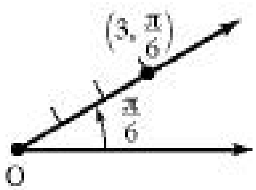
Chapter 10

Polar Coordinates; Vectors

10.R Chapter Review

1. $\left(3, \frac{\pi}{6}\right)$

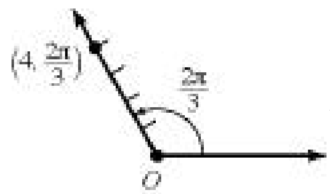


$x = 3\cos\frac{\pi}{6} = \frac{3\sqrt{3}}{2}$

$y = 3\sin\frac{\pi}{6} = \frac{3}{2}$

$\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

2. $\left(4, \frac{2\pi}{3}\right)$

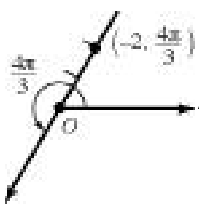


$x = 4\cos\frac{2\pi}{3} = -2$

$y = 4\sin\frac{2\pi}{3} = 2\sqrt{3}$

$\left(-2, 2\sqrt{3}\right)$

3. $\left(-2, \frac{4\pi}{3}\right)$

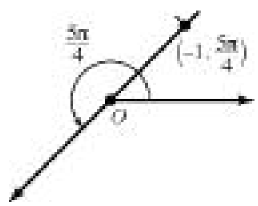


$x = -2\cos\frac{4\pi}{3} = 1$

$y = -2\sin\frac{4\pi}{3} = \sqrt{3}$

$\left(1, \sqrt{3}\right)$

4. $\left(-1, \frac{5\pi}{4}\right)$

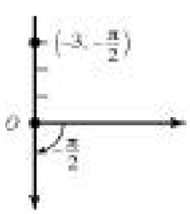


$x = -1\cos\frac{5\pi}{4} = \frac{\sqrt{2}}{2}$

$y = -1\sin\frac{5\pi}{4} = \frac{\sqrt{2}}{2}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

5. $\left(-3, -\frac{\pi}{2}\right)$

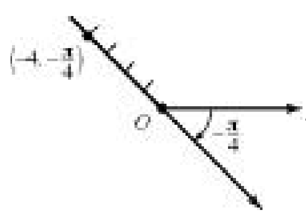


$x = -3\cos\left(-\frac{\pi}{2}\right) = 0$

$y = -3\sin\left(-\frac{\pi}{2}\right) = 3$

(0, 3)

6. $\left(-4, -\frac{\pi}{4}\right)$



$x = -4\cos\frac{-\pi}{4} = -2\sqrt{2}$

$y = -4\sin\frac{-\pi}{4} = 2\sqrt{2}$

$\left(-2\sqrt{2}, 2\sqrt{2}\right)$

7. The point (–3, 3) lies in quadrant II.

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 3^2} = 3\sqrt{2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{3}{-3}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

Polar coordinates of the point $(-3, 3)$ are $\left(-3\sqrt{2}, -\frac{\pi}{4}\right)$ or $\left(3\sqrt{2}, \frac{3\pi}{4}\right)$.

8. The point (1, –1) lies in quadrant IV.

$$r=\sqrt{x^2+y^2}=\sqrt{1^2+(-1)^2}=\sqrt{2} \qquad \theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{-1}{1}\right)=\tan^{-1}(-1)=-\frac{\pi}{4}$$

Polar coordinates of the point $(1,\ -1)$ are $\left(\sqrt{2},\ -\frac{\pi}{4}\right)$ and $\left(-\sqrt{2},\ \frac{3\pi}{4}\right)$.

9. The point (0, –2) lies on the negative y-axis.

$$r=\sqrt{x^2+y^2}=\sqrt{0^2+(-2)^2}=2 \qquad \theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{-2}{0}\right)=-\frac{\pi}{2}$$

Polar coordinates of the point $(0,\ -2)$ are $\left(2,\ -\frac{\pi}{2}\right)$ or $\left(-2,\ \frac{\pi}{2}\right)$.

10. The point (2, 0) lies on the positive x-axis.

$$r=\sqrt{x^2+y^2}=\sqrt{2^2+0^2}=\sqrt{4}=2 \qquad \theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{0}{2}\right)=\tan^{-1}0=0$$

Polar coordinates of the point (2, 0) are (2, 0) and (–2, π).

11. The point (3, 4) lies in quadrant I.

$$r=\sqrt{x^2+y^2}=\sqrt{3^2+4^2}=5 \qquad \theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{4}{3}\right)\approx 0.93$$

Polar coordinates of the point $(3,\ 4)$ are $(5,\ 0.93)$ or $(-5,\ 4.07)$.

12. The point (–5, 12) lies in quadrant II.

$$r=\sqrt{x^2+y^2}=\sqrt{(-5)^2+12^2}=13 \qquad \theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{12}{-5}\right)\approx -1.18$$

Polar coordinates of the point $(-5,\ 12)$ are $(13,\ 1.96)$ or $(-13,\ 5.10)$.

13.
$$\begin{aligned}3x^2+3y^2&=6y\\ x^2+y^2&=2y\\ r^2&=2r\sin\theta\\ r^2-2r\sin\theta&=0\end{aligned}$$

14.
$$\begin{aligned}2x^2-2y^2&=5y\\ 2r^2\cos^2\theta-2r^2\sin^2\theta&=5r\sin\theta\\ 2r^2\left(\cos^2\theta-\sin^2\theta\right)&=5r\sin\theta\\ 2r\cos(2\theta)&=5\sin\theta\end{aligned}$$

15.
$$\begin{aligned}2x^2-y^2&=\frac{y}{x}\\ 2x^2+2y^2-3y^2&=\frac{y}{x}\\ 2\left(x^2+y^2\right)-3y^2&=\frac{y}{x}\\ 2r^2-3(r\sin\theta)^2&=\tan\theta\\ 2r^2-3r^2\sin^2\theta-\tan\theta&=0\\ r^2(2-3\sin\theta)-\tan\theta&=0\end{aligned}$$

16.
$$\begin{aligned}x^2+2y^2&=\frac{y}{x}\\ 2x^2+2y^2-x^2&=\frac{y}{x}\\ 2\left(x^2+y^2\right)-x^2&=\frac{y}{x}\\ 2r^2-(r\cos\theta)^2&=\tan\theta\\ 2r^2-r^2\cos^2\theta-\tan\theta&=0\\ r^2\left(2-\cos^2\theta\right)-\tan\theta&=0\end{aligned}$$

17. $$\begin{aligned} x\left(x^2+y^2\right) &= 4 \\ r\cos\theta\left(r^2\right) &= 4 \\ r^3\cos\theta &= 4 \end{aligned}$$

