Chapter 11

Analytic Geometry

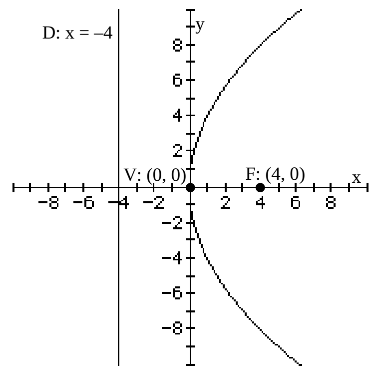
11.2 The Parabola

1.	B	2.	G	3.	E	4.	D
5.	H	6.	A	7.	C	8.	F

9. The focus is (4, 0) and the vertex is (0, 0). Both lie on the horizontal line $y = 0$. $a = 4$ and since (4, 0) is to the right of (0, 0), the parabola opens to the right. The equation of the parabola is:

$$y^2 = 4ax$$
$$y^2 = 4 \; 4 \; x$$
$$y^2 = 16x$$

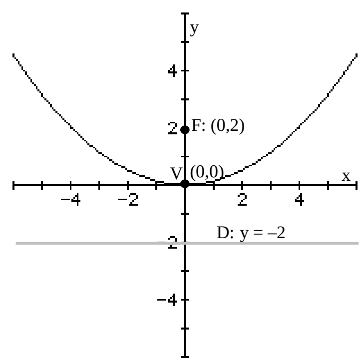
Letting $x = 4$, we find $y^2 = 64$ or $y = \pm 8$.
The points (4, 8) and (4, –8) define the latus rectum.



10. The focus is (0, 2) and the vertex is (0, 0). Both lie on the vertical line $x = 0$. $a = 2$ and since (0, 2) is above (0, 0), the parabola opens up The equation of the parabola is:

$$x^2 = 4ay$$
$$x^2 = 4 \; 2 \; y$$
$$x^2 = 8y$$

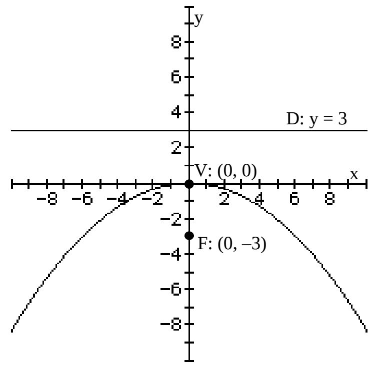
Letting $y = 2$, we find $x^2 = 16$ or $x = \pm 4$.
The points (–4, 2) and (4, 2) define the latus rectum.



11. The focus is (0, –3) and the vertex is (0, 0). Both lie on the vertical line $x = 0$. $a = 3$ and since (0, –3) is below (0, 0), the parabola opens down. The equation of the parabola is:

$$x^2 = -4ay$$
$$x^2 = -4\ 3\ y$$
$$x^2 = -12y$$

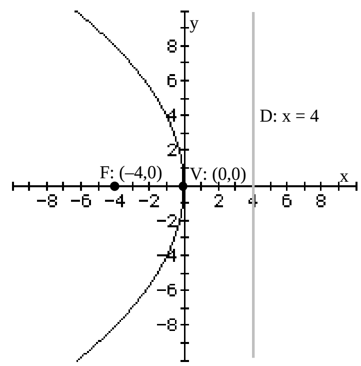
Letting $y = -3$, we find $x^2 = 36$ or $x = \pm 6$.
The points (6, 3) and (6, –3) define the latus rectum.



12. The focus is (–4, 0) and the vertex is (0, 0). Both lie on the horizontal line $y = 0$. $a = 4$ and since (–4, 0) is to the left of (0, 0), the parabola opens to the left. The equation of the parabola is:

$$y^2 = -4ax$$
$$y^2 = -4\ 4\ x$$
$$y^2 = -16x$$

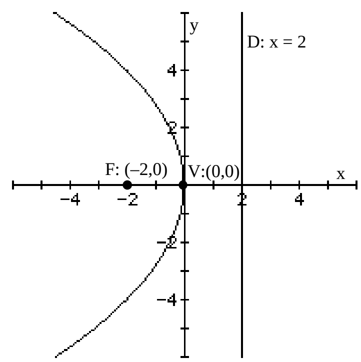
Letting $x = -4$, we find $y^2 = 64$ or $y = \pm 8$. The points (–4, 8) and (–4, –8) define the latus rectum.



13. The focus is (–2, 0) and the directrix is $x = 2$. The vertex is (0, 0). $a = 2$ and since (–2, 0) is to the left of (0, 0), the parabola opens to the left. The equation of the parabola is:

$$y^2 = -4ax$$
$$y^2 = -4\ 2\ x$$
$$y^2 = -8x$$

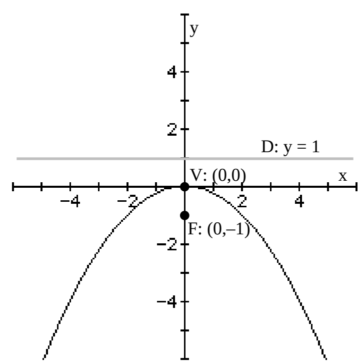
Letting $x = -2$, we find $y^2 = 16$ or $y = \pm 4$. The points (–2, 4) and (–2, –4) define the latus rectum.



