Chapter 12

Systems of Equations and Inequalities

12.1 Systems of Linear Equations: Two Equations Containing Two Variables

1. Substituting the values of the variables:

$$\begin{cases} 2x - y = 5 & \rightarrow \quad 2(2) - (-1) = 4 + 1 = 5 \\ 5x + 2y = 8 & \rightarrow \quad 5(2) + 2(-1) = 10 - 2 = 8 \end{cases}$$

Each equation is satisfied, so $x = 2,\ y = -1$ is a solution to the system of equations.

2. Substituting the values of the variables:

$$\begin{cases} 3x + 2y = 2 & \rightarrow \quad 3(-2) + 2(4) = -6 + 8 = 2 \\ x - 7y = -30 & \rightarrow \quad (-2) - 7(4) = -2 - 28 = -30 \end{cases}$$

Each equation is satisfied, so $x = -2,\ y = 4$ is a solution to the system of equations.

3. Substituting the values of the variables:

$$\begin{cases} 3x - 4y = 4 & \rightarrow \quad 3(2) - 4\left(\frac{1}{2}\right) = 6 - 2 = 4 \\ \frac{1}{2}x - 3y = -\frac{1}{2} & \rightarrow \quad \frac{1}{2}(2) - 3\left(\frac{1}{2}\right) = 1 - \frac{3}{2} = -\frac{1}{2} \end{cases}$$

Each equation is satisfied, so $x = 2,\ y = \frac{1}{2}$ is a solution to the system of equations.

4. Substituting the values of the variables:

$$\begin{cases} 2x + \frac{1}{2}y = 0 & \rightarrow \quad 2\left(-\frac{1}{2}\right) + \frac{1}{2}(2) = -1 + 1 = 0 \\ 3x - 4y = -\frac{19}{2} & \rightarrow \quad 3\left(-\frac{1}{2}\right) - 4(2) = -\frac{3}{2} - 8 = -\frac{19}{2} \end{cases}$$

Each equation is satisfied, so $x = -\frac{1}{2},\ y = 2$ is a solution to the system of equations.

5. Substituting the values of the variables:

$$\begin{cases} x^2 - y^2 = 3 & \rightarrow \quad 2^2 - 1^2 = 4 - 1 = 3 \\ xy = 2 & \rightarrow \quad (2)(1) = 2 \end{cases}$$

Each equation is satisfied, so $x = 2,\ y = 1$ is a solution to the system of equations.

6. Substituting the values of the variables:

$$\begin{cases} x^2 - y^2 = 3 & \rightarrow \quad (-2)^2 - (-1)^2 = 4 - 1 = 3 \\ xy = 2 & \rightarrow \quad (-2)(-1) = 2 \end{cases}$$

Each equation is satisfied, so $x = -2, y = -1$ is a solution to the system of equations.

7. Substituting the values of the variables:

$$\begin{cases} \dfrac{x}{1+x} + 3y = 6 & \rightarrow \quad \dfrac{0}{1+0} + 3(2) = 0 + 6 = 6 \\ x + 9y^2 = 36 & \rightarrow \quad 0 + 9(2)^2 = 0 + 9\ 4 = 36 \end{cases}$$

Each equation is satisfied, so $x = 0, y = 2$ is a solution to the system.

8. Substituting the values of the variables:

$$\begin{cases} \dfrac{x}{x-1} + y = 5 & \rightarrow \quad \dfrac{2}{2-1} + 3 = 2 + 3 = 5 \\ 3x - y = 3 & \rightarrow \quad 3(2) - 3 = 6 - 3 = 3 \end{cases}$$

Each equation is satisfied, so $x = 2, y = 3$ is a solution to the system.

9. Solve the first equation for y, substitute into the second equation and solve:

$$\begin{cases} x + y = 8 & \rightarrow \quad y = 8 - x \\ x - y = 4 \end{cases}$$

$$x - (8 - x) = 4 \rightarrow x - 8 + x = 4$$

$$2x = 12 \rightarrow x = 6$$

Since $x = 6,\ y = 8 - 6 = 2$

The solution of the system is $x = 6,\ y = 2$.

