(\frac{15\pi}{16}\right)+i\sin\left(\frac{15\pi}{16}\right)\right)$$

$$\frac{z}{w}=\frac{4\left(\cos\left(\frac{3\pi}{8}\right)+i\sin\left(\frac{3\pi}{8}\right)\right)}{2\left(\cos\left(\frac{9\pi}{16}\right)+i\sin\left(\frac{9\pi}{16}\right)\right)}=\frac{4}{2}\left(\cos\left(\frac{3\pi}{8}-\frac{9\pi}{16}\right)+i\sin\left(\frac{3\pi}{8}-\frac{9\pi}{16}\right)\right)$$

$$=2\left(\cos\left(-\frac{3\pi}{16}\right)+i\sin\left(-\frac{3\pi}{16}\right)\right)=2\left(\cos\left(\frac{29\pi}{16}\right)+i\sin\left(\frac{29\pi}{16}\right)\right)$$

29. $z=2+2i \qquad r=\sqrt{2^2+2^2}=\sqrt{8}=2\sqrt{2} \qquad \tan\theta=\frac{2}{2}=1 \qquad \theta=45°$

$z=2\sqrt{2}\left(\cos(45°)+i\sin(45°)\right)$

$w=\sqrt{3}-i \qquad r=\sqrt{\left(\sqrt{3}\right)^2+(-1)^2}=\sqrt{4}=2 \qquad \tan\theta=\frac{-1}{\sqrt{3}}=-\frac{\sqrt{3}}{3} \qquad \theta=330°$

$w=2\left(\cos(330°)+i\sin(330°)\right)$

$$z\cdot w=2\sqrt{2}\left(\cos(45°)+i\sin(45°)\right)\cdot 2\left(\cos(330°)+i\sin(330°)\right)$$

$$=2\sqrt{2}\cdot 2\left(\cos(45°+330°)+i\sin(45°+330°)\right)=4\sqrt{2}\left(\cos(375°)+i\sin(375°)\right)$$

$$=4\sqrt{2}\left(\cos(375°-360°)+i\sin(375°-360°)\right)=4\sqrt{2}\left(\cos(15°)+i\sin(15°)\right)$$

$$\frac{z}{w}=\frac{2\sqrt{2}\left(\cos(45°)+i\sin(45°)\right)}{2\left(\cos(330°)+i\sin(330°)\right)}=\frac{2\sqrt{2}}{2}\left(\cos(45°-330°)+i\sin(45°-330°)\right)$$

$$=\sqrt{2}\left(\cos(-285°)+i\sin(-285°)\right)=\sqrt{2}\left(\cos(75°)+i\sin(75°)\right)$$

30. $z=1-i \qquad r=\sqrt{1^2+(-1)^2}=\sqrt{2} \qquad \tan\theta=\frac{-1}{1}=-1 \qquad \theta=315°$

$z=\sqrt{2}\left(\cos(315°)+i\sin(315°)\right)$

$w=1-\sqrt{3}i \qquad r=\sqrt{1^2+\left(-\sqrt{3}\right)^2}=\sqrt{4}=2 \qquad \tan\theta=\frac{-\sqrt{3}}{1}=-\sqrt{3} \qquad \theta=300°$

$w=2\left(\cos(300°)+i\sin(300°)\right)$

$$z\cdot w=\sqrt{2}\left(\cos(315°)+i\sin(315°)\right)\cdot 2\left(\cos(300°)+i\sin(300°)\right)$$

$$=\sqrt{2}\cdot 2\left(\cos((315°)+(300°))+i\sin(315°+300°)\right)=2\sqrt{2}\left(\cos 615°+i\sin 615°\right)$$

$$=2\sqrt{2}\left(\cos(615°-360°)+i\sin(615°-360°)\right)=2\sqrt{2}\left(\cos(255°)+i\sin(255°)\right)$$

$$\frac{z}{w}=\frac{\sqrt{2}\left(\cos 315°+i\sin 315°\right)}{2\left(\cos 300°+i\sin 300°\right)}=\frac{\sqrt{2}}{2}\left(\cos(315°-300°)+i\sin(315°-300°)\right)$$

$$=\frac{\sqrt{2}}{2}\left(\cos(15°)+i\sin(15°)\right)$$

31. $\left[4\left(\cos(40°)+i\sin(40°)\right)\right]^3=4^3\left(\cos(3\cdot 40°)+i\sin(3\cdot 40°)\right)=64\left(\cos(120°)+i\sin(120°)\right)$

$$=64\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=-32+32\sqrt{3}\,i$$

32. $$\left[3\left(\cos\left(80^\circ\right)+i\sin\left(80^\circ\right)\right)\right]^3 = 3^3\left(\cos(3\cdot 80^\circ)+i\sin(3\cdot 80^\circ)\right) = 27\left(\cos\left(240^\circ\right)+i\sin\left(240^\circ\right)\right)$$
$$= 27\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) = -\frac{27}{2}-\frac{27\sqrt{3}}{2}i$$

