Chapter 11

Analytic Geometry

11.3 The Ellipse

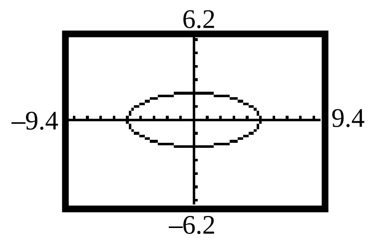
1. C 2. D 3. B 4. A

5. $\dfrac{x^2}{25}+\dfrac{y^2}{4}=1$

The center of the ellipse is at the origin.
$a = 5$; $b = 2$. The vertices are (5, 0) and (–5, 0).
Find the value of c:

$$c^2 = a^2 - b^2 = 25 - 4 = 21$$
$$c = \sqrt{21}$$

The foci are $\left(\sqrt{21}, 0\right)$ and $\left(-\sqrt{21}, 0\right)$.
To graph, enter:
$y_1 = 2\sqrt{(1 - x^2/25)}$; $y_1 = -2\sqrt{(1 - x^2/25)}$

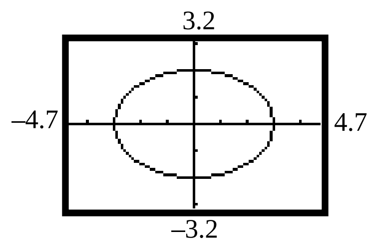


6. $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$

The center of the ellipse is at the origin.
$a = 3$; $b = 2$. The vertices are (3, 0) and (–3, 0).
Find the value of c:

$$c^2 = a^2 - b^2 = 9 - 4 = 5$$
$$c = \sqrt{5}$$

The foci are $\left(\sqrt{5}, 0\right)$ and $\left(-\sqrt{5}, 0\right)$.
To graph, enter:
$y_1 = 2\sqrt{(1 - x^2/9)}$; $y_1 = -2\sqrt{(1 - x^2/9)}$

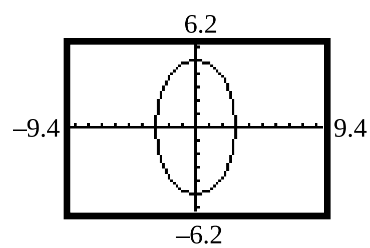


7. $\dfrac{x^2}{9}+\dfrac{y^2}{25}=1$

The center of the ellipse is at the origin.
$a = 5$; $b = 3$. The vertices are (0, 5) and (0, –5).
Find the value of c:

$$c^2 = a^2 - b^2 = 25 - 9 = 16$$
$$c = 4$$

The foci are (0, 4) and (0, –4).
To graph, enter:
$y_1 = 5\sqrt{(1 - x^2/9)}$; $y_1 = -5\sqrt{(1 - x^2/9)}$



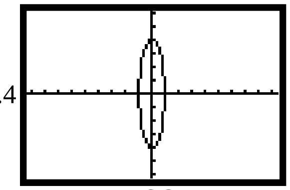
8. $x^2 + \frac{y^2}{16} = 1$

The center of the ellipse is at the origin.
$a = 4$, $b = 1$. The vertices are (0, 4) and (0, –4).
Find the value of c:

$$c^2 = a^2 - b^2 = 16 - 1 = 15$$
$$c = \sqrt{15}$$

The foci are $\left(0, \sqrt{15}\right)$ and $\left(0, -\sqrt{15}\right)$

To graph, enter: $y_1 = 4\sqrt{(1-x^2)}$; $y_1 = -4\sqrt{(1-x^2)}$



9. $4x^2 + y^2 = 16$

Divide by 16 to put in standard form:

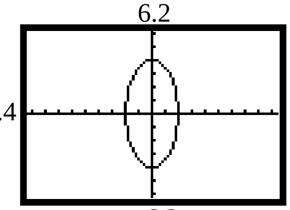
$$\frac{4x^2}{16} + \frac{y^2}{16} = \frac{16}{16} \quad \rightarrow \quad \frac{x^2}{4} + \frac{y^2}{16} = 1$$

The center of the ellipse is at the origin.
$a = 4$, $b = 2$. The vertices are (0, 4) and (0, –4).
Find the value of c:

$$c^2 = a^2 - b^2 = 16 - 4 = 12$$
$$c = \sqrt{12} = 2\sqrt{3}$$

The foci are $\left(0, 2\sqrt{3}\right)$ and $\left(0, -2\sqrt{3}\right)$.

To graph, enter:
$y_1 = \sqrt{(16-4x^2)}$; $y_1 = -\sqrt{(16-4x^2)}$



10. $x^2 + 9y^2 = 18$

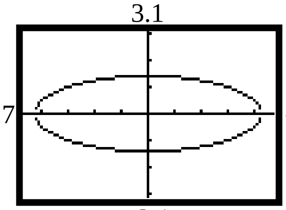
Divide by 18 to put in standard form:

$$\frac{x^2}{18} + \frac{9y^2}{18} = \frac{18}{18} \quad \rightarrow \quad \frac{x^2}{18} + \frac{y^2}{2} = 1$$

The center of the ellipse is at the origin.
$a = 3\sqrt{2}$, $b = \sqrt{2}$. The vertices are $\left(3\sqrt{2}, 0\right)$ and $\left(-3\sqrt{2}, 0\right)$. Find the value of c:

$$c^2 = a^2 - b^2 = 18 - 2 = 16$$
$$c = 4$$

The foci are (4, 0) and (–4, 0).
To graph, enter:
$y_1 = \sqrt{(18-x^2)}\ /\ 3$; $y_1 = -\sqrt{(18-x^2)}\ /\ 3$



11. $4y^2 + x^2 = 8$

Divide by 8 to put in standard form:

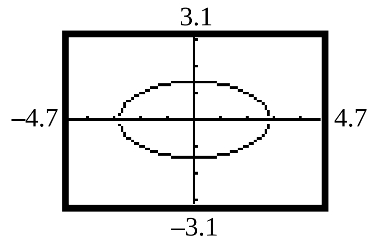
$$\frac{4y^2}{8} + \frac{x^2}{8} = \frac{8}{8} \quad \to \quad \frac{x^2}{8} + \frac{y^2}{2} = 1$$

The center of the ellipse is at the origin.
$a = \sqrt{8} = 2\sqrt{2}$, $b = \sqrt{2}$. The vertices are $\left(2\sqrt{2}, 0\right)$ and $\left(-2\sqrt{2}, 0\right)$. Find the value of c:

$$c^2 = a^2 - b^2 = 8 - 2 = 6$$
$$c = \sqrt{6}$$

The foci are $\left(\sqrt{6}, 0\right)$ and $\left(-\sqrt{6}, 0\right)$.

To graph, enter:
$y_1 = \sqrt{(2 - x^2 / 4)}$; $y_1 = -\sqrt{(2 - x^2 / 4)}$



12. $4y^2 + 9x^2 = 36$

Divide by 36 to put in standard form:

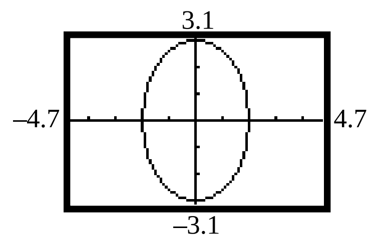
$$\frac{4y^2}{36} + \frac{9x^2}{36} = \frac{36}{36} \quad \to \quad \frac{x^2}{4} + \frac{y^2}{9} = 1$$

The center of the ellipse is at the origin.
$a = 3$, $b = 2$. The vertices are (0, 3) and (0, –3).
Find the value of c:

$$c^2 = a^2 - b^2 = 9 - 4 = 5$$
$$c = \sqrt{5}$$

The foci are $\left(0, \sqrt{5}\right)$ and $\left(0, -\sqrt{5}\right)$.

