Chapter 12

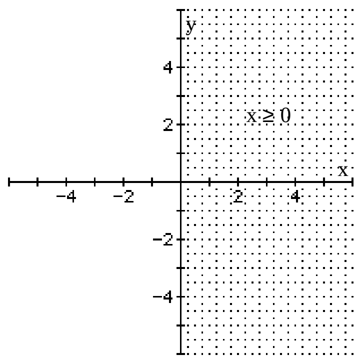
Systems of Equations and Inequalities

12.8 Systems of Inequalities

1. $x \geq 0$

Graph the line $x = 0$. Use a solid line since the inequality uses ≥.

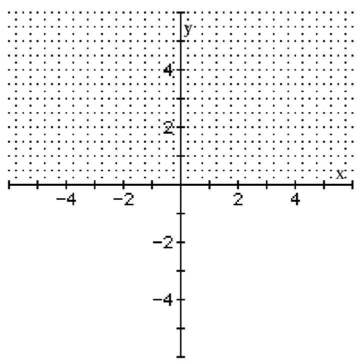
Choose a test point not on the line, such as (2, 0). Since $2 \geq 0$ is true, shade the side of the line containing (2, 0).



2. $y \geq 0$

Graph the line $y = 0$. Use a solid line since the inequality uses ≥.

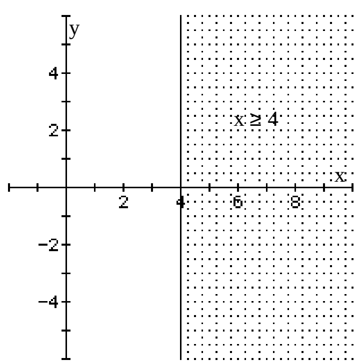
Choose a test point not on the line, such as (0, 2). Since $2 \geq 0$ is true, shade the side of the line containing (0, 2).



3. $x \geq 4$

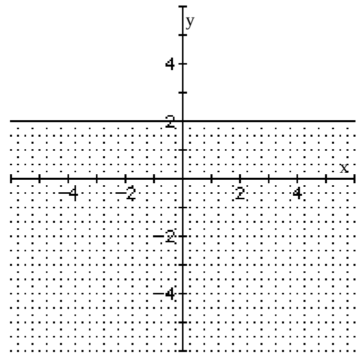
Graph the line $x = 4$. Use a solid line since the inequality uses ≥.

Choose a test point not on the line, such as (5, 0). Since $5 \geq 0$ is true, shade the side of the line containing (5, 0).



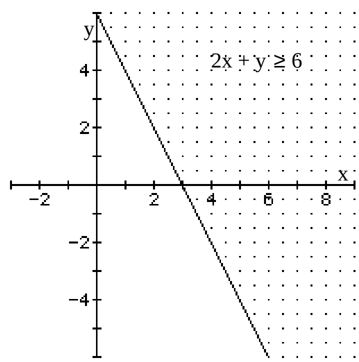
4. $y \le 2$

Graph the line $y = 2$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (5, 0).
Since 0≤ 2 is true, shade the side of the line containing (5, 0).



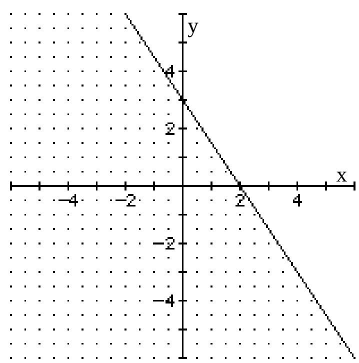
5. $2x + y \ge 6$

Graph the line $2x + y = 6$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $2(0)+0 \ge 6$ is false, shade the opposite side of the line from (0, 0).