33. $$\left[2\left(\cos\left(\frac{\pi}{10}\right)+i\sin\left(\frac{\pi}{10}\right)\right)\right]^5 = 2^5\left(\cos\left(5\cdot\frac{\pi}{10}\right)+i\sin\left(5\cdot\frac{\pi}{10}\right)\right) = 32\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)$$
$$= 32(0+1\,i) = 0+32\,i$$

34. $$\left[\sqrt{2}\left(\cos\left(\frac{5\pi}{16}\right)+i\sin\left(\frac{5\pi}{16}\right)\right)\right]^4 = \sqrt{2}^{\,4}\left(\cos\left(4\cdot\frac{5\pi}{16}\right)+i\sin\left(4\cdot\frac{5\pi}{16}\right)\right)$$
$$= 4\left(\cos\left(\frac{5\pi}{4}\right)+i\sin\left(\frac{5\pi}{4}\right)\right) = 4\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right) = -2\sqrt{2}-2\sqrt{2}\,i$$

35. $$\left[\sqrt{3}\left(\cos\left(10^\circ\right)+i\sin\left(10^\circ\right)\right)\right]^6 = \sqrt{3}^{\,6}\left(\cos(6\cdot 10^\circ)+i\sin(6\cdot 10^\circ)\right) = 27\left(\cos\left(60^\circ\right)+i\sin\left(60^\circ\right)\right)$$
$$= 27\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = \frac{27}{2}+\frac{27\sqrt{3}}{2}i$$

36. $$\left[\frac{1}{2}\left(\cos\left(72^\circ\right)+i\sin\left(72^\circ\right)\right)\right]^5 = \left(\frac{1}{2}\right)^5\left(\cos(5\cdot 72^\circ)+i\sin(5\cdot 72^\circ)\right)$$
$$= \frac{1}{32}\left(\cos\left(360^\circ\right)+i\sin\left(360^\circ\right)\right) = \frac{1}{32}(1+0\,i) = \frac{1}{32}$$

37. $$\left[\sqrt{5}\left(\cos\left(\frac{3\pi}{16}\right)+i\sin\left(\frac{3\pi}{16}\right)\right)\right]^4 = \left(\sqrt{5}\right)^4\left(\cos\left(4\cdot\frac{3\pi}{16}\right)+i\sin\left(4\cdot\frac{3\pi}{16}\right)\right)$$
$$= 25\left(\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right) = 25\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right) = -\frac{25\sqrt{2}}{2}+\frac{25\sqrt{2}}{2}i$$

38. $$\left[\sqrt{3}\left(\cos\left(\frac{5\pi}{18}\right)+i\sin\left(\frac{5\pi}{18}\right)\right)\right]^6 = \left(\sqrt{3}\right)^6\left(\cos\left(6\cdot\frac{5\pi}{18}\right)+i\sin\left(6\cdot\frac{5\pi}{18}\right)\right)$$
$$= 27\left(\cos\left(\frac{5\pi}{3}\right)+i\sin\left(\frac{5\pi}{3}\right)\right) = 27\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) = \frac{27}{2}-\frac{27\sqrt{3}}{2}i$$

39. $1-i$ $\quad r = \sqrt{1^2+(-1)^2} = \sqrt{2}$ $\quad \tan\theta = \frac{-1}{1} = -1$ $\quad \theta = \frac{7\pi}{4}$

$$1-i=\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right)$$

$$(1-i)^5=\left[\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right)\right]^5=\left(\sqrt{2}\right)^5\left(\cos\left(5\cdot\frac{7\pi}{4}\right)+i\sin\left(5\cdot\frac{7\pi}{4}\right)\right)$$

$$=4\sqrt{2}\left(\cos\left(\frac{35\pi}{4}\right)+i\sin\left(\frac{35\pi}{4}\right)\right)=4\sqrt{2}\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\,i\right)=-4+4\,i$$