18. $$\begin{aligned} y\left(x^2-y^2\right) &= 3 \\ r\sin\theta\left(r^2\cos^2\theta - r^2\sin^2\theta\right) &= 3 \\ r^3\sin\theta\left(\cos^2\theta-\sin^2\theta\right) &= 3 \\ r^3\sin\theta\cos\left(2\theta\right) &= 3 \end{aligned}$$

19. $$\begin{aligned} r &= 2\sin\theta \\ r^2 &= 2r\sin\theta \\ x^2+y^2 &= 2y \\ x^2+y^2-2y &= 0 \end{aligned}$$

20. $$\begin{aligned} 3r &= \sin\theta \\ 3r^2 &= r\sin\theta \\ 3(x^2+y^2) &= y \\ 3x^2+3y^2-y &= 0 \end{aligned}$$

21. $$\begin{aligned} r &= 5 \\ r^2 &= 25 \\ x^2+y^2 &= 25 \end{aligned}$$

22. $$\begin{aligned} \theta &= \frac{\pi}{4} \\ \tan\theta &= \tan\left(\frac{\pi}{4}\right) \\ \frac{y}{x} &= 1 \\ y &= x \end{aligned}$$

23. $$\begin{aligned} r\cos\theta + 3r\sin\theta &= 6 \\ x+3y &= 6 \end{aligned}$$

24. $$\begin{aligned} r^2\tan\theta &= 1 \\ \left(x^2+y^2\right)\left(\frac{y}{x}\right) &= 1 \\ x^2y+y^3 &= x \\ y^3+x^2y-x &= 0 \end{aligned}$$

25. $r = 4\cos\theta$ The graph will be a circle. Check for symmetry:

Polar axis: Replace θ by $-\theta$. The result is $r = 4\cos(-\theta) = 4\cos\theta$.

The graph is symmetric with respect to the polar axis.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi-\theta$.

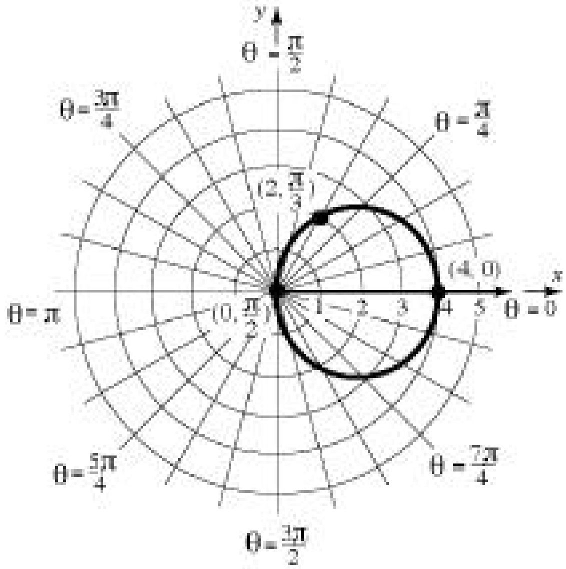
$$\begin{aligned} r &= 4\cos(\pi-\theta) = 4(\cos(\pi)\cos\theta + \sin(\pi)\sin\theta) \\ &= 4(-\cos\theta+0) = -4\cos\theta \end{aligned}$$

The test fails.

The pole: Replace r by $-r$. $-r = 4\cos\theta$. The test fails.

Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 4\cos\theta$	4	$2\sqrt{3} \approx 3.5$	2	0	-2	$-2\sqrt{3} \approx -3.5$	-4



26. $r = 3\sin\theta$ The graph will be a circle. Check for symmetry:

Polar axis: Replace θ by $-\theta$. The result is $r = 3\sin(-\theta) = -3\sin\theta$.
The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

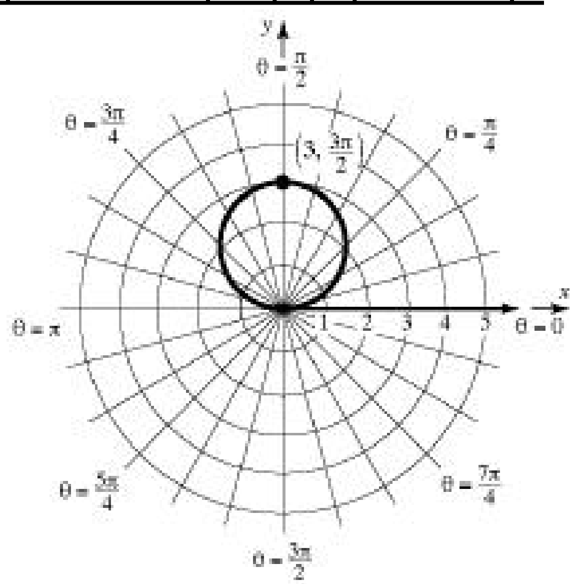
$$\begin{aligned} r &= 3\sin(\pi - \theta) = 3(\sin(\pi)\cos\theta - \cos(\pi)\sin\theta) \\ &= 3(0 + \sin\theta) = 3\sin\theta \end{aligned}$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

The pole: Replace r by $-r$. $-r = 3\sin\theta$. The test fails.

Due to symmetry to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 3\sin\theta$	-3	$-\frac{3\sqrt{3}}{2} \approx -2.6$	$-\frac{3}{2}$	0	$\frac{3}{2}$	$\frac{3\sqrt{3}}{2} \approx 2.6$	3

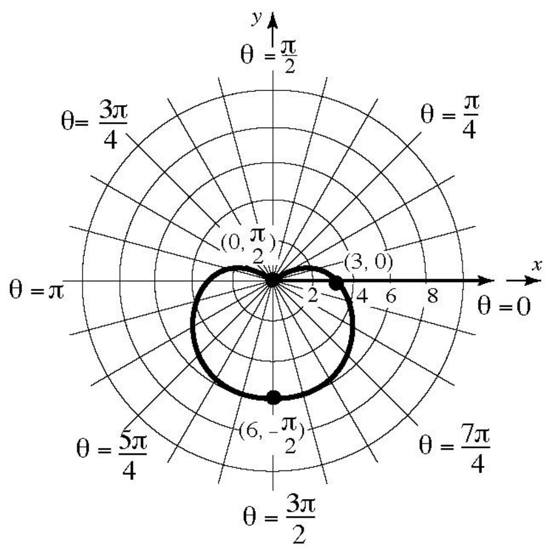


27. $r = 3 - 3\sin\theta$ The graph will be a cardioid. Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 3 - 3\sin(-\theta) = 3 + 3\sin\theta$.
The test fails.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 3 - 3\sin(\pi - \theta) = 3 - 3(\sin(\pi)\cos\theta - \cos(\pi)\sin\theta)$$
$$= 3 - 3(0 + \sin\theta) = 3 - 3\sin\theta$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.
The pole: Replace r by $-r$. $-r = 3 - 3\sin\theta$. The test fails.
Due to symmetry to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 3 - 3\sin\theta$	6	$3 + \frac{3\sqrt{3}}{2} \approx 5.6$	$\frac{9}{2}$	3	$\frac{3}{2}$	$3 - \frac{3\sqrt{3}}{2} \approx 0.4$	0



