# Chapter 11

# Analytic Geometry

## 11.4 The Hyperbola

1. B  2. C  3. A  4. D

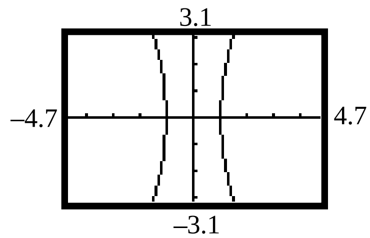
5. Center: (0, 0); Focus: (3, 0); Vertex: (1, 0);
Transverse axis is the x-axis; $a = 1$; $c = 3$. Find b:

$$b^2 = c^2 - a^2 = 9 - 1 = 8$$

$$b = \sqrt{8} = 2\sqrt{2}$$

Write the equation: $\dfrac{x^2}{1} - \dfrac{y^2}{8} = 1$

To graph, enter: $y_1 = \sqrt{8(x^2 - 1)}$; $y_2 = -\sqrt{8(x^2 - 1)}$



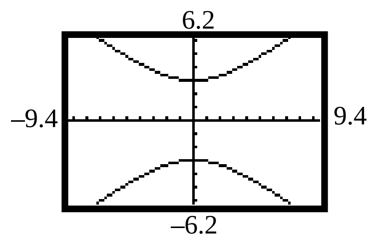

6. Center: (0, 0); Focus: (0, 5); Vertex: (0, 3);
Transverse axis is the y-axis; $a = 3$; $c = 5$. Find b:

$$b^2 = c^2 - a^2 = 25 - 9 = 16$$

$$b = 4$$

Write the equation: $\dfrac{y^2}{9} - \dfrac{x^2}{16} = 1$

To graph, enter:
$y_1 = 3\sqrt{1 + x^2 / 16}$; $y_2 = -3\sqrt{1 + x^2 / 16}$



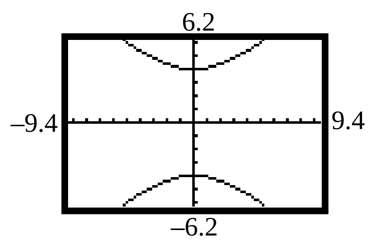

7. Center: (0, 0); Focus: (0, –6); Vertex: (0, 4)
Transverse axis is the y-axis; $a = 4$; $c = 6$. Find b:

$$b^2 = c^2 - a^2 = 36 - 16 = 20$$

$$b = \sqrt{20} = 2\sqrt{5}$$

Write the equation: $\dfrac{y^2}{16} - \dfrac{x^2}{20} = 1$

To graph, enter:
$y_1 = 4\sqrt{1 + x^2 / 20}$; $y_2 = -4\sqrt{1 + x^2 / 20}$



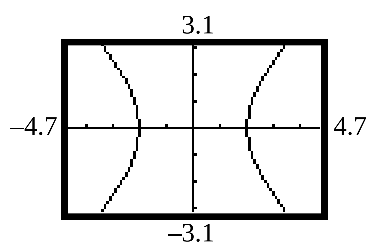

8. Center: (0, 0); Focus: (–3, 0); Vertex: (2, 0)
Transverse axis is the x-axis; $a = 2$; $c = 3$. Find b:

$$b^2 = c^2 - a^2 = 9 - 4 = 5$$

$$b = \sqrt{5}$$

Write the equation: $\dfrac{x^2}{4} - \dfrac{y^2}{5} = 1$

To graph, enter:
$y_1 = \sqrt{5(x^2 / 4 - 1)}$; $y_2 = -\sqrt{5(x^2 / 4 - 1)}$



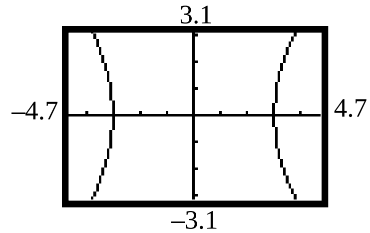
9. Foci: (–5, 0), (5, 0); Vertex: (3, 0) Center: (0, 0);
Transverse axis is the x-axis; $a = 3$; $c = 5$. Find b:

$$b^2 = c^2 - a^2 = 25 - 9 = 16 \quad \rightarrow \quad b = 4$$

Write the equation: $\frac{x^2}{9} - \frac{y^2}{16} = 1$

To graph, enter:

$$y_1 = 4\sqrt{x^2/9 - 1};\ y_2 = -4\sqrt{x^2/9 - 1}$$



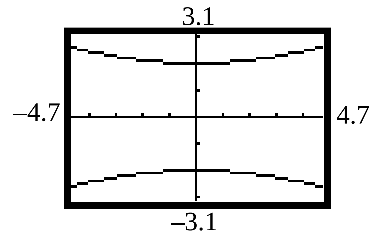

10. Focus: (0, 6); Vertices: (0, –2), (0, 2)
Center: (0, 0); Transverse axis is the y-axis;
$a = 2$; $c = 6$. Find b:

$$b^2 = c^2 - a^2 = 36 - 4 = 32 \quad \rightarrow \quad b = 4\sqrt{2}$$

Write the equation: $\frac{y^2}{4} - \frac{x^2}{32} = 1$

To graph, enter:

$$y_1 = 2\sqrt{x^2/32 + 1};\ y_2 = -2\sqrt{x^2/32 + 1}$$



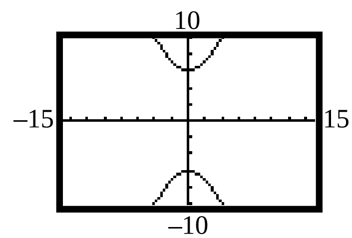

11. Vertices: (0, –6), (0, 6); Asymptote: $y = 2x$;
Center: (0, 0); Transverse axis is the y-axis;
$a = 6$. Find b using the slope of the asymptote:

$$\frac{a}{b} = \frac{6}{b} = 2 \quad \rightarrow \quad 2b = 6 \quad \rightarrow \quad b = 3$$

Write the equation: $\frac{y^2}{36} - \frac{x^2}{9} = 1$

To graph, enter:

$$y_1 = 6\sqrt{1 + x^2/9};\ y_2 = -6\sqrt{1 + x^2/9}$$



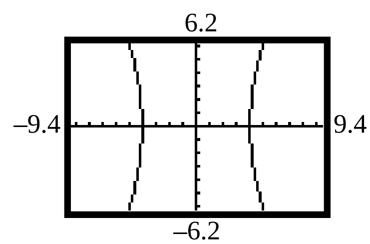

12. Vertices: (–4, 0), (4, 0); Asymptote: $y = 2x$;
Center: (0, 0); Transverse axis is the x-axis;
$a = 4$. Find b using the slope of the asymptote:

$$\frac{b}{a} = \frac{b}{4} = 2 \quad \rightarrow \quad b = 8$$

Write the equation: $\frac{x^2}{16} - \frac{y^2}{64} = 1$

To graph, enter:

$$y_1 = 8\sqrt{x^2/16 - 1};\ y_2 = -8\sqrt{x^2/16 - 1}$$



13. Foci: (–4, 0), (4, 0); Asymptote: $y = -x$;
Center: (0, 0); Transverse axis is the x-axis; $c = 4$.
Using the slope of the asymptote:

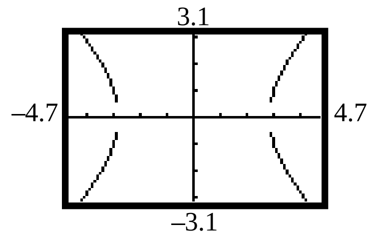
$$-\frac{b}{a} = -1 \rightarrow -b = -a \rightarrow b = a$$