14. The focus is (0, –1) and the directrix is $y = 1$. The vertex is (0, 0). $a = 1$ and since (0, –1) is below (0, 0), the parabola opens down. The equation of the parabola is:

$$x^2 = -4ay$$
$$x^2 = -4\ 1\ y$$
$$x^2 = -4y$$

Letting $y = -1$, we find $x^2 = 4$ or $x = \pm 2$. The points (–2, –1) and (2, –1) define the latus rectum.



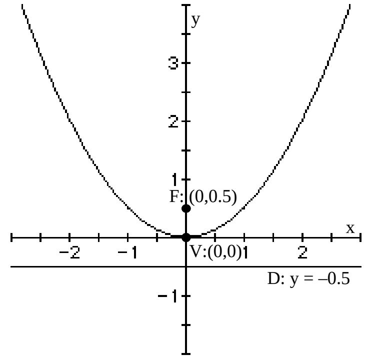
15. The directrix is $y = -\frac{1}{2}$ and the vertex is (0, 0). The focus is $\left(0, \frac{1}{2}\right)$. $a = \frac{1}{2}$ and since $\left(0, \frac{1}{2}\right)$ is above (0, 0), the parabola opens up. The equation of the parabola is:

$$x^2 = 4ay$$

$$x^2 = 4 \cdot \frac{1}{2} \cdot y \rightarrow x^2 = 2y$$

Letting $y = \frac{1}{2}$, we find $x^2 = 1$ or $x = \pm 1$.

The points $\left(1, \frac{1}{2}\right)$ and $\left(-1, \frac{1}{2}\right)$ define the latus rectum.

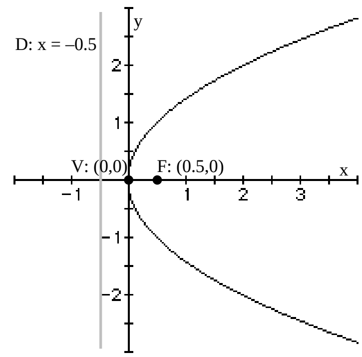


16. The directrix is $x = -\frac{1}{2}$ and the vertex is (0, 0). The focus is $\left(\frac{1}{2}, 0\right)$. $a = \frac{1}{2}$ and since $\left(\frac{1}{2}, 0\right)$ is to the right of (0, 0), the parabola opens to the right. The equation of the parabola is:

$$y^2 = 4ax$$

$$y^2 = 4 \cdot \frac{1}{2} \cdot x \rightarrow y^2 = 2x$$

Letting $x = \frac{1}{2}$, we find $y^2 = 1$ or $y = \pm 1$. The points $\left(\frac{1}{2}, -1\right)$ and $\left(\frac{1}{2}, 1\right)$ define the latus rectum.



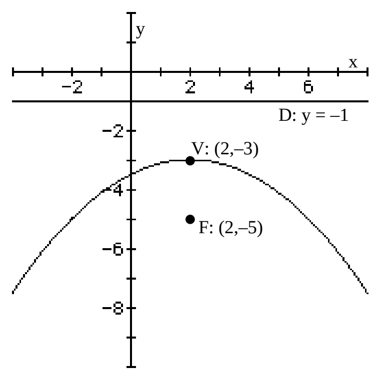
17. The focus is (2, –5) and the vertex is (2, –3). Both lie on the vertical line $x = 2$. $a = 2$ and since (2, –5) is below (2, –3), the parabola opens down. The equation of the parabola is:

$$(x-h)^2 = -4a(y-k)$$

$$(x-2)^2 = -4 \cdot 2 \cdot (y-(-3))$$

$$(x-2)^2 = -8(y+3)$$

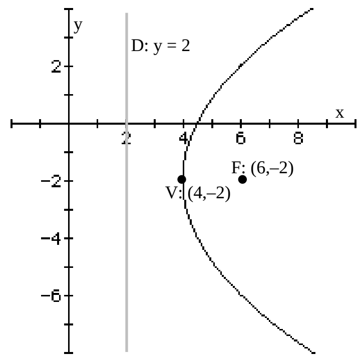
Letting $y = -5$, we find $(x-2)^2 = 16$ or $x - 2 = \pm 4$. So, $x = 6$ or $x = -2$. The points (6, –5) and (–2, –5) define the latus rectum.



18. The focus is (6, –2) and the vertex is (4, –2). Both lie on the horizontal line $y = -2$. $a = 2$ and since (6,–2) is to the right of (4, –2), the parabola opens to the right. The equation of the parabola is:

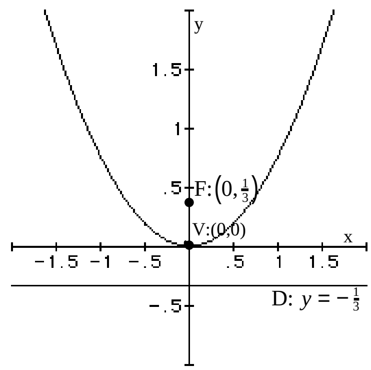
$$(y-k)^2 = 4a(x-h)$$
$$(y-(-2))^2 = 4\cdot 2\cdot(x-4)$$
$$(y+2)^2 = 8(x-4)$$

Letting $x = 6$, we find $(y+2)^2 = 16$ or $y + 2 = \pm 4$. So, $y = -6$ or $y = 2$. The points (6, –6) and (6, 2) define the latus rectum.



