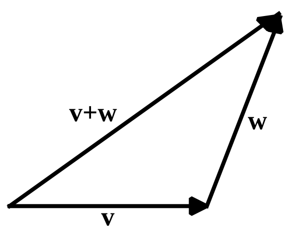
Chapter 10

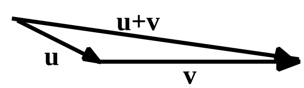
Polar Coordinates; Vectors

10.4 Vectors

1. **v + w**

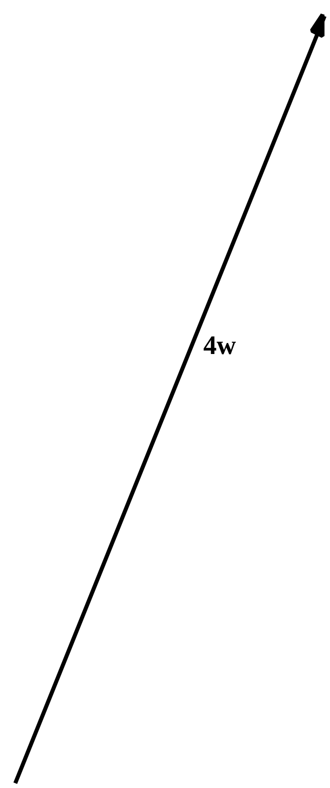


2. **u + v**

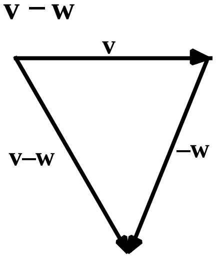


3. **3v**

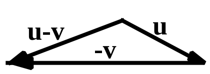
4. **4 w**



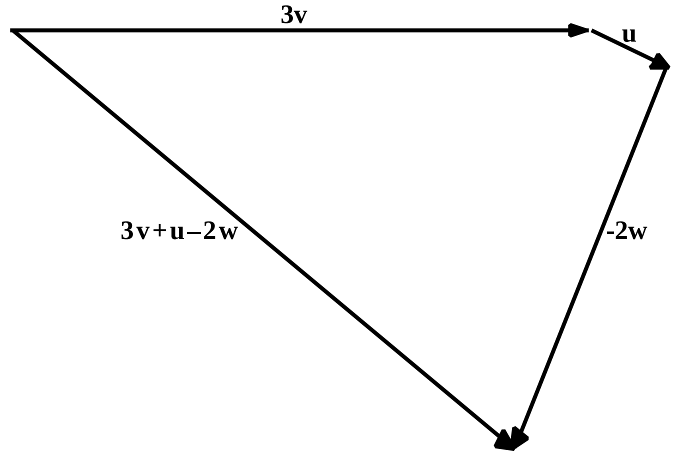
5. **v − w**



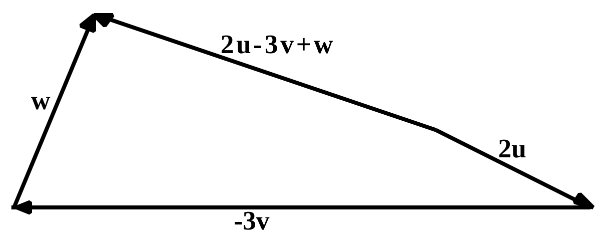
6. **u − v**



7. $\mathbf{3v + u - 2w}$



8. $\mathbf{2u - 3v + w}$



9. True

10. False $\mathbf{K + G = -F}$

11. False $\mathbf{C = -F + E - D}$

12. True

13. False $\mathbf{D - E = H + G}$

14. False $\mathbf{C + H = -G - F}$

15. True

16. True

17. If $\|\mathbf{v}\| = 4$, then $\|3\mathbf{v}\| = |3|\|\mathbf{v}\| = 3(4) = 12$.

18. If $\|\mathbf{v}\| = 2$, then $\|-4\mathbf{v}\| = |-4|\|\mathbf{v}\| = 4(2) = 8$.

19. $P = (0, 0), Q = (3, 4)$ $\quad \mathbf{v} = (3-0)\mathbf{i} + (4-0)\mathbf{j} = 3\mathbf{i} + 4\mathbf{j}$