Find b:

$$\begin{aligned} b^2 &= c^2 - a^2 \\ a^2 + b^2 &= c^2 \qquad (c = 4) \\ b^2 + b^2 &= 16 \\ 2b^2 &= 16 \\ b^2 &= 8 \rightarrow b = \sqrt{8} = 2\sqrt{2} \\ a &= \sqrt{8} = 2\sqrt{2} \qquad (a = b) \end{aligned}$$

Write the equation: $\frac{x^2}{8} - \frac{y^2}{8} = 1$

To graph, enter: $y_1 = \sqrt{x^2 - 8}$; $y_2 = -\sqrt{x^2 - 8}$




14. Foci: (0, –2), (0, 2); Asymptote: $y = -x$;
Center: (0, 0); Transverse axis is the y-axis; $c = 2$.
Using the slope of the asymptote:

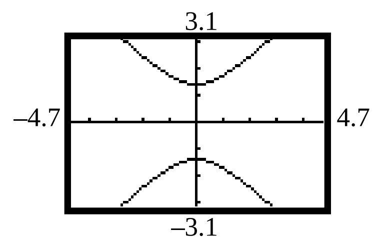
$$-\frac{a}{b} = -1 \rightarrow -b = -a \rightarrow b = a$$

Find b:

$$\begin{aligned} b^2 &= c^2 - a^2 \\ a^2 + b^2 &= c^2 \qquad (c = 2) \\ b^2 + b^2 &= 4 \\ 2b^2 &= 4 \\ b^2 &= 2 \rightarrow b = \sqrt{2} \\ a &= \sqrt{2} \qquad (a = b) \end{aligned}$$

Write the equation: $\frac{y^2}{2} - \frac{x^2}{2} = 1$

To graph, enter: $y_1 = \sqrt{x^2 + 2}$; $y_2 = -\sqrt{x^2 + 2}$



15. $\dfrac{x^2}{25}-\dfrac{y^2}{9}=1$

The center of the hyperbola is at (0, 0).

$a=5, \quad b=3$. The vertices are (5, 0) and (–5, 0).

Find the value of c:

$$c^2=a^2+b^2=25+9=34$$

$$c=\sqrt{34}$$

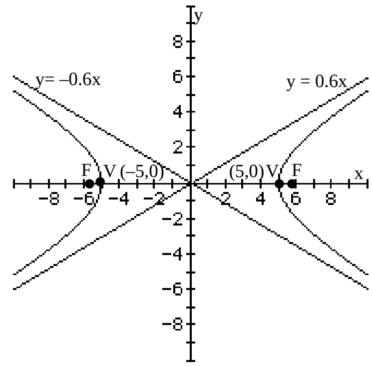
The foci are $\left(\sqrt{34},\,0\right)$ and $\left(-\sqrt{34},\,0\right)$.

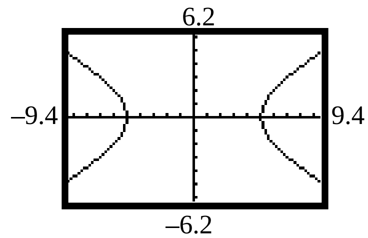
The transverse axis is $y=0$.

The asymptotes are $y=\dfrac{3}{5}x$ and $y=-\dfrac{3}{5}x$.

To graph, enter:

$y_1=3\sqrt{(x^2/25-1)}$; $y_2=-3\sqrt{(x^2/25-1)}$




16. $\dfrac{y^2}{16}-\dfrac{x^2}{4}=1$

The center of the hyperbola is at (0, 0).

$a=4, \quad b=2$. The vertices are (0, 4) and (0, –4).

Find the value of c:

$$c^2=a^2+b^2=16+4=20$$

$$c=\sqrt{20}=2\sqrt{5}$$

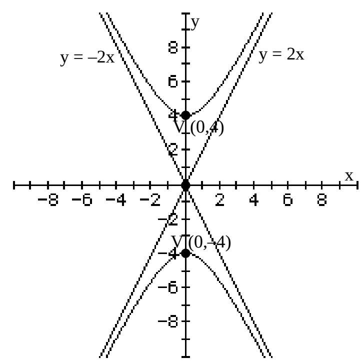
The foci are $\left(0,\,2\sqrt{5}\right)$ and $\left(0,\,-2\sqrt{5}\right)$.

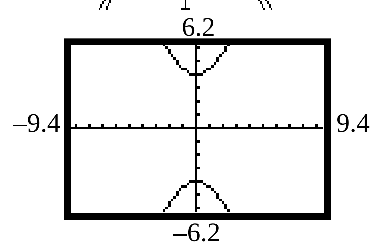
The transverse axis is $x=0$.

The asymptotes are $y=2x$ and $y=-2x$.

To graph, enter:

$y_1=4\sqrt{(x^2/4+1)}$; $y_2=-4\sqrt{(x^2/4+1)}$



17. $4x^2 - y^2 = 16$

Divide both sides by 16 to put in standard form:

$$\frac{4x^2}{16} - \frac{y^2}{16} = \frac{16}{16}$$

$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$

The center of the hyperbola is at (0, 0).

$a = 2$, $b = 4$. The vertices are (2, 0) and (–2, 0).

Find the value of c:

$$c^2 = a^2 + b^2 = 4 + 16 = 20$$

$$c = \sqrt{20} = 2\sqrt{5}$$

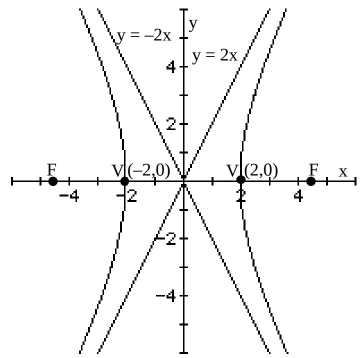
The foci are $\left(2\sqrt{5},\ 0\right)$ and $\left(-2\sqrt{5},\ 0\right)$.

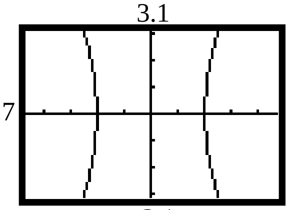
The transverse axis is $y = 0$.

The asymptotes are $y = 2x$ and $y = -2x$.

To graph, enter:

$y_1 = 4\sqrt{(x^2/4-1)}$; $y_2 = -4\sqrt{(x^2/4-1)}$




18. $y^2 - 4x^2 = 16$

Divide both sides by 16 to put in standard form:

$$\frac{y^2}{16} - \frac{4x^2}{16} = \frac{16}{16}$$

$$\frac{y^2}{16} - \frac{x^2}{4} = 1$$

The center of the hyperbola is at (0, 0).

$a = 4$, $b = 2$. The vertices are (0, 4) and (0, –4).

Find the value of c:

$$c^2 = a^2 + b^2 = 16 + 4 = 20$$

$$c = \sqrt{20} = 2\sqrt{5}$$

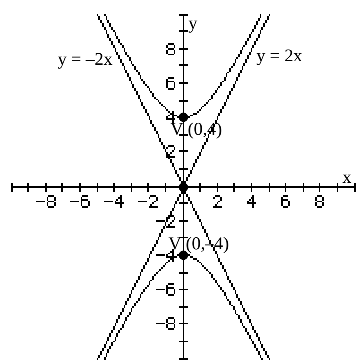
The foci are $\left(0,\ 2\sqrt{5}\right)$ and $\left(0,\ -2\sqrt{5}\right)$.