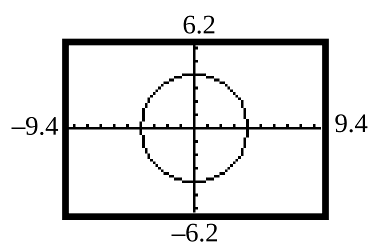
To graph, enter:
$y_1 = \sqrt{\left((36 - 9x^2) / 4\right)}$; $y_1 = -\sqrt{\left((36 - 9x^2) / 4\right)}$



13. $x^2 + y^2 = 16$

This is the equation of a circle whose center is at (0, 0) and radius = 4.

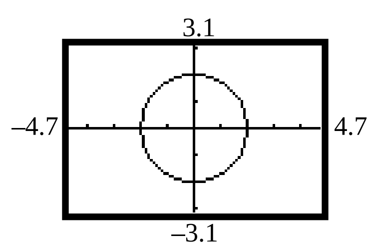
To graph, enter: $y_1 = \sqrt{(16 - x^2)}$; $y_1 = -\sqrt{(16 - x^2)}$



14. $x^2 + y^2 = 4$

This is the equation of a circle whose center is at (0, 0) and radius = 2.

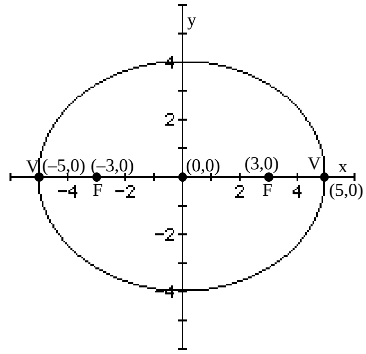
To graph, enter: $y_1 = \sqrt{(4 - x^2)}$; $y_1 = -\sqrt{(4 - x^2)}$



15. Center: (0, 0); Focus: (3, 0); Vertex: (5, 0);
Major axis is the x-axis; $a = 5$; $c = 3$. Find b:

$$b^2 = a^2 - c^2 = 25 - 9 = 16$$
$$b = 4$$

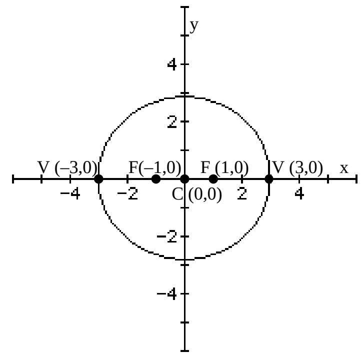
Write the equation: $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$



16. Center: (0, 0); Focus: (–1, 0); Vertex: (3, 0);
Major axis is the x-axis; $a = 3$; $c = 1$. Find b:

$$b^2 = a^2 - c^2 = 9 - 1 = 8$$
$$b = 2\sqrt{2}$$

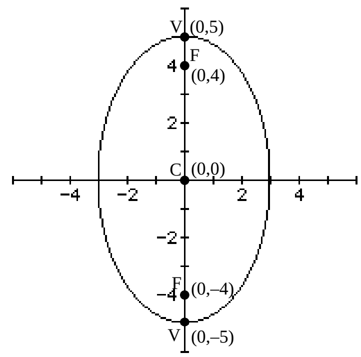
Write the equation: $\dfrac{x^2}{9} + \dfrac{y^2}{8} = 1$



17. Center: (0, 0); Focus: (0, –4); Vertex: (0, 5);
Major axis is the y-axis; $a = 5$; $c = 4$. Find b:

$$b^2 = a^2 - c^2 = 25 - 16 = 9$$
$$b = 3$$

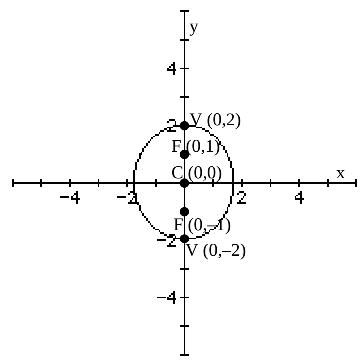
Write the equation: $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$