40. $\sqrt{3}-i \qquad r=\sqrt{\left(\sqrt{3}\right)^2+(-1)^2}=\sqrt{4}=2 \qquad \tan\theta=\frac{-1}{\sqrt{3}}=-\frac{\sqrt{3}}{3} \qquad \theta=330^\circ$

$$\sqrt{3}-i=2\left(\cos\left(330^\circ\right)+i\sin\left(330^\circ\right)\right)$$

$$\left(\sqrt{3}-i\right)^6=\left[2\left(\cos\left(330^\circ\right)+i\sin\left(330^\circ\right)\right)\right]^6=2^6\left(\cos\left(6\cdot 330^\circ\right)+i\sin\left(6\cdot 330^\circ\right)\right)$$

$$=64\left(\cos\left(1980^\circ\right)+i\sin\left(1980^\circ\right)\right)=64\left(-1+0\,i\right)=-64$$

41. $\sqrt{2}-i \qquad r=\sqrt{\left(\sqrt{2}\right)^2+(-1)^2}=\sqrt{3} \qquad \tan\theta=\frac{-1}{\sqrt{2}}=-\frac{\sqrt{2}}{2} \qquad \theta\approx 324.7^\circ$

$$\sqrt{2}-i\approx\sqrt{3}\left(\cos\left(324.7^\circ\right)+i\sin\left(324.7^\circ\right)\right)$$

$$\left(\sqrt{2}-i\right)^6\approx\left[\sqrt{3}\left(\cos\left(324.7^\circ\right)+i\sin\left(324.7^\circ\right)\right)\right]^6=\left(\sqrt{3}\right)^6\left(\cos\left(6\cdot 324.7^\circ\right)+i\sin\left(6\cdot 324.7^\circ\right)\right)$$

$$=27\left(\cos\left(1948.2^\circ\right)+i\sin\left(1948.2^\circ\right)\right)\approx 27\left(-0.8499+0.5270\,i\right)$$

$$=-22.95+14.23\,i$$

42. $1-\sqrt{5}\,i \qquad r=\sqrt{1^2+\left(-\sqrt{5}\right)^2}=\sqrt{6} \qquad \tan\theta=\frac{-\sqrt{5}}{1}=-\sqrt{5} \qquad \theta\approx 294.1^\circ$

$$1-\sqrt{5}\,i\approx\sqrt{6}\left(\cos\left(294.1^\circ\right)+i\sin\left(294.1^\circ\right)\right)$$

$$\left(1-\sqrt{5}\,i\right)^8\approx\left[\sqrt{6}\left(\cos\left(294.1^\circ\right)+i\sin\left(294.1^\circ\right)\right)\right]^8=\left(\sqrt{6}\right)^8\left(\cos\left(8\cdot 294.1^\circ\right)+i\sin\left(8\cdot 294.1^\circ\right)\right)$$

$$=1296\left(\cos\left(2352.8^\circ\right)+i\sin\left(2352.8^\circ\right)\right)\approx 1296\left(-0.9751-0.2215\,i\right)$$

$$=-1263.7-287.1\,i$$

43. $1+i \qquad r=\sqrt{1^2+1^2}=\sqrt{2} \qquad \tan\theta=\frac{1}{1}=1 \qquad \theta=45^\circ$

$$1+i=\sqrt{2}\left(\cos 45^\circ+i\sin 45^\circ\right)$$

The three complex cube roots of $1+i=\sqrt{2}\left(\cos\left(45^\circ\right)+i\sin\left(45^\circ\right)\right)$ are:

$$z_k = \sqrt[3]{\sqrt{2}}\left[\cos\left(\frac{45^\circ}{3} + \frac{360^\circ k}{3}\right) + i\sin\left(\frac{45^\circ}{3} + \frac{360^\circ k}{3}\right)\right]$$
$$= \sqrt[6]{2}\left[\cos\left(15^\circ + 120^\circ k\right) + i\sin\left(15^\circ + 120^\circ k\right)\right]$$
$$z_0 = \sqrt[6]{2}\left[\cos\left(15^\circ + 120^\circ \cdot 0\right) + i\sin\left(15^\circ + 120^\circ \cdot 0\right)\right] = \sqrt[6]{2}\left(\cos\left(15^\circ\right) + i\sin\left(15^\circ\right)\right)$$
$$z_1 = \sqrt[6]{2}\left[\cos\left(15^\circ + 120^\circ \cdot 1\right) + i\sin\left(15^\circ + 120^\circ \cdot 1\right)\right] = \sqrt[6]{2}\left(\cos\left(135^\circ\right) + i\sin\left(135^\circ\right)\right)$$
$$z_2 = \sqrt[6]{2}\left[\cos\left(15^\circ + 120^\circ \cdot 2\right) + i\sin\left(15^\circ + 120^\circ \cdot 2\right)\right] = \sqrt[6]{2}\left(\cos\left(255^\circ\right) + i\sin\left(255^\circ\right)\right)$$