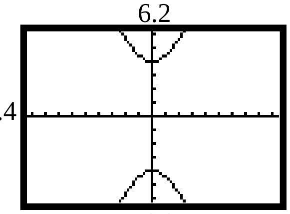
The transverse axis is $x = 0$.

The asymptotes are $y = 2x$ and $y = -2x$.

To graph, enter:

$y_1 = 4\sqrt{(x^2/4+1)}$; $y_2 = -4\sqrt{(x^2/4+1)}$



19. $y^2 - 9x^2 = 9$

Divide both sides by 9 to put in standard form:

$$\frac{y^2}{9} - \frac{9x^2}{9} = \frac{9}{9}$$

$$\frac{y^2}{9} - \frac{x^2}{1} = 1$$

The center of the hyperbola is at (0, 0).

$a = 3$, $b = 1$. The vertices are (0, 3) and (0, –3).

Find the value of c:

$$c^2 = a^2 + b^2 = 9 + 1 = 10$$

$$c = \sqrt{10}$$

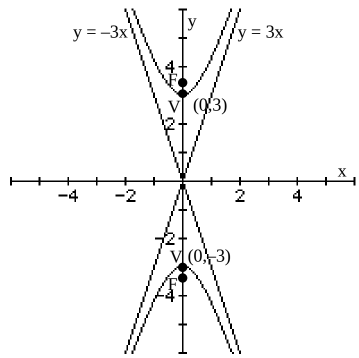
The foci are $\left(0, \sqrt{10}\right)$ and $\left(0, -\sqrt{10}\right)$.

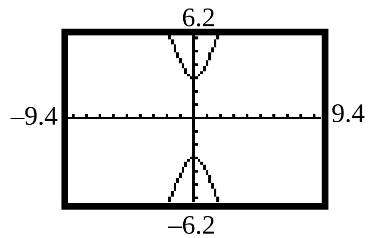
The transverse axis is $x = 0$.

The asymptotes are $y = 3x$ and $y = -3x$.

To graph, enter:

$y_1 = \sqrt{(9x^2 + 9)}$; $y_2 = -\sqrt{(9x^2 + 9)}$




20. $x^2 - y^2 = 4$

Divide both sides by 4 to put in standard form:

$$\frac{x^2}{4} - \frac{y^2}{4} = \frac{4}{4}$$

$$\frac{x^2}{4} - \frac{y^2}{4} = 1$$

The center of the hyperbola is at (0, 0).

$a = 2$, $b = 2$. The vertices are (2, 0) and (–2, 0).

Find the value of c:

$$c^2 = a^2 + b^2 = 4 + 4 = 8$$

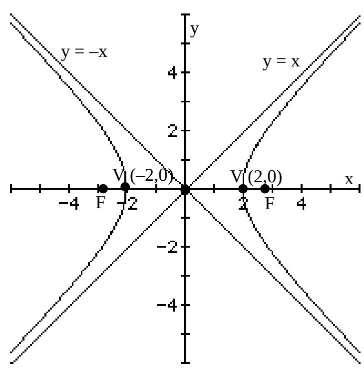
$$c = \sqrt{8} = 2\sqrt{2}$$

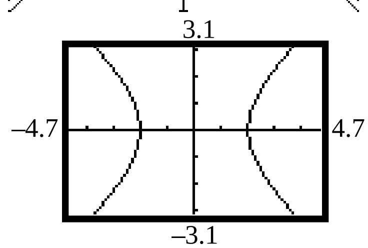
The foci are $\left(2\sqrt{2}, 0\right)$ and $\left(-2\sqrt{2}, 0\right)$.

The transverse axis is $y = 0$.

The asymptotes are $y = x$ and $y = -x$.

To graph, enter: $y_1 = \sqrt{(x^2 - 4)}$; $y_2 = -\sqrt{(x^2 - 4)}$



21. $y^2 - x^2 = 25$

Divide both sides by 25 to put in standard form:

$$\frac{y^2}{25} - \frac{x^2}{25} = 1$$

The center of the hyperbola is at (0, 0).

$a = 5$; $b = 5$. The vertices are (0, 5) and (0, –5).

Find the value of c:

$$c^2 = a^2 + b^2 = 25 + 25 = 50$$

$$c = \sqrt{50} = 5\sqrt{2}$$

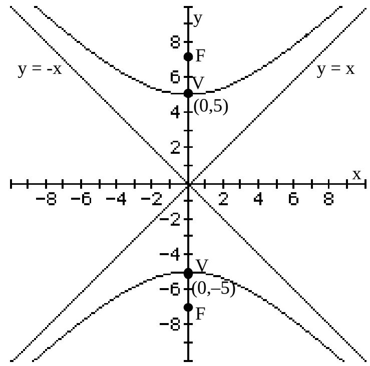
The foci are $\left(0, 5\sqrt{2}\right)$ and $\left(0, -5\sqrt{2}\right)$.

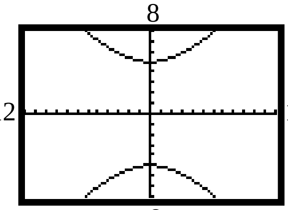
The transverse axis is $x = 0$.

The asymptotes are $y = x$ and $y = -x$.

To graph, enter:

$y_1 = \sqrt{(x^2 + 25)}$; $y_2 = -\sqrt{(x^2 + 25)}$




22. $2x^2 - y^2 = 4$

Divide both sides by 4 to put in standard form:

$$\frac{x^2}{2} - \frac{y^2}{4} = 1$$

The center of the hyperbola is at (0, 0).

$a = \sqrt{2}$, $b = 2$. The vertices are $\left(\sqrt{2}, 0\right)$ and $\left(-\sqrt{2}, 0\right)$. Find the value of c:

$$c^2 = a^2 + b^2 = 2 + 4 = 6$$

$$c = \sqrt{6}$$

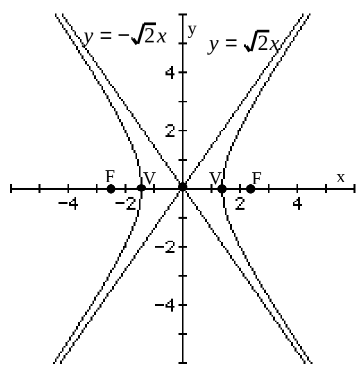
The foci are $\left(\sqrt{6}, 0\right)$ and $\left(-\sqrt{6}, 0\right)$.

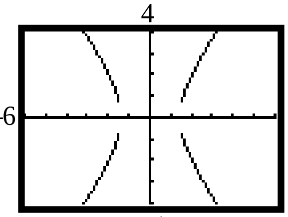
The transverse axis is $y = 0$.

The asymptotes are $y = \sqrt{2}x$ and $y = -\sqrt{2}\,x$.