18. Center: (0, 0); Focus: (0, 1); Vertex: (0, –2);
Major axis is the y-axis; $a = 2$; $c = 1$. Find b:

$$b^2 = a^2 - c^2 = 4 - 1 = 3$$
$$b = \sqrt{3}$$

Write the equation: $\dfrac{x^2}{3} + \dfrac{y^2}{4} = 1$

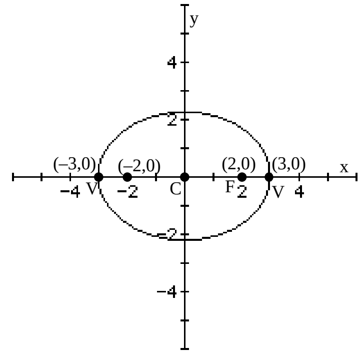


19. Foci: (±2, 0); Length of major axis is 6.
Center: (0, 0); Major axis is the x-axis;
$a = 3;\ \ c = 2$. Find b:

$$b^2 = a^2 - c^2 = 9 - 4 = 5$$

$$b = \sqrt{5}$$

Write the equation: $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$

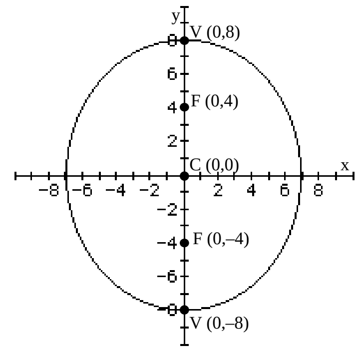


20. Focus: (0, –4); Vertices: (0, ±8).
Center: (0, 0); Major axis is the y-axis;
$a = 8;\ \ c = 4$. Find b:

$$b^2 = a^2 - c^2 = 64 - 16 = 48$$

$$b = 4\sqrt{3}$$

Write the equation: $\dfrac{x^2}{48} + \dfrac{y^2}{64} = 1$

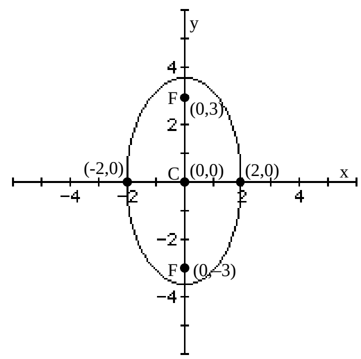


21. Foci: (0, ±3); x-intercepts are ±2. Center: (0, 0);
Major axis is the y-axis; $c = 3;\ \ b = 2$. Find a:

$$a^2 = b^2 + c^2 = 4 + 9 = 13$$

$$a = \sqrt{13}$$

Write the equation: $\dfrac{x^2}{4} + \dfrac{y^2}{13} = 1$

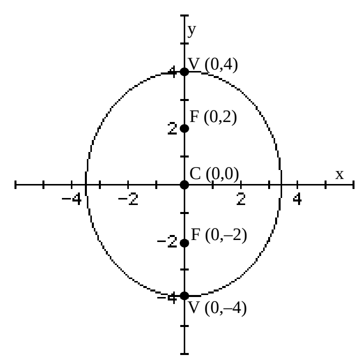


22. Foci: (0, ±2); length of the major axis is 8.
Center: (0, 0); Major axis is the y-axis;
$a = 4;\ \ c = 2$. Find b:

$$b^2 = a^2 - c^2 = 16 - 4 = 12$$

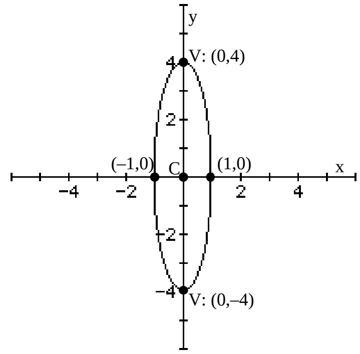
$$b = 2\sqrt{3}$$

Write the equation: $\dfrac{x^2}{12} + \dfrac{y^2}{16} = 1$



23. Center: (0, 0); Vertex: (0, 4); $b = 1$; Major axis is the y-axis; $a = 4$; $b = 1$.

Write the equation: $\frac{x^2}{1} + \frac{y^2}{16} = 1$

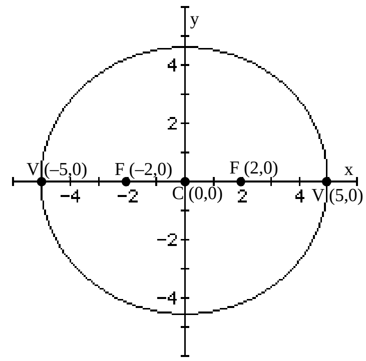


24. Vertices: (±5, 0); $c = 2$; Major axis is the x-axis; $a = 5$; Find b:

$$b^2 = a^2 - c^2 = 25 - 4 = 21$$

$$b = \sqrt{21}$$

Write the equation: $\frac{x^2}{25} + \frac{y^2}{21} = 1$



25. $\frac{(x+1)^2}{4} + \frac{(y-1)^2}{1} = 1$

26. $\frac{(x+1)^2}{1} + \frac{(y+1)^2}{4} = 1$

27. $\frac{(x-1)^2}{1} + \frac{y^2}{4} = 1$

28. $\frac{x^2}{4} + \frac{(y-1)^2}{1} = 1$

29. The equation $\frac{(x-3)^2}{4} + \frac{(y+1)^2}{9} = 1$ is in the form $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ (major axis parallel to the y-axis) where $a = 3$, $b = 2$, $h = 3$, and $k = -1$. Solving for c:

$$c^2 = a^2 - b^2 = 9 - 4 = 5$$

$$c = \sqrt{5}$$

Thus, we have:

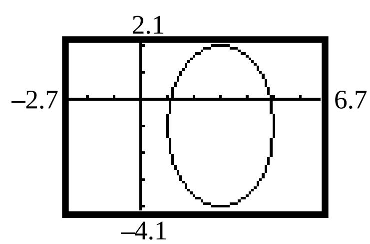
Center: (3, –1)

Foci: $\left(3, -1+\sqrt{5}\right)$, $\left(3, -1-\sqrt{5}\right)$

Vertices: (3, 2), (3, –4)

To graph, enter: $y_1 = -1 + 3\sqrt{1-(x-3)^2/4}$;

$y_2 = -1 - 3\sqrt{1-(x-3)^2/4}$



30. The equation $\dfrac{(x+4)^2}{9}+\dfrac{(y+2)^2}{4}=1$ is in the form $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$ (major axis parallel to the x-axis) where $a=3$, $b=2$, $h=-4$, and $k=-2$. Solving for c:

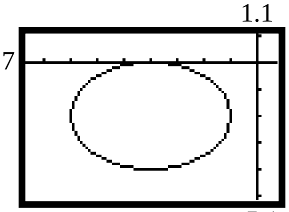
$$c^2=a^2-b^2=9-4=5$$
$$c=\sqrt{5}$$

Thus, we have:

Center: (–4, –2)

Foci: $\left(-4+\sqrt{5},-2\right)$, $\left(-4-\sqrt{5},-2\right)$

Vertices: (–7, –2), (–1, –2)



To graph, enter: $y_1=-2+2\sqrt{1-(x+4)^2/9}$;

$y_2=-2-2\sqrt{1-(x+4)^2/9}$

31. Divide by 16 to put the equation in standard form:

$$(x+5)^2+4(y-4)^2=16$$
$$\frac{(x+5)^2}{16}+\frac{4(y-4)^2}{16}=\frac{16}{16}$$
$$\frac{(x+5)^2}{16}+\frac{(y-4)^2}{4}=1$$

The equation is in the form $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$ (major axis parallel to the x-axis) where $a=4$, $b=2$, $h=-5$, and $k=4$. Solving for c:

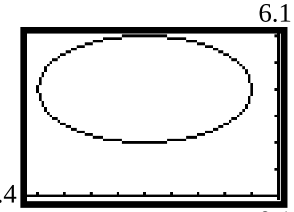
$$c^2=a^2-b^2=16-4=12$$
$$c=\sqrt{12}=2\sqrt{3}$$

Thus, we have:

Center: (–5, 4)

Foci: $\left(-5-2\sqrt{3},4\right)$, $\left(-5+2\sqrt{3},4\right)$

Vertices: (–9, 4), (–1, 4)



To graph, enter: $y_1=4+2\sqrt{1-(x+5)^2/16}$;

$y_2=4-2\sqrt{1-(x+5)^2/16}$

32. Divide by 18 to put the equation in standard form:

$$9(x-3)^2+(y+2)^2=18$$
$$\frac{9(x-3)^2}{18}+\frac{(y+2)^2}{18}=\frac{18}{18}$$
$$\frac{(x-3)^2}{2}+\frac{(y+2)^2}{18}=1$$

The equation is in the form $\dfrac{(x-h)^2}{b^2}+\dfrac{(y-k)^2}{a^2}=1$ (major axis parallel to the y-axis) where $a=3\sqrt{2}$, $b=\sqrt{2}$, $h=3$, and $k=-2$. Solving for c:

$$c^2=a^2-b^2=18-2=16$$
$$c=4$$

Thus, we have:

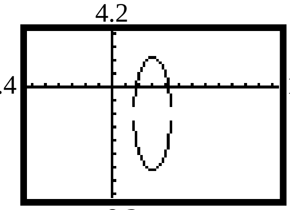
Center: (3, –2)

Foci: (3, 2), (3, –6)

Vertices: $\left(3, -2+3\sqrt{2}\right)$, $\left(3, -2-3\sqrt{2}\right)$

To graph, enter: $y_1 = -2+\sqrt{18-9(x-3)^2}$;

$y_2 = -2-\sqrt{18-9(x-3)^2}$



33. Complete the square to put the equation in standard form:

$$x^2+4x+4y^2-8y+4=0$$
$$(x^2+4x+4)+4(y^2-2y+1)=-4+4+4$$
$$(x+2)^2+4(y-1)^2=4$$
$$\frac{(x+2)^2}{4}+\frac{4(y-1)^2}{4}=\frac{4}{4}$$
$$\frac{(x+2)^2}{4}+\frac{(y-1)^2}{1}=1$$