44. $\sqrt{3} - i \qquad r = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = \sqrt{4} = 2 \qquad \tan\theta = \dfrac{-1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3} \qquad \theta = 330^\circ$

$\sqrt{3} - i = 2\left(\cos\left(330^\circ\right) + i\sin\left(330^\circ\right)\right)$

The four complex fourth roots of $\sqrt{3} - i = 2\left(\cos 330^\circ + i\sin 330^\circ\right)$ are:

$$z_k = \sqrt[4]{2}\left[\cos\left(\frac{330^\circ}{4} + \frac{360^\circ k}{4}\right) + i\sin\left(\frac{330^\circ}{4} + \frac{360^\circ k}{4}\right)\right]$$
$$= \sqrt[4]{2}\left[\cos\left(82.5^\circ + 90^\circ k\right) + i\sin\left(82.5^\circ + 90^\circ k\right)\right]$$
$$z_0 = \sqrt[4]{2}\left[\cos\left(82.5^\circ + 90^\circ \cdot 0\right) + i\sin\left(82.5^\circ + 90^\circ \cdot 0\right)\right] = \sqrt[4]{2}\left(\cos\left(82.5^\circ\right) + i\sin\left(82.5^\circ\right)\right)$$
$$z_1 = \sqrt[4]{2}\left[\cos\left(82.5^\circ + 90^\circ \cdot 1\right) + i\sin\left(82.5^\circ + 90^\circ \cdot 1\right)\right] = \sqrt[4]{2}\left(\cos\left(172.5^\circ\right) + i\sin\left(172.5^\circ\right)\right)$$
$$z_2 = \sqrt[4]{2}\left[\cos\left(82.5^\circ + 90^\circ \cdot 2\right) + i\sin\left(82.5^\circ + 90^\circ \cdot 2\right)\right] = \sqrt[4]{2}\left(\cos\left(262.5^\circ\right) + i\sin\left(262.5^\circ\right)\right)$$
$$z_3 = \sqrt[4]{2}\left[\cos\left(82.5^\circ + 90^\circ \cdot 3\right) + i\sin\left(82.5^\circ + 90^\circ \cdot 3\right)\right] = \sqrt[4]{2}\left(\cos\left(352.5^\circ\right) + i\sin\left(352.5^\circ\right)\right)$$

45. $4 - 4\sqrt{3}i \qquad r = \sqrt{4^2 + \left(-4\sqrt{3}\right)^2} = \sqrt{64} = 8 \qquad \tan\theta = \dfrac{-4\sqrt{3}}{4} = -\sqrt{3} \qquad \theta = 300^\circ$

$4 - 4\sqrt{3}\,i = 8\left(\cos\left(300^\circ\right) + i\sin\left(300^\circ\right)\right)$

The four complex fourth roots of $4 - 4\sqrt{3}\,i = 8\left(\cos\left(300^\circ\right) + i\sin\left(300^\circ\right)\right)$ are:

$$z_k = \sqrt[4]{8}\left[\cos\left(\frac{300^\circ}{4} + \frac{360^\circ k}{4}\right) + i\sin\left(\frac{300^\circ}{4} + \frac{360^\circ k}{4}\right)\right]$$
$$= \sqrt[4]{8}\left[\cos\left(75^\circ + 90^\circ k\right) + i\sin\left(75^\circ + 90^\circ k\right)\right]$$
$$z_0 = \sqrt[4]{8}\left[\cos\left(75^\circ + 90^\circ \cdot 0\right) + i\sin\left(75^\circ + 90^\circ \cdot 0\right)\right] = \sqrt[4]{8}\left(\cos\left(75^\circ\right) + i\sin\left(75^\circ\right)\right)$$
$$z_1 = \sqrt[4]{8}\left[\cos\left(75^\circ + 90^\circ \cdot 1\right) + i\sin\left(75^\circ + 90^\circ \cdot 1\right)\right] = \sqrt[4]{8}\left(\cos\left(165^\circ\right) + i\sin\left(165^\circ\right)\right)$$
$$z_2 = \sqrt[4]{8}\left[\cos\left(75^\circ + 90^\circ \cdot 2\right) + i\sin\left(75^\circ + 90^\circ \cdot 2\right)\right] = \sqrt[4]{8}\left(\cos\left(255^\circ\right) + i\sin\left(255^\circ\right)\right)$$
$$z_3 = \sqrt[4]{8}\left[\cos\left(75^\circ + 90^\circ \cdot 3\right) + i\sin\left(75^\circ + 90^\circ \cdot 3\right)\right] = \sqrt[4]{8}\left(\cos\left(345^\circ\right) + i\sin\left(345^\circ\right)\right)$$