To graph, enter:

$y_1 = \sqrt{(2x^2 - 4)}$; $y_2 = -\sqrt{(2x^2 - 4)}$



23. $x^2 - y^2 = 1$

24. $y^2 - x^2 = 1$

25. $\dfrac{y^2}{36} - \dfrac{x^2}{9} = 1$

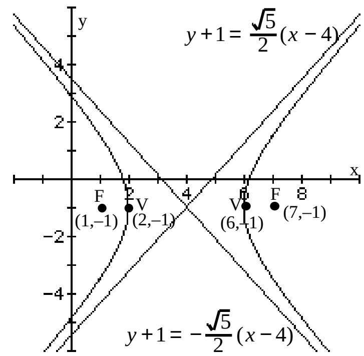
26. $\dfrac{x^2}{4} - \dfrac{y^2}{16} = 1$

27. Center: (4, –1); Focus: (7, –1); Vertex: (6, –1);
Transverse axis is parallel to the x-axis;
$a = 2$; $c = 3$. Find b:

$$b^2 = c^2 - a^2 = 9 - 4 = 5$$

$$b = \sqrt{5}$$

Write the equation: $\dfrac{(x-4)^2}{4} - \dfrac{(y+1)^2}{5} = 1$

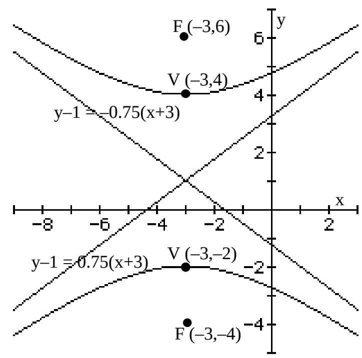



28. Center: (–3, 1); Focus: (–3, 6); Vertex: (–3, 4);
Transverse axis is parallel to the y-axis;
$a = 3$; $c = 5$. Find b:

$$b^2 = c^2 - a^2 = 25 - 9 = 16$$

$$b = 4$$

Write the equation: $\dfrac{(y-1)^2}{9} - \dfrac{(x+3)^2}{16} = 1$

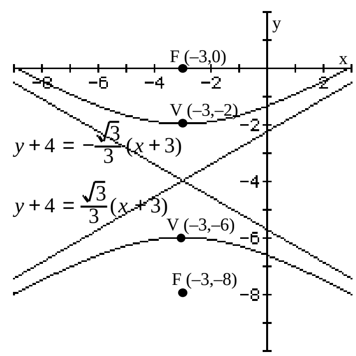



29. Center: (–3, –4); Focus: (–3, –8);
Vertex: (–3, –2); Transverse axis is parallel to the y-axis; $a = 2$; $c = 4$. Find b:

$$b^2 = c^2 - a^2 = 16 - 4 = 12$$

$$b = \sqrt{12} = 2\sqrt{3}$$

Write the equation: $\dfrac{(y+4)^2}{4} - \dfrac{(x+3)^2}{12} = 1$

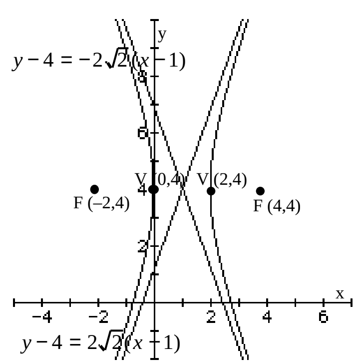



30. Center: (1, 4); Focus: (–2, 4); Vertex: (0, 4);
Transverse axis is parallel to the x-axis;
$a = 1$; $c = 3$. Find b:

$$b^2 = c^2 - a^2 = 9 - 1 = 8$$

$$b = \sqrt{8} = 2\sqrt{2}$$

Write the equation: $\dfrac{(x-1)^2}{1} - \dfrac{(y-4)^2}{8} = 1$

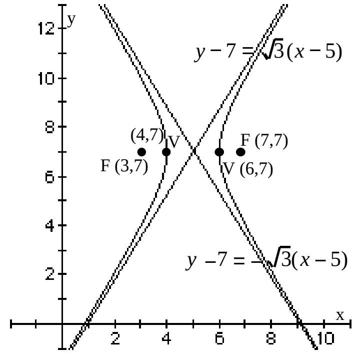


31. Foci: (3, 7), (7, 7); Vertex: (6, 7); Center: (5, 7); Transverse axis is parallel to the x-axis; $a = 1$; $c = 2$. Find b:

$$b^2 = c^2 - a^2 = 4 - 1 = 3$$

$$b = \sqrt{3}$$

Write the equation: $\dfrac{(x-5)^2}{1} - \dfrac{(y-7)^2}{3} = 1$

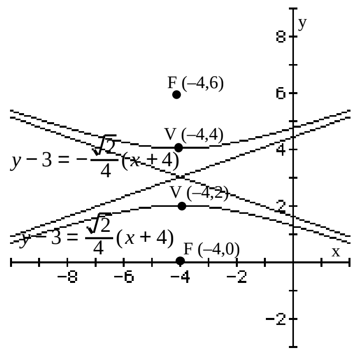



32. Focus: (–4, 0); Vertices: (–4, 4), (–4, 2); Center: (–4, 3); Transverse axis is parallel to the y-axis; $a = 1$; $c = 3$. Find b:

$$b^2 = c^2 - a^2 = 9 - 1 = 8$$

$$b = \sqrt{8} = 2\sqrt{2}$$

Write the equation: $\dfrac{(y-3)^2}{1} - \dfrac{(x+4)^2}{8} = 1$

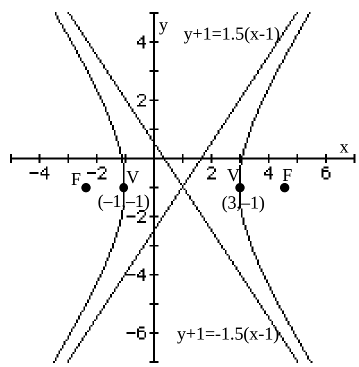



33. Vertices: (–1, –1), (3, –1); Center: (1, –1); Transverse axis is parallel to the x-axis; $a = 2$.

Asymptote: $\dfrac{x-1}{2} = \dfrac{y+1}{3}$ Using the slope of the asymptote:

$$\frac{b}{a} = \frac{b}{2} = \frac{3}{2} \rightarrow b = 3$$

Write the equation: $\dfrac{(x-1)^2}{4} - \dfrac{(y+1)^2}{9} = 1$

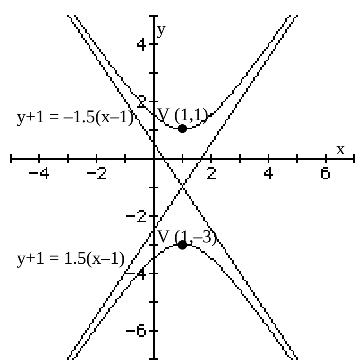



34. Vertices: (1, –3), (1, 1); Center: (1, –1); Transverse axis is parallel to the y-axis; $a = 2$.

Asymptote: $\dfrac{x-1}{2} = \dfrac{y+1}{3}$ Using the slope of the asymptote:

$$\frac{a}{b} = \frac{2}{b} = \frac{3}{2} \rightarrow 3b = 4 \rightarrow b = \frac{4}{3}$$

Write the equation: $\dfrac{(y+1)^2}{4} - \dfrac{(x-1)^2}{\frac{16}{9}} = 1$



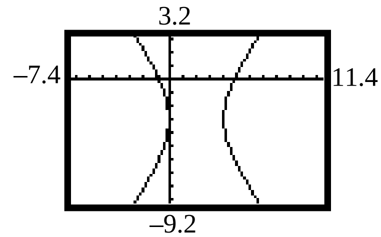
35. $\dfrac{(x-2)^2}{4}-\dfrac{(y+3)^2}{9}=1$