The equation is in the form $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ (major axis parallel to the x-axis) where $a=2,\ b=1,\ h=-2,$ and $k=1$. Solving for c:

$$c^2=a^2-b^2=4-1=3$$
$$c=\sqrt{3}$$

Thus, we have:

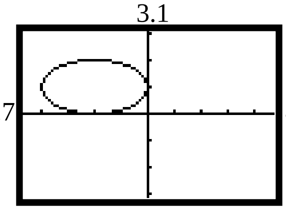
Center: (–2, 1)

Foci: $\left(-2-\sqrt{3}, 1\right)$, $\left(-2+\sqrt{3}, 1\right)$

Vertices: (–4, 1), (0, 1)

To graph, enter: $y_1 = 1+\sqrt{1-(x+2)^2/4}$;

$y_2 = 1-\sqrt{1-(x+2)^2/4}$



34. Complete the square to put the equation in standard form:

$$x^2+3y^2-12y+9=0$$
$$x^2+3(y^2-4y+4)=-9+12$$
$$x^2+3(y-2)^2=3$$
$$\frac{x^2}{3}+\frac{3(y-2)^2}{3}=\frac{3}{3}$$
$$\frac{x^2}{3}+\frac{(y-2)^2}{1}=1$$

The equation is in the form $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ (major axis parallel to the x-axis) where $a=\sqrt{3},\ b=1,\ h=0,$ and $k=2$. Solving for c:

$$c^2=a^2-b^2=3-1=2$$
$$c=\sqrt{2}$$

Thus, we have:

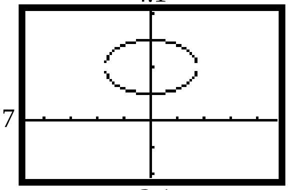
Center: (0, 2)

Foci: $\left(-\sqrt{2}, 2\right)$, $\left(\sqrt{2}, 2\right)$

Vertices: $\left(-\sqrt{3}, 2\right)$, $\left(\sqrt{3}, 2\right)$

To graph, enter: $y_1 = 2 + \sqrt{1 - x^2/3}$;

$y_2 = 2 - \sqrt{1 - x^2/3}$



35. Complete the square to put the equation in standard form:

$$2x^2 + 3y^2 - 8x + 6y + 5 = 0$$
$$2(x^2 - 4x) + 3(y^2 + 2y) = -5$$
$$2(x^2 - 4x + 4) + 3(y^2 + 2y + 1) = -5 + 8 + 3$$
$$2(x-2)^2 + 3(y+1)^2 = 6$$
$$\frac{2(x-2)^2}{6} + \frac{3(y+1)^2}{6} = \frac{6}{6}$$
$$\frac{(x-2)^2}{3} + \frac{(y+1)^2}{2} = 1$$

The equation is in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (major axis parallel to the x-axis) where $a = \sqrt{3}$, $b = \sqrt{2}$, $h = 2$, and $k = -1$. Solving for c:

$$c^2 = a^2 - b^2 = 3 - 2 = 1 \quad \rightarrow \quad c = 1$$

Thus, we have:

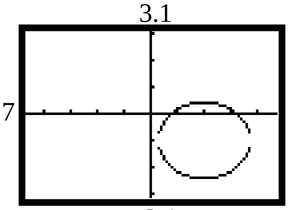
Center: (2, –1)

Foci: (1, –1), (3, –1)

Vertices: $\left(2 - \sqrt{3}, -1\right)$, $\left(2 + \sqrt{3}, -1\right)$

To graph, enter: $y_1 = -1 + \sqrt{2 - 2(x-2)^2/3}$;

$y_2 = -1 - \sqrt{2 - 2(x-2)^2/3}$



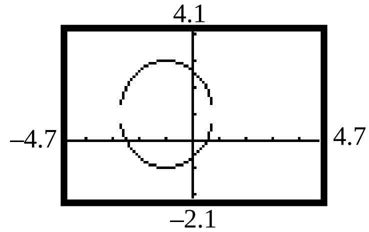
36. Complete the square to put the equation in standard form:

$$4x^2 + 3y^2 + 8x - 6y = 5$$
$$4(x^2 + 2x) + 3(y^2 - 2y) = 5$$
$$4(x^2 + 2x + 1) + 3(y^2 - 2y + 1) = 5 + 4 + 3$$
$$4(x+1)^2 + 3(y-1)^2 = 12$$
$$\frac{4(x+1)^2}{12} + \frac{3(y-1)^2}{12} = \frac{12}{12}$$
$$\frac{(x+1)^2}{3} + \frac{(y-1)^2}{4} = 1$$

The equation is in the form $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ (major axis parallel to the y-axis) where $a = 2$, $b = \sqrt{3}$, $h = -1$, and $k = 1$. Solving for c:

$$c^2 = a^2 - b^2 = 4 - 3 = 1 \quad \rightarrow \quad c = 1$$

Thus, we have:
Center: $(-1, 1)$
Foci: $(-1, 0)$, $(-1, 2)$
Vertices: $(-1, -1)$, $(-1, 3)$
To graph, enter: $y_1 = 1 + 2\sqrt{1-(x+1)^2/3}$;
$y_2 = 1 - 2\sqrt{1-(x+1)^2/3}$



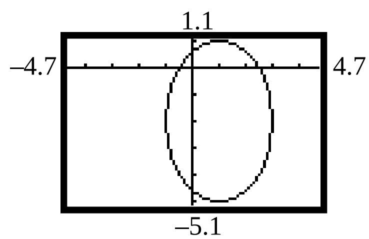
37. Complete the square to put the equation in standard form:

$$9x^2 + 4y^2 - 18x + 16y - 11 = 0$$
$$9(x^2 - 2x) + 4(y^2 + 4y) = 11$$
$$9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = 11 + 9 + 16$$
$$9(x-1)^2 + 4(y+2)^2 = 36$$
$$\frac{9(x-1)^2}{36} + \frac{4(y+2)^2}{36} = \frac{36}{36}$$
$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$

The equation is in the form $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ (major axis parallel to the y-axis) where $a = 3$, $b = 2$, $h = 1$, and $k = -2$. Solving for c:

$$c^2 = a^2 - b^2 = 9 - 4 = 5$$
$$c = \sqrt{5}$$

Thus, we have:
Center: $(1, -2)$
Foci: $\left(1, -2+\sqrt{5}\right)$, $\left(1, -2-\sqrt{5}\right)$
Vertices: $(1, 1)$, $(1, -5)$
To graph, enter: $y_1 = -2 + 3\sqrt{1-(x-1)^2/4}$;
$y_2 = -2 - 3\sqrt{1-(x-1)^2/4}$



38. Complete the square to put the equation in standard form:

$$x^2 + 9y^2 + 6x - 18y + 9 = 0$$
$$(x^2 + 6x) + 9(y^2 - 2y) = -9$$
$$(x^2 + 6x + 9) + 9(y^2 - 2y + 1) = -9 + 9 + 9$$
$$(x+3)^2 + 9(y-1)^2 = 9$$
$$\frac{(x+3)^2}{9} + \frac{9(y-1)^2}{9} = \frac{9}{9}$$
$$\frac{(x+3)^2}{9} + \frac{(y-1)^2}{1} = 1$$