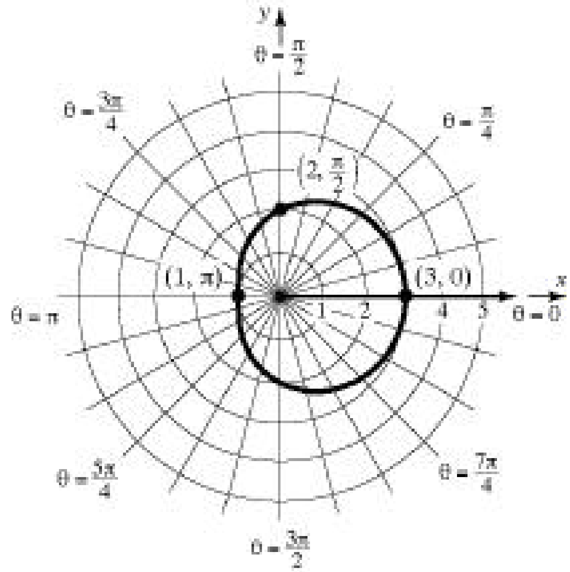
28. $r = 2 + \cos\theta$ The graph will be a limacon without an inner loop.
Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r = 2 + \cos(-\theta) = 2 + \cos\theta$.
The graph is symmetric with respect to the polar axis.
The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

$$r = 2 + \cos(\pi - \theta) = 2 + (\cos(\pi)\cos\theta + \sin(\pi)\sin\theta)$$
$$= 2 + (-\cos\theta + 0) = 2 - \cos\theta$$

The test fails.

The pole: Replace r by $-r$. $-r = 2 + \cos\theta$. The test fails.
Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r=2+\cos\theta$	3	$2+\frac{\sqrt{3}}{2}\approx 2.9$	$\frac{5}{2}$	2	$\frac{3}{2}$	$2-\frac{\sqrt{3}}{2}\approx 1.1$	1

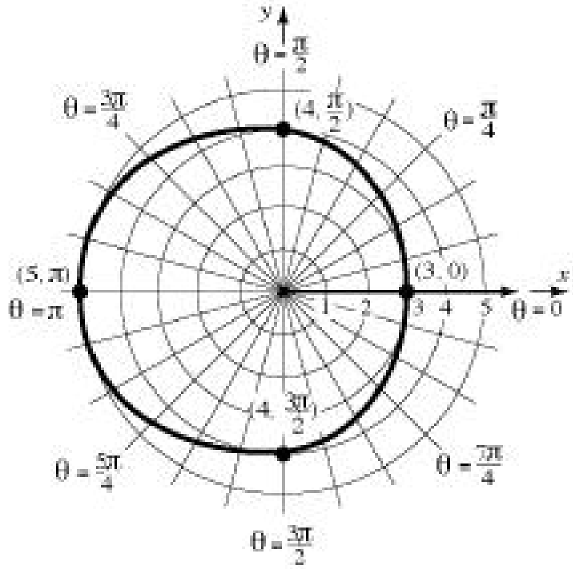


29. $r=4-\cos\theta$ The graph will be a limacon without an inner loop.
Check for symmetry:
Polar axis: Replace θ by $-\theta$. The result is $r=4-\cos(-\theta)=4-\cos\theta$.
The graph is symmetric with respect to the polar axis.
The line $\theta=\frac{\pi}{2}$: Replace θ by $\pi-\theta$.

$$r=4-\cos(\pi-\theta)=4-(\cos(\pi)\cos\theta+\sin(\pi)\sin\theta)$$
$$=4-(-\cos\theta+0)=4+\cos\theta$$

The test fails.
The pole: Replace r by $-r$. $-r=4-\cos\theta$. The test fails.
Due to symmetry to the polar axis, assign values to θ from 0 to π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r=4-\cos\theta$	3	$4-\frac{\sqrt{3}}{2}\approx 3.1$	$\frac{7}{2}$	4	$\frac{9}{2}$	$4+\frac{\sqrt{3}}{2}\approx 4.9$	5



30. $r = 1 - 2\sin\theta$ The graph will be a limacon with an inner loop.

Check for symmetry:

Polar axis: Replace θ by $-\theta$. The result is $r = 1 - 2\sin(-\theta) = 1 + 2\sin\theta$.
The test fails.

The line $\theta = \frac{\pi}{2}$: Replace θ by $\pi - \theta$.

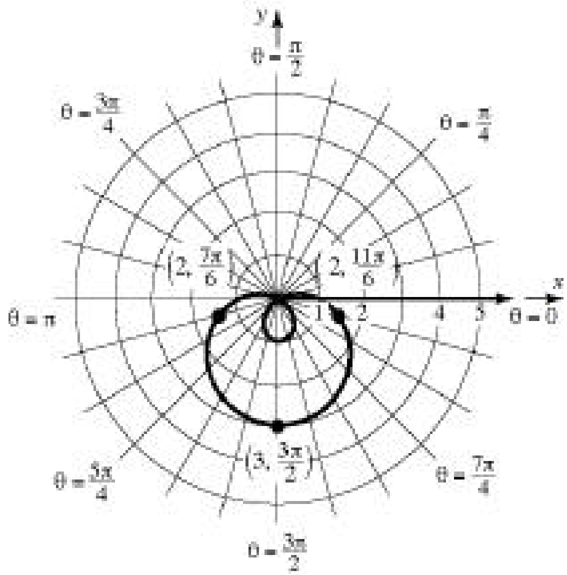
$$r = 1 - 2\sin(\pi - \theta) = 1 - 2(\sin(\pi)\cos\theta - \cos(\pi)\sin\theta)$$
$$= 1 - 2(0 + \sin\theta) = 1 - 2\sin\theta$$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

The pole: Replace r by $-r$. $-r = 1 - 2\sin\theta$. The test fails.

Due to symmetry to the line $\theta = \frac{\pi}{2}$, assign values to θ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$r = 1 - 2\sin\theta$	3	$1 + \sqrt{3} \approx 2.7$	2	1	0	$1 - \sqrt{3} \approx -0.7$	-1



31. $r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-1}{-1} = 1 \quad \rightarrow \quad \theta = 225^\circ$

The polar form of $z = -1 - i$ is $z = r(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos(225^\circ) + i\sin(225^\circ)\right)$.