20. $P = (0, 0), Q = (-3, -5)$ $\quad \mathbf{v} = (-3-0)\mathbf{i} + (-5-0)\mathbf{j} = -3\mathbf{i} - 5\mathbf{j}$

21. $P = (3, 2), Q = (5, 6)$ $\quad \mathbf{v} = (5-3)\mathbf{i} + (6-2)\mathbf{j} = 2\mathbf{i} + 4\mathbf{j}$

22. $P = (-3, 2), Q = (6, 5)$ $\quad \mathbf{v} = (6-(-3))\mathbf{i} + (5-2)\mathbf{j} = 9\mathbf{i} + 3\mathbf{j}$

23. $P = (-2, -1), Q = (6, -2)$ $\quad \mathbf{v} = (6-(-2))\mathbf{i} + (-2-(-1))\mathbf{j} = 8\mathbf{i} - \mathbf{j}$

24. $P = (-1, 4), Q = (6, 2)$ $\quad \mathbf{v} = (6-(-1))\mathbf{i} + (2-4)\mathbf{j} = 7\mathbf{i} - 2\mathbf{j}$

25. $P = (1,\ 0),\ Q = (0, 1)$ $\quad \mathbf{v} = (0-1)\mathbf{i} + (1-0)\mathbf{j} = -\mathbf{i} + \mathbf{j}$

26. $P = (1,\ 1),\ Q = (2, 2)$ $\quad \mathbf{v} = (2-1)\mathbf{i} + (2-1)\mathbf{j} = \mathbf{i} + \mathbf{j}$

27. For $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$, $\|\mathbf{v}\| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$

28. For $\mathbf{v} = -5\mathbf{i} + 12\mathbf{j}$, $\|\mathbf{v}\| = \sqrt{(-5)^2 + 12^2} = \sqrt{169} = 13$

29. For $\mathbf{v} = \mathbf{i} - \mathbf{j}$, $\|\mathbf{v}\| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

30. For $\mathbf{v} = -\mathbf{i} - \mathbf{j}$, $\|\mathbf{v}\| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

31. For $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$, $\|\mathbf{v}\| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$

32. For $\mathbf{v} = 6\mathbf{i} + 2\mathbf{j}$, $\|\mathbf{v}\| = \sqrt{6^2 + 2^2} = \sqrt{40} = 2\sqrt{10}$

33. $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j},\ \mathbf{w} = -2\mathbf{i} + 3\mathbf{j}$
$$2\mathbf{v} + 3\mathbf{w} = 2(3\mathbf{i} - 5\mathbf{j}) + 3(-2\mathbf{i} + 3\mathbf{j}) = 6\mathbf{i} - 10\mathbf{j} - 6\mathbf{i} + 9\mathbf{j} = -\mathbf{j}$$

34. $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j},\ \mathbf{w} = -2\mathbf{i} + 3\mathbf{j}$
$$3\mathbf{v} - 2\mathbf{w} = 3(3\mathbf{i} - 5\mathbf{j}) - 2(-2\mathbf{i} + 3\mathbf{j}) = 9\mathbf{i} - 15\mathbf{j} + 4\mathbf{i} - 6\mathbf{j} = 13\mathbf{i} - 21\mathbf{j}$$

35. $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j},\ \mathbf{w} = -2\mathbf{i} + 3\mathbf{j}$
$$\|\mathbf{v} - \mathbf{w}\| = \|(3\mathbf{i} - 5\mathbf{j}) - (-2\mathbf{i} + 3\mathbf{j})\| = \|5\mathbf{i} - 8\mathbf{j}\| = \sqrt{5^2 + (-8)^2} = \sqrt{89}$$

36. $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j},\ \mathbf{w} = -2\mathbf{i} + 3\mathbf{j}$
$$\|\mathbf{v} + \mathbf{w}\| = \|(3\mathbf{i} - 5\mathbf{j}) + (-2\mathbf{i} + 3\mathbf{j})\| = \|\mathbf{i} - 2\mathbf{j}\| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$