6. $3x + 2y \le 6$

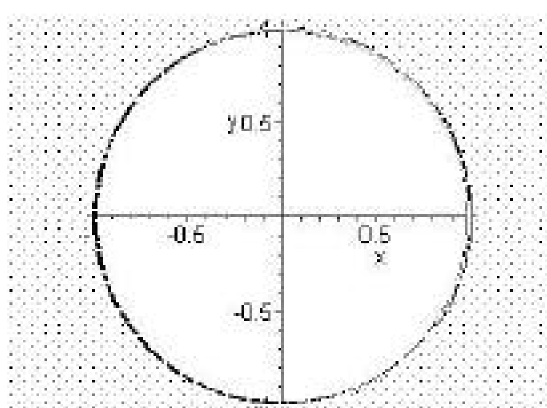
Graph the line $3x + 2y = 6$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $3(0) + 2(0) \le 6$ is true, shade the side of the line containing (0, 0).



7. $x^2 + y^2 > 1$

Graph the circle $x^2 + y^2 > 1$. Use a dashed line since the inequality uses >.

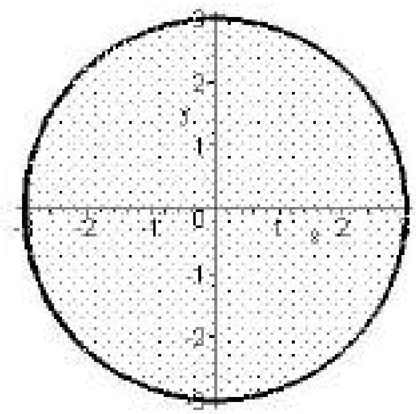
Choose a test point not on the circle, such As (0, 0). Since $0^2 + 0^2 > 1$ is false, shade the opposite side of the circle from (0, 0).



8. $x^2 + y^2 \le 9$

Graph the circle $x^2 + y^2 = 9$. Use a solid Line since the inequality uses $\le$.

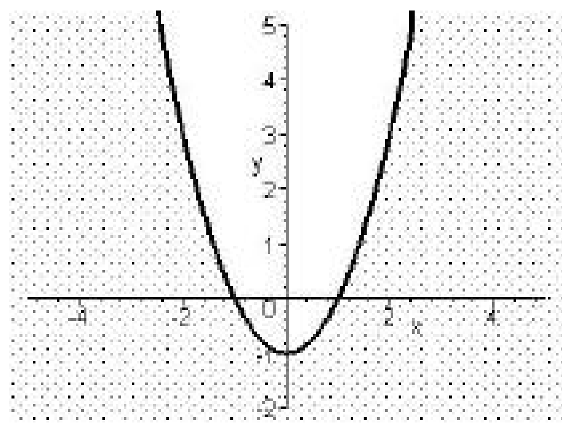
Choose a test point not on the circle, such as (0, 0). Since $0^2 + 0^2 \le 9$ is true, shade the same side of the circle as (0, 0).



9. $y \le x^2 - 1$

Graph the parabola $y = x^2 - 1$. Use a solid line since the inequality uses $\le$.

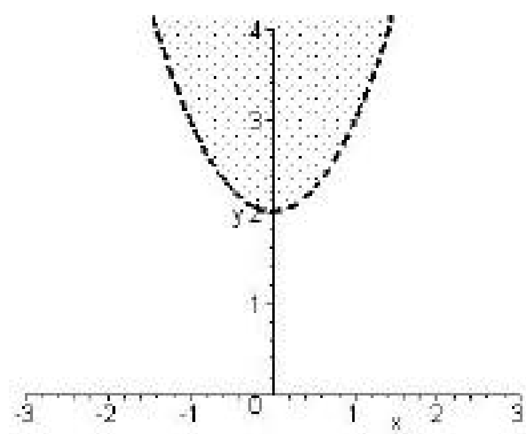
Choose a test point not on the parabola, such as (0, 0). Since $0 \le 0^2 - 1$ is false, shade the opposite side of the parabola from (0, 0).



10. $y > x^2 + 2$

Graph the line $y = x^2 + 2$. Use a dashed line since the inequality uses >.