The equation is in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (major axis parallel to the x-axis) where $a = 3$, $b = 1$, $h = -3$, and $k = 1$. Solving for c:

$$c^2 = a^2 - b^2 = 9 - 1 = 8$$
$$c = 2\sqrt{2}$$

Thus, we have:

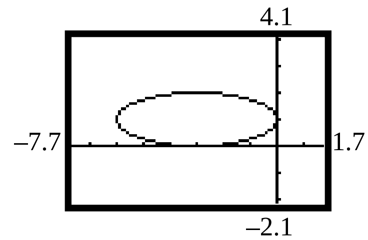
Center: (–3, 1))

Foci: $\left(-3+2\sqrt{2}, 1\right), \ \left(-3-2\sqrt{2}, 1\right)$

Vertices: (0, 1), (–6, 1)

To graph, enter: $y_1 = 1 + \sqrt{1-(x+3)^2/9}$;

$y_2 = 1 - \sqrt{1-(x+3)^2/9}$



39. Complete the square to put the equation in standard form:

$$4x^2 + y^2 + 4y = 0$$
$$4x^2 + y^2 + 4y + 4 = 4$$
$$4x^2 + (y+2)^2 = 4$$
$$\frac{4x^2}{4} + \frac{(y+2)^2}{4} = \frac{4}{4}$$
$$\frac{x^2}{1} + \frac{(y+2)^2}{4} = 1$$

The equation is in the form $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ (major axis parallel to the y-axis) where $a = 2, \ b = 1, \ h = 0$, and $k = -2$. Solving for c:

$$c^2 = a^2 - b^2 = 4 - 1 = 3 \quad \rightarrow \quad c = \sqrt{3}$$

Thus, we have:

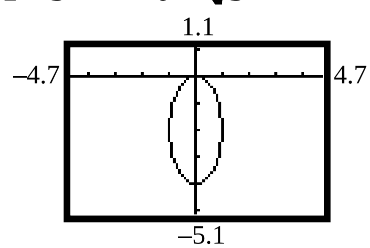
Center: (0,–2)

Foci: $\left(0, -2+\sqrt{3}\right), \ \left(0, -2-\sqrt{3}\right)$

Vertices: (0, 0), (0, –4)

To graph, enter: $y_1 = -2 + 2\sqrt{1-x^2}$;

$y_2 = -2 - 2\sqrt{1-x^2}$



40. Complete the square to put the equation in standard form:

$$9x^2 + y^2 - 18x = 0$$
$$9(x^2 - 2x + 1) + y^2 = 9$$
$$9(x-1)^2 + y^2 = 9$$
$$\frac{9(x-1)^2}{9} + \frac{y^2}{9} = \frac{9}{9}$$
$$\frac{(x-1)^2}{1} + \frac{y^2}{9} = 1$$

The equation is in the form $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ (major axis parallel to the y-axis) where $a = 3, \ b = 1, \ h = 1$, and $k = 0$. Solving for c:

$$c^2 = a^2 - b^2 = 9 - 1 = 8 \quad \rightarrow \quad c = 2\sqrt{2}$$

Thus, we have:

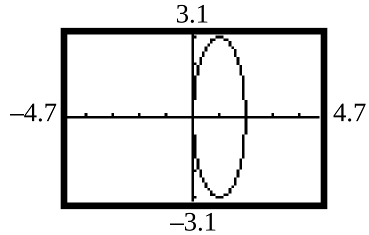
Center: (1, 0)

Foci: $\left(1, 2\sqrt{2}\right)$, $\left(1, -2\sqrt{2}\right)$

Vertices: (1, 3), (1, –3)

To graph, enter: $y_1 = 3\sqrt{1-(x-1)^2}$;

$y_2 = -3\sqrt{1-(x-1)^2}$

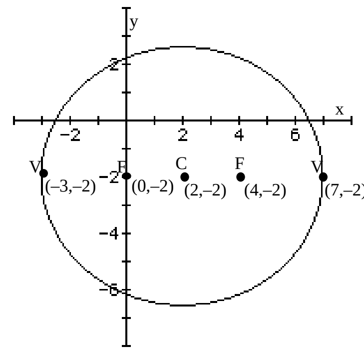


41. Center: (2, –2); Vertex: (7, –2); Focus: (4, –2); Major axis parallel to the x-axis; $a = 5$; $c = 2$.
Find b:

$$b^2 = a^2 - c^2 = 25 - 4 = 21$$

$$b = \sqrt{21}$$

Write the equation: $\dfrac{(x-2)^2}{25} + \dfrac{(y+2)^2}{21} = 1$

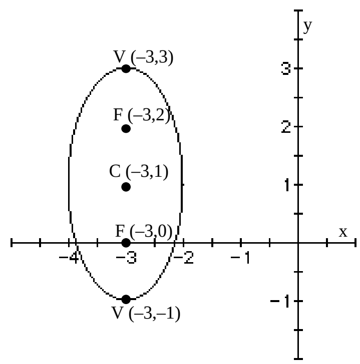


42. Center: (–3, 1); Vertex: (–3, 3); Focus: (–3, 0); Major axis parallel to the y-axis; $a = 2$; $c = 1$.
Find b:

$$b^2 = a^2 - c^2 = 4 - 1 = 3$$

$$b = \sqrt{3}$$

Write the equation: $\dfrac{(x+3)^2}{3} + \dfrac{(y-1)^2}{4} = 1$

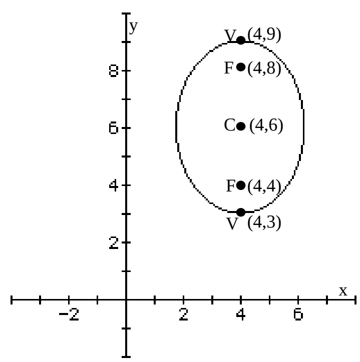


43. Vertices: (4, 3), (4, 9); Focus: (4, 8); Center: (4, 6); Major axis parallel to the y-axis; $a = 3$; $c = 2$. Find b:

$$b^2 = a^2 - c^2 = 9 - 4 = 5$$

$$b = \sqrt{5}$$

Write the equation: $\dfrac{(x-4)^2}{5} + \dfrac{(y-6)^2}{9} = 1$

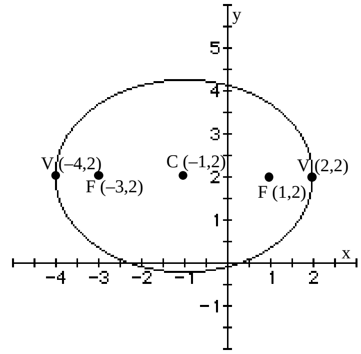


44. Foci: (1, 2), (–3, 2); Vertex: (–4, 2);
Center: (–1, 2); Major axis parallel to the x-axis;
$a = 3$; $c = 2$. Find b:

$$b^2 = a^2 - c^2 = 9 - 4 = 5$$

$$b = \sqrt{5}$$

Write the equation: $\dfrac{(x+1)^2}{9} + \dfrac{(y-2)^2}{5} = 1$

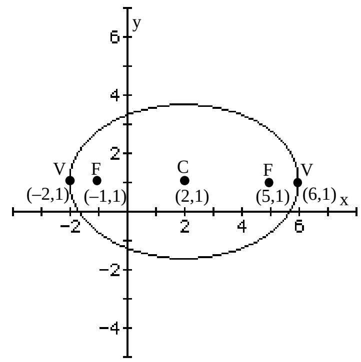


45. Foci: (5, 1), (–1, 1); length of the major axis = 8;
Center: (2, 1); Major axis parallel to the x-axis;
$a = 4$; $c = 3$. Find b:

$$b^2 = a^2 - c^2 = 16 - 9 = 7$$

$$b = \sqrt{7}$$

Write the equation: $\dfrac{(x-2)^2}{16} + \dfrac{(y-1)^2}{7} = 1$

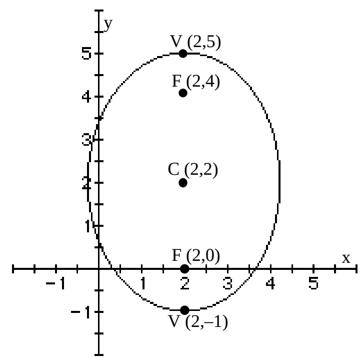


46. Vertices: (2, 5), (2, –1); $c = 2$;
Center: (2, 2); Major axis parallel to the y-axis;
$a = 3$; $c = 2$. Find b:

$$b^2 = a^2 - c^2 = 9 - 4 = 5$$

$$b = \sqrt{5}$$

Write the equation: $\dfrac{(x-2)^2}{5} + \dfrac{(y-2)^2}{9} = 1$



47. Center: (1, 2); Focus: (4, 2); contains the point (1, 3); Major axis parallel to the x-axis; $c = 3$.

The equation has the form: $\dfrac{(x-1)^2}{a^2} + \dfrac{(y-2)^2}{b^2} = 1$

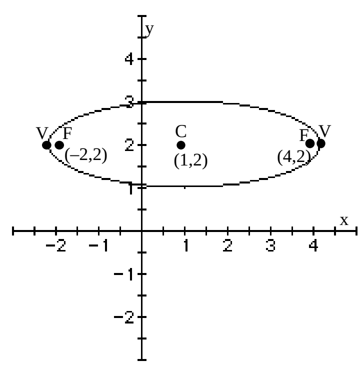
Since the point (1, 3) is on the curve:

$$\frac{0}{a^2} + \frac{1}{b^2} = 1 \rightarrow \frac{1}{b^2} = 1 \rightarrow b^2 = 1 \rightarrow b = 1$$

Find a:

$$a^2 = b^2 + c^2 = 1 + 9 = 10 \rightarrow a = \sqrt{10}$$

Write the equation: $\dfrac{(x-1)^2}{10} + \dfrac{(y-2)^2}{1} = 1$



48. Center: (1, 2); Focus: (1, 4); contains the point (2, 2); Major axis parallel to the y-axis; $c = 2$.

The equation has the form: $\dfrac{(x-1)^2}{b^2} + \dfrac{(y-2)^2}{a^2} = 1$

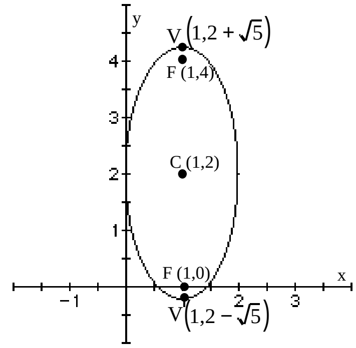
Since the point (2, 2) is on the curve:

$$\frac{1}{b^2} + \frac{0}{a^2} = 1 \rightarrow \frac{1}{b^2} = 1 \rightarrow b^2 = 1 \rightarrow b = 1$$

Find a:

$$a^2 = b^2 + c^2 = 1 + 4 = 5 \rightarrow a = \sqrt{5}$$

Write the equation: $\dfrac{(x-1)^2}{1} + \dfrac{(y-2)^2}{5} = 1$



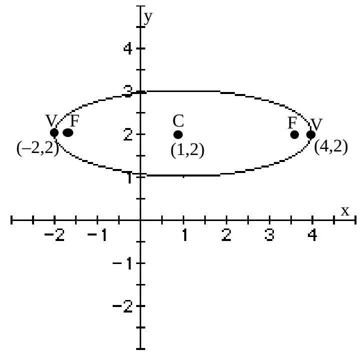
49. Center: (1, 2); Vertex: (4, 2); contains the point (1, 3); Major axis parallel to the x-axis; $a = 3$.

The equation has the form: $\dfrac{(x-1)^2}{a^2} + \dfrac{(y-2)^2}{b^2} = 1$

Since the point (1, 3) is on the curve:

$$\frac{0}{9} + \frac{1}{b^2} = 1 \rightarrow \frac{1}{b^2} = 1 \rightarrow b^2 = 1 \rightarrow b = 1$$

Write the equation: $\dfrac{(x-1)^2}{9} + \dfrac{(y-2)^2}{1} = 1$



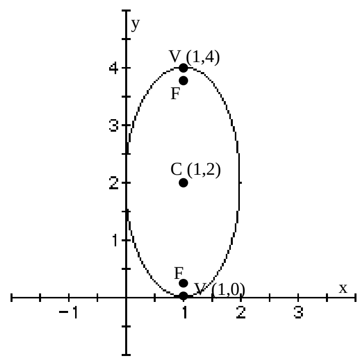
50. Center: (1, 2); Vertex: (1, 4); contains the point (2, 2); Major axis parallel to the y-axis; $a = 2$.

The equation has the form: $\dfrac{(x-1)^2}{b^2} + \dfrac{(y-2)^2}{a^2} = 1$

Since the point (2, 2) is on the curve:

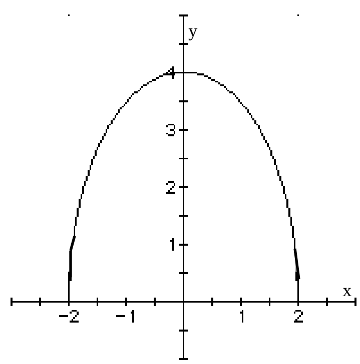
$$\frac{1}{b^2} + \frac{0}{a^2} = 1 \rightarrow \frac{1}{b^2} = 1 \rightarrow b^2 = 1 \rightarrow b = 1$$

Write the equation: $\dfrac{(x-1)^2}{1} + \dfrac{(y-2)^2}{4} = 1$



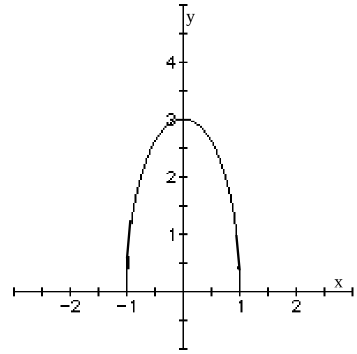
51. Rewrite the equation:

$$y = \sqrt{16 - 4x^2}$$
$$y^2 = 16 - 4x^2, \quad y \ge 0$$
$$4x^2 + y^2 = 16, \quad y \ge 0$$
$$\frac{x^2}{4} + \frac{y^2}{16} = 1, \quad y \ge 0$$