37. $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j},\ \mathbf{w} = -2\mathbf{i} + 3\mathbf{j}$
$$\|\mathbf{v}\| - \|\mathbf{w}\| = \|3\mathbf{i} - 5\mathbf{j}\| - \|-2\mathbf{i} + 3\mathbf{j}\| = \sqrt{3^2 + (-5)^2} - \sqrt{(-2)^2 + 3^2} = \sqrt{34} - \sqrt{13}$$

38. $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j},\ \mathbf{w} = -2\mathbf{i} + 3\mathbf{j}$
$$\|\mathbf{v}\| + \|\mathbf{w}\| = \|3\mathbf{i} - 5\mathbf{j}\| + \|-2\mathbf{i} + 3\mathbf{j}\| = \sqrt{3^2 + (-5)^2} + \sqrt{(-2)^2 + 3^2} = \sqrt{34} + \sqrt{13}$$

39. $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{5\mathbf{i}}{\|5\mathbf{i}\|} = \dfrac{5\mathbf{i}}{\sqrt{25+0}} = \dfrac{5\mathbf{i}}{5} = \mathbf{i}$

40. $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{-3\mathbf{j}}{\|-3\mathbf{j}\|} = \dfrac{-3\mathbf{j}}{\sqrt{0+9}} = \dfrac{-3\mathbf{j}}{3} = -\mathbf{j}$

41. $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{3\mathbf{i}-4\mathbf{j}}{\|3\mathbf{i}-4\mathbf{j}\|} = \dfrac{3\mathbf{i}-4\mathbf{j}}{\sqrt{3^2+(-4)^2}} = \dfrac{3\mathbf{i}-4\mathbf{j}}{\sqrt{25}} = \dfrac{3\mathbf{i}-4\mathbf{j}}{5} = \dfrac{3}{5}\mathbf{i} - \dfrac{4}{5}\mathbf{j}$

42. $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{-5\mathbf{i}+12\mathbf{j}}{\|-5\mathbf{i}+12\mathbf{j}\|} = \dfrac{-5\mathbf{i}+12\mathbf{j}}{\sqrt{(-5)^2+12^2}} = \dfrac{-5\mathbf{i}+12\mathbf{j}}{\sqrt{169}} = \dfrac{-5\mathbf{i}+12\mathbf{j}}{13} = \dfrac{-5}{13}\mathbf{i} + \dfrac{12}{13}\mathbf{j}$

43. $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{\mathbf{i}-\mathbf{j}}{\|\mathbf{i}-\mathbf{j}\|} = \dfrac{\mathbf{i}-\mathbf{j}}{\sqrt{1^2+(-1)^2}} = \dfrac{\mathbf{i}-\mathbf{j}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\mathbf{i} - \dfrac{1}{\sqrt{2}}\mathbf{j} = \dfrac{\sqrt{2}}{2}\mathbf{i} - \dfrac{\sqrt{2}}{2}\mathbf{j}$

44. $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{2\mathbf{i}-\mathbf{j}}{\|2\mathbf{i}-\mathbf{j}\|} = \dfrac{2\mathbf{i}-\mathbf{j}}{\sqrt{2^2+(-1)^2}} = \dfrac{2\mathbf{i}-\mathbf{j}}{\sqrt{5}} = \dfrac{2}{\sqrt{5}}\mathbf{i} - \dfrac{1}{\sqrt{5}}\mathbf{j} = \dfrac{2\sqrt{5}}{5}\mathbf{i} - \dfrac{\sqrt{5}}{5}\mathbf{j}$

45. Let $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$. We want $\|\mathbf{v}\| = 4$ and $a = 2b$.

$\|\mathbf{v}\| = \sqrt{a^2+b^2} = \sqrt{(2b)^2+b^2} = \sqrt{5b^2}$

$\sqrt{5b^2} = 4 \quad \rightarrow \quad 5b^2 = 16 \quad \rightarrow \quad b^2 = \dfrac{16}{5} \quad \rightarrow \quad b = \pm\sqrt{\dfrac{16}{5}} = \pm\dfrac{4}{\sqrt{5}} = \pm\dfrac{4\sqrt{5}}{5}$