46. $-8 - 8i \qquad r = \sqrt{(-8)^2 + (-8)^2} = 8\sqrt{2} \qquad \tan\theta = \dfrac{-8}{-8} = 1 \qquad \theta = 225^\circ$

$-8 - 8i = 8\sqrt{2}\left(\cos\left(225^\circ\right) + i\sin 225^\circ\right)$

The three complex cube roots of $-8 - 8i = 8\sqrt{2}\left(\cos\left(225^\circ\right) + i\sin\left(225^\circ\right)\right)$ are:

$$z_k = \sqrt[3]{8\sqrt{2}}\left[\cos\left(\frac{225^\circ}{3} + \frac{360^\circ k}{3}\right) + i\sin\left(\frac{225^\circ}{3} + \frac{360^\circ k}{3}\right)\right]$$

$$= 2\sqrt[6]{2}\left[\cos\left(75^\circ + 120^\circ k\right) + i\sin\left(75^\circ + 120^\circ k\right)\right]$$

$$z_0 = 2\sqrt[6]{2}\left[\cos\left(75^\circ + 120^\circ\cdot 0\right) + i\sin\left(75^\circ + 120^\circ\cdot 0\right)\right] = 2\sqrt[6]{2}\left(\cos\left(75^\circ\right) + i\sin\left(75^\circ\right)\right)$$

$$z_1 = 2\sqrt[6]{2}\left[\cos\left(75^\circ + 120^\circ\cdot 1\right) + i\sin\left(75^\circ + 120^\circ\cdot 1\right)\right] = 2\sqrt[6]{2}\left(\cos\left(195^\circ\right) + i\sin\left(195^\circ\right)\right)$$

$$z_2 = 2\sqrt[6]{2}\left[\cos\left(75^\circ + 120^\circ\cdot 2\right) + i\sin\left(75^\circ + 120^\circ\cdot 2\right)\right] = 2\sqrt[6]{2}\left(\cos\left(315^\circ\right) + i\sin\left(315^\circ\right)\right)$$

47. $-16i$ $\quad r = \sqrt{0^2 + (-16)^2} = \sqrt{256} = 16 \quad \tan\theta = \frac{-16}{0} = \text{undefined} \quad \theta = 270^\circ$

$-16i = 16\left(\cos\left(270^\circ\right) + i\sin\left(270^\circ\right)\right)$

The four complex fourth roots of $-16i = 16\left(\cos 270^\circ + i\sin 270^\circ\right)$ are:

$$z_k = \sqrt[4]{16}\left[\cos\left(\frac{270^\circ}{4} + \frac{360^\circ k}{4}\right) + i\sin\left(\frac{270^\circ}{4} + \frac{360^\circ k}{4}\right)\right]$$

$$= 2\left[\cos\left(67.5^\circ + 90^\circ k\right) + i\sin\left(67.5^\circ + 90^\circ k\right)\right]$$

$$z_0 = 2\left[\cos\left(67.5^\circ + 90^\circ\cdot 0\right) + i\sin\left(67.5^\circ + 90^\circ\cdot 0\right)\right] = 2\left(\cos\left(67.5^\circ\right) + i\sin\left(67.5^\circ\right)\right)$$

$$z_1 = 2\left[\cos\left(67.5^\circ + 90^\circ\cdot 1\right) + i\sin\left(67.5^\circ + 90^\circ\cdot 1\right)\right] = 2\left(\cos\left(157.5^\circ\right) + i\sin\left(157.5^\circ\right)\right)$$

$$z_2 = 2\left[\cos\left(67.5^\circ + 90^\circ\cdot 2\right) + i\sin\left(67.5^\circ + 90^\circ\cdot 2\right)\right] = 2\left(\cos\left(247.5^\circ\right) + i\sin\left(247.5^\circ\right)\right)$$

$$z_3 = 2\left[\cos\left(67.5^\circ + 90^\circ\cdot 3\right) + i\sin\left(67.5^\circ + 90^\circ\cdot 3\right)\right] = 2\left(\cos\left(337.5^\circ\right) + i\sin\left(337.5^\circ\right)\right)$$