32. $r = \sqrt{x^2 + y^2} = \sqrt{\left(-\sqrt{3}\right)^2 + 1^2} = \sqrt{4} = 2$

$\tan\theta = \dfrac{y}{x} = \dfrac{1}{-\sqrt{3}} = -\dfrac{\sqrt{3}}{3} \quad \rightarrow \quad \theta = 150^\circ$

The polar form of $z = -\sqrt{3} + i$ is $z = r(\cos\theta + i\sin\theta) = 2\left(\cos(150^\circ) + i\sin(150^\circ)\right)$.

33. $r = \sqrt{x^2 + y^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$

$\tan\theta = \dfrac{y}{x} = -\dfrac{3}{4} \quad \rightarrow \quad \theta \approx 323.1^\circ$

The polar form of $z = 4 - 3i$ is $z = r(\cos\theta + i\sin\theta) = 5\left(\cos(323.1^\circ) + i\sin(323.1^\circ)\right)$.

34. $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + (-2)^2} = \sqrt{13}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{2}{3} \quad \rightarrow \quad \theta \approx 326.3^\circ$

The polar form of $z = 3 - 2i$ is $z = r(\cos\theta + i\sin\theta) = \sqrt{13}\left(\cos(326.3^\circ) + i\sin(326.3^\circ)\right)$.

35. $2\left(\cos(150^\circ) + i\sin(150^\circ)\right) = 2\left(-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right) = -\sqrt{3} + i$

36. $3\left(\cos(60^\circ) + i\sin(60^\circ)\right) = 3\left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right) = \dfrac{3}{2} + \dfrac{3\sqrt{3}}{2}i$

37. $3\left(\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right)=3\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=-\frac{3}{2}+\frac{3\sqrt{3}}{2}i$

38. $4\left(\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right)=4\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)=-2\sqrt{2}+2\sqrt{2}\,i$

39. $0.1\left(\cos\left(350^\circ\right)+i\sin\left(350^\circ\right)\right)\approx 0.1\left(0.9848-0.1736i\right)=0.0985-0.0174i$

40. $0.5\left(\cos\left(160^\circ\right)+i\sin\left(160^\circ\right)\right)\approx 0.5\left(-0.9397+0.3420i\right)=-0.4699+0.1710i$

41. $z\ w=\left(\cos\left(80^\circ\right)+i\sin\left(80^\circ\right)\right)\ \left(\cos\left(50^\circ\right)+i\sin\left(50^\circ\right)\right)$

$=1\ 1\left(\cos(80^\circ+50^\circ)+i\sin(80^\circ+50^\circ)\right)=\cos\left(130^\circ\right)+i\sin\left(130^\circ\right)$

$\frac{z}{w}=\frac{\left(\cos\left(80^\circ\right)+i\sin\left(80^\circ\right)\right)}{\left(\cos\left(50^\circ\right)+i\sin\left(50^\circ\right)\right)}=\frac{1}{1}\left(\cos(80^\circ-50^\circ)+i\sin(80^\circ-50^\circ)\right)$

$=\cos\left(30^\circ\right)+i\sin\left(30^\circ\right)$

42. $z\ w=\left(\cos\left(205^\circ\right)+i\sin\left(205^\circ\right)\right)\ \left(\cos\left(85^\circ\right)+i\sin\left(85^\circ\right)\right)$

$=1\ 1\left(\cos(205^\circ+85^\circ)+i\sin(205^\circ+85^\circ)\right)=\cos\left(290^\circ\right)+i\sin\left(290^\circ\right)$

$\frac{z}{w}=\frac{\left(\cos\left(205^\circ\right)+i\sin\left(205^\circ\right)\right)}{\left(\cos\left(85^\circ\right)+i\sin\left(85^\circ\right)\right)}=\frac{1}{1}\left(\cos(205^\circ-85^\circ)+i\sin(205^\circ-85^\circ)\right)$

$=\cos\left(120^\circ\right)+i\sin\left(120^\circ\right)$

43. $z\ w=3\left(\cos\left(\frac{9\pi}{5}\right)+i\sin\left(\frac{9\pi}{5}\right)\right)\ 2\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right)=3\ 2\left(\cos\left(\frac{9\pi}{5}+\frac{\pi}{5}\right)+i\sin\left(\frac{9\pi}{5}+\frac{\pi}{5}\right)\right)$

$=6\ \left(\cos\left(2\pi\right)+i\sin\left(2\pi\right)\right)=6\left(\cos\left(0\right)+i\sin\left(0\right)\right)$

$\frac{z}{w}=\frac{3\left(\cos\left(\frac{9\pi}{5}\right)+i\sin\left(\frac{9\pi}{5}\right)\right)}{2\left(\cos\left(\frac{\pi}{5}\right)+i\sin\left(\frac{\pi}{5}\right)\right)}=\frac{3}{2}\left(\cos\left(\frac{9\pi}{5}-\frac{\pi}{5}\right)+i\sin\left(\frac{9\pi}{5}-\frac{\pi}{5}\right)\right)=\frac{3}{2}\left(\cos\left(\frac{8\pi}{5}\right)+i\sin\left(\frac{8\pi}{5}\right)\right)$

44. $z\ w=2\left(\cos\left(\frac{5\pi}{3}\right)+i\sin\left(\frac{5\pi}{3}\right)\right)\ 3\left(\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right)=2\ 3\left(\cos\left(\frac{5\pi}{3}+\frac{\pi}{3}\right)+i\sin\left(\frac{5\pi}{3}+\frac{\pi}{3}\right)\right)$

$=6\ \left(\cos\left(2\pi\right)+i\sin\left(2\pi\right)\right)=6\ \left(\cos\left(0\right)+i\sin\left(0\right)\right)$

$\frac{z}{w}=\frac{2\left(\cos\left(\frac{5\pi}{3}\right)+i\sin\left(\frac{5\pi}{3}\right)\right)}{3\left(\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right)}=\frac{2}{3}\left(\cos\left(\frac{5\pi}{3}-\frac{\pi}{3}\right)+i\sin\left(\frac{5\pi}{3}-\frac{\pi}{3}\right)\right)=\frac{2}{3}\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$