10. Solve the first equation for x, substitute into the second equation and solve:

$$\begin{cases} x + 2y = 5 & \rightarrow \quad x = 5 - 2y \\ x + y = 3 \end{cases}$$

$$(5 - 2y) + y = 3 \rightarrow 5 - y = 3$$

$$-y = -2 \rightarrow y = 2$$

Since $y = 2,\ x = 5 - 2(2) = 1$ The solution of the system is $x = 1,\ y = 2$.

11. Multiply each side of the first equation by 3 and add the equations:

$$\begin{cases} 5x - y = 13 & \xrightarrow{3} \quad 15x - 3y = 39 \\ 2x + 3y = 12 & \longrightarrow \quad \underline{2x + 3y = 12} \end{cases}$$

$$\begin{aligned} 17x &= 51 \\ x &= 3 \end{aligned}$$

Substitute and solve for y:

$$5(3) - y = 13 \rightarrow 15 - y = 13 \rightarrow -y = -2 \rightarrow y = 2$$

The solution of the system is $x = 3,\ y = 2$.

12. Add the equations:

$$\begin{cases} x+3y=5 \\ 2x-3y=-8 \end{cases}$$
$$\begin{aligned} 3x &= -3 \\ x &= -1 \end{aligned}$$

Substitute and solve for y:

$$-1+3y=5 \rightarrow 3y=6 \rightarrow y=2$$

The solution of the system is $x=-1,\ y=2$.

13. Solve the first equation for x and substitute into the second equation:

$$\begin{cases} 3x=24 \quad \rightarrow \quad x=8 \\ x+2y=0 \end{cases}$$

$$8+2y=0 \rightarrow 2y=-8 \rightarrow y=-4$$

The solution of the system is $x=8,\ y=-4$.

14. Solve the second equation for y and substitute into the first equation:

$$\begin{cases} 4x+5y=-3 \\ -2y=-4 \quad \rightarrow \quad y=2 \end{cases}$$

$$4x+5(2)=-3 \rightarrow 4x+10=-3 \rightarrow 4x=-13$$

$$x=-\frac{13}{4}$$

The solution of the system is $x=-\frac{13}{4},\ y=2$.

15. Multiply each side of the first equation by 2 and each side of the second equation by 3 to eliminate y:

$$\begin{cases} 3x-6y=2 \xrightarrow{\cdot 2} 6x-12y=4 \\ 5x+4y=1 \xrightarrow{\cdot 3} 15x+12y=3 \end{cases}$$
$$\begin{aligned} 21x &= 7 \\ x &= \frac{1}{3} \end{aligned}$$

Substitute and solve for y:

$$3\left(\frac{1}{3}\right)-6y=2 \rightarrow 1-6y=2 \rightarrow -6y=1 \rightarrow y=-\frac{1}{6}$$

The solution of the system is $x=\frac{1}{3},\ y=-\frac{1}{6}$.

16. Multiply each side of the first equation by 5 and each side of the second equation by 4 to eliminate y:

$$\begin{cases} 2x+4y=\dfrac{2}{3} \\ 3x-5y=-10 \end{cases} \begin{array}{l} \xrightarrow{\times 5} \\ \xrightarrow{\times 4} \end{array} \begin{array}{rcl} 10x+20y &=& \dfrac{10}{3} \\ 12x-20y &=& -40 \\ \hline 22x &=& -\dfrac{110}{3} \\ x &=& -\dfrac{5}{3} \end{array}$$

Substitute and solve for y:

$$3\left(-\frac{5}{3}\right)-5y=-10 \rightarrow -5-5y=-10 \rightarrow -5y=-5 \rightarrow y=1$$

The solution of the system is $x=-\dfrac{5}{3},\ y=1$.

17. Solve the first equation for y, substitute into the second equation and solve:

$$\begin{cases} 2x+y=1 & \rightarrow \quad y=1-2x \\ 4x+2y=3 \end{cases}$$

$$4x+2(1-2x)=3 \rightarrow 4x+2-4x=3 \rightarrow 0x=1$$

This has no solution, so the system is inconsistent.

18. Solve the first equation for x, substitute into the second equation and solve:

$$\begin{cases} x-y=5 & \rightarrow \quad x=y+5 \\ -3x+3y=2 \end{cases}$$

$$-3(y+5)+3y=2 \rightarrow -3y-15+3y=2 \rightarrow 0y=17$$

This has no solution, so the system is inconsistent.