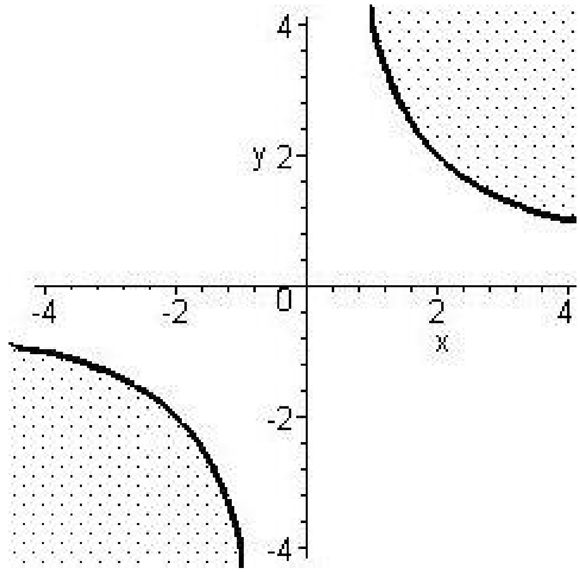
Choose a test point not on the line, such as (0, 0). Since $0 > 0^2 + 2$ is false, shade the opposite side of the parabola from (0, 0).



11. $xy \ge 4$

Graph the hyperbola $xy = 4$. Use a solid line since the inequality uses $\ge$.

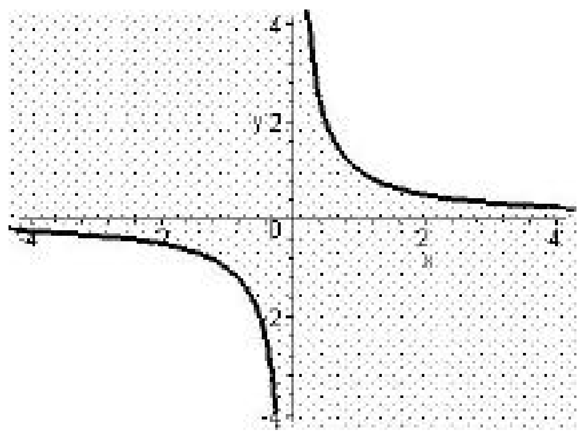
Choose a test point not on the hyperbola, such as (0, 0). Since $0\ \ 0 \ge 4$ is false, shade the opposite side of the hyperbola from (0, 0).



12. $xy \le 1$

Graph the hyperbola $xy = 1$. Use a solid line since the inequality uses $\ge$.

Choose a test point not on the hyperbola, such as (0, 0). Since $0\ \ 0 \le 1$ is true, shade the same side of the hyperbola as (0, 0).

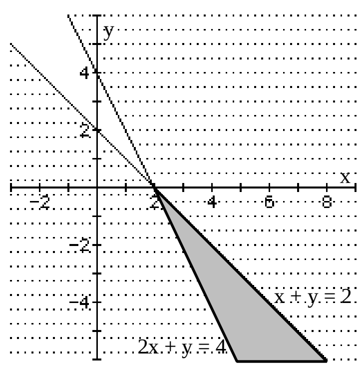


13. $\begin{cases} x + y \le 2 \\ 2x + y \ge 4 \end{cases}$

(a) Graph the line $x + y = 2$. Use a solid line since the inequality uses $\le$.
Choose a test point not on the line, such as (0, 0). Since $0 + 0 \le 2$ is true, shade the side of the line containing (0, 0).

(b) Graph the line $2x + y = 4$. Use a solid line since the inequality uses $\ge$.
Choose a test point not on the line, such as (0, 0). Since $2(0) + 0 \ge 4$ is false, shade the opposite side of the line from (0, 0).

(c) The overlapping region is the solution

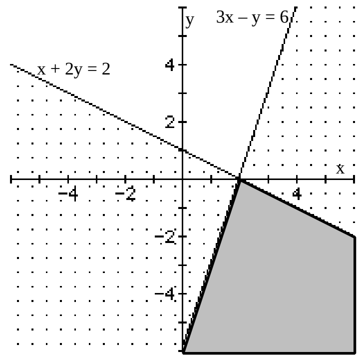


14. $\begin{cases} 3x - y \geq 6 \\ x + 2y \leq 2 \end{cases}$

(a) Graph the line $3x - y = 6$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $3(0) - 0 \geq 6$ is false, shade the opposite side of the line from (0, 0).