45. $z\ w=5\left(\cos\left(10^\circ\right)+i\sin\left(10^\circ\right)\right)\ \left(\cos\left(355^\circ\right)+i\sin\left(355^\circ\right)\right)$

$$= 5 \cdot 1\left(\cos(10^\circ + 355^\circ) + i\sin(10^\circ + 355^\circ)\right) = 5\left(\cos(365^\circ) + i\sin(365^\circ)\right)$$
$$= 5\left(\cos(5^\circ) + i\sin(5^\circ)\right)$$
$$\frac{z}{w} = \frac{5\left(\cos(10^\circ) + i\sin(10^\circ)\right)}{\left(\cos(355^\circ) + i\sin(355^\circ)\right)} = \frac{5}{1}\left(\cos(10^\circ - 355^\circ) + i\sin(10^\circ - 355^\circ)\right)$$
$$= 5\left(\cos(-345^\circ) + i\sin(-345^\circ)\right) = 5\left(\cos(15^\circ) + i\sin(15^\circ)\right)$$

46. $z \cdot w = 4\left(\cos(50^\circ) + i\sin(50^\circ)\right) \cdot \left(\cos(340^\circ) + i\sin(340^\circ)\right)$
$$= 4 \cdot 1\left(\cos(50^\circ + 340^\circ) + i\sin(50^\circ + 340^\circ)\right) = 4\left(\cos(390^\circ) + i\sin(390^\circ)\right)$$
$$= 4\left(\cos(30^\circ) + i\sin(30^\circ)\right)$$
$$\frac{z}{w} = \frac{4\left(\cos(50^\circ) + i\sin 50^\circ\right)}{\left(\cos(340^\circ) + i\sin(340^\circ)\right)} = \frac{4}{1}\left(\cos(50^\circ - 340^\circ) + i\sin(50^\circ - 340^\circ)\right)$$
$$= 4\left(\cos(-290^\circ) + i\sin(-290^\circ)\right) = 4\left(\cos(70^\circ) + i\sin(70^\circ)\right)$$

47. $\left[3\left(\cos(20^\circ) + i\sin(20^\circ)\right)\right]^3 = 3^3\left(\cos(3 \cdot 20^\circ) + i\sin(3 \cdot 20^\circ)\right) = 27\left(\cos(60^\circ) + i\sin(60^\circ)\right)$
$$= 27\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \frac{27}{2} + \frac{27\sqrt{3}}{2}i$$

48. $\left[2\left(\cos(50^\circ) + i\sin(50^\circ)\right)\right]^3 = 2^3\left(\cos(3 \cdot 50^\circ) + i\sin(3 \cdot 50^\circ)\right) = 8\left(\cos(150^\circ) + i\sin(150^\circ)\right)$
$$= 8\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -4\sqrt{3} + 4i$$

49. $\left[\sqrt{2}\left(\cos\left(\frac{5\pi}{8}\right) + i\sin\left(\frac{5\pi}{8}\right)\right)\right]^4 = \left(\sqrt{2}\right)^4\left(\cos\left(4 \cdot \frac{5\pi}{8}\right) + i\sin\left(4 \cdot \frac{5\pi}{8}\right)\right)$
$$= 4\left(\cos\left(\frac{5\pi}{2}\right) + i\sin\left(\frac{5\pi}{2}\right)\right) = 4(0 + 1i) = 4i$$

50. $\left[2\left(\cos\left(\frac{5\pi}{16}\right) + i\sin\left(\frac{5\pi}{16}\right)\right)\right]^4 = 2^4\left(\cos\left(4 \cdot \frac{5\pi}{16}\right) + i\sin\left(4 \cdot \frac{5\pi}{16}\right)\right)$
$$= 16\left(\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right) = 16\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = -8\sqrt{2} - 8\sqrt{2}\,i$$

51. $1 - \sqrt{3}i \qquad r = \sqrt{1^2 + \left(-\sqrt{3}\right)^2} = 2 \qquad \tan\theta = \frac{-\sqrt{3}}{1} = -\sqrt{3} \qquad \theta = 300^\circ$
$$1 - \sqrt{3}i = 2\left(\cos(300^\circ) + i\sin(300^\circ)\right)$$
$$\left(1 - \sqrt{3}i\right)^6 = \left[2\left(\cos(300^\circ) + i\sin(300^\circ)\right)\right]^6 = 2^6\left(\cos(6 \cdot 300^\circ) + i\sin(6 \cdot 300^\circ)\right)$$
$$= 64\left(\cos(1800^\circ) + i\sin(1800^\circ)\right) = 64\left(\cos(0^\circ) + i\sin(0^\circ)\right)$$
$$= 64 + 0i = 64$$

52. $2-2i \quad r=\sqrt{2^2+(-2)^2}=2\sqrt{2} \quad \tan\theta=\frac{-2}{2}=-1 \quad \theta=315^\circ$

$$2-2i=2\sqrt{2}\left(\cos(315^\circ)+i\sin(315^\circ)\right)$$

$$(2-2i)^8=\left[2\sqrt{2}\left(\cos(315^\circ)+i\sin(315^\circ)\right)\right]^8=\left(2\sqrt{2}\right)^8\left(\cos(8\cdot 315^\circ)+i\sin(8\cdot 315^\circ)\right)$$
$$=4096\left(\cos(2520^\circ)+i\sin(2520^\circ)\right)=4096\left(\cos(0^\circ)+i\sin(0^\circ)\right)$$
$$=4096(1+0\,i)=4096$$

53. $3+4i \quad r=\sqrt{3^2+4^2}=5 \quad \tan\theta=\frac{4}{3} \quad \theta\approx 53.1^\circ$

$$3+4i=5\left(\cos(53.1^\circ)+i\sin(53.1^\circ)\right)$$

$$(3+4i)^4=\left[5^4\left(\cos(4\cdot 53.1^\circ)+i\sin(4\cdot 53.1^\circ)\right)\right]^4=625\left(\cos(212.4^\circ)+i\sin(212.4^\circ)\right)$$
$$\approx 625\left(-0.8443+i(-0.5358)\right)=-527.7-334.9i$$

54. $1-2i \quad r=\sqrt{1^2+(-2)^2}=\sqrt{5} \quad \tan\theta=\frac{-2}{1}=-2 \quad \theta\approx 296.6^\circ$

$$1-2i=\sqrt{5}\left(\cos(296.6^\circ)+i\sin(296.6^\circ)\right)$$

$$(1-2i)^4=\left[\left(\sqrt{5}\right)^4\left(\cos(4\cdot 296.6^\circ)+i\sin(4\cdot 296.6^\circ)\right)\right]^4$$
$$=25\left(\cos(1186.4^\circ)+i\sin(1186.4^\circ)\right)$$
$$\approx 25\left(-0.2823+i(0.9593)\right)=-7.0575+23.9825i$$