48. -8 $\quad r = \sqrt{(-8)^2 + 0^2} = 8 \quad \tan\theta = \frac{0}{-8} = 0 \quad \theta = 180^\circ$

$-8 = 8\left(\cos\left(180^\circ\right) + i\sin\left(180^\circ\right)\right)$

The three complex cube roots of $-8 = 8\left(\cos\left(180^\circ\right) + i\sin\left(180^\circ\right)\right)$ are:

$$z_k = \sqrt[3]{8}\left[\cos\left(\frac{180^\circ}{3} + \frac{360^\circ k}{3}\right) + i\sin\left(\frac{180^\circ}{3} + \frac{360^\circ k}{3}\right)\right]$$

$$= 2\left[\cos\left(60^\circ + 120^\circ k\right) + i\sin\left(60^\circ + 120^\circ k\right)\right]$$

$$z_0 = 2\left[\cos\left(60^\circ + 120^\circ\cdot 0\right) + i\sin\left(60^\circ + 120^\circ\cdot 0\right)\right] = 2\left(\cos\left(60^\circ\right) + i\sin\left(60^\circ\right)\right)$$

$$z_1 = 2\left[\cos\left(60^\circ + 120^\circ\cdot 1\right) + i\sin\left(60^\circ + 120^\circ\cdot 1\right)\right] = 2\left(\cos\left(180^\circ\right) + i\sin\left(180^\circ\right)\right)$$

$$z_2 = 2\left[\cos\left(60^\circ + 120^\circ\cdot 2\right) + i\sin\left(60^\circ + 120^\circ\cdot 2\right)\right] = 2\left(\cos\left(300^\circ\right) + i\sin\left(300^\circ\right)\right)$$

49. i $\quad r = \sqrt{0^2 + 1^2} = \sqrt{1} = 1 \quad \tan\theta = \frac{1}{0} = \text{undefined} \quad \theta = 90^\circ$

$i = 1\left(\cos\left(90^\circ\right) + i\sin\left(90^\circ\right)\right)$

The five complex fifth roots of $i = 1\left(\cos\left(90^\circ\right) + i\sin\left(90^\circ\right)\right)$ are:

$$z_k = \sqrt[5]{1}\left[\cos\left(\frac{90^\circ}{5} + \frac{360^\circ k}{5}\right) + i\sin\left(\frac{90^\circ}{5} + \frac{360^\circ k}{5}\right)\right]$$
$$= 1\left[\cos\left(18^\circ + 72^\circ k\right) + i\sin\left(18^\circ + 72^\circ k\right)\right]$$
$$z_0 = 1\left[\cos\left(18^\circ + 72^\circ \cdot 0\right) + i\sin\left(18^\circ + 72^\circ \cdot 0\right)\right] = \cos\left(18^\circ\right) + i\sin\left(18^\circ\right)$$
$$z_1 = 1\left[\cos\left(18^\circ + 72^\circ \cdot 1\right) + i\sin\left(18^\circ + 72^\circ \cdot 1\right)\right] = \cos\left(90^\circ\right) + i\sin\left(90^\circ\right)$$
$$z_2 = 1\left[\cos\left(18^\circ + 72^\circ \cdot 2\right) + i\sin\left(18^\circ + 72^\circ \cdot 2\right)\right] = \cos\left(162^\circ\right) + i\sin\left(162^\circ\right)$$
$$z_3 = 1\left[\cos\left(18^\circ + 72^\circ \cdot 3\right) + i\sin\left(18^\circ + 72^\circ \cdot 3\right)\right] = \cos\left(234^\circ\right) + i\sin\left(234^\circ\right)$$
$$z_4 = 1\left[\cos\left(18^\circ + 72^\circ \cdot 4\right) + i\sin\left(18^\circ + 72^\circ \cdot 4\right)\right] = \cos\left(306^\circ\right) + i\sin\left(306^\circ\right)$$

50. $-i \qquad r = \sqrt{0^2 + (-1)^2} = \sqrt{1} = 1 \qquad \tan\theta = \frac{-1}{0} = \text{undefined} \qquad \theta = 270^\circ$