19. Vertex: (0,0). Since the axis of symmetry is vertical, the parabola opens up or down. Since (2, 3) is above (0, 0), the parabola opens up. The equation has the form $x^2 = 4ay$. Substitute the coordinates of (2, 3) into the equation to find a:

$$2^2 = 4a\cdot 3$$
$$4 = 12a \rightarrow a = \frac{1}{3}$$

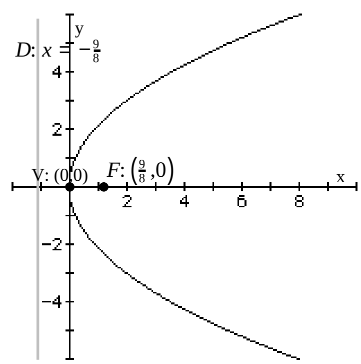


The equation of the parabola is: $x^2 = \frac{4}{3}y$. The focus is $\left(0, \frac{1}{3}\right)$. Letting $y = \frac{1}{3}$, we find $x^2 = \frac{4}{9}$ or $x = \pm\frac{2}{3}$. The points $\left(\frac{2}{3}, \frac{1}{3}\right)$ and $\left(-\frac{2}{3}, \frac{1}{3}\right)$ define the latus rectum.

20. Vertex: (0,0). Since the axis of symmetry is horizontal, the parabola opens left or right. Since (2, 3) is to the right of (0, 0), the parabola opens to the right. The equation has the form $y^2 = 4ax$. Substitute the coordinates of (2, 3) into the equation to find a:

$$3^2 = 4a\cdot 2$$
$$9 = 8a \rightarrow a = \frac{9}{8}$$

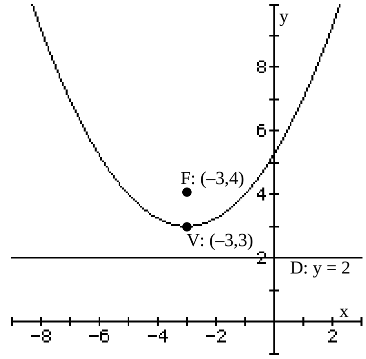
The equation of the parabola is: $y^2 = \frac{9}{2}x$. The focus is $\left(\frac{9}{8}, 0\right)$. Letting $x = \frac{9}{8}$, we find $y^2 = \frac{81}{16}$ or $y = \pm\frac{9}{4}$. The points $\left(\frac{9}{8}, \frac{9}{4}\right)$ and $\left(\frac{9}{8}, -\frac{9}{4}\right)$ define the latus rectum.



21. The directrix is $y = 2$ and the focus is (–3, 4). This is a vertical case, so the vertex is (–3, 3). $a = 1$ and since (–3, 4) is above $y = 2$, the parabola opens up. The equation of the parabola is:

$$(x-h)^2 = 4a(y-k)$$
$$(x-(-3))^2 = 4\;1\;(y-3)$$
$$(x+3)^2 = 4(y-3)$$

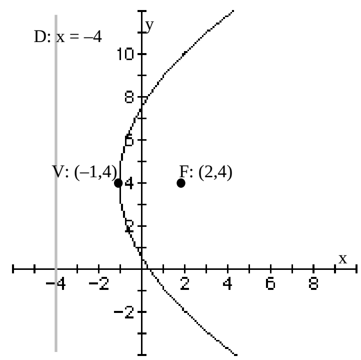
Letting $y = 4$, we find $(x+3)^2 = 4$ or $x+3 = \pm 2$. So, $x = -1$ or $x = -5$. The points (–1, 4) and (–5, 4) define the latus rectum.



22. The directrix is $x = -4$ and the focus is (2, 4). This is a horizontal case, so the vertex is (–1, 4). $a = 3$ and since (2, 4) is to the right of $x = -4$, the parabola opens to the right. The equation of the parabola is:

$$(y-k)^2 = 4a(x-h)$$
$$(y-4)^2 = 4\;3\;(x-(-1))$$
$$(y-4)^2 = 12(x+1)$$

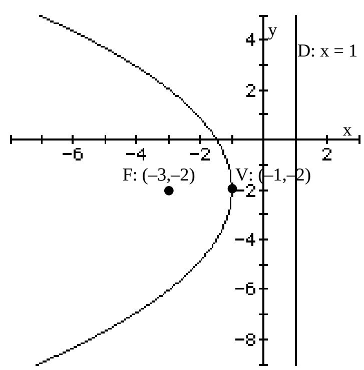
Letting $x = 2$, we find $(y-4)^2 = 36$ or $y-4 = \pm 6$. So, $y = -2$ or $y = 10$. The points (2, –2) and (2, 10) define the latus rectum.



23. The directrix is $x = 1$ and the focus is (–3, –2). This is a horizontal case, so the vertex is (–1, –2). $a = 2$ and since (–3, –2) is to the left of $x = 1$, the parabola opens to the left. The equation of the parabola is:

$$(y-k)^2 = -4a(x-h)$$
$$(y-(-2))^2 = -4\;2\;(x-(-1))$$
$$(y+2)^2 = -8(x+1)$$

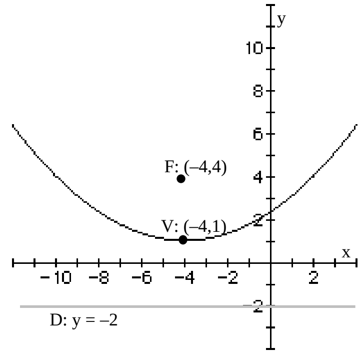
Letting $x = -3$, we find $(y+2)^2 = 16$ or $y+2 = \pm 4$. So, $y = 2$ or $y = -6$. The points (–3, 2) and (–3, –6) define the latus rectum.