19. Solve the first equation for y, substitute into the second equation and solve:

$$\begin{cases} 2x-y=0 & \rightarrow \quad 2x=y \\ 3x+2y=7 \end{cases}$$

$$3x+2(2x)=7 \rightarrow 3x+4x=7 \rightarrow 7x=7 \rightarrow x=1$$

Since $x=1,\ y=2(1)=2$ The solution of the system is $x=1,\ y=2$.

20. Solve the second equation for y, substitute into the first equation and solve:

$$\begin{cases} 3x+3y=-1 \\ 4x+y=\dfrac{8}{3} & \rightarrow \quad y=\dfrac{8}{3}-4x \end{cases}$$

$$3x+3\left(\frac{8}{3}-4x\right)=-1 \rightarrow 3x+8-12x=-1 \rightarrow -9x=-9 \rightarrow x=1$$

Since $x=1,\ y=\dfrac{8}{3}-4(1)=\dfrac{8}{3}-4=-\dfrac{4}{3}$

The solution of the system is $x=1,\ y=-\dfrac{4}{3}$.

21. Solve the first equation for x, substitute into the second equation and solve:

$$\begin{cases} x+2y=4 & \rightarrow \quad x=4-2y \\ 2x+4y=8 \end{cases}$$

$$2(4-2y)+4y=8 \rightarrow 8-4y+4y=8 \rightarrow 0y=0$$

These equations are dependent. Any real number is a solution for y.
The solution of the system is $x=4-2y$, where y is any real number.

22. Solve the first equation for y, substitute into the second equation and solve:

$$\begin{cases} 3x-y=7 & \rightarrow \quad y=3x-7 \\ 9x-3y=21 \end{cases}$$

$$9x-3(3x-7)=21 \rightarrow 9x-9x+21=21 \rightarrow 0x=0$$

These equations are dependent. Any real number is a solution for x.
The solution of the system is $y=3x-7$, where x is any real number.

23. Multiply each side of the first equation by –5, and add the equations to eliminate x:

$$\begin{cases} 2x-3y=-1 & \xrightarrow{-5} & -10x+15y=5 \\ 10x+y=11 & \longrightarrow & \underline{10x+y=11} \end{cases}$$

$$16y=16$$
$$y=1$$

Substitute and solve for x:

$$2x-3(1)=-1 \rightarrow 2x-3=-1 \rightarrow 2x=2 \rightarrow x=1$$

The solution of the system is $x=1,\ y=1$.

24. Multiply each side of the first equation by 5, and add the equations to eliminate y:

$$\begin{cases} 3x-2y=0 & \xrightarrow{5} & 15x-10y=0 \\ 5x+10y=4 & \longrightarrow & \underline{5x+10y=4} \end{cases}$$

$$20x\qquad=4$$
$$x=\frac{1}{5}$$

Substitute and solve for y:

$$5\left(\frac{1}{5}\right)+10y=4 \rightarrow 1+10y=4 \rightarrow 10y=3 \rightarrow y=\frac{3}{10}$$

The solution of the system is $x=\frac{1}{5},\ y=\frac{3}{10}$.

25. Solve the second equation for x, substitute into the first equation and solve:

$$\begin{cases} 2x + 3y = 6 \\ x - y = \dfrac{1}{2} \quad \rightarrow \quad x = y + \dfrac{1}{2} \end{cases}$$

$$2\left(y + \frac{1}{2}\right) + 3y = 6 \rightarrow 2y + 1 + 3y = 6 \rightarrow 5y = 5 \rightarrow y = 1$$

Since $y = 1$, $x = 1 + \frac{1}{2} = \frac{3}{2}$ The solution of the system is $x = \frac{3}{2},\ y = 1$.

26. Solve the second equation for x, substitute into the first equation and solve:

$$\begin{cases} \dfrac{1}{2}x + y = -2 \\ x - 2y = 8 \quad \rightarrow \quad x = 2y + 8 \end{cases}$$

$$\frac{1}{2}(2y + 8) + y = -2 \rightarrow y + 4 + y = -2 \rightarrow 2y = -6 \rightarrow y = -3$$

Since $y = -3$, $x = 2(-3) + 8 = -6 + 8 = 2$ The solution of the system is $x = 2,\ y = -3$.