(b) Graph the line $x + 2y = 2$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0 + 2(0) \leq 2$ is true, shade the side of the line containing (0, 0).

(c) The overlapping region is the solution

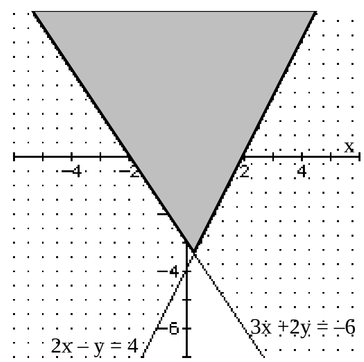


15. $\begin{cases} 2x - y \leq 4 \\ 3x + 2y \geq -6 \end{cases}$

(a) Graph the line $2x - y = 4$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $2(0) - 0 \leq 4$ is true, shade the side of the line containing (0, 0).

(b) Graph the line $3x + 2y = -6$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $3(0) + 2(0) \geq -6$ is true, shade the side of the line containing (0, 0).

(c) The overlapping region is the solution

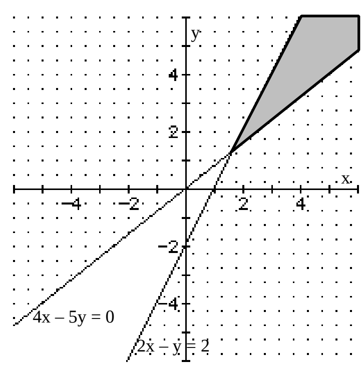


16. $\begin{cases} 4x - 5y \leq 0 \\ 2x - y \geq 2 \end{cases}$

(a) Graph the line $4x - 5y = 0$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (2, 0). Since $4(2) - 5(0) \leq 0$ is false, shade the opposite side of the line from (2, 0).

(b) Graph the line $2x - y = 2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $2(0) - 0 \geq 2$ is false, shade the opposite side of the line from (0, 0).

(c) The overlapping region is the solution

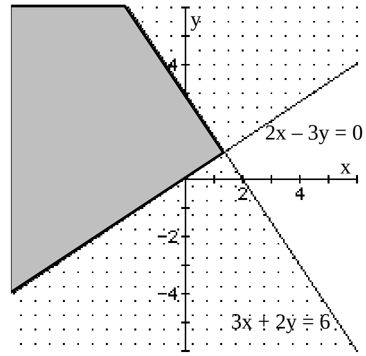


17. $\begin{cases} 2x - 3y \le 0 \\ 3x + 2y \le 6 \end{cases}$

(a) Graph the line $2x - 3y = 0$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 3). Since $2(0) - 3(3) \le 0$ is true, shade the side of the line containing (0, 3).

(b) Graph the line $3x + 2y = 6$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $3(0) + 2(0) \le 6$ is true, shade the side of the line containing (0, 0).

(c) The overlapping region is the solution

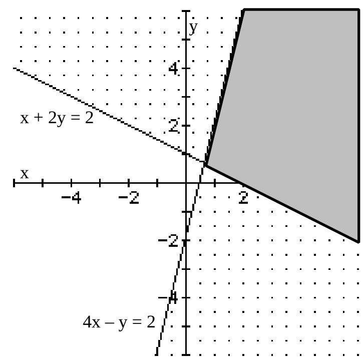


18. $\begin{cases} 4x - y \ge 2 \\ x + 2y \ge 2 \end{cases}$

(a) Graph the line $4x - y = 2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $4(0) - 0 \ge 2$ is false, shade the opposite side of the line from (0, 0).

(b) Graph the line $x + 2y = 2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $0 + 2(0) \ge 2$ is false, shade the opposite side of the line from (0, 0).