$a = 2b = \pm\dfrac{8\sqrt{5}}{5}$

$\mathbf{v} = \dfrac{8\sqrt{5}}{5}\mathbf{i} + \dfrac{4\sqrt{5}}{5}\mathbf{j}$ or $\mathbf{v} = -\dfrac{8\sqrt{5}}{5}\mathbf{i} - \dfrac{4\sqrt{5}}{5}\mathbf{j}$

46. Let $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$. We want $\|\mathbf{v}\| = 3$ and $a = b$.

$\|\mathbf{v}\| = \sqrt{a^2+b^2} = \sqrt{b^2+b^2} = \sqrt{2b^2}$

$\sqrt{2b^2} = 3 \quad \rightarrow \quad 2b^2 = 9 \quad \rightarrow \quad b^2 = \dfrac{9}{2} \quad \rightarrow \quad b = \pm\sqrt{\dfrac{9}{2}} = \pm\dfrac{3}{\sqrt{2}} = \pm\dfrac{3\sqrt{2}}{2}$

$a = b = \pm\dfrac{3\sqrt{2}}{2}$

$\mathbf{v} = \dfrac{3\sqrt{2}}{2}\mathbf{i} + \dfrac{3\sqrt{2}}{2}\mathbf{j}$ or $\mathbf{v} = -\dfrac{3\sqrt{2}}{2}\mathbf{i} - \dfrac{3\sqrt{2}}{2}\mathbf{j}$

47. $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$, $\mathbf{w} = x\mathbf{i} + 3\mathbf{j}$ $\quad \|\mathbf{v}+\mathbf{w}\| = 5$

$$\|\mathbf{v}+\mathbf{w}\| = \|2\mathbf{i}-\mathbf{j}+x\mathbf{i}+3\mathbf{j}\| = \|(2+x)\mathbf{i}+2\mathbf{j}\| = \sqrt{(2+x)^2+2^2}$$
$$= \sqrt{x^2+4x+4+4} = \sqrt{x^2+4x+8}$$

Solve for x:

$$\sqrt{x^2+4x+8} = 5 \rightarrow x^2+4x+8 = 25 \rightarrow x^2+4x-17 = 0$$

$$x = \frac{-4 \pm \sqrt{16-4(1)(-17)}}{2(1)} = \frac{-4\pm\sqrt{84}}{2} = \frac{-4\pm 2\sqrt{21}}{2} = -2\pm\sqrt{21}$$

$$x = -2+\sqrt{21} \approx 2.58 \text{ or } x = -2-\sqrt{21} \approx -6.58$$

48. $P = (-3, 1)$, $Q = (x, 4)$ $\quad \mathbf{v} = (x-(-3))\mathbf{i} + (4-1)\mathbf{j} = (x+3)\mathbf{i} + 3\mathbf{j}$

$$\|\mathbf{v}\| = \sqrt{(x+3)^2 + 3^2} = \sqrt{x^2 + 6x + 9 + 9} = \sqrt{x^2 + 6x + 18}$$

$$\sqrt{x^2 + 6x + 18} = 5 \rightarrow x^2 + 6x + 18 = 25 \rightarrow x^2 + 6x - 7 = 0$$

$$(x+7)(x-1) = 0 \rightarrow x = -7 \text{ or } x = 1$$

49. $\|\mathbf{v}\| = 5, \quad \alpha = 60^\circ$

$$\mathbf{v} = \|\mathbf{v}\|(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}) = 5(\cos(60^\circ)\mathbf{i} + \sin(60^\circ)\mathbf{j}) = 5\left(\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right) = \frac{5}{2}\mathbf{i} + \frac{5\sqrt{3}}{2}\mathbf{j}$$

50. $\|\mathbf{v}\| = 8, \quad \alpha = 45^\circ$

$$\mathbf{v} = \|\mathbf{v}\|(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}) = 8(\cos(45^\circ)\mathbf{i} + \sin(45^\circ)\mathbf{j}) = 8\left(\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}\right) = 4\sqrt{2}\mathbf{i} + 4\sqrt{2}\mathbf{j}$$