55. $27+0i \quad r=\sqrt{27^2+0^2}=27 \quad \tan\theta=\frac{0}{27}=0 \quad \theta=0^\circ$

$$27+0i=27\left(\cos(0^\circ)+i\sin(0^\circ)\right)$$

The three complex cube roots of $27=27(\cos 0^\circ+i\sin 0^\circ)$ are:

$$z_k=\sqrt[3]{27}\left[\cos\left(\frac{0^\circ}{3}+\frac{360^\circ k}{3}\right)+i\sin\left(\frac{0^\circ}{3}+\frac{360^\circ k}{3}\right)\right]$$
$$=3\left[\cos(120^\circ k)+i\sin(120^\circ k)\right]$$
$$z_0=3\left[\cos(120^\circ\cdot 0)+i\sin(120^\circ\cdot 0)\right]=3\left(\cos(0^\circ)+i\sin(0^\circ)\right)=3$$
$$z_1=3\left[\cos(120^\circ\cdot 1)+i\sin(120^\circ\cdot 1)\right]=3\left(\cos(120^\circ)+i\sin(120^\circ)\right)=-\frac{3}{2}+\frac{3\sqrt{3}}{2}i$$
$$z_2=3\left[\cos(120^\circ\cdot 2)+i\sin(120^\circ\cdot 2)\right]=3\left(\cos(240^\circ)+i\sin(240^\circ)\right)=-\frac{3}{2}-\frac{3\sqrt{3}}{2}i$$

56. $-16 \quad r=\sqrt{(-16)^2+0^2}=\sqrt{256}=16 \quad \tan\theta=\frac{0}{-16}=0 \quad \theta=180^\circ$

$$-16=16\left(\cos(180^\circ)+i\sin(180^\circ)\right)$$

The four complex fourth roots of $-16=16\left(\cos(180^\circ)+i\sin(180^\circ)\right)$ are:

$$z_k = \sqrt[4]{16}\left[\cos\left(\frac{180^\circ}{4} + \frac{360^\circ\ k}{4}\right) + i\sin\left(\frac{180^\circ}{4} + \frac{360^\circ\ k}{4}\right)\right]$$

$$= 2\left[\cos\left(45^\circ + 90^\circ k\right) + i\sin\left(45^\circ + 90^\circ\ k\right)\right]$$

$$z_0 = 2\left[\cos\left(45^\circ + 90^\circ\ \ 0\right) + i\sin\left(45^\circ + 90^\circ\ \ 0\right)\right] = 2\ \left(\cos\left(45^\circ\right) + i\sin\left(45^\circ\right)\right) = \sqrt{2} + \sqrt{2}\, i$$

$$z_1 = 2\left[\cos\left(45^\circ + 90^\circ\ \ 1\right) + i\sin\left(45^\circ + 90^\circ\ \ 1\right)\right] = 2\ \left(\cos\left(135^\circ\right) + i\sin\left(135^\circ\right)\right) = -\sqrt{2} + \sqrt{2}\, i$$

$$z_2 = 2\left[\cos\left(45^\circ + 90^\circ\ \ 2\right) + i\sin\left(45^\circ + 90^\circ\ \ 2\right)\right] = 2\ \left(\cos\left(225^\circ\right) + i\sin\left(225^\circ\right)\right) = -\sqrt{2} - \sqrt{2}\, i$$

$$z_3 = 2\left[\cos\left(45^\circ + 90^\circ\ \ 3\right) + i\sin\left(45^\circ + 90^\circ\ \ 3\right)\right] = 2\ \left(\cos\left(315^\circ\right) + i\sin\left(315^\circ\right)\right) = \sqrt{2} - \sqrt{2}\, i$$

57. $P = (1, -2),\ Q = (3, -6)$ $\quad \mathbf{v} = (3-1)\mathbf{i} + (-6-(-2))\mathbf{j} = 2\mathbf{i} - 4\mathbf{j}$

$$\|\mathbf{v}\| = \sqrt{2^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$$

58. $P = (-3, 1),\ Q = (4, -2)$ $\quad \mathbf{v} = (4-(-3))\mathbf{i} + (-2-1)\mathbf{j} = 7\mathbf{i} - 3\mathbf{j}$

$$\|\mathbf{v}\| = \sqrt{7^2 + (-3)^2} = \sqrt{58}$$

59. $P = (0, -2),\ Q = (-1, 1)$ $\quad \mathbf{v} = (-1-0)\mathbf{i} + (1-(-2))\mathbf{j} = -1\mathbf{i} + 3\mathbf{j}$

$$\|\mathbf{v}\| = \sqrt{(-1)^2 + 3^2} = \sqrt{10}$$

60. $P = (3, -4),\ Q = (-2, 0)$ $\quad \mathbf{v} = (-2-3)\mathbf{i} + (0-(-4))\mathbf{j} = -5\mathbf{i} + 4\mathbf{j}$

$$\|\mathbf{v}\| = \sqrt{(-5)^2 + 4^2} = \sqrt{41}$$

61. $\mathbf{v} = -2\mathbf{i} + \mathbf{j},\ \ \mathbf{w} = 4\mathbf{i} - 3\mathbf{j}$

$$4\mathbf{v} - 3\mathbf{w} = 4\left(-2\mathbf{i} + \mathbf{j}\right) - 3\left(4\mathbf{i} - 3\mathbf{j}\right) = -8\mathbf{i} + 4\mathbf{j} - 12\mathbf{i} + 9\mathbf{j} = -20\mathbf{i} + 13\mathbf{j}$$

62. $\mathbf{v} = -2\mathbf{i} + \mathbf{j},\ \ \mathbf{w} = 4\mathbf{i} - 3\mathbf{j}$

$$-\mathbf{v} + 2\mathbf{w} = -\left(-2\mathbf{i} + \mathbf{j}\right) + 2\left(4\mathbf{i} - 3\mathbf{j}\right) = 2\mathbf{i} - \mathbf{j} + 8\mathbf{i} - 6\mathbf{j} = 10\mathbf{i} - 7\mathbf{j}$$

63. $\mathbf{v} = -2\mathbf{i} + \mathbf{j}$

$$\|\mathbf{v}\| = \|-2\mathbf{i} + \mathbf{j}\| = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$$

64. $\mathbf{v} = -2\mathbf{i} + \mathbf{j},\ \ \mathbf{w} = 4\mathbf{i} - 3\mathbf{j}$

$$\|\mathbf{v} + \mathbf{w}\| = \left\|\left(-2\mathbf{i} + \mathbf{j}\right) + \left(4\mathbf{i} - 3\mathbf{j}\right)\right\| = \|2\mathbf{i} - 2\mathbf{j}\| = \sqrt{2^2 + \left(-2\right)^2} = \sqrt{8} = 2\sqrt{2}$$