(c) The overlapping region is the solution

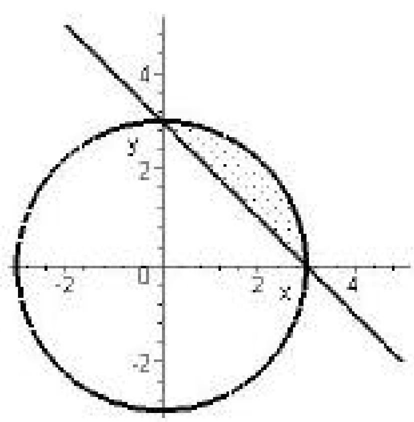


19. $\begin{cases} x^2 + y^2 \le 9 \\ x + y \ge 3 \end{cases}$

(a) Graph the circle $x^2 + y^2 = 9$. Use a solid line since the inequality uses ≥. Choose a test point not on the circle, such as (0, 0). Since $0^2 + 0^2 \le 9$ is true, shade the same side of the circle as (0, 0).

(b) Graph the line $x + y = 3$. Use a solid line since the inequality uses ≥. Choose a test point not on the line, such as (0, 0). Since $0 + 0 \ge 3$ is false, shade the opposite side of the line from (0, 0).

(c) The overlapping region is the solution

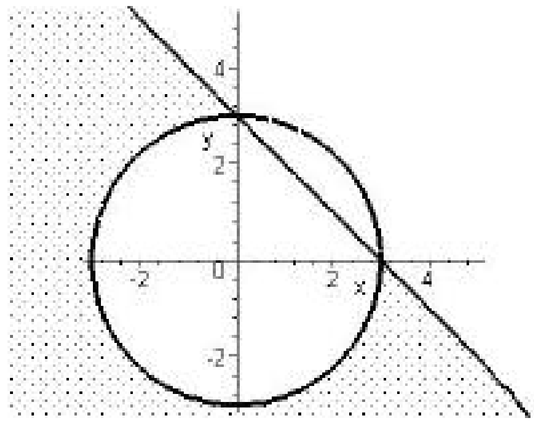


20. $\begin{cases} x^2 + y^2 \geq 9 \\ x + y \leq 3 \end{cases}$

(a) Graph the circle $x^2 + y^2 = 9$.
Use a solid line since the inequality uses ≥.
Choose a test point not on the circle, such as (0, 0). Since $0^2 + 0^2 \geq 9$ is false, shade the opposite side of the circle as (0, 0).

(b) Graph the line $x + y = 3$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0 + 0 \leq 3$ is true, shade the same side of the line as (0, 0).

(c) The overlapping region is the solution

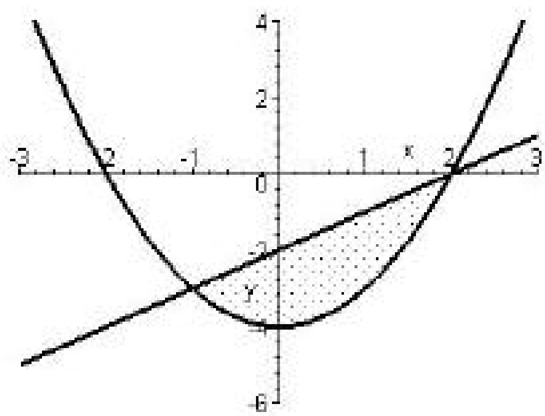


21. $\begin{cases} y \geq x^2 - 4 \\ y \leq x - 2 \end{cases}$

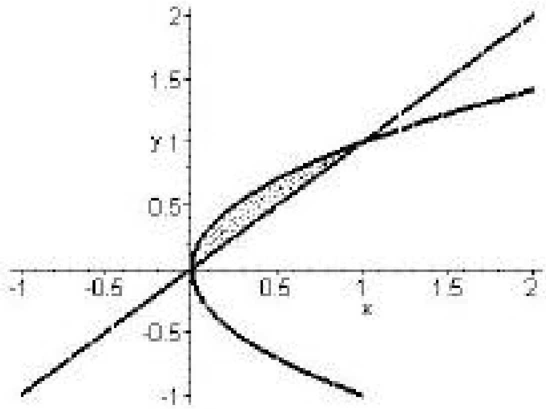
(a) Graph the parabola $y = x^2 - 4$.
Use a solid line since the inequality uses ≥.
Choose a test point not on the parabola, such as (0, 0). Since $0 \geq 0^2 - 4$ is true, shade the same side of the parabola as (0, 0).