51. $\|\mathbf{v}\| = 14, \quad \alpha = 120^\circ$

$$\mathbf{v} = \|\mathbf{v}\|(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}) = 14(\cos(120^\circ)\mathbf{i} + \sin(120^\circ)\mathbf{j}) = 14\left(-\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right) = -7\mathbf{i} + 7\sqrt{3}\mathbf{j}$$

52. $\|\mathbf{v}\| = 3, \quad \alpha = 240^\circ$

$$\mathbf{v} = \|\mathbf{v}\|(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}) = 3(\cos(240^\circ)\mathbf{i} + \sin(240^\circ)\mathbf{j}) = 3\left(-\frac{1}{2}\mathbf{i} - \frac{\sqrt{3}}{2}\mathbf{j}\right) = -\frac{3}{2}\mathbf{i} - \frac{3\sqrt{3}}{2}\mathbf{j}$$

53. $\|\mathbf{v}\| = 25, \quad \alpha = 330^\circ$

$$\mathbf{v} = \|\mathbf{v}\|(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}) = 25(\cos(330^\circ)\mathbf{i} + \sin(330^\circ)\mathbf{j}) = 25\left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right) = \frac{25\sqrt{3}}{2}\mathbf{i} - \frac{25}{2}\mathbf{j}$$

54. $\|\mathbf{v}\| = 15, \quad \alpha = 315^\circ$

$$\mathbf{v} = \|\mathbf{v}\|(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}) = 15(\cos 315^\circ\mathbf{i} + \sin 315^\circ\mathbf{j}) = 15\left(\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}\right)$$

$$= \frac{15\sqrt{2}}{2}\mathbf{i} - \frac{15\sqrt{2}}{2}\mathbf{j}$$

55. $\mathbf{F} = 40(\cos(30^\circ)\mathbf{i} + \sin(30^\circ)\mathbf{j}) = 40\left(\frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}\right) = 20\sqrt{3}\mathbf{i} + 20\mathbf{j}$

56. $\mathbf{F} = 100(\cos(20^\circ)\mathbf{i} + \sin(20^\circ)\mathbf{j}) \approx 100(0.9397\mathbf{i} + 0.3420\mathbf{j}) = 93.97\mathbf{i} + 34.20\mathbf{j}$

57. $\mathbf{F}_1 = 40(\cos(30^\circ)\mathbf{i} + \sin(30^\circ)\mathbf{j}) = 40\left(\frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}\right) = 20\sqrt{3}\mathbf{i} + 20\mathbf{j}$

$$\mathbf{F}_2 = 60(\cos(-45^\circ)\mathbf{i} + \sin(-45^\circ)\mathbf{j}) = 60\left(\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}\right) = 30\sqrt{2}\mathbf{i} - 30\sqrt{2}\mathbf{j}$$

$$\mathbf{F}_1 + \mathbf{F}_2 = 20\sqrt{3}\mathbf{i} + 20\mathbf{j} + 30\sqrt{2}\mathbf{i} - 30\sqrt{2}\mathbf{j} = \left(20\sqrt{3} + 30\sqrt{2}\right)\mathbf{i} + \left(20 - 30\sqrt{2}\right)\mathbf{j}$$

58. $\mathbf{F}_1 = 30\left(\cos(45^\circ)\mathbf{i} + \sin(45^\circ)\mathbf{j}\right) = 30\left(\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}\right) = 15\sqrt{2}\,\mathbf{i} + 15\sqrt{2}\,\mathbf{j}$

$\mathbf{F}_2 = 70\left(\cos(120^\circ)\mathbf{i} + \sin(120^\circ)\mathbf{j}\right) = 70\left(-\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right) = -35\,\mathbf{i} + 35\sqrt{3}\,\mathbf{j}$

$\mathbf{F}_1 + \mathbf{F}_2 = 15\sqrt{2}\,\mathbf{i} + 15\sqrt{2}\,\mathbf{j} + (-35)\mathbf{i} + 35\sqrt{3}\,\mathbf{j} = \left(15\sqrt{2} - 35\right)\mathbf{i} + \left(15\sqrt{2} + 35\sqrt{3}\right)\mathbf{j}$

59. Let $\mathbf{F_1}$ be the tension on the left cable and $\mathbf{F}_2$ be the tension on the right cable. Let $\mathbf{F_3}$ represent the force of the weight of the box.