27. Multiply each side of the first equation by –6 and each side of the second equation by 12 to eliminate x:

$$\begin{cases} \dfrac{1}{2}x + \dfrac{1}{3}y = 3 \quad \xrightarrow{-6} \quad -3x - 2y = -18 \\ \dfrac{1}{4}x - \dfrac{2}{3}y = -1 \quad \xrightarrow{12} \quad \underline{3x - 8y = -12} \end{cases}$$

$$-10y = -30$$

$$y = 3$$

Substitute and solve for x:

$$\frac{1}{2}x + \frac{1}{3}(3) = 3 \rightarrow \frac{1}{2}x + 1 = 3 \rightarrow \frac{1}{2}x = 2 \rightarrow x = 4$$

The solution of the system is $x = 4,\ y = 3$.

28. Multiply each side of the first equation by 6 and each side of the second equation by 12 to eliminate the fractions:

$$\begin{cases} \dfrac{1}{3}x - \dfrac{3}{2}y = -5 \quad \xrightarrow{6} \quad 2x - 9y = -30 \quad \xrightarrow{-9} \quad -18x + 81y = 270 \\ \dfrac{3}{4}x + \dfrac{1}{3}y = 11 \quad \xrightarrow{12} \quad 9x + 4y = 132 \quad \xrightarrow{2} \quad \underline{18x + 8y = 264} \end{cases}$$

$$89y = 534$$

$$y = 6$$

Substitute and solve for x:

$$\frac{3}{4}x + \frac{1}{3}(6) = 11 \rightarrow \frac{3}{4}x + 2 = 11 \rightarrow \frac{3}{4}x = 9 \rightarrow x = 12$$

The solution of the system is $x = 12,\ y = 6$.

29. Add the equations to eliminate y and solve for x:

$$\begin{cases} 3x - 5y = 3 \\ 15x + 5y = 21 \end{cases}$$
$$18x = 24$$
$$x = \frac{4}{3}$$

Substitute and solve for y:

$$3\left(\frac{4}{3}\right) - 5y = 3 \rightarrow 4 - 5y = 3 \rightarrow -5y = -1 \rightarrow y = \frac{1}{5}$$

The solution of the system is $x = \frac{4}{3},\ y = \frac{1}{5}$.

30. Multiply each side of the second equation by 2, add the equations to eliminate y and solve for x:

$$\begin{cases} 2x - y = -1 & \longrightarrow & 2x - y = -1 \\ x + \frac{1}{2}y = \frac{3}{2} & \xrightarrow{2} & 2x + y = 3 \end{cases}$$
$$4x = 2$$
$$x = \frac{1}{2}$$

Substitute and solve for y:

$$2\left(\frac{1}{2}\right) - y = -1 \rightarrow 1 - y = -1 \rightarrow -y = -2 \rightarrow y = 2$$

The solution of the system is $x = \frac{1}{2},\ y = 2$.

31. Rewrite letting $a = \frac{1}{x},\ b = \frac{1}{y}$:

$$\begin{cases} \frac{1}{x} + \frac{1}{y} = 8 & \longrightarrow & a + b = 8 \\ \frac{3}{x} - \frac{5}{y} = 0 & \longrightarrow & 3a - 5b = 0 \end{cases}$$

Solve the first equation for a, substitute into the second equation and solve:

$$\begin{cases} a + b = 8 & \rightarrow \quad a = 8 - b \\ 3a - 5b = 0 \end{cases}$$

$$3(8 - b) - 5b = 0 \rightarrow 24 - 3b - 5b = 0 \rightarrow -8b = -24 \rightarrow b = 3$$

Since $b = 3$, $a = 8 - 3 = 5$

Thus, $x = \frac{1}{a} = \frac{1}{5},\ y = \frac{1}{b} = \frac{1}{3}$

The solution of the system is $x = \frac{1}{5},\ y = \frac{1}{3}$.

32. Rewrite letting $a = \frac{1}{x},\ b = \frac{1}{y}$:

$$\begin{cases} \frac{4}{x} - \frac{3}{y} = 0 & \longrightarrow & 4a - 3b = 0 & \longrightarrow & 4a - 3b = 0 \\ \frac{6}{x} + \frac{3}{2y} = 2 & \longrightarrow & 6a + \frac{3}{2}b = 2 & \xrightarrow{2} & \underline{12a + 3b = 4} \end{cases}$$

$$16a = 4$$

$$a = \frac{1}{4}$$

Substitute and solve for b:

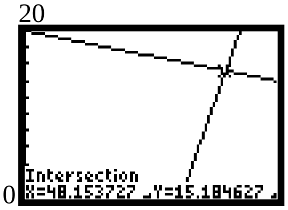
$$4\left(\frac{1}{4}\right) - 3b = 0 \rightarrow 1 - 3b = 0 \rightarrow -3b = -1 \rightarrow b = \frac{1}{3}$$

Thus, $x = \frac{1}{a} = 4,\ y = \frac{1}{b} = 3$ The solution of the system is $x = 4,\ y = 3$.