65. $\mathbf{v} = -2\mathbf{i} + \mathbf{j},\ \ \mathbf{w} = 4\mathbf{i} - 3\mathbf{j}$

$$\|\mathbf{v}\| + \|\mathbf{w}\| = \|-2\mathbf{i} + \mathbf{j}\| + \|4\mathbf{i} - 3\mathbf{j}\| = \sqrt{(-2)^2 + 1^2} + \sqrt{4^2 + (-3)^2} = \sqrt{5} + 5$$

66. $\mathbf{v} = -2\mathbf{i} + \mathbf{j},\ \ \mathbf{w} = 4\mathbf{i} - 3\mathbf{j}$

$$\|2\mathbf{v}\| - 3\|\mathbf{w}\| = \|2(-2\mathbf{i} + \mathbf{j})\| - 3\|4\mathbf{i} - 3\mathbf{j}\| = \|-4\mathbf{i} + 2\mathbf{j}\| - 3\|4\mathbf{i} - 3\mathbf{j}\|$$

$$= \sqrt{(-4)^2 + 2^2} - 3\sqrt{4^2 + (-3)^2} = \sqrt{20} - 3\ \ 5 = 2\sqrt{5} - 15$$

67. $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{-2\mathbf{i}+\mathbf{j}}{\|-2\mathbf{i}+\mathbf{j}\|} = \dfrac{-2\mathbf{i}+\mathbf{j}}{\sqrt{(-2)^2+1^2}} = \dfrac{-2\mathbf{i}+\mathbf{j}}{\sqrt{5}} = -\dfrac{2\sqrt{5}}{5}\mathbf{i} + \dfrac{\sqrt{5}}{5}\mathbf{j}$

68. $\mathbf{u} = \dfrac{-\mathbf{w}}{\|\mathbf{w}\|} = \dfrac{-(4\mathbf{i}-3\mathbf{j})}{\|4\mathbf{i}-3\mathbf{j}\|} = \dfrac{-4\mathbf{i}+3\mathbf{j}}{\sqrt{4^2+(-3)^2}} = \dfrac{-4\mathbf{i}+3\mathbf{j}}{\sqrt{25}} = -\dfrac{4}{5}\mathbf{i} + \dfrac{3}{5}\mathbf{j}$

69. $\mathbf{v} = -2\mathbf{i}+\mathbf{j}, \quad \mathbf{w} = 4\mathbf{i}-3\mathbf{j}$

$$\mathbf{v}\cdot\mathbf{w} = -2(4)+1(-3) = -8-3 = -11$$

$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|} = \frac{-11}{\sqrt{(-2)^2+1^2}\sqrt{4^2+(-3)^2}} = \frac{-11}{\sqrt{5}\ \ 5} = \frac{-11}{5\sqrt{5}} \approx -0.9839$$

$$\theta \approx 169.7^\circ$$

70. $\mathbf{v} = 3\mathbf{i}-\mathbf{j}, \quad \mathbf{w} = \mathbf{i}+\mathbf{j}$

$$\mathbf{v}\cdot\mathbf{w} = 3\ \ 1+(-1)(1) = 3-1 = 2$$

$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|} = \frac{2}{\sqrt{3^2+(-1)^2}\sqrt{1^2+1^2}} = \frac{2}{\sqrt{10}\sqrt{2}} = \frac{1}{\sqrt{5}} \approx 0.4472$$

$$\theta \approx 63.4^\circ$$

71. $\mathbf{v} = \mathbf{i}-3\mathbf{j}, \quad \mathbf{w} = -\mathbf{i}+\mathbf{j}$

$$\mathbf{v}\cdot\mathbf{w} = 1(-1)+(-3)(1) = -1-3 = -4$$

$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|} = \frac{-4}{\sqrt{1^2+(-3)^2}\sqrt{(-1)^2+1^2}} = \frac{-4}{\sqrt{10}\sqrt{2}} = \frac{-2}{\sqrt{5}} \approx -0.8944$$

$$\theta \approx 153.4^\circ$$

72. $\mathbf{v} = \mathbf{i}+4\mathbf{j}, \quad \mathbf{w} = 3\mathbf{i}-2\mathbf{j}$

$$\mathbf{v}\cdot\mathbf{w} = 1(3)+(4)(-2) = 3-8 = -5$$

$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|} = \frac{-5}{\sqrt{1^2+4^2}\sqrt{3^2+(-2)^2}} = \frac{-5}{\sqrt{17}\sqrt{13}} = \frac{-5}{\sqrt{221}} \approx -0.3363$$

$$\theta \approx 109.7^\circ$$

73. $\text{proj}_{\mathbf{w}}\mathbf{v} = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w} = \dfrac{(2\mathbf{i}+3\mathbf{j})\cdot(3\mathbf{i}+\mathbf{j})}{\left(\sqrt{3^2+1^2}\right)^2}(3\mathbf{i}+\mathbf{j}) = \dfrac{2\ \ 3+3\ \ 1}{10}(3\mathbf{i}+\mathbf{j}) = \dfrac{27}{10}\mathbf{i} + \dfrac{9}{10}\mathbf{j}$

74. $\text{proj}_{\mathbf{w}}\mathbf{v} = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w} = \dfrac{(-\mathbf{i}+2\mathbf{j})\cdot(3\mathbf{i}-\mathbf{j})}{\left(\sqrt{3^2+(-1)^2}\right)^2}(3\mathbf{i}-\mathbf{j}) = \dfrac{-1\ \ 3+2(-1)}{10}(3\mathbf{i}-\mathbf{j})$

$$= \frac{-15}{10}\mathbf{i} + \frac{5}{10}\mathbf{j} = -\frac{3}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$$

75. Let the positive x-axis point downstream, so that the velocity of the current is $\mathbf{v_c} = 2\mathbf{i}$.
Let $\mathbf{v_w}$ = the velocity of the swimmer in the water.
Let $\mathbf{v_g}$ = the velocity of the swimmer relative to the land.
Then $\mathbf{v_g} = \mathbf{v_w} + \mathbf{v_c}$
The speed of the swimmer is $\|\mathbf{v_w}\| = 5$; its direction is directly across the river, so

Let $\mathbf{v_w} = 5\mathbf{j}$.

$\mathbf{v_g} = \mathbf{v_w} + \mathbf{v_c} = 5\mathbf{j} + 2\mathbf{i} = 2\mathbf{i} + 5\mathbf{j}$

Let $\|\mathbf{v_g}\| = \sqrt{2^2 + 5^2} = \sqrt{29} \approx 5.4$ miles per hour .

Since the river is 1 mile wide, it takes the swimmer 0.2 hours to cross the river. The swimmer will end up (0.2)(2) = 0.4 miles downstream.

76. Let $\mathbf{v_a}$ = the velocity of the plane in still air.

$\mathbf{v_w}$ = the velocity of the wind.