$\mathbf{F_1} = \|\mathbf{F_1}\|\left(\cos(155^\circ)\mathbf{i} + \sin(155^\circ)\mathbf{j}\right) \approx \|\mathbf{F_1}\|\left(-0.9063\mathbf{i} + 0.4226\mathbf{j}\right)$

$\mathbf{F}_2 = \|\mathbf{F}_2\|\left(\cos(40^\circ)\mathbf{i} + \sin(40^\circ)\mathbf{j}\right) \approx \|\mathbf{F}_2\|\left(0.7660\mathbf{i} + 0.6428\mathbf{j}\right)$

$\mathbf{F}_3 = -1000\mathbf{j}$

For equilibrium, the sum of the force vectors must be zero.

$$\begin{aligned}\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} &= -0.9063\|\mathbf{F_1}\|\mathbf{i} + 0.4226\|\mathbf{F_1}\|\mathbf{j} + 0.7660\|\mathbf{F}_2\|\mathbf{i} + 0.6428\|\mathbf{F}_2\|\mathbf{j} - 1000\mathbf{j}\\ &= \left(-0.9063\|\mathbf{F_1}\| + 0.7660\|\mathbf{F}_2\|\right)\mathbf{i} + \left(0.4226\|\mathbf{F_1}\| + 0.6428\|\mathbf{F}_2\| - 1000\right)\mathbf{j}\\ &= 0\end{aligned}$$

Set the $\mathbf{i}$ and $\mathbf{j}$ components equal to zero and solve:

$$\begin{cases} -0.9063\|\mathbf{F_1}\| + 0.7660\|\mathbf{F}_2\| = 0 \quad \rightarrow \quad \|\mathbf{F}_2\| = \dfrac{0.9063}{0.7660}\|\mathbf{F_1}\| = 1.1832\,\|\mathbf{F_1}\| \\ 0.4226\|\mathbf{F_1}\| + 0.6428\|\mathbf{F}_2\| - 1000 = 0 \end{cases}$$

$0.4226\|\mathbf{F_1}\| + 0.6428\left(1.1832\|\mathbf{F_1}\|\right) - 1000 = 0$

$1.1832\|\mathbf{F_1}\| = 1000$

$\|\mathbf{F_1}\| = 845.2$ pounds

$\|\mathbf{F}_2\| = 1.1832(845.2) = 1000$ pounds

The tension in the left cable is about 845.2 pounds and the tension in the right cable is about 1000 pounds.

60. Let $\mathbf{F_1}$ be the tension on the left cable and $\mathbf{F}_2$ be the tension on the right cable. Let $\mathbf{F_3}$ represent the force of the weight of the box.

$\mathbf{F_1} = \|\mathbf{F_1}\|\left(\cos(145^\circ)\mathbf{i} + \sin(145^\circ)\mathbf{j}\right) \approx \|\mathbf{F_1}\|\left(-0.8192\mathbf{i} + 0.5736\mathbf{j}\right)$

$\mathbf{F}_2 = \|\mathbf{F}_2\|\left(\cos(50^\circ)\mathbf{i} + \sin(50^\circ)\mathbf{j}\right) \approx \|\mathbf{F}_2\|\left(0.6428\mathbf{i} + 0.7660\mathbf{j}\right)$

$\mathbf{F}_3 = -800\mathbf{j}$

For equilibrium, the sum of the force vectors must be zero.

$$\begin{aligned}\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} &= -0.8192\|\mathbf{F_1}\|\mathbf{i} + 0.5736\|\mathbf{F_1}\|\mathbf{j} + 0.6428\|\mathbf{F}_2\|\mathbf{i} + 0.7660\|\mathbf{F}_2\|\mathbf{j} - 800\mathbf{j}\\ &= \left(-0.8192\|\mathbf{F_1}\| + 0.6428\|\mathbf{F}_2\|\right)\mathbf{i} + \left(0.5736\|\mathbf{F_1}\| + 0.7660\|\mathbf{F}_2\| - 800\right)\mathbf{j}\\ &= 0\end{aligned}$$