(b) Graph the line $y = x - 2$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0 \leq 0 - 2$ is false, shade the opposite side of the line from (0, 0).

(c) The overlapping region is the solution



22. $\begin{cases} y^2 \le x \\ y \ge x \end{cases}$

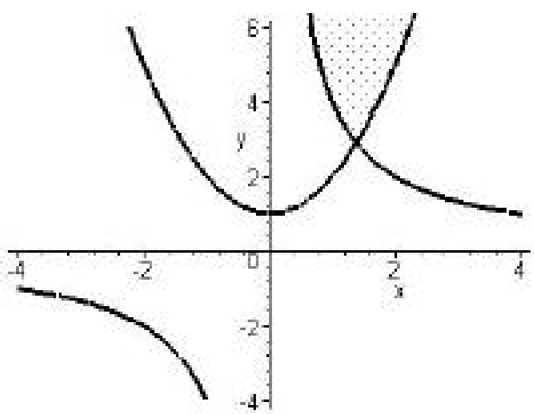


(a) Graph the parabola $y^2 = x$.
Use a solid line since the inequality uses $\le$.
Choose a test point not on the parabola, such as (1, 2). Since $2^2 \le 1$ is false, shade the opposite side of the parabola from (1, 2).

(b) Graph the line $y = x$. Use a solid line since the inequality uses $\ge$.
Choose a test point not on the line, such as (1, 2). Since $2 \ge 1$ is true, shade the same side of the line as (1, 2).

(c) The overlapping region is the solution

23. $\begin{cases} xy \ge 4 \\ y \ge x^2 + 1 \end{cases}$

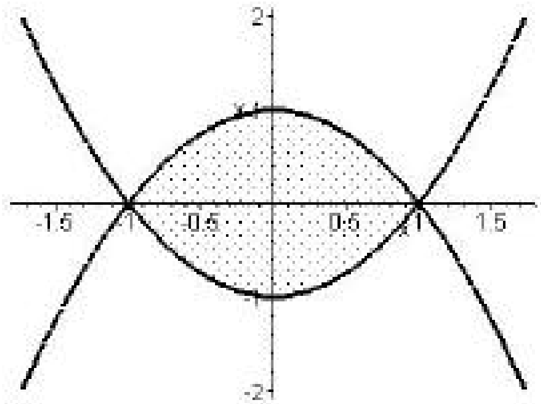


(a) Graph the hyperbola $xy = 4$.
Use a solid line since the inequality uses $\ge$.
Choose a test point not on the parabola, such as (0, 0). Since $0 \quad 0 \ge 4$ is false, shade the opposite side of the hyperbola from (0, 0).

(b) Graph the parabola $y = x^2 + 1$. Use a solid line since the inequality uses $\ge$.
Choose a test point not on the parabola, such as (0, 0). Since $0 \ge 0^2 + 1$ is false, shade the opposite side of the parabola from (0, 0).

(c) The overlapping region is the solution

24. $\begin{cases} y + x^2 \le 1 \\ y \ge x^2 - 1 \end{cases}$

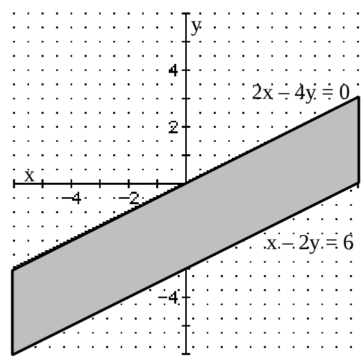


(a) Graph the parabola $y + x^2 = 1$.
Use a solid line since the inequality uses $\le$.
Choose a test point not on the parabola, such as (0, 0). Since $0 + 0^2 \le 1$ is true, shade the same side of the parabola as (0, 0).