33. Graph the two equations as y_1 and y_2, and use INTERSECT to solve:

$$\begin{cases} y_1 = \sqrt{2}x - 20\sqrt{7} \\ y_2 = -0.1x + 20 \end{cases}$$

20



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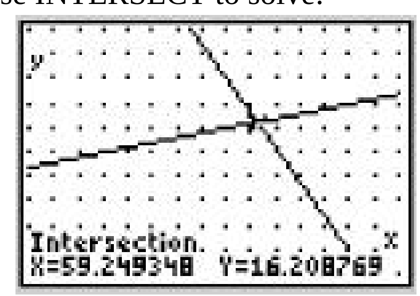
The solution of the system is $x = 48.15,\quad y = 15.18$.

34. Graph the two equations as y_1 and y_2, and use INTERSECT to solve:

$$\begin{cases} y_1 = -\sqrt{2}x + 100 \\ y_2 = 0.2x + \sqrt{19} \end{cases}$$



The solution of the system is $x = 59.25,\ y = 16.21$.

35. Solve for y in each equation, graph the two equations as y_1 and y_2, and use INTERSECT to solve:

$$\begin{cases} \sqrt{2}x + \sqrt{3}y + \sqrt{6} = 0 \\ \sqrt{3}x - \sqrt{2}y + 60 = 0 \end{cases}$$

$$\begin{cases} y_1 = \dfrac{-\sqrt{2}x - \sqrt{6}}{\sqrt{3}} \\ y_2 = \dfrac{\sqrt{3}x + 60}{\sqrt{2}} \end{cases}$$

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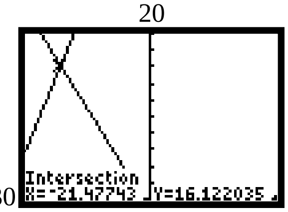

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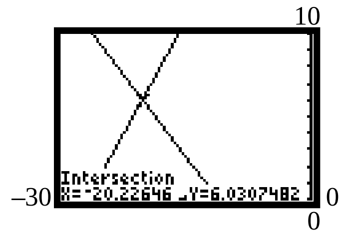
The solution of the system is $x = -21.48,\ y = 16.12$.

36. Solve for y in each equation, graph the two equations as y_1 and y_2, and use INTERSECT to solve:

$$\begin{cases} \sqrt{5}x - \sqrt{6}y + 60 = 0 \\ 0.2x + 0.3y + \sqrt{5} = 0 \end{cases}$$

$$\begin{cases} y_1 = \dfrac{\sqrt{5}x + 60}{\sqrt{6}} \\ y_2 = \dfrac{-0.2x - \sqrt{5}}{0.3} \end{cases}$$

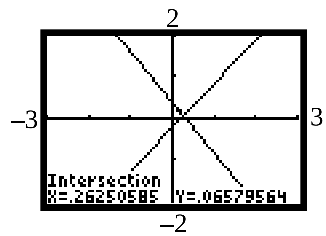


The solution of the system is $x = -20.23,\ y = 6.03$.

37. Solve for y in each equation, graph the two equations as y_1 and y_2, and use INTERSECT to solve:

$$\begin{cases} \sqrt{3}x + \sqrt{2}y = \sqrt{0.3} \\ 100x - 95y = 20 \end{cases}$$

$$\begin{cases} y_1 = \dfrac{-\sqrt{3}x + \sqrt{0.3}}{\sqrt{2}} \\ y_2 = \dfrac{100x - 20}{95} \end{cases}$$

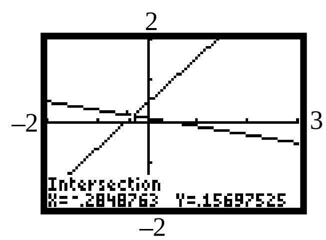


The solution of the system is $x = 0.26,\ y = 0.07$.