The center of the hyperbola is at (2, –3).
$a=2,\ \ b=3$. The vertices are (0, –3) and (4, –3).
Find the value of c:

$$c^2=a^2+b^2=4+9=13$$
$$c=\sqrt{13}$$

Foci: $\left(2-\sqrt{13},-3\right)$ and $\left(2+\sqrt{13},-3\right)$.
Transverse axis: $y=-3$.
Asymptotes: $y+3=\dfrac{3}{2}(x-2),\ \ y+3=-\dfrac{3}{2}(x-2)$.
To graph, enter: $y_1=-3+3\sqrt{((x-2)^2/4-1)}$;

$$y_2=-3-3\sqrt{((x-2)^2/4-1)}$$



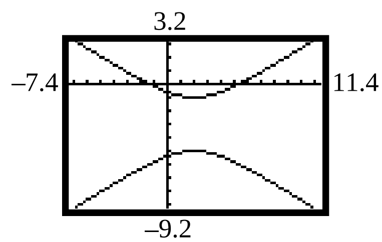

36. $\dfrac{(y+3)^2}{4}-\dfrac{(x-2)^2}{9}=1$

The center of the hyperbola is at (2, –3).
$a=2,\ \ b=3$. The vertices are (2, –1) and (2, –5).
Find the value of c:

$$c^2=a^2+b^2=4+9=13$$
$$c=\sqrt{13}$$

Foci: $\left(2,-3-\sqrt{13}\right)$ and $\left(2,-3+\sqrt{13}\right)$.
Transverse axis: $x=2$.
Asymptotes: $y+3=\dfrac{2}{3}(x-2),\ \ y+3=-\dfrac{2}{3}(x-2)$.
To graph, enter: $y_1=-3+2\sqrt{((x-2)^2/9+1)}$;

$$y_2=-3-2\sqrt{((x-2)^2/9+1)}$$




37. $(y-2)^2-4(x+2)^2=4$

Divide both sides by 4 to put in standard form:

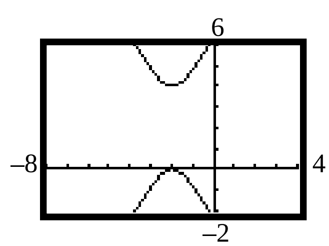
$$\frac{(y-2)^2}{4}-\frac{(x+2)^2}{1}=1$$

The center of the hyperbola is at (–2, 2).
$a=2,\ \ b=1$. The vertices are (–2, 4) and (–2, 0).
Find the value of c:

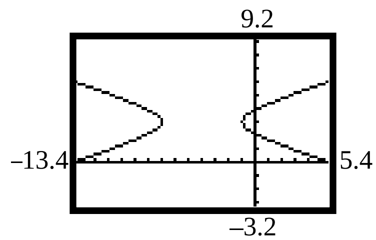
$$c^2=a^2+b^2=4+1=5$$
$$c=\sqrt{5}$$

Foci: $\left(-2,2-\sqrt{5}\right)$ and $\left(-2,2+\sqrt{5}\right)$.
Transverse axis: $x=-2$.
Asymptotes: $y-2=2(x+2),\ \ y-2=-2(x+2)$.
To graph, enter: $y_1=2+2\sqrt{((x+2)^2+1)}$;

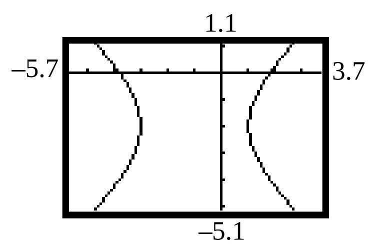
$$y_2=2-2\sqrt{((x+2)^2+1)}$$



38. $(x+4)^2 - 9(y-3)^2 = 9$
Divide both sides by 9 to put in standard form:
$$\frac{(x+4)^2}{9} - \frac{(y-3)^2}{1} = 1$$
The center of the hyperbola is at (–4, 3).
$a = 3$, $b = 1$. The vertices are (–7, 3) and (–1, 3).
Find the value of c:
$$c^2 = a^2 + b^2 = 9 + 1 = 10$$
$$c = \sqrt{10}$$
Foci: $\left(-4-\sqrt{10},\, 3\right)$ and $\left(-4+\sqrt{10},\, 3\right)$.
Transverse axis: $y = 3$.
Asymptotes: $y-3 = \frac{1}{3}(x+4),\ y-3 = -\frac{1}{3}(x+4)$.
To graph, enter: $y_1 = 3 + \sqrt{((x+4)^2 / 9 - 1)}$;
$$y_2 = 3 - \sqrt{((x+4)^2 / 9 - 1)}$$




39. $(x+1)^2 - (y+2)^2 = 4$
Divide both sides by 4 to put in standard form:
$$\frac{(x+1)^2}{4} - \frac{(y+2)^2}{4} = 1$$
The center of the hyperbola is at (–1, –2).
$a = 2$, $b = 2$. The vertices are (–3, –2) and (1, –2).
Find the value of c:
$$c^2 = a^2 + b^2 = 4 + 4 = 8$$
$$c = \sqrt{8} = 2\sqrt{2}$$
Foci: $\left(-1-2\sqrt{2}, -2\right)$ and $\left(-1+2\sqrt{2}, -2\right)$.
Transverse axis: $y = -2$.
Asymptotes: $y+2 = x+1$, $y+2 = -(x+1)$.
To graph, enter: $y_1 = -2 + 2\sqrt{((x+1)^2 / 4 - 1)}$;
$$y_2 = -2 - 2\sqrt{((x+1)^2 / 4 - 1)}$$



40. $(y-3)^2-(x+2)^2=4$

Divide both sides by 4 to put in standard form:

$$\frac{(y-3)^2}{4}-\frac{(x+2)^2}{4}=1$$

The center of the hyperbola is at (–2, 3).

$a=2,\ b=2$. The vertices are (–2, 5) and (–2, 1).

Find the value of c:

$$c^2=a^2+b^2=4+4=8$$

$$c=\sqrt{8}=2\sqrt{2}$$

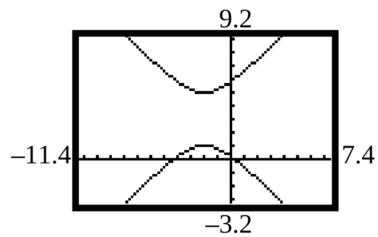
Foci: $\left(-2,\,3-2\sqrt{2}\right)$ and $\left(-2,\,3+2\sqrt{2}\right)$.

Transverse axis: $x=-2$.

Asymptotes: $y-3=x+2,\ y-3=-(x+2)$.

To graph, enter: $y_1=3+2\sqrt{((x+2)^2/4+1)}$;

$$y_2=3-2\sqrt{((x+2)^2/4+1)}$$




41. Complete the square to put in standard form:

$$x^2-y^2-2x-2y-1=0$$

$$(x^2-2x+1)-(y^2+2y+1)=1+1-1$$

$$(x-1)^2-(y+1)^2=1$$

The center of the hyperbola is at (1, –1).

$a=1,\ b=1$. The vertices are (0, –1) and (2, –1).

Find the value of c:

$$c^2=a^2+b^2=1+1=2$$

$$c=\sqrt{2}$$

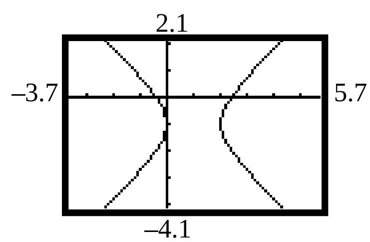
Foci: $\left(1-\sqrt{2},-1\right)$ and $\left(1+\sqrt{2},-1\right)$.