24. The directrix is $y = -2$ and the focus is (–4, 4). This is a vertical case, so the vertex is (–4, 1). $a = 3$ and since (–4, 4) is above $y = -2$, the parabola opens up. The equation of the parabola is:

$$(x-h)^2 = 4a(y-k)$$
$$(x-(-4))^2 = 4\ 3\ (y-1)$$
$$(x+4)^2 = 12(y-1)$$

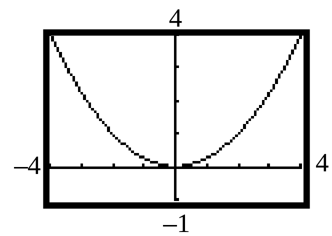
Letting $y = 4$, we find $(x+4)^2 = 36$ or $x + 4 = \pm 6$. So, $x = -10$ or $x = 2$. The points (–10, 4) and (2, 4) define the latus rectum.



25. The equation $x^2 = 4y$ is in the form $x^2 = 4ay$ where $4a = 4$ or $a = 1$. Thus, we have:

Vertex: (0, 0)
Focus: (0, 1)
Directrix: $y = -1$

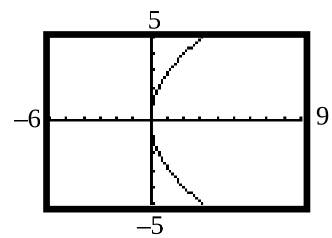
To graph, enter: $y_1 = x^2 / 4$



26. The equation $y^2 = 8x$ is in the form $y^2 = 4ax$ where $4a = 8$ or $a = 2$. Thus, we have:

Vertex: (0, 0)
Focus: (2, 0)
Directrix: $x = -2$

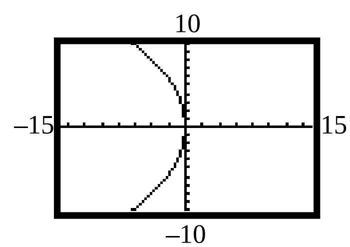
To graph, enter: $y_1 = \sqrt{8x}$; $y_2 = -\sqrt{8x}$



27. The equation $y^2 = -16x$ is in the form $y^2 = -4ax$ where $-4a = -16$ or $a = 4$. Thus, we have:

Vertex: (0, 0)
Focus: (–4, 0)
Directrix: $x = 4$

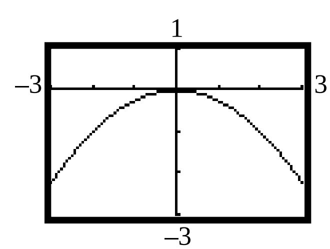
To graph, enter: $y_1 = \sqrt{-16x}$; $y_2 = -\sqrt{-16x}$



28. The equation $x^2 = -4y$ is in the form $x^2 = -4ay$ where $-4a = -4$ or $a = 1$. Thus, we have:

Vertex: (0, 0)
Focus: (0, –1)
Directrix: $y = 1$

To graph, enter: $y_1 = x^2 / (-4)$

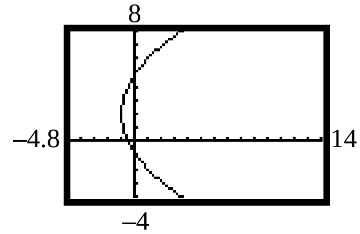


29. The equation $(y-2)^2 = 8(x+1)$ is in the form $(y-k)^2 = 4a(x-h)$ where $4a = 8$ or $a = 2$, $h = -1$, and $k = 2$. Thus, we have:

Vertex: (–1, 2)
Focus: (1, 2)
Directrix: $x = -3$

To graph, enter:

$y_1 = 2 + \sqrt{8(x+1)}$; $y_2 = 2 - \sqrt{8(x+1)}$

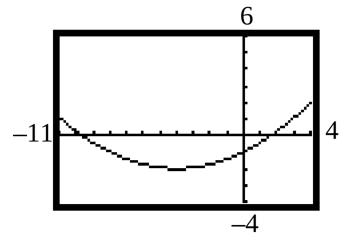


30. The equation $(x+4)^2 = 16(y+2)$ is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 16$ or $a = 4$, $h = -4$, and $k = -2$. Thus, we have:

Vertex: (–4, –2)
Focus: (–4, 2)
Directrix: $y = -6$

To graph, enter:

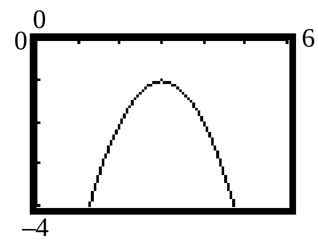
$y_1 = -2 + (x+4)^2 / 16$



31. The equation $(x-3)^2 = -(y+1)$ is in the form $(x-h)^2 = -4a(y-k)$ where $-4a = -1$ or $a = \frac{1}{4}$, $h = 3$, and $k = -1$. Thus, we have:

Vertex: (3, –1)
Focus: $\left(3, -\frac{5}{4}\right)$
Directrix: $y = -\frac{3}{4}$

To graph, enter: $y_1 = -1 - (x-3)^2$

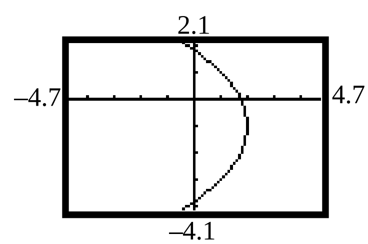


32. The equation $(y+1)^2 = -4(x-2)$ is in the form $(y-k)^2 = -4a(x-h)$ where $-4a = -4$ or $a = 1$, $h = 2$, and $k = -1$. Thus, we have:

Vertex: (2, –1)
Focus: $(1, -1)$
Directrix: $x = 3$

To graph, enter:

$y_1 = -1 + \sqrt{-4(x-2)}$; $y_2 = -1 - \sqrt{-4(x-2)}$

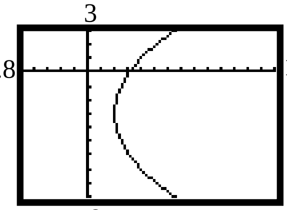


33. The equation $(y+3)^2 = 8(x-2)$ is in the form $(y-k)^2 = 4a(x-h)$ where $4a = 8$ or $a = 2$, $h = 2$, and $k = -3$. Thus, we have:

Vertex: (2, –3)
Focus: (4, –3)
Directrix: $x = 0$

To graph, enter:

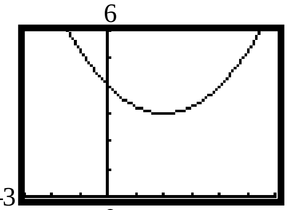
$y_1 = -3 + \sqrt{8(x-2)}$; $y_2 = -3 - \sqrt{8(x-2)}$



34. The equation $(x-2)^2 = 4(y-3)$ is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 4$ or $a = 1$, $h = 2$, and $k = 3$. Thus, we have:

Vertex: (2, 3)
Focus: (2, 4)
Directrix: $y = 2$

To graph, enter: $y_1 = 3 + (x-2)^2 / 4$



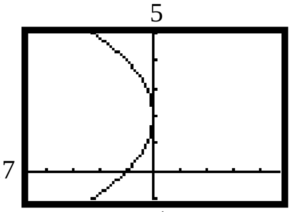
35. Complete the square to put in standard form:

$$y^2 - 4y + 4x + 4 = 0$$
$$y^2 - 4y + 4 = -4x$$
$$(y-2)^2 = -4x$$

The equation is in the form $(y-k)^2 = -4a(x-h)$ where $-4a = -4$ or $a = 1$, $h = 0$, and $k = 2$. Thus, we have:

Vertex: (0, 2)
Focus: (–1, 2)
Directrix: $x = 1$

To graph, enter: $y_1 = 2 + \sqrt{-4x}$; $y_2 = 2 - \sqrt{-4x}$



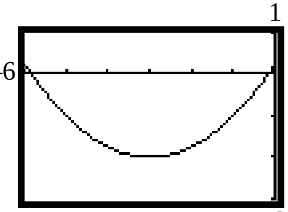
36. Complete the square to put in standard form:

$$x^2 + 6x - 4y + 1 = 0$$
$$x^2 + 6x + 9 = 4y - 1 + 9$$
$$(x+3)^2 = 4(y+2)$$

The equation is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 4$ or $a = 1$, $h = -3$, and $k = -2$. Thus, we have:

Vertex: (–3, –2)
Focus: (–3, –1)
Directrix: $y = -3$

To graph, enter: $y_1 = -2 + (x+3)^2 / 4$



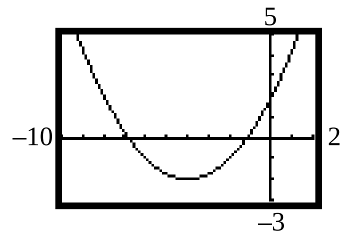
37. Complete the square to put in standard form:

$$x^2 + 8x = 4y - 8$$
$$x^2 + 8x + 16 = 4y - 8 + 16$$
$$(x+4)^2 = 4(y+2)$$

The equation is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 4$ or $a = 1$, $h = -4$, and $k = -2$. Thus, we have:

Vertex: (–4, –2)
Focus: (–4, –1)
Directrix: $y = -3$

To graph, enter: $y_1 = -2 + (x+4)^2 / 4$



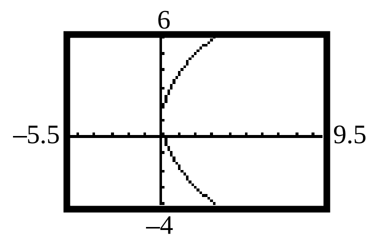
38. Complete the square to put in standard form:

$$y^2 - 2y = 8x - 1$$
$$y^2 - 2y + 1 = 8x - 1 + 1$$
$$(y-1)^2 = 8x$$

The equation is in the form $(y-k)^2 = 4a(x-h)$ where $4a = 8$ or $a = 2$, $h = 0$, and $k = 1$. Thus, we have:

Vertex: (0, 1)
Focus: (2, 1)
Directrix: $x = -2$

To graph, enter: $y_1 = 1 + \sqrt{8x}$; $y_1 = 1 - \sqrt{8x}$



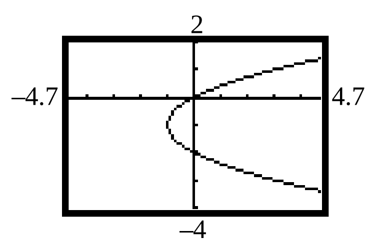
39. Complete the square to put in standard form:

$$y^2 + 2y - x = 0$$
$$y^2 + 2y + 1 = x + 1$$
$$(y+1)^2 = x + 1$$

The equation is in the form $(y-k)^2 = 4a(x-h)$ where $4a = 1$ or $a = \frac{1}{4}$, $h = -1$, and $k = -1$. Thus, we have:

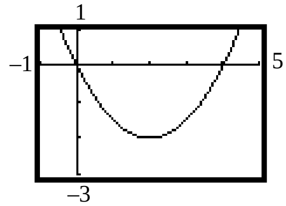
Vertex: (–1, –1)
Focus: $\left(-\frac{3}{4}, -1\right)$
Directrix: $x = -\frac{5}{4}$

To graph, enter: $y_1 = -1 + \sqrt{x+1}$; $y_2 = -1 - \sqrt{x+1}$



40. Complete the square to put in standard form:

$$x^2 - 4x = 2y$$
$$x^2 - 4x + 4 = 2y + 4$$
$$(x-2)^2 = 2(y+2)$$



The equation is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 2$ or $a = \frac{1}{2}$, $h = 2$, and $k = -2$.

Thus, we have:

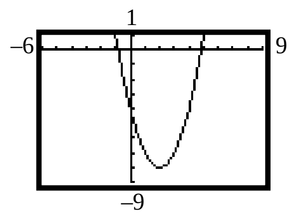
Vertex: (2, –2)

Focus: $\left(2, -\frac{3}{2}\right)$

Directrix: $y = -\frac{5}{2}$ To graph, enter: $y_1 = -2 + (x-2)^2/2$

41. Complete the square to put in standard form:

$$x^2 - 4x = y + 4$$
$$x^2 - 4x + 4 = y + 4 + 4$$
$$(x-2)^2 = y + 8$$



The equation is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 1$ or $a = \frac{1}{4}$, $h = 2$, and $k = -8$.

Thus, we have:

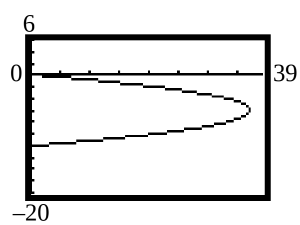
Vertex: (2, –8)

Focus: $\left(2, -\frac{31}{4}\right)$

Directrix: $y = -\frac{33}{4}$ To graph, enter: $y_1 = -8 + (x-2)^2$

42. Complete the square to put in standard form:

$$y^2 + 12y = -x + 1$$
$$y^2 + 12y + 36 = -x + 1 + 36$$
$$(y+6)^2 = -(x-37)$$



The equation is in the form $(y-k)^2 = -4a(x-h)$ where $-4a = -1$ or $a = \frac{1}{4}$, $h = 37$, and $k = -6$.

Thus, we have:

Vertex: (37, –6)

Focus: $\left(\frac{147}{4}, -6\right)$

Directrix: $x = \frac{149}{4}$ To graph, enter: $y_1 = -6 + \sqrt{-(x-37)}$; $y_2 = -6 - \sqrt{-(x-37)}$

43. $(y-1)^2 = c(x-0)$
$(y-1)^2 = cx$
$(2-1)^2 = c(1) \rightarrow 1 = c$
$(y-1)^2 = x$

44. $(x-1)^2 = c(y-2)$
$(2-1)^2 = c(1-2)$
$1 = -c \rightarrow c = -1$
$(x-1)^2 = -(y-2)$

45. $(y-1)^2 = c(x-2)$
$(0-1)^2 = c(1-2)$
$1 = -c \rightarrow c = -1$
$(y-1)^2 = -(x-2)$

46. $(x-0)^2 = c(y-(-1))$
$x^2 = c(y+1)$
$2^2 = c(0+1) \rightarrow 4 = c$
$x^2 = 4(y+1)$

47. $(x-0)^2 = c(y-1)$
$(2-0)^2 = c(2-1) \rightarrow 4 = c$
$x^2 = 4(y-1)$

48. $(x-1)^2 = c(y-(-1))$
$(0-1)^2 = c(1+1) \rightarrow 1 = 2c \rightarrow c = \frac{1}{2}$
$(x-1)^2 = \frac{1}{2}(y+1)$

49. $(y-0)^2 = c(x-(-2))$
$y^2 = c(x+2)$
$1^2 = c(0+2) \rightarrow 1 = 2c \rightarrow c = \frac{1}{2}$
$y^2 = \frac{1}{2}(x+2)$

50. $(y-0)^2 = c(x-1)$
$(1-0)^2 = c(0-1)$
$1 = -c \rightarrow c = -1$
$y^2 = -(x-1)$

51. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the parabola is 10 feet across and 4 feet deep, the points (5, 4) and (–5, 4) are on the parabola.
Substitute and solve for a:

$$5^2 = 4a(4) \rightarrow 25 = 16a \rightarrow a = \frac{25}{16}$$

a is the distance from the vertex to the focus. Thus, the receiver (located at the focus) is $\frac{25}{16} = 1.5625$ feet, or 18.75 inches from the base of the dish, along the axis of the parabola.

52. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the parabola is 6 feet across and 2 feet deep, the points (3, 2) and (–3, 2) are on the parabola.
Substitute and solve for a:

$$3^2 = 4a(2) \rightarrow 9 = 8a \rightarrow a = \frac{9}{8}$$

a is the distance from the vertex to the focus. Thus, the receiver (located at the focus) is $\frac{9}{8} = 1.125$ feet, or 13.5 inches from the base of the dish, along the axis of the parabola.

53. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the parabola is 4 inches across and 1 inch deep, the points (2, 1) and (–2, 1) are on the parabola.
Substitute and solve for a:

$$2^2 = 4a(1) \quad \rightarrow \quad 4 = 4a \quad \rightarrow \quad a = 1$$

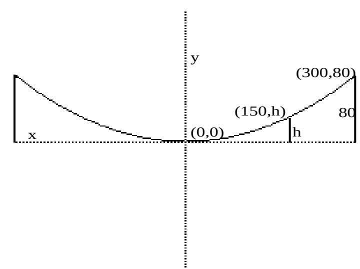
a is the distance from the vertex to the focus. Thus, the bulb (located at the focus) should be 1 inch, from the vertex.

54. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the focus is 1 inch from the vertex and the depth is 2 inches, $a = 1$ and the points $(x, 2)$ and $(-x, 2)$ are on the parabola.
Substitute and solve for x:

$$x^2 = 4(1)(2) \rightarrow x^2 = 8 \rightarrow x = \pm 2\sqrt{2}$$

The diameter of the headlight is $4\sqrt{2}$ inches.

55. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = cy$.
The point (300, 80) is a point on the parabola.
Solve for c and find the equation:



$$300^2 = c(80) \rightarrow c = 1125$$

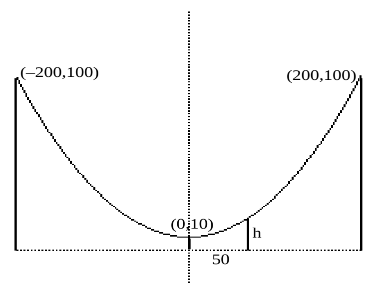
$$x^2 = 1125y$$

Since the height of the cable, 150 feet from the center, is to be found, the point (150, h) is a point on the parabola. Solve for h:

$$150^2 = 1125h \rightarrow 22500 = 1125h \rightarrow h = 20$$

The height of the cable, 150 feet from the center, is 20 feet.

56. Set up the problem so that the vertex of the parabola is at (0, 10) and it opens up. Then the equation of the parabola has the form: $x^2 = c(y-10)$.
The point (200, 100) is a point on the parabola.
Solve for c and find the equation:



$$200^2 = c(100-10)$$

$$40000 = 90c \rightarrow c = 444.44$$

$$x^2 = 444.44(y-10)$$

Since the height of the cable, 50 feet from the center, is to be found, the point (50, h) is a point on the parabola. Solve for h:

$$50^2 = 444.44(h-10) \rightarrow 2500 = 444.44h - 4444.4$$

$$444.44h = 6944.4 \rightarrow h = 15.625$$

The height of the cable, 50 feet from the center, is 15.625 feet.

57. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. a is the distance from the vertex to the focus (where the source is located), so $a = 2$. Since the opening is 5 feet across, there is a point (2.5, y) on the parabola. Solve for y:

$$x^2 = 8y \rightarrow 2.5^2 = 8y \rightarrow 6.25 = 8y \rightarrow y = 0.78125 \text{ feet}$$

The depth of the searchlight should be 0.78125 feet.

58. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. a is the distance from the vertex to the focus (where the source is located), so $a = 2$. Since the depth is 4 feet, there is a point (x, 4) on the parabola. Solve for x:

$$x^2 = 8y \rightarrow x^2 = 8\ \ 4 \rightarrow x^2 = 32 \rightarrow x = \pm 4\sqrt{2}$$

The width of the opening of the searchlight should be $8\sqrt{2}$ feet.

59. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the parabola is 20 feet across and 6 feet deep, the points (10, 6) and (–10, 6) are on the parabola.
Substitute and solve for a:

$$10^2 = 4a(6) \rightarrow 100 = 24a \rightarrow a \approx 4.17 \text{ feet}$$

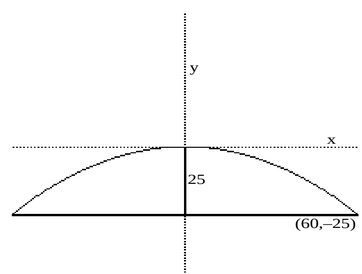
The heat source will be concentrated 4.17 feet from the base, along the axis of symmetry.

60. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the parabola is 4 inches across and 3 feet deep, the points (2, 36) and (–2, 36) are on the parabola.
Substitute and solve for a:

$$2^2 = 4a(36) \rightarrow 4 = 144a \rightarrow a = \frac{1}{36} \approx 0.0278 \text{ inches}$$

The collected light will be concentrated 0.0278 inches from the vertex, along the axis of symmetry.

61. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens down. Then the equation of the parabola has the form: $x^2 = cy$.
The point (60, –25) is a point on the parabola.
Solve for c and find the equation:



$$60^2 = c(-25) \rightarrow c = -144$$

$$x^2 = -144y$$

To find the height of the bridge, 10 feet from the center, the point (10, y) is a point on the parabola. Solve for y:

$$10^2 = -144y \rightarrow 100 = -144y \rightarrow y = -0.69$$

The height of the bridge, 10 feet from the center, is 25 – 0.69 = 24.31 feet.
To find the height of the bridge, 30 feet from the center, the point (30, y) is a point on the parabola. Solve for y:

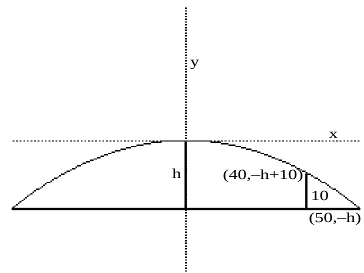
$$30^2 = -144y \rightarrow 900 = -144y \rightarrow y = -6.25$$

The height of the bridge, 30 feet from the center, is 25 – 6.25 = 18.75 feet.
To find the height of the bridge, 50 feet from the center, the point (50, y) is a point on the parabola. Solve for y:

$$50^2 = -144y \to 2500 = -144y \to y = -17.36$$

The height of the bridge, 50 feet from the center, is 25 – 17.36 = 7.64 feet.

62. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens down. Then the equation of the parabola has the form: $x^2 = cy$.
The points (50, –h) and (40, –h+10) are points on the parabola. Substitute and solve for c and h:



$$50^2 = c(-h) \qquad 40^2 = c(-h+10)$$
$$ch = -2500 \qquad 1600 = -ch + 10c$$
$$1600 = -(-2500) + 10c \to -900 = 10c \to c = -90$$
$$-90h = -2500 \to h = 27.78$$

The height of the bridge at the center is 27.78 feet.

63. $Ax^2 + Ey = 0 \quad A \neq 0,\ E \neq 0$

$$Ax^2 = -Ey \to x^2 = \frac{-E}{A}y$$

This is the equation of a parabola with vertex at (0, 0) and axis of symmetry being the y-axis. The focus is $\left(0, \frac{-E}{4A}\right)$. The directrix is $y = \frac{E}{4A}$.

64. $Cy^2 + Dx = 0 \quad C \neq 0,\ D \neq 0$

$$Cy^2 = -Dx \to y^2 = \frac{-D}{C}x$$

This is the equation of a parabola with vertex at (0, 0) and axis of symmetry being the x-axis. The focus is $\left(\frac{-D}{4C}, 0\right)$. The directrix is $x = \frac{D}{4C}$.

65. $Ax^2 + Dx + Ey + F = 0 \qquad A \neq 0$

(a) If $E \neq 0$, then:

$$Ax^2 + Dx = -Ey - F \to A\left(x^2 + \frac{D}{A}x + \frac{D^2}{4A^2}\right) = -Ey - F + \frac{D^2}{4A}$$

$$\left(x + \frac{D}{2A}\right)^2 = \frac{1}{A}\left(-Ey - F + \frac{D^2}{4A}\right) \to \left(x + \frac{D}{2A}\right)^2 = \frac{-E}{A}\left(y + \frac{F}{E} - \frac{D^2}{4AE}\right)$$

$$\left(x + \frac{D}{2A}\right)^2 = \frac{-E}{A}\left(y - \frac{D^2 - 4AF}{4AE}\right)$$

This is the equation of a parabola whose vertex is $\left(\frac{-D}{2A}, \frac{D^2 - 4AF}{4AE}\right)$.

(b) If $E = 0$, then

$$Ax^2 + Dx + F = 0 \rightarrow x = \frac{-D \pm \sqrt{D^2 - 4AF}}{2A}$$

If $D^2 - 4AF = 0$, then $x = \frac{-D}{2A}$ is a vertical line.

(c) If $E = 0$, then

$$Ax^2 + Dx + F = 0 \rightarrow x = \frac{-D \pm \sqrt{D^2 - 4AF}}{2A}$$

If $D^2 - 4AF > 0$,

then $x = \frac{-D + \sqrt{D^2 - 4AF}}{2A}$ or $x = \frac{-D - \sqrt{D^2 - 4AF}}{2A}$ are two vertical lines.

(d) If $E = 0$, then

$$Ax^2 + Dx + F = 0 \rightarrow x = \frac{-D \pm \sqrt{D^2 - 4AF}}{2A}$$

If $D^2 - 4AF < 0$, there is no real solution. The graph contains no points.

66. $Cy^2 + Dx + Ey + F = 0 \qquad C \neq 0$

(a) If $D \neq 0$, then:

$$Cy^2 + Ey = -Dx - F \rightarrow C\left(y^2 + \frac{E}{C}y + \frac{E^2}{4C^2}\right) = -Dx - F + \frac{E^2}{4C}$$

$$\left(y + \frac{E}{2C}\right)^2 = \frac{1}{C}\left(-Dx - F + \frac{E^2}{4C}\right)$$

$$\left(y + \frac{E}{2C}\right)^2 = \frac{-D}{C}\left(x + \frac{F}{D} - \frac{E^2}{4CD}\right) \rightarrow \left(y + \frac{E}{2C}\right)^2 = \frac{-D}{C}\left(x - \frac{E^2 - 4CF}{4CD}\right)$$

This is the equation of a parabola whose vertex is $\left(\frac{-E}{2C}, \frac{E^2 - 4CF}{4CD}\right)$.

(b) If $D = 0$, then

$$Cy^2 + Ey + F = 0 \rightarrow y = \frac{-E \pm \sqrt{E^2 - 4CF}}{2C}$$

If $E^2 - 4CF = 0$, then $y = \frac{-E}{2C}$ is a horizontal line.

(c) If $D = 0$, then

$$Cy^2 + Ey + F = 0 \rightarrow y = \frac{-E \pm \sqrt{E^2 - 4CF}}{2C}$$

If $E^2 - 4CF > 0$,

then $y = \frac{-E + \sqrt{E^2 - 4CF}}{2C}$ or $y = \frac{-E - \sqrt{E^2 - 4CF}}{2C}$ are two horizontal lines.

(d) If $D = 0$, then

$$Cy^2 + Ey + F = 0 \rightarrow x = \frac{-E \pm \sqrt{E^2 - 4CF}}{2C}$$

If $E^2 - 4CF < 0$, there is no real solution. The graph contains no points.