Set the $\mathbf{i}$ and $\mathbf{j}$ components equal to zero and solve:

$$\begin{cases} -0.8192\|\mathbf{F_1}\| + 0.6428\|\mathbf{F}_2\| = 0 \quad \rightarrow \quad \|\mathbf{F}_2\| = \dfrac{0.8192}{0.6428}\|\mathbf{F_1}\| = 1.2744\|\mathbf{F_1}\| \\ 0.5736\|\mathbf{F_1}\| + 0.7660\|\mathbf{F}_2\| - 800 = 0 \end{cases}$$

$$0.5736\|\mathbf{F_1}\| + 0.7660\left(1.2744\|\mathbf{F_1}\|\right) - 800 = 0$$

$$1.5498\|\mathbf{F_1}\| = 800$$

$$\|\mathbf{F_1}\| = 516.2 \text{ pounds}$$

$$\|\mathbf{F}_2\| = 1.2744(516.2) = 657.8 \text{ pounds}$$

The tension in the left cable is about 516.2 pounds and the tension in the right cable is about 657.8 pounds.

61. Let $\mathbf{F_1}$ be the tension on the left end of the rope and $\mathbf{F}_2$ be the tension on the right end of the rope. Let $\mathbf{F_3}$ represent the force of the weight of the tightrope walker.

$$\mathbf{F_1} = \|\mathbf{F_1}\|\left(\cos(175.8^\circ)\mathbf{i} + \sin(175.8^\circ)\mathbf{j}\right) \approx \|\mathbf{F_1}\|\left(-0.9973\mathbf{i} + 0.0732\mathbf{j}\right)$$

$$\mathbf{F}_2 = \|\mathbf{F}_2\|\left(\cos(3.7^\circ)\mathbf{i} + \sin(3.7^\circ)\mathbf{j}\right) \approx \|\mathbf{F}_2\|\left(0.9979\mathbf{i} + 0.0645\mathbf{j}\right)$$

$$\mathbf{F}_3 = -150\mathbf{j}$$

For equilibrium, the sum of the force vectors must be zero.

$$\begin{aligned} \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} &= -0.9973\|\mathbf{F_1}\|\mathbf{i} + 0.0732\|\mathbf{F_1}\|\mathbf{j} + 0.9979\|\mathbf{F}_2\|\mathbf{i} + 0.0645\|\mathbf{F}_2\|\mathbf{j} - 150\mathbf{j} \\ &= \left(-0.9973\|\mathbf{F_1}\| + 0.9979\|\mathbf{F}_2\|\right)\mathbf{i} + \left(0.0732\|\mathbf{F_1}\| + 0.0645\|\mathbf{F}_2\| - 150\right)\mathbf{j} \\ &= 0 \end{aligned}$$

Set the $\mathbf{i}$ and $\mathbf{j}$ components equal to zero and solve:

$$\begin{cases} -0.9973\|\mathbf{F}_1\| + 0.9979\|\mathbf{F}_2\| = 0 \quad \rightarrow \quad \|\mathbf{F}_2\| = \dfrac{0.9973}{0.9979}\|\mathbf{F_1}\| = 0.9994\|\mathbf{F_1}\| \\ 0.0732\|\mathbf{F_1}\| + 0.0645\|\mathbf{F}_2\| - 150 = 0 \end{cases}$$

$$0.0732\|\mathbf{F_1}\| + 0.0645\left(0.9994\|\mathbf{F_1}\|\right) - 150 = 0$$

$$0.1377\|\mathbf{F_1}\| = 150$$

$$\|\mathbf{F_1}\| = 1089.3 \text{ pounds}$$

$$\|\mathbf{F}_2\| = 0.9994(1089.3) = 1088.6 \text{ pounds}$$

The tension in the left end of the rope is about 1089.3 pounds and the tension in the right end of the rope is about 1088.6 pounds.