38. Solve for y in each equation, graph the two equations as y_1 and y_2, and use INTERSECT to solve:

$$\begin{cases} \sqrt{6}x - \sqrt{5}y + \sqrt{1.1} = 0 \\ y = -0.2x + 0.1 \end{cases}$$

$$\begin{cases} y_1 = \dfrac{\sqrt{6}x + \sqrt{1.1}}{\sqrt{5}} \\ y_2 = -0.2x + 0.1 \end{cases}$$



The solution of the system is $x = -0.28,\ y = 0.16$.

39. Solve the system by substitution:

$$Q_s = Q_d$$

$$-200 + 50p = 1000 - 25p \rightarrow 75p = 1200 \rightarrow p = 16$$

Therefore, $Q_s = -200 + 50(16) = -200 + 800 = 600$

The equilibrium price is \$16 and the equilibrium quantity is 600 T-shirts.

40. Solve the system by substitution:

$$Q_s = Q_d$$

$$-2000 + 3000p = 10000 - 1000p \rightarrow 4000p = 12000 \rightarrow p = 3$$

Therefore, $Q_s = -2000 + 3000(3) = -2000 + 9000 = 7000$

The equilibrium price is \$3 and the equilibrium quantity is 7000 hot dogs.

41. Let l be the length of the rectangle and w be the width of the rectangle. Then:

$$2l + 2w = 90$$
$$l = 2w$$

Solve by substitution:

$$2(2w) + 2w = 90$$
$$4w + 2w = 90 \rightarrow 6w = 90 \rightarrow w = 15 \text{ feet}$$
$$l = 2(15) = 30 \text{ feet}$$

The dimensions of the floor are 15 feet by 30 feet.

42. Let l be the length of the rectangle and w be the width of the rectangle. Then:

$$2l + 2w = 3000$$
$$l = w + 50$$

Solve by substitution:

$$2(w + 50) + 2w = 3000 \rightarrow 2w + 100 + 2w = 3000 \rightarrow 4w = 2900 \rightarrow w = 725 \text{ meters}$$
$$l = 725 + 50 = 775 \text{ meters}$$

The dimensions of the field are 725 meters by 775 meters.

43. Let x = the cost of one cheeseburger and y = the cost of one shake. Then:

$$4x + 2y = 790$$
$$2y = x + 15$$

Solve by substitution:

$$4x + x + 15 = 790 \rightarrow 5x = 775 \rightarrow x = 155$$
$$2y = 155 + 15 \rightarrow 2y = 170 \rightarrow y = 85$$

A cheeseburger cost \$1.55 and a shake costs \$0.85.

44. Let x = the number of adult tickets sold and y = the number of senior tickets sold. Then:

$$x + y = 325 \quad \rightarrow \quad y = 325 - x$$
$$9x + 7y = 2495$$

Solve by substitution:

$$9x + 7(325 - x) = 2495$$
$$9x + 2275 - 7x = 2495$$
$$2x = 220 \rightarrow x = 110 \rightarrow y = 325 - 110 = 215$$

There were 110 adult tickets sold and 215 senior citizen tickets sold.

45. Let x = the number of pounds of cashews.
Then $x + 30$ is the number of pounds in the mixture.
The value of the cashews is $5x$.
The value of the peanuts is 1.50(30) = 45.
The value of the mixture is $3(x + 30)$.
Setting up a value equation:

$$5x + 45 = 3(x + 30)$$
$$5x + 45 = 3x + 90$$
$$2x = 45$$
$$x = 22.5$$

22.5 pounds of cashews should be used in the mixture.

46. Let x = the amount invested in AA bonds.
Let y = the amount invested in the Bank Certificate.

(a) Then $x + y = 150{,}000$ represents the total investment.
$0.10x + 0.05y = 12{,}000$ represents the earnings on the investment.
Solve by substitution:

$$\begin{aligned} 0.10(150{,}000 - y) + 0.05y &= 12{,}000 \\ 15{,}000 - 0.10y + 0.05y &= 12{,}000 \\ -0.05y &= -3000 \\ y &= 60{,}000 \\ x &= 150{,}000 - 60{,}000 = 90{,}000 \end{aligned}$$

$90,000 should be invested in AA Bonds and $60,000 in a Bank Certificate.