$-i = 1\left(\cos\left(270^\circ\right) + i\sin\left(270^\circ\right)\right)$

The five complex fifth roots of $-i = 1\left(\cos\left(270^\circ\right) + i\sin\left(270^\circ\right)\right)$ are:

$$z_k = \sqrt[5]{1}\left[\cos\left(\frac{270^\circ}{5} + \frac{360^\circ k}{5}\right) + i\sin\left(\frac{270^\circ}{5} + \frac{360^\circ k}{5}\right)\right]$$
$$= 1\left[\cos\left(54^\circ + 72^\circ k\right) + i\sin\left(54^\circ + 72^\circ k\right)\right]$$
$$z_0 = 1\left[\cos\left(54^\circ + 72^\circ \cdot 0\right) + i\sin\left(54^\circ + 72^\circ \cdot 0\right)\right] = \cos\left(54^\circ\right) + i\sin\left(54^\circ\right)$$
$$z_1 = 1\left[\cos\left(54^\circ + 72^\circ \cdot 1\right) + i\sin\left(54^\circ + 72^\circ \cdot 1\right)\right] = \cos\left(126^\circ\right) + i\sin\left(126^\circ\right)$$
$$z_2 = 1\left[\cos\left(54^\circ + 72^\circ \cdot 2\right) + i\sin\left(54^\circ + 72^\circ \cdot 2\right)\right] = \cos\left(198^\circ\right) + i\sin\left(198^\circ\right)$$
$$z_3 = 1\left[\cos\left(54^\circ + 72^\circ \cdot 3\right) + i\sin\left(54^\circ + 72^\circ \cdot 3\right)\right] = \cos\left(270^\circ\right) + i\sin\left(270^\circ\right)$$
$$z_4 = 1\left[\cos\left(54^\circ + 72^\circ \cdot 4\right) + i\sin\left(54^\circ + 72^\circ \cdot 4\right)\right] = \cos\left(342^\circ\right) + i\sin\left(342^\circ\right)$$

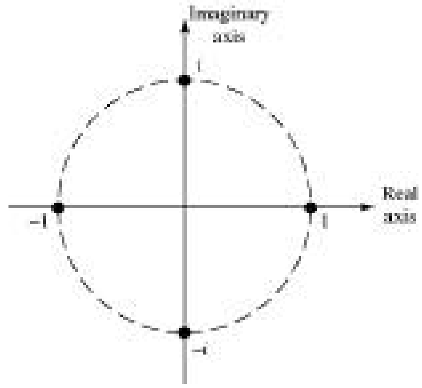
51. $1 = 1 + 0i \qquad r = \sqrt{1^2 + 0^2} = \sqrt{1} = 1 \qquad \tan\theta = \frac{0}{1} = 0 \qquad \theta = 0^\circ$

$1 + 0i = 1\left(\cos\left(0^\circ\right) + i\sin\left(0^\circ\right)\right)$

The four complex fourth roots of $1 + 0i = 1\left(\cos\left(0^\circ\right) + i\sin\left(0^\circ\right)\right)$ are:

$$z_k = \sqrt[4]{1}\left[\cos\left(\frac{0^\circ}{4} + \frac{360^\circ k}{4}\right) + i\sin\left(\frac{0^\circ}{4} + \frac{360^\circ k}{4}\right)\right]$$
$$= 1\left[\cos\left(90^\circ k\right) + i\sin\left(90^\circ k\right)\right]$$
$$z_0 = \cos\left(90^\circ \cdot 0\right) + i\sin\left(90^\circ \cdot 0\right) = \cos\left(0^\circ\right) + i\sin\left(0^\circ\right) = 1 + 0i = 1$$
$$z_1 = \cos\left(90^\circ \cdot 1\right) + i\sin\left(90^\circ \cdot 1\right) = \cos\left(90^\circ\right) + i\sin\left(90^\circ\right) = 0 + 1i = i$$
$$z_2 = \cos\left(90^\circ \cdot 2\right) + i\sin\left(90^\circ \cdot 2\right) = \cos\left(180^\circ\right) + i\sin\left(180^\circ\right) = -1 + 0i = -1$$
$$z_3 = \cos\left(90^\circ \cdot 3\right) + i\sin\left(90^\circ \cdot 3\right) = \cos\left(270^\circ\right) + i\sin\left(270^\circ\right) = 0 - 1i = -i$$

The complex roots are: $1,\ i,\ -1,\ -i$.