(b) Graph the parabola $y = x^2 - 1$. Use a solid line since the inequality uses $\ge$.
Choose a test point not on the parabola, such as (0, 0). Since $0 \ge 0^2 - 1$ is false, shade the opposite side of the parabola from (0, 0).

(c) The overlapping region is the solution

25. $\begin{cases} x - 2y \le 6 \\ 2x - 4y \ge 0 \end{cases}$

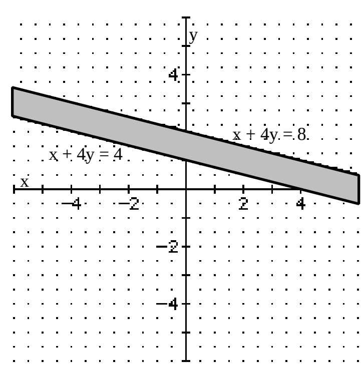


(a) Graph the line $x - 2y = 6$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0 - 2(0) \le 6$ is true, shade the side of the line containing (0, 0).

(b) Graph the line $2x - 4y = 0$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 2). Since $2(0) - 4(2) \ge 0$ is false, shade the opposite side of the line from (0, 2).

(c) The overlapping region is the solution

26. $\begin{cases} x + 4y \le 8 \\ x + 4y \ge 4 \end{cases}$



(a) Graph the line $x + 4y = 8$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0 + 4(0) \le 8$ is true, shade the side of the line containing (0, 0).

(b) Graph the line $x + 4y = 4$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $0 + 4(0) \ge 4$ is false, shade the opposite side of the line from (0, 0).

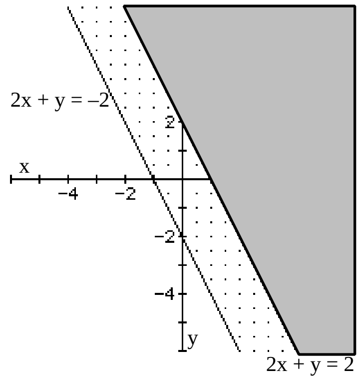
(c) The overlapping region is the solution

27. $\begin{cases} 2x + y \geq -2 \\ 2x + y \geq \ \ 2 \end{cases}$

(a) Graph the line $2x + y = -2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $2(0) + 0 \geq -2$ is true, shade the side of the line containing (0, 0).

(b) Graph the line $2x + y = 2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $2(0) + 0 \geq 2$ is false, shade the opposite side of the line from (0, 0).

(c) The overlapping region is the solution.

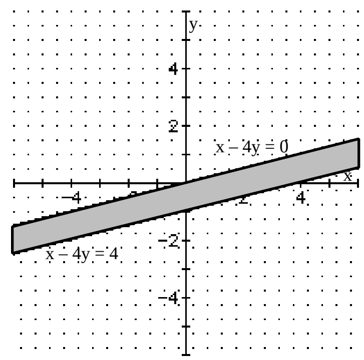


28. $\begin{cases} x - 4y \leq 4 \\ x - 4y \geq 0 \end{cases}$

(a) Graph the line $x - 4y = 4$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0 - 4(0) \leq 4$ is true, shade the side of the line containing (0, 0).

(b) Graph the line $x - 4y = 0$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (1, 0). Since $1 - 4(0) \geq 0$ is true, shade the side of the line containing (1, 0).

(c) The overlapping region is the solution.

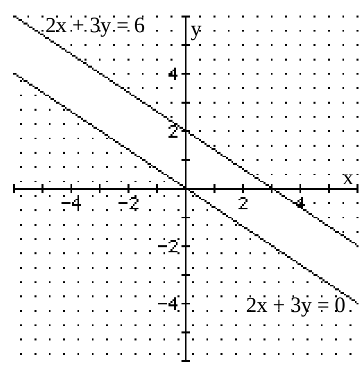


29. $\begin{cases} 2x + 3y \geq 6 \\ 2x + 3y \leq 0 \end{cases}$

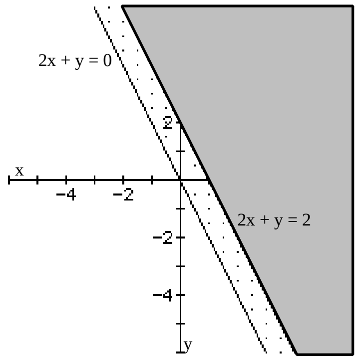
(a) Graph the line $2x + 3y = 6$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $2(0) + 3(0) \geq 6$ is false, shade the opposite side of the line from (0, 0).