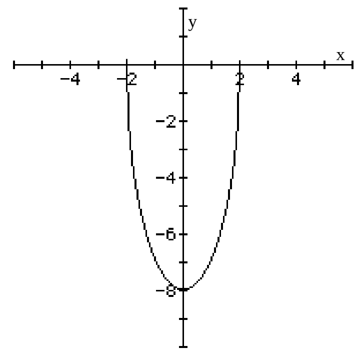
52. Rewrite the equation:

$$\begin{aligned} y &= \sqrt{9 - 9x^2} \\ y^2 &= 9 - 9x^2, \quad && y \geq 0 \\ 9x^2 + y^2 &= 9, && y \geq 0 \\ \frac{x^2}{1} + \frac{y^2}{9} &= 1, && y \geq 0 \end{aligned}$$



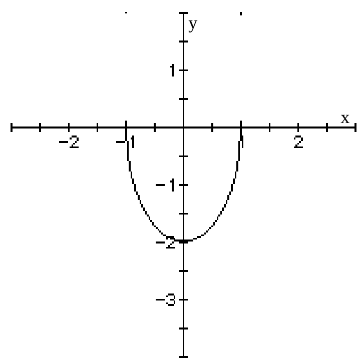
53. Rewrite the equation:

$$\begin{aligned} y &= -\sqrt{64 - 16x^2} \\ y^2 &= 64 - 16x^2, \quad && y \leq 0 \\ 16x^2 + y^2 &= 64, && y \leq 0 \\ \frac{x^2}{4} + \frac{y^2}{64} &= 1, && y \leq 0 \end{aligned}$$



54. Rewrite the equation:

$$\begin{aligned} y &= -\sqrt{4 - 4x^2} \\ y^2 &= 4 - 4x^2, \quad && y \leq 0 \\ 4x^2 + y^2 &= 4, && y \leq 0 \\ \frac{x^2}{1} + \frac{y^2}{4} &= 1, && y \leq 0 \end{aligned}$$



55. The center of the ellipse is (0, 0). The length of the major axis is 20, so $a = 10$. The length of half the minor axis is 6, so $b = 6$. The ellipse is situated with its major axis on the x-axis. The equation is: $\frac{x^2}{100} + \frac{y^2}{36} = 1$.

56. The center of the ellipse is (0, 0). The length of the major axis is 30, so $a = 15$. The length of half the minor axis is 10, so $b = 10$. The ellipse is situated with its major axis on the x-axis. The equation is: $\frac{x^2}{225} + \frac{y^2}{100} = 1$.
The roadway is 12 feet above the axis of the ellipse.
At the center ($x = 0$), the roadway is 2 feet above the arch.
At a point 5 feet either side of the center, evaluate the equation at $x = 5$:

$$\frac{5^2}{225} + \frac{y^2}{100} = 1 \;\rightarrow\; \frac{y^2}{100} = 1 - \frac{25}{225} = \frac{200}{225} \;\rightarrow\; y = 10\sqrt{\frac{200}{225}} \approx 9.43$$

The vertical distance from the roadway to the arch is $12 - 9.43 \approx 2.57$ feet.
At a point 10 feet either side of the center, evaluate the equation at $x = 10$:

$$\frac{10^2}{225}+\frac{y^2}{100}=1 \;\rightarrow\; \frac{y^2}{100}=1-\frac{100}{225}=\frac{125}{225} \;\rightarrow\; y=10\sqrt{\frac{125}{225}}\approx 7.45$$

The vertical distance from the roadway to the arch is $12-7.45\approx 4.55$ feet.
At a point 15 feet either side of the center, the roadway is 12 feet above the arch.

57. Assume that the half ellipse formed by the gallery is centered at (0, 0). Since the hall is 100 feet long, $2a=100$ or $a=50$. The distance from the center to the foci is 25 feet, so $c=25$. Find the height of the gallery which is b:

$$b^2=a^2-c^2=2500-625=1875 \;\rightarrow\; b=\sqrt{1875}\approx 43.3$$

The ceiling will be 43.3 feet high in the center.

58. Assume that the half ellipse formed by the gallery is centered at (0, 0). Since the distance between the foci is 100 feet and Jim is 6 feet from the nearest wall, the length of the gallery is 112 feet. $2a=112$ or $a=56$. The distance from the center to the foci is 50 feet, so $c=50$. Find the height of the gallery which is b:

$$b^2=a^2-c^2=3136-2500=636 \;\rightarrow\; b=\sqrt{636}\approx 25.2$$

The ceiling will be 25.2 feet high in the center.

59. Place the semielliptical arch so that the x-axis coincides with the water and the y-axis passes through the center of the arch. Since the bridge has a span of 120 feet, the length of the major axis is 120, or $2a=120$ or $a=60$. The maximum height of the bridge is 25 feet, so $b=25$. The equation is: $\frac{x^2}{3600}+\frac{y^2}{625}=1$.

The height 10 feet from the center:

$$\frac{10^2}{3600}+\frac{y^2}{625}=1 \rightarrow \frac{y^2}{625}=1-\frac{100}{3600} \rightarrow y^2=625\left(\frac{3500}{3600}\right) \rightarrow y\approx 24.65 \text{ feet}$$

The height 30 feet from the center:

$$\frac{30^2}{3600}+\frac{y^2}{625}=1 \rightarrow \frac{y^2}{625}=1-\frac{900}{3600} \rightarrow y^2=625\left(\frac{2700}{3600}\right) \rightarrow y\approx 21.65 \text{ feet}$$

The height 50 feet from the center:

$$\frac{50^2}{3600}+\frac{y^2}{625}=1 \rightarrow \frac{y^2}{625}=1-\frac{2500}{3600} \rightarrow y^2=625\left(\frac{1100}{3600}\right) \rightarrow y\approx 13.82 \text{ feet}$$

60. Place the semielliptical arch so that the x-axis coincides with the water and the y-axis passes through the center of the arch. Since the bridge has a span of 100 feet, the length of the major axis is 100, or $2a=100$ or $a=50$. Let h be the maximum height of the bridge. The equation is: $\frac{x^2}{2500}+\frac{y^2}{h^2}=1$.

The height of the arch 40 feet from the center is 10 feet. So (40, 10) is a point on the ellipse. Substitute and solve for h:

$$\frac{40^2}{2500}+\frac{10^2}{h^2}=1 \rightarrow \frac{10^2}{h^2}=1-\frac{1600}{2500}=\frac{9}{25} \rightarrow 9h^2=2500 \rightarrow h=\frac{50}{3}\approx 16.67$$

The height of the arch at its center is 16.67 feet.

61. Place the semielliptical arch so that the x-axis coincides with the major axis and the y-axis passes through the center of the arch. Since the ellipse is 40 feet wide, the length of the major axis is 40, or $2a = 40$ or $a = 20$. The height is 15 feet at the center, so $b = 15$. The equation is: $\frac{x^2}{400} + \frac{y^2}{225} = 1$.