Transverse axis: $y=-1$.

Asymptotes: $y+1=x-1,\ y+1=-(x-1)$.

To graph, enter: $y_1=-1+\sqrt{((x-1)^2-1)}$;

$$y_2=-1-\sqrt{((x-1)^2-1)}$$



42. Complete the square to put in standard form:

$$y^2 - x^2 - 4y + 4x - 1 = 0$$
$$(y^2 - 4y + 4) - (x^2 - 4x + 4) = 1 + 4 - 4$$
$$(y-2)^2 - (x-2)^2 = 1$$

The center of the hyperbola is at (2, 2).

$a = 1, \quad b = 1$. The vertices are (2, 1) and (2, 3).

Find the value of c:

$$c^2 = a^2 + b^2 = 1 + 1 = 2$$
$$c = \sqrt{2}$$

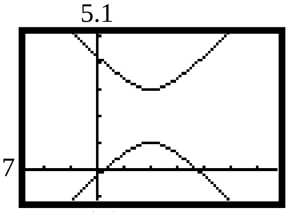
Foci: $\left(2,\ 2-\sqrt{2}\right)$ and $\left(2,\ 2+\sqrt{2}\right)$.

Transverse axis: $x = 2$.

Asymptotes: $y - 2 = x - 2, \quad y - 2 = -(x-2)$.

To graph, enter: $y_1 = 2 + \sqrt{((x-2)^2 + 1)}$;

$y_2 = 2 - \sqrt{((x-2)^2 + 1)}$




43. Complete the square to put in standard form:

$$y^2 - 4x^2 - 4y - 8x - 4 = 0$$
$$(y^2 - 4y + 4) - 4(x^2 + 2x + 1) = 4 + 4 - 4$$
$$(y-2)^2 - 4(x+1)^2 = 4$$
$$\frac{(y-2)^2}{4} - \frac{(x+1)^2}{1} = 1$$

The center of the hyperbola is at (–1, 2).

$a = 2, \quad b = 1$. The vertices are (–1, 4) and (–1, 0).

Find the value of c:

$$c^2 = a^2 + b^2 = 4 + 1 = 5$$
$$c = \sqrt{5}$$

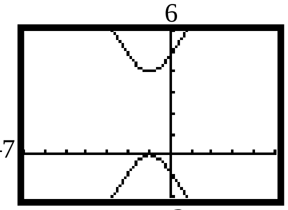
Foci: $\left(-1,\ 2-\sqrt{5}\right)$ and $\left(-1,\ 2+\sqrt{5}\right)$.

Transverse axis: $x = -1$.

Asymptotes: $y - 2 = 2(x+1), \quad y - 2 = -2(x+1)$.

To graph, enter: $y_1 = 2 + 2\sqrt{((x+1)^2 + 1)}$;

$y_2 = 2 - 2\sqrt{((x+1)^2 + 1)}$




44. Complete the square to put in standard form:

$$2x^2 - y^2 + 4x + 4y - 4 = 0$$
$$2(x^2 + 2x + 1) - (y^2 - 4y + 4) = 4 + 2 - 4$$
$$2(x+1)^2 - (y-2)^2 = 2$$
$$\frac{(x+1)^2}{1} - \frac{(y-2)^2}{2} = 1$$

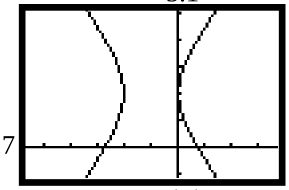
The center of the hyperbola is at (–1, 2).
$a = 1$, $b = \sqrt{2}$. The vertices are (–2, 2) and (0, 2).
Find the value of c:

$$c^2 = a^2 + b^2 = 1 + 2 = 3$$

$$c = \sqrt{3}$$

Foci: $\left(-1-\sqrt{3},\, 2\right)$ and $\left(-1+\sqrt{3},\, 2\right)$.
Transverse axis: $y = 2$.
Asymptotes:
$y - 2 = \sqrt{2}(x+1)$, $y - 2 = -\sqrt{2}(x+1)$.
To graph, enter: $y_1 = 2 + \sqrt{2}\sqrt{((x+1)^2 - 1)}$;

$$y_2 = 2 - \sqrt{2}\sqrt{((x+1)^2 - 1)}$$




45. Complete the square to put in standard form:

$$4x^2 - y^2 - 24x - 4y + 16 = 0$$

$$4(x^2 - 6x + 9) - (y^2 + 4y + 4) = -16 + 36 - 4$$

$$4(x-3)^2 - (y+2)^2 = 16$$

$$\frac{(x-3)^2}{4} - \frac{(y+2)^2}{16} = 1$$

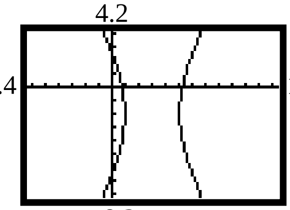
The center of the hyperbola is at (3, –2).
$a = 2$, $b = 4$. The vertices are (1, –2) and (5, –2).
Find the value of c:

$$c^2 = a^2 + b^2 = 4 + 16 = 20$$

$$c = \sqrt{20} = 2\sqrt{5}$$

Foci: $\left(3 - 2\sqrt{5}, -2\right)$ and $\left(3 + 2\sqrt{5}, -2\right)$.
Transverse axis: $y = -2$.
Asymptotes: $y + 2 = 2(x-3)$, $y + 2 = -2(x-3)$.
To graph, enter: $y_1 = -2 + 4\sqrt{((x-3)^2 / 4 - 1)}$;

$$y_2 = -2 - 4\sqrt{((x-3)^2 / 4 - 1)}$$




46. Complete the square to put in standard form:

$$2y^2 - x^2 + 2x + 8y + 3 = 0$$

$$2(y^2 + 4y + 4) - (x^2 - 2x + 1) = -3 + 8 - 1$$

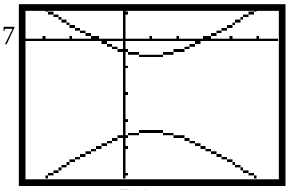
$$2(y+2)^2 - (x-1)^2 = 4$$

$$\frac{(y+2)^2}{2} - \frac{(x-1)^2}{4} = 1$$

The center of the hyperbola is at (1, –2).
$a=\sqrt{2}$, $b=2$. The vertices are
$\left(1,\ -2-\sqrt{2}\right)$ and $\left(1,\ -2+\sqrt{2}\right)$ Find the value of c:

$$c^2 = a^2 + b^2 = 2 + 4 = 6$$
$$c = \sqrt{6}$$

Foci: $\left(1,\ -2-\sqrt{6}\right)$ and $\left(1,\ -2+\sqrt{6}\right)$.
Transverse axis: $x = 1$.
Asymptotes: $y+2=\dfrac{\sqrt{2}}{2}(x-1)$, $y+2=-\dfrac{\sqrt{2}}{2}(x-1)$
To graph, enter: $y_1 = -2+\sqrt{2}\sqrt{((x-1)^2/4+1)}$;
$y_2 = -2-\sqrt{2}\sqrt{((x-1)^2/4+1)}$



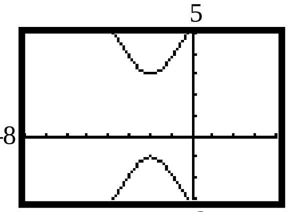

47. Complete the square to put in standard form:

$$y^2-4x^2-16x-2y-19=0$$
$$(y^2-2y+1)-4(x^2+4x+4)=19+1-16$$
$$(y-1)^2-4(x+2)^2=4$$
$$\frac{(y-1)^2}{4}-\frac{(x+2)^2}{1}=1$$