(b) Then $x + y = 150{,}000$ represents the total investment.
$0.10x + 0.05y = 14{,}000$ represents the earnings on the investment.
Solve by substitution:

$$\begin{aligned} 0.10(150{,}000 - y) + 0.05y &= 14{,}000 \\ 15{,}000 - 0.10y + 0.05y &= 14{,}000 \\ -0.05y &= -1000 \\ y &= 20{,}000 \\ x &= 150{,}000 - 20{,}000 = 130{,}000 \end{aligned}$$

$130,000 should be invested in AA Bonds and $20,000 in a Bank Certificate.

47. Let x = the plane's airspeed and y = the wind speed.

	Rate	Time	Distance
With Wind	$x + y$	3	600
Against	$x - y$	4	600

$$(x + y)(3) = 600 \quad \rightarrow \quad x + y = 200$$
$$(x - y)(4) = 600 \quad \rightarrow \quad x - y = 150$$

Solving by elimination:

$$\begin{aligned} 2x &= 350 \\ x &= 175 \\ y &= 200 - x = 200 - 175 = 25 \end{aligned}$$

The airspeed of the plane is 175 mph, and the wind speed is 25 mph.

48. Let y = the wind speed.

	Rate	Time	Distance
With Wind	$150 + y$	2	$300 + 2y$
Against	$150 - y$	3	$450 - 3y$

The distances are equal, so equate and solve:

$$\begin{aligned} 300 + 2y &= 450 - 3y \\ 5y &= 150 \\ y &= 30 \end{aligned}$$

The wind speed is 30 mph.

49. Let x = the number of one design.
Let y = the number of the second design.
Then $x+y$ = the total number of sets of dishes.
$25x+45y$ = the cost of the dishes.
Setting up the equations and solving by substitution:

$$\begin{cases} x+y=200 \rightarrow y=200-x \\ 25x+45y=7400 \end{cases}$$

$$\begin{aligned} 25x+45(200-x)&=7400 \\ 25x+9000-45x&=7400 \\ -20x&=-1600 \\ x&=80 \\ y&=200-80=120 \end{aligned}$$

80 sets of the \$25 dishes and 120 sets of the \$45 dishes should be ordered.

50. Let x = the cost of a hot dog.
Let y = the cost of a soft drink.
Setting up the equations and solving by substitution:

$$\begin{cases} 10x+5y=12.50 \xrightarrow{5} 2x+y=2.50 \rightarrow y=2.50-2x \\ 7x+4y=9.00 \end{cases}$$

$$\begin{aligned} 7x+4(2.50-2x)&=9.00 \\ 7x+10.00-8x&=9.00 \\ -x&=-1 \\ x&=1.00 \\ y&=2.50-2(1)=0.50 \end{aligned}$$

A hot dog costs \$1.00 and a soft drink costs \$0.50.

51. Let x = the cost per package of bacon.
Let y = the cost of a carton of eggs.
Set up a system of equations for the problem:

$$\begin{cases} 3x+2y=7.45 \\ 2x+3y=6.45 \end{cases}$$

Multiply each side of the first equation by 3 and each side of the second equation by –2 and solve by elimination:

$$\begin{cases} 3x+2y=7.45 \xrightarrow{3} & 9x+6y=22.35 \\ 2x+3y=6.45 \xrightarrow{-2} & -4x-6y=-12.90 \end{cases}$$

$$\begin{aligned} 5x&=9.45 \\ x&=1.89 \end{aligned}$$

Substitute and solve for y:

$$3(1.89)+2y=7.45$$

$$5.67+2y=7.45 \rightarrow 2y=1.78 \rightarrow y=0.89$$

A package of bacon costs \$1.89 and a carton of eggs cost \$0.89.
The refund for 2 packages of bacon and 2 cartons of eggs will be \$5.56.

52. Let x = Pamela's speed in still water.
Let y = the speed of the current.

	Rate	Time	Distance
Downstream	$x+y$	3	15
Upstream	$x-y$	5	15

Set up a system of equations for the problem:

$$\begin{cases} 3(x+y)=15 \xrightarrow{3} x+y=5 \rightarrow y=5-x \\ 5(x-y)=15 \xrightarrow{5} x-y=3 \end{cases}$$

Solve by substitution:

$$x-(5-x)=3 \rightarrow x-5+x=3 \rightarrow 2x=8 \rightarrow x=4$$

$$y=5-4=1$$

Pamela's speed is 4 miles per hour and the speed of the current is 1 mile per hour.