$\mathbf{v_g}$ = the velocity of the plane relative to the ground.

$\mathbf{v_g} = \mathbf{v_a} + \mathbf{v_w}$

$\mathbf{v_a} = 500\,\mathbf{j}$

$$\mathbf{v_w} = 60\left(\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}\right) = 30\sqrt{2}\,\mathbf{i} - 30\sqrt{2}\,\mathbf{j}$$

$$\mathbf{v_g} = \mathbf{v_a} + \mathbf{v_w} = 500\,\mathbf{j} + 30\sqrt{2}\,\mathbf{i} - 30\sqrt{2}\,\mathbf{j} = \left(30\sqrt{2}\right)\mathbf{i} + \left(500 - 30\sqrt{2}\right)\mathbf{j}$$

The speed of the plane relative to the ground is:

$$\|\mathbf{v_g}\| = \sqrt{\left(30\sqrt{2}\right)^2 + \left(500 - 30\sqrt{2}\right)^2} = \sqrt{1800 + 250000 - 30000\sqrt{2} + 1800}$$

$$= \sqrt{211{,}173.6} \approx 459.5 \text{ miles per hour}$$

To find the direction, find the angle between $\mathbf{v_g}$ and a convenient vector such as due north, $\mathbf{j}$.

$$\cos\theta = \frac{\mathbf{v}_g \cdot \mathbf{j}}{\|\mathbf{v}_g\|\|-\mathbf{j}\|} = \frac{\left(30\sqrt{2}\right)\ 0 + \left(500 - 30\sqrt{2}\right)(1)}{459.5\sqrt{0^2 + 1^2}} = \frac{500 - 30\sqrt{2}}{459.5} \approx 0.9958$$

$$\theta \approx 5.3^\circ$$

The plane is traveling with a ground speed of about 459.5 miles per hour in a direction of 5.3° east of north.

77. Let $\mathbf{F_1}$ be the tension on the left cable and $\mathbf{F}_2$ be the tension on the right cable. Let $\mathbf{F_3}$ represent the force of the weight of the box.

$$\mathbf{F_1} = \|\mathbf{F_1}\|\left(\cos(140^\circ)\mathbf{i} + \sin(140^\circ)\mathbf{j}\right) \approx \|\mathbf{F_1}\|\left(-0.7660\mathbf{i} + 0.6428\mathbf{j}\right)$$

$$\mathbf{F}_2 = \|\mathbf{F}_2\|\left(\cos(30^\circ)\mathbf{i} + \sin(30^\circ)\mathbf{j}\right) \approx \|\mathbf{F}_2\|\left(0.8660\mathbf{i} + 0.5000\mathbf{j}\right)$$

$$\mathbf{F}_3 = -2000\mathbf{j}$$

For equilibrium, the sum of the force vectors must be zero.

$$\begin{aligned}\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} &= -0.7660\|\mathbf{F_1}\|\mathbf{i} + 0.6428\|\mathbf{F_1}\|\mathbf{j} + 0.8660\|\mathbf{F}_2\|\mathbf{i} + 0.5000\|\mathbf{F}_2\|\mathbf{j} - 2000\mathbf{j}\\ &= \left(-0.7660\|\mathbf{F_1}\| + 0.8660\|\mathbf{F}_2\|\right)\mathbf{i} + \left(0.6428\|\mathbf{F_1}\| + 0.5000\|\mathbf{F}_2\| - 2000\right)\mathbf{j}\\ &= 0\end{aligned}$$

Set the $\mathbf{i}$ and $\mathbf{j}$ components equal to zero and solve:

$$\begin{cases} -0.7660\|\mathbf{F}_1\| + 0.8660\|\mathbf{F}_2\| = 0 \quad \rightarrow \quad \|\mathbf{F}_2\| = \dfrac{0.7660}{0.8660}\|\mathbf{F}_1\| = 0.8845\|\mathbf{F}_1\| \\ 0.6428\|\mathbf{F}_1\| + 0.5000\|\mathbf{F}_2\| - 2000 = 0 \end{cases}$$

$$0.6428\|\mathbf{F}_1\| + 0.5000\left(0.8845\|\mathbf{F}_1\|\right) - 2000 = 0$$

$$1.0851\|\mathbf{F}_1\| = 2000$$

$$\|\mathbf{F}_1\| \approx 1843 \text{ pounds}$$

$$\|\mathbf{F}_2\| = 0.8845(1843) = 1630 \text{ pounds}$$

The tension in the left cable is about 1843 pounds and the tension in the right cable is about 1630 pounds.

78.

79. Let the positive x-axis point downstream, so that the velocity of the current is $\mathbf{v_c} = 5\mathbf{i}$.
Let $\mathbf{v_w}$ = the velocity of the boat in the water.
Let $\mathbf{v}_L$ = the velocity of the boat relative to the land.
Then $\mathbf{v}_L = \mathbf{v_w} + \mathbf{v_c}$
The speed of the boat is $\|\mathbf{v_w}\| = 15$; we need to find the direction.

Let $\mathbf{v_w} = a\mathbf{i} + b\mathbf{j}$ so $\|\mathbf{v_w}\| = \sqrt{a^2 + b^2} = 15 \rightarrow a^2 + b^2 = 225$.
Let $\mathbf{v}_L = k\mathbf{j}$.

Since $\mathbf{v}_L = \mathbf{v_w} + \mathbf{v_c}$, $k\mathbf{j} = a\mathbf{i} + b\mathbf{j} + 5\mathbf{i} \rightarrow k\mathbf{j} = (a+5)\mathbf{i} + b\mathbf{j}$
$a + 5 = 0$ and $k = b \rightarrow a = -5$

$a^2 + b^2 = 225 \rightarrow 25 + b^2 = 225 \rightarrow b^2 = 200 \rightarrow k = b = \sqrt{200}$

$\mathbf{v_w} = -5\mathbf{i} + \sqrt{200}\mathbf{j}$ and $\mathbf{v}_L = \sqrt{200}\mathbf{j}$

Find the angle between $\mathbf{v}_w$ and $\mathbf{v}_L$:

$$\cos\theta = \frac{\mathbf{v}_w \cdot \mathbf{v}_L}{\|\mathbf{v}_w\|\|\mathbf{v}_L\|} = \frac{-5\ \ 0 + \left(\sqrt{200}\right)^2}{(15)\left(\sqrt{200}\right)} = \frac{200}{15\sqrt{200}} = \frac{\sqrt{200}}{15} \approx 0.9428$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{200}}{15}\right) \approx 19.47^{\circ}$$

The heading of the boat needs to be 19.47° upstream, in other words, the boat should head at an angle of $\alpha = 70.53^{\circ}$ to the shore.