The center of the hyperbola is at (–2, 1).
$a=2$, $b=1$. The vertices are (–2, 3) and (–2, –1).
Find the value of c:

$$c^2=a^2+b^2=4+1=5$$
$$c=\sqrt{5}$$

Foci: $\left(-2, 1-\sqrt{5}\right)$ and $\left(-2, 1+\sqrt{5}\right)$.
Transverse axis: $x=-2$.
Asymptotes: $y-1=2(x+2)$, $y-1=-2(x+2)$.
To graph, enter: $y_1 = 1+2\sqrt{((x+2)^2+1)}$;
$y_2 = 1-2\sqrt{((x+2)^2+1)}$




48. Complete the square to put in standard form:

$$x^2-3y^2+8x-6y+4=0$$
$$(x^2+8x+16)-3(y^2+2y+1)=-4+16-3$$
$$(x+4)^2-3(y+1)^2=9$$
$$\frac{(x+4)^2}{9}-\frac{(y+1)^2}{3}=1$$

The center of the hyperbola is at (–4, –1).
$a = 3; \; b = \sqrt{3}$. The vertices are (–7, –1) and (–1, –1). Find the value of c:

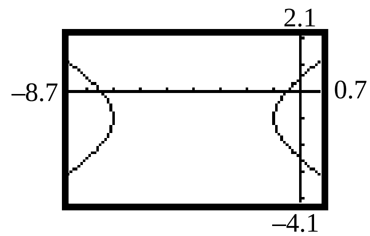
$$c^2 = a^2 + b^2 = 9 + 3 = 12$$
$$c = \sqrt{12} = 2\sqrt{3}$$

Foci: $\left(-4 - 2\sqrt{3}, -1\right)$ and $\left(-4 + 2\sqrt{3}, -1\right)$.

Transverse axis: $y = -1$.

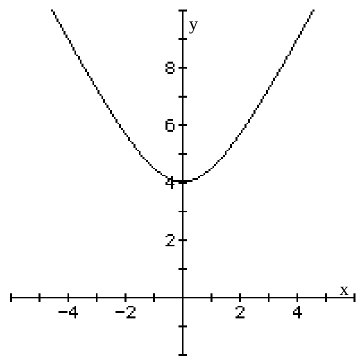
Asymptotes: $y + 1 = \frac{\sqrt{3}}{3}(x+4)$, $y + 1 = -\frac{\sqrt{3}}{3}(x+4)$

To graph, enter: $y_1 = -1 + \sqrt{3}\sqrt{((x+4)^2/9 - 1)}$;
$y_2 = -1 - \sqrt{3}\sqrt{((x+4)^2/9 - 1)}$



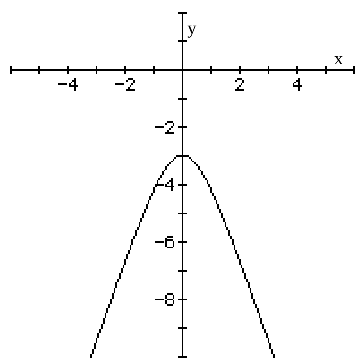

49. Rewrite the equation:

$$y = \sqrt{16 + 4x^2}$$
$$y^2 = 16 + 4x^2, \quad y \ge 0$$
$$y^2 - 4x^2 = 16, \quad y \ge 0$$
$$\frac{y^2}{16} - \frac{x^2}{4} = 1, \quad y \ge 0$$



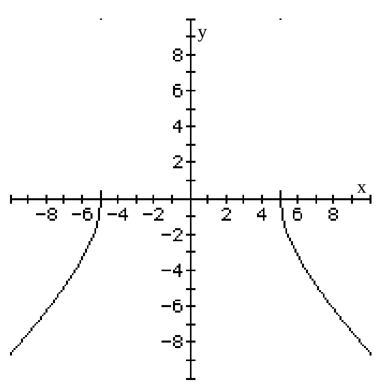

50. Rewrite the equation:

$$y = -\sqrt{9 + 9x^2}$$
$$y^2 = 9 + 9x^2, \quad y \le 0$$
$$y^2 - 9x^2 = 9, \quad y \le 0$$
$$\frac{y^2}{9} - \frac{x^2}{1} = 1, \quad y \le 0$$



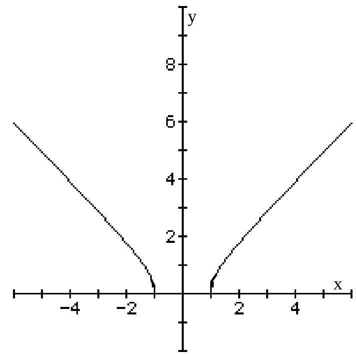

51. Rewrite the equation:

$$y = -\sqrt{-25 + x^2}$$
$$y^2 = -25 + x^2, \quad y \le 0$$
$$x^2 - y^2 = 25, \quad y \le 0$$
$$\frac{x^2}{25} - \frac{y^2}{25} = 1, \quad y \le 0$$

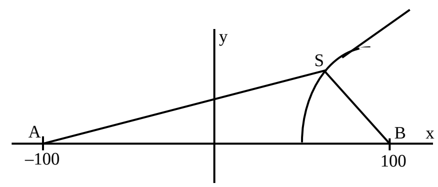


52. Rewrite the equation:

$$y = \sqrt{-1 + x^2}$$
$$y^2 = -1 + x^2, \quad y \ge 0$$
$$x^2 - y^2 = 1, \quad y \ge 0$$




53. (a) Set up a coordinate system so that the two stations lie on the x-axis and the origin is midway between them. The ship lies on a hyperbola whose foci are the locations of the two stations. Since the time difference is 0.00038 seconds and the speed of the signal is 186,000 miles per second, the difference in the distances of the ships from each station is:




distance = (186,000)(0.00038) = 70.68 miles

The difference of the distances from the ship to each station, 70.68, equals $2a$, so $a = 35.34$ and the vertex of the corresponding hyperbola is at (35.34, 0). Since the focus is at (100, 0), following this hyperbola, the ship would reach shore 64.66 miles from the master station.

(b) The ship should follow a hyperbola with a vertex at (80, 0). For this hyperbola, $a = 80$, so the constant difference of the distances from the ship to each station is 160. The time difference the ship should look for is:

$$\text{time} = \frac{160}{186,000} = 0.00086 \text{ seconds}$$

(c) Find the equation of the hyperbola with vertex at (80, 0) and a focus at (100, 0). The form of the equation of the hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{where } a = 80.$$

Since $c = 100$ and $b^2 = c^2 - a^2 \;\rightarrow\; b^2 = 100^2 - 80^2 = 3600$.

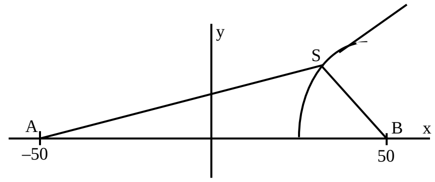
The equation of the hyperbola is: $\dfrac{x^2}{6400} - \dfrac{y^2}{3600} = 1$.