The height 10 feet either side of the center:

$$\frac{10^2}{400} + \frac{y^2}{225} = 1 \rightarrow \frac{y^2}{225} = 1 - \frac{100}{400} \rightarrow y^2 = 225\ \ \frac{3}{4} \rightarrow y \approx 12.99 \text{ feet}$$

The height 20 feet either side of the center:

$$\frac{20^2}{400} + \frac{y^2}{225} = 1 \rightarrow \frac{y^2}{225} = 1 - \frac{400}{400} \rightarrow y^2 = 225\ \ 0 \rightarrow y \approx 0 \text{ feet}$$

62. Place the semielliptical arch so that the x-axis coincides with the major axis and the y-axis passes through the center of the arch. Since the height of the arch at the center is 20 feet, $b = 20$. The length of the major axis is to be found, so it is necessary to solve for a. The equation is: $\frac{x^2}{a^2} + \frac{y^2}{400} = 1$.

The height of the arch 28 feet from the center is to be 13 feet, so the point (28, 13) is on the ellipse. Substitute and solve for a:

$$\frac{28^2}{a^2} + \frac{13^2}{400} = 1 \rightarrow \frac{784}{a^2} = 1 - \frac{169}{400} = \frac{231}{400} \rightarrow 231a^2 = 313600 \rightarrow a^2 = 1357.6 \rightarrow a = 36.8$$

The span of the bridge is 73.6 feet.

63. Since the mean distance is 93 million miles, $a = 93$ million. The length of the major axis is 186 million. The perihelion is 186 million – 94.5 million = 91.5 million miles. The distance from the center of the ellipse to the sun (focus) is 93 million – 91.5 million = 1.5 million miles; therefore, $c = 1.5$ million. Find b:

$$b^2 = a^2 - c^2 = \left(93 \times 10^6\right)^2 - \left(1.5 \times 10^6\right)^2 = 8.64675 \times 10^{15} \rightarrow b = 92.99 \times 10^6$$

The equation of the orbit is: $\frac{x^2}{\left(93 \times 10^6\right)^2} + \frac{y^2}{\left(92.99 \times 10^6\right)^2} = 1$.

64. Since the mean distance is 142 million miles, $a = 142$ million. The length of the major axis is 284 million. The aphelion is 284 million – 128.5 million = 155.5 million miles. The distance from the center of the ellipse to the sun (focus) is 142 million – 128.5 million = 13.5 million miles; therefore, $c = 13.5$ million. Find b:

$$b^2 = a^2 - c^2 = \left(142 \times 10^6\right)^2 - \left(13.5 \times 10^6\right)^2 = 1.9982 \times 10^{16} \rightarrow b = 141.4 \times 10^6$$

The equation of the orbit is: $\frac{x^2}{\left(142 \times 10^6\right)^2} + \frac{y^2}{\left(141.4 \times 10^6\right)^2} = 1$.

65. The mean distance is 507 million – 23.2 million = 483.8 million miles.
The perihelion is 483.8 million – 23.2 million = 460.6 million miles.
Since $a = 483.8 \times 10^6$ and $c = 23.2 \times 10^6$, we can find b:

$$b^2 = a^2 - c^2 = \left(483.8 \times 10^6\right)^2 - \left(23.2 \times 10^6\right)^2 = 2.335242 \times 10^{17} \rightarrow b = 483.2 \times 10^6$$

The equation of the orbit of Jupiter is: $\frac{x^2}{\left(483.8 \times 10^6\right)^2} + \frac{y^2}{\left(483.2 \times 10^6\right)^2} = 1$.

66. The mean distance is 4551 million + 897.5 million = 5448.5 million miles.
The aphelion is 5448.5 million + 897.5 million = 6346 million miles.
Since $a = 5448.5 \times 10^6$ and $c = 897.5 \times 10^6$, we can find b:

$$b^2 = a^2 - c^2 = \left(5448.5 \times 10^6\right)^2 - \left(897.5 \times 10^6\right)^2 = 2.88806 \times 10^{19}$$
$$b = 5374.1 \times 10^6$$

The equation of the orbit of Pluto is: $\dfrac{x^2}{\left(5448.5 \times 10^6\right)^2} + \dfrac{y^2}{\left(5374.1 \times 10^6\right)^2} = 1$.

67. If the x-axis is placed along the 100 foot length and the y-axis is placed along the 50 foot length, the equation for the ellipse is: $\dfrac{x^2}{50^2} + \dfrac{y^2}{25^2} = 1$.
Find y when x = 40:

$$\frac{40^2}{50^2} + \frac{y^2}{25^2} = 1 \to \frac{y^2}{625} = 1 - \frac{1600}{2500} \to y^2 = 625\left(\frac{9}{25}\right) \to y \approx 15 \text{ feet}$$

The width 10 feet from the side is 30 feet.

68. If the x-axis is placed along the 80 foot length and the y-axis is placed along the 40 foot width, the equation for the ellipse is: $\dfrac{x^2}{40^2} + \dfrac{y^2}{20^2} = 1$.
Find y when x = 30:

$$\frac{30^2}{40^2} + \frac{y^2}{20^2} = 1 \to \frac{y^2}{400} = 1 - \frac{900}{1600} \to y^2 = 400\left(\frac{7}{16}\right) = 175 \to y \approx 13.2 \text{ feet}$$

The width 10 feet from the side is 26.4 feet.

69. (a) Put the equation in standard ellipse form:

$$Ax^2 + Cy^2 + F = 0 \qquad A \neq 0,\ C \neq 0,\ F \neq 0$$
$$Ax^2 + Cy^2 = -F$$
$$\frac{Ax^2}{-F} + \frac{Cy^2}{-F} = 1$$
$$\frac{x^2}{(-F/A)} + \frac{y^2}{(-F/C)} = 1 \qquad \text{where } -F/A \text{ and } -F/C \text{ are positive}$$

This is the equation of an ellipse with center at (0, 0).

(b) If $A = C$, the equation becomes:

$$Ax^2 + Ay^2 = -F \quad \to \quad x^2 + y^2 = \frac{-F}{A}$$

This is the equation of a circle with center at (0, 0) and radius of $\sqrt{\dfrac{-F}{A}}$.

70. Complete the square on the given equation:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0, \qquad A \neq 0, \; C \neq 0$$

$$A\left(x^2 + \frac{D}{A}x\right) + C\left(y^2 + \frac{E}{C}y\right) = -F$$

$$A\left(x^2 + \frac{D}{A}x + \frac{D^2}{4A^2}\right) + C\left(y^2 + \frac{E}{C}y + \frac{E^2}{4C^2}\right) = \frac{D^2}{4A^2} + \frac{E^2}{4C^2} - F$$

$$A\left(x + \frac{D}{2A}\right)^2 + C\left(y + \frac{E}{2C}\right)^2 = \frac{D^2}{4A^2} + \frac{E^2}{4C^2} - F$$

(a) If $\frac{D^2}{4A^2} + \frac{E^2}{4C^2} - F$ is of the same sign as A (and C), this is the equation of an ellipse whose center is $\left(\frac{-D}{2A}, \frac{-E}{2C}\right)$.

(b) If $\frac{D^2}{4A^2} + \frac{E^2}{4C^2} - F = 0$, the graph is a single point $\left(\frac{-D}{2A}, \frac{-E}{2C}\right)$.

(c) If $\frac{D^2}{4A^2} + \frac{E^2}{4C^2} - F$ is of the opposite sign as A (and C), this graph contains no points since the left side has the opposite sign of the right side.

71. Answers will vary.