62. Let $\mathbf{F_1}$ be the tension on the left end of the rope and $\mathbf{F}_2$ be the tension on the right end of the rope. Let $\mathbf{F_3}$ represent the force of the weight of the tightrope walker.

$$\mathbf{F_1} = \|\mathbf{F_1}\|\left(\cos(176.2^\circ)\mathbf{i} + \sin(176.2^\circ)\mathbf{j}\right) \approx \|\mathbf{F_1}\|\left(-0.9978\mathbf{i} + 0.0663\mathbf{j}\right)$$

$$\mathbf{F}_2 = \|\mathbf{F}_2\|\left(\cos(2.6^\circ)\mathbf{i} + \sin(2.6^\circ)\mathbf{j}\right) \approx \|\mathbf{F}_2\|\left(0.9990\mathbf{i} + 0.0454\mathbf{j}\right)$$

$$\mathbf{F}_3 = -135\mathbf{j}$$

For equilibrium, the sum of the force vectors must be zero.

$$\begin{aligned} \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} &= -0.9978\|\mathbf{F}_1\|\mathbf{i} + 0.0663\|\mathbf{F_1}\|\mathbf{j} + 0.9990\|\mathbf{F}_2\|\mathbf{i} + 0.0454\|\mathbf{F}_2\|\mathbf{j} - 135\mathbf{j} \\ &= \left(-0.9978\|\mathbf{F}_1\| + 0.9990\|\mathbf{F}_2\|\right)\mathbf{i} + \left(0.0663\|\mathbf{F}_1\| + 0.0454\|\mathbf{F}_2\| - 135\right)\mathbf{j} \\ &= 0 \end{aligned}$$

Set the $\mathbf{i}$ and $\mathbf{j}$ components equal to zero and solve:

$$\begin{cases} -0.9978\|\mathbf{F_1}\| + 0.9990\|\mathbf{F}_2\| = 0 & \rightarrow \quad \|\mathbf{F}_2\| = \dfrac{0.9978}{0.9990}\|\mathbf{F_1}\| = 0.9988\|\mathbf{F_1}\| \\ 0.0663\|\mathbf{F_1}\| + 0.0454\|\mathbf{F}_2\| - 135 = 0 \end{cases}$$

$$0.0663\|\mathbf{F_1}\| + 0.0454\left(0.9988\|\mathbf{F_1}\|\right) - 135 = 0$$

$$0.1116\|\mathbf{F_1}\| = 135$$

$$\|\mathbf{F_1}\| = 1209.7 \text{ pounds}$$

$$\|\mathbf{F}_2\| = 0.9988(1209.7) = 1208.2 \text{ pounds}$$

The tension in the left end of the rope is about 1209.7 pounds and the tension in the right end of the rope is about 1208.2 pounds.

63. The given forces are:

$$\mathbf{F_1} = -3\mathbf{i}$$

$$\mathbf{F_2} = -\mathbf{i} + 4\mathbf{j}$$

$$\mathbf{F}_3 = 4\mathbf{i} - 2\mathbf{j}$$

$$\mathbf{F}_4 = -4\mathbf{j}$$

A vector $\mathbf{x} = a\,\mathbf{i} + b\,\mathbf{j}$ needs to be added for equilibrium. Find vector $\mathbf{x} = a\,\mathbf{i} + b\,\mathbf{j}$:

$$\begin{aligned} \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 + \mathbf{x} &= \mathbf{0} \\ -3\mathbf{i} + (-\mathbf{i} + 4\,\mathbf{j}) + (4\mathbf{i} - 2\,\mathbf{j}) + (-4\,\mathbf{j}) + (a\mathbf{i} + b\,\mathbf{j}) &= \mathbf{0} \\ 0\mathbf{i} - 2\,\mathbf{j} + (a\mathbf{i} + b\,\mathbf{j}) &= \mathbf{0} \\ a\mathbf{i} + (-2 + b)\,\mathbf{j} &= \mathbf{0} \\ a &= 0 \\ -2 + b &= 0 \quad \rightarrow \quad b = 2 \end{aligned}$$

Therefore, $\mathbf{x} = 2\,\mathbf{j}$.