Since the ship is 50 miles off shore, we have $y = 50$. Solve the equation for $x$:

$$\frac{x^2}{6400} - \frac{50^2}{3600} = 1 \;\rightarrow\; \frac{x^2}{6400} = 1 + \frac{2500}{3600} = \frac{61}{36} \;\rightarrow\; x^2 = 6400\left(\frac{61}{36}\right)$$
$$x \approx 104 \text{ miles}$$

The ship's location is (104, 50).

54. (a) Set up a coordinate system so that the two stations lie on the x-axis and the origin is midway between them. The ship lies on a hyperbola whose foci are the locations of the two stations. Since the time difference is 0.00032 seconds and the speed of the signal is 186,000 miles per second, the difference in the distances of the ships from each station is:




distance = (186,000)(0.00032) = 59.52 miles

The difference of the distances from the ship to each station, 59.52, equals $2a$, so $a = 29.76$ and the vertex of the corresponding hyperbola is at (29.76, 0). Since the focus is at (50, 0), following this hyperbola, the ship would reach shore 20.24 miles from the master station.

(b) The ship should follow a hyperbola with a vertex at (40, 0). For this hyperbola, $a = 40$, so the constant difference of the distances from the ship to each station is 80. The time difference the ship should look for is:

$$\text{time} = \frac{80}{186,000} = 0.00043 \text{ seconds}$$

(c) Find the equation of the hyperbola with vertex at (40, 0) and a focus at (50, 0). The form of the equation of the hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{where } a = 40.$$

Since $c = 50$ and $b^2 = c^2 - a^2 \rightarrow b^2 = 50^2 - 40^2 = 900$.

The equation of the hyperbola is: $\dfrac{x^2}{1600} - \dfrac{y^2}{900} = 1$.

Since the ship is 20 miles off shore, we have $y = 20$. Solve the equation for $x$:

$$\frac{x^2}{1600} - \frac{20^2}{900} = 1 \rightarrow \frac{x^2}{1600} = 1 + \frac{400}{900} = \frac{13}{9} \rightarrow x^2 = 1600\ \frac{13}{9}$$

$$x \approx 48 \text{ miles}$$

The ship's location is (48, 20).

55. (a) Set up a rectangular coordinate system so that the two devices lie on the x-axis and the origin is midway between them. The devices serve as foci to the hyperbola so $c = \dfrac{2000}{2} = 1000$. Since the explosion occurs 200 feet from point B, the vertex of the hyperbola is (800, 0); therefore, $a = 800$. Finding b:

$$b^2 = c^2 - a^2 \rightarrow b^2 = 1000^2 - 800^2 = 360000 \rightarrow b = 600$$

The equation of the hyperbola is:

$$\frac{x^2}{800^2} - \frac{y^2}{600^2} = 1$$

If $x = 1000$, find $y$:

$$\frac{1000^2}{800^2} - \frac{y^2}{600^2} = 1 \rightarrow \frac{y^2}{600^2} = \frac{1000^2}{800^2} - 1 = \frac{600^2}{800^2} \rightarrow y^2 = 600^2\ \frac{600^2}{800^2}$$

$$y = 450 \text{ feet}$$

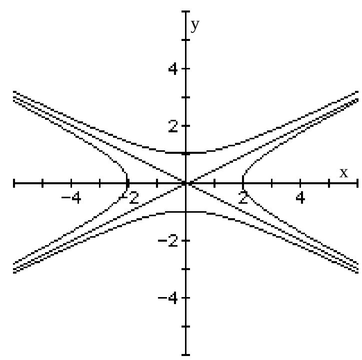
The second detonation should take place 450 feet north of point B.

56. Answers will vary.

57. If the eccentricity is close to 1, then $c \approx a$ and $b \approx 0$. When b is close to 0, the hyperbola is very narrow, because the slopes of the asymptotes are close to 0.
If the eccentricity is very large, then $c$ is much larger than $a$ and $b$ is very large. The result is a hyperbola that is very wide.

58. If $a = b$, then $c^2 = a^2 + a^2 = 2a^2$
Thus, $\frac{c^2}{a^2} = 2$ or $\frac{c}{a} = \sqrt{2}$
The eccentricity of an equilateral hyperbola is $\sqrt{2}$.

59. $\frac{x^2}{4} - y^2 = 1 \quad (a = 2,\ b = 1)$
is a hyperbola with horizontal transverse axis, centered at (0, 0) and has asymptotes: $y = \pm\frac{1}{2}x$
$y^2 - \frac{x^2}{4} = 1 \quad (a = 1,\ b = 2)$
is a hyperbola with vertical transverse axis, centered at (0, 0) and has asymptotes: $y = \pm\frac{1}{2}x$
Since the two hyperbolas have the same asymptotes, they are conjugate.




60. $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Solve for y:

$$\frac{y^2}{a^2} = 1 + \frac{x^2}{b^2}$$

$$y^2 = a^2\left(1 + \frac{x^2}{b^2}\right)$$

$$y^2 = \frac{a^2x^2}{b^2}\left(\frac{b^2}{x^2} + 1\right)$$

$$y = \pm\frac{ax}{b}\sqrt{\frac{b^2}{x^2} + 1}$$

As $x \to -\infty$ or as $x \to \infty$, the term $\frac{b^2}{x^2}$ gets close to 0, so the expression under the radical gets closer to 1. Thus, the graph of the hyperbola gets closer to the lines $y = -\frac{a}{b}x$ and $y = \frac{a}{b}x$. These lines are the asymptotes of the hyperbola.

61. Put the equation in standard hyperbola form:

$$Ax^2 + Cy^2 + F = 0 \qquad A \neq 0,\ C \neq 0,\ F \neq 0$$

$$Ax^2 + Cy^2 = -F$$

$$\frac{Ax^2}{-F} + \frac{Cy^2}{-F} = 1$$

$$\frac{x^2}{-F/A} + \frac{y^2}{-F/C} = 1$$

Since $-F/A$ and $-F/C$ have opposite signs, this is a hyperbola with center at (0, 0).

62. Complete the square on the given equation:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0, \qquad \text{where } A \text{ and } C \text{ have opposite signs.}$$

$$A\left(x^2 + \frac{D}{A}x\right) + C\left(y^2 + \frac{E}{C}y\right) = -F$$

$$A\left(x^2 + \frac{D}{A}x + \frac{D^2}{4A^2}\right) + C\left(y^2 + \frac{E}{C}y + \frac{E^2}{4C^2}\right) = \frac{D^2}{4A} + \frac{E^2}{4C} - F$$

$$A\left(x + \frac{D}{2A}\right)^2 + C\left(y + \frac{E}{2C}\right)^2 = \frac{D^2}{4A} + \frac{E^2}{4C} - F$$

(a) If $\frac{D^2}{4A} + \frac{E^2}{4C} - F \neq 0$, this is the equation of a hyperbola whose center is $\left(\frac{-D}{2A}, \frac{-E}{2C}\right)$ if $C < 0$ or $\left(\frac{-D}{2A}, \frac{E}{2C}\right)$ if $A < 0$.

(b) If $\frac{D^2}{4A^2} + \frac{E^2}{4C^2} - F = 0$, then

$$A\left(x + \frac{D}{2A}\right)^2 + C\left(y + \frac{E}{2C}\right)^2 = 0 \quad \rightarrow \quad \left(y + \frac{E}{2C}\right)^2 = \frac{-A}{C}\left(x + \frac{D}{2A}\right)^2$$

$$\rightarrow \quad y + \frac{E}{2C} = \pm\sqrt{\frac{-A}{C}}\left(x + \frac{D}{2A}\right)$$

which is the graph of two intersecting lines.