53. Let x = the # of mg of liquid 1.
Let y = the # of mg of liquid 2.
Setting up the equations and solving by substitution:

$$\begin{cases} 0.2x+0.4y=40 \text{ vitamin C} \\ 0.3x+0.2y=30 \text{ vitamin D} \end{cases}$$

multiplying each equation by 10 yields

$$\begin{cases} 2x+4y=400 \longrightarrow 2x+4y=400 \\ 3x+2y=300 \xrightarrow{2} 6x+4y=600 \end{cases}$$

subtracting the bottom equation from the top equation yields

$$2x+4y-(6x+4y)=-200$$

$$2x-6x=-200 \rightarrow -4x=-200 \rightarrow x=50$$

$$\rightarrow 2(50)+4y=400 \rightarrow 100+4y=400$$

$$\rightarrow 4y=300 \rightarrow y=\frac{300}{4}=75$$

So 50 mg of liquid 1 should be mixed with 75 mg of liquid 2.

54. Let x = the # of units of powder 1.
Let y = the # of units of powder 2.
Setting up the equations and solving by substitution

$$\begin{cases} 0.2x+0.4y=12 \text{ vitamin } B_{12} \\ 0.3x+0.2y=12 \text{ vitamin E} \end{cases}$$

multiplying each equation by 10 yields

$$\begin{cases} 2x+4y=120 \longrightarrow 2x+4y=120 \\ 3x+2y=120 \xrightarrow{2} 6x+4y=240 \end{cases}$$

subtracting the bottom equation from the top equation yields

$2x + 4y - (6x + 4y) = -120 \rightarrow 2x - 6x = -120 \rightarrow -4x = -120 \rightarrow x = 30$

$\rightarrow 2(30) + 4y = 120 \rightarrow 60 + 4y = 120$

$\rightarrow 4y = 60$

$y = \dfrac{60}{4} = 15$

So 30 units of powder 1 should be mixed with 15 units of powder 2.

55. Solve the system by substitution:

$R = C$

$8x = 4.5x + 17500 \rightarrow 3.5x = 17500 \rightarrow x = 5000$

5000 units must be produced and sold for the firm to break-even.

56. Solve the system by substitution:

$R = C$

$12x = 10x + 15000 \rightarrow 2x = 15000 \rightarrow x = 7500$

7500 units must be produced and sold for the firm to break-even.

57. $\begin{cases} y = m_1 x + b_1 \\ y = m_2 x + b_2 \end{cases}$ subtracting the 2 equations yields

$0 = m_1 x + b_1 - m_2 x + b_2$

$-b_2 = x(m_1 - m_2)$

$\dfrac{-b_2}{m_1 - m_2} = x$, provided $m_1 \neq m_2$

so $y = m_1\left(\dfrac{-b_2}{m_1 - m_2}\right) + b_1 = \left(\dfrac{-b_2 m_1}{m_1 - m_2}\right) + b_1 = \dfrac{-b_2 m_1 + b_1(m_1 - m_2)}{m_1 - m_2}$

Therefore the solution set is $\left\{ x = \dfrac{-b_2}{m_1 - m_2}, y = \dfrac{-b_2 m_1 + b_1(m_1 - m_2)}{m_1 - m_2} \right\}$, provided $m_1 \neq m_2$.

58. $\begin{cases} y = m_1 x + b_1 \\ y = m_2 x + b_2 \end{cases}$ where $m_1 = m_2 = m$ and $b_1 \neq b_2$

$\begin{cases} y = mx + b_1 \\ y = mx + b_2 \end{cases}$ subtracting the 2 equations yields

$0 = 0 + b_1 - b_2$

$0 = b_1 - b_2$

$b_2 = b_1$

But this contradicts the assumption that $b_1 \neq b_2$, so there is no solution to the system. That is, the system is inconsistent.

59. $\begin{cases} y = m_1 x + b_1 \\ y = m_2 x + b_2 \end{cases}$ where $m_1 = m_2 = m$ and $b_1 = b_2$

$\begin{cases} y = mx + b_1 \\ y = mx + b_1 \end{cases}$ subtracting the 2 equations yields

$0 = 0$

Since this statement is always true, there are infinitely many solutions to the system. That is, the system is dependent.

60. Answers will vary. 61. Answers will vary. 62. Answers will vary.