52. $1 = 1 + 0i \qquad r = \sqrt{1^2 + 0^2} = \sqrt{1} = 1 \qquad \tan\theta = \frac{0}{1} = 0 \qquad \theta = 0^{\circ}$

$1 + 0i = 1\left(\cos\left(0^{\circ}\right) + i\sin\left(0^{\circ}\right)\right)$

The six complex sixth roots of $1 + 0i = 1\left(\cos 0^{\circ} + i\sin 0^{\circ}\right)$ are:

$$z_k = \sqrt[6]{1}\left[\cos\left(\frac{0^{\circ}}{6} + \frac{360^{\circ} k}{6}\right) + i\sin\left(\frac{0^{\circ}}{6} + \frac{360^{\circ} k}{6}\right)\right]$$

$$= 1\left[\cos\left(60^{\circ} k\right) + i\sin\left(60^{\circ} k\right)\right]$$

$$z_0 = \cos\left(60^{\circ}\cdot 0\right) + i\sin\left(60^{\circ}\cdot 0\right) = \cos\left(0^{\circ}\right) + i\sin\left(0^{\circ}\right) = 1 + 0i = 1$$

$$z_1 = \cos\left(60^{\circ}\cdot 1\right) + i\sin\left(60^{\circ}\cdot 1\right) = \cos\left(60^{\circ}\right) + i\sin\left(60^{\circ}\right) = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

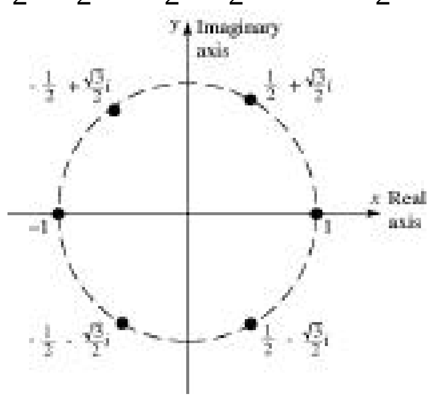
$$z_2 = \cos\left(60^{\circ}\cdot 2\right) + i\sin\left(60^{\circ}\cdot 2\right) = \cos\left(120^{\circ}\right) + i\sin\left(120^{\circ}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z_3 = \cos\left(60^{\circ}\cdot 3\right) + i\sin\left(60^{\circ}\cdot 3\right) = \cos\left(180^{\circ}\right) + i\sin\left(180^{\circ}\right) = -1 + 0i = -1$$

$$z_4 = \cos\left(60^{\circ}\cdot 4\right) + i\sin\left(60^{\circ}\cdot 4\right) = \cos\left(240^{\circ}\right) + i\sin\left(240^{\circ}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$z_5 = \cos\left(60^{\circ}\cdot 5\right) + i\sin\left(60^{\circ}\cdot 5\right) = \cos\left(300^{\circ}\right) + i\sin\left(300^{\circ}\right) = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

The complex roots are: $1,\ \frac{1}{2} + \frac{\sqrt{3}}{2}i,\ -\frac{1}{2} + \frac{\sqrt{3}}{2}i,\ -1,\ -\frac{1}{2} - \frac{\sqrt{3}}{2}i,\ \frac{1}{2} - \frac{\sqrt{3}}{2}i$.



53. Let $w = r\left(\cos\theta + i\sin\theta\right)$ be a complex number. If $w \neq 0$, there are n distinct nth roots of w, given by the formula:

$$z_k = \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)\right), \text{ where } k = 0, 1, 2, \ldots, n-1$$

$$|z_k| = \sqrt[n]{r} \text{ for all } k$$

54. Since $|z_k| = \sqrt[n]{r}$ for all k, each of the complex n[th] roots lies on a circle with center at the origin and radius $\sqrt[n]{r}$.

55. Examining the formula for the distinct complex nth roots of the complex number $w = r(\cos\theta + i\sin\theta)$,

$$z_k = \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)\right), \text{ where } k = 0, 1, 2, \ldots, n-1$$

we see that the z_k are spaced apart by an angle of $\frac{2\pi}{n}$.

56. Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$$\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} \cdot \frac{(\cos\theta_2 - i\sin\theta_2)}{(\cos\theta_2 - i\sin\theta_2)}$$

$$= \frac{r_1}{r_2} \cdot \frac{\cos\theta_1\cos\theta_2 - i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2}{\cos^2\theta_2 + \sin^2\theta_2}$$

$$= \frac{r_1}{r_2} \cdot \frac{\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 + i(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)}{1}$$

$$= \frac{r_1}{r_2}\left(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right)$$