(b) Graph the line $2x + 3y = 0$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 2). Since $2(0) + 3(2) \leq 0$ is false, shade the opposite side of the line from (0, 2).

(c) Since the regions do not overlap, the solution is an empty set.



30. $\begin{cases} 2x + y \ge 0 \\ 2x + y \ge 2 \end{cases}$



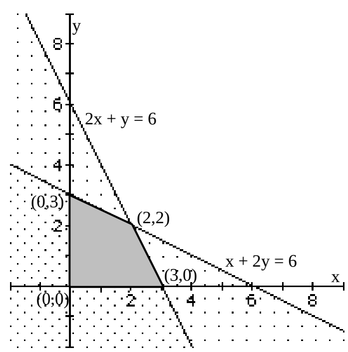
(a) Graph the line $2x + y = 0$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (1, 0). Since $2(1) + 0 \ge 0$ is true, shade the side of the line containing (1, 0).

(b) Graph the line $2x + y = 2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $2(0) + 0 \ge 2$ is false, shade the opposite side of the line from (0, 0).

(c) The overlapping region is the solution.

31. Graph the system of linear inequalities:

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ 2x + y \le 6 \\ x + 2y \le 6 \end{cases}$$



(a) Graph $x \ge 0; y \ge 0$. Shaded region is the first quadrant.

(b) Graph the line $2x + y = 6$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $2(0) + 0 \le 6$ is true, shade the side of the line containing (0, 0).

(c) Graph the line $x + 2y = 6$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0 + 2(0) \le 6$ is true, shade the side of the line containing (0, 0).

(d) The overlapping region is the solution.

(e) The graph is bounded.

(f) Find the vertices:
The x-axis and y-axis intersect at (0, 0).
The intersection of $x + 2y = 6$ and the y-axis is (0, 3).
The intersection of $2x + y = 6$ and the x-axis is (3, 0).
To find the intersection of $x + 2y = 6$ and $2x + y = 6$, solve the system:

$$\begin{cases} x + 2y = 6 & \rightarrow \quad x = 6 - 2y \\ 2x + y = 6 \end{cases}$$

Substitute and solve:

$$2(6 - 2y) + y = 6 \rightarrow 12 - 4y + y = 6 \rightarrow -3y = -6 \rightarrow y = 2$$

$$x = 6 - 2(2) = 6 - 4 = 2$$

The point of intersection is (2, 2).
The four corner points are (0, 0), (0, 3), (3, 0), and (2, 2

32. Graph the system of linear inequalities:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \geq 4 \\ 2x + 3y \geq 6 \end{cases}$$

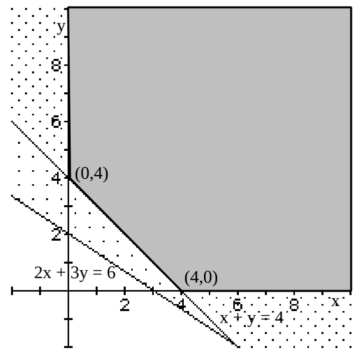
(a) Graph $x \geq 0;\ y \geq 0$. Shaded region is the first quadrant.

(b) Graph the line $x + y = 4$. Use a solid line since the inequality uses ≥.

Choose a test point not on the line, such as (0, 0). Since $0 + 0 \geq 4$ is false, shade the opposite side of the line from (0, 0).

(c) Graph the line $2x + 3y = 6$. Use a solid line since the inequality uses ≥.

Choose a test point not on the line, such as (0, 0). Since $2(0) + 3(0) \geq 6$ is false, shade the opposite side of the line from (0, 0).



(d) The overlapping region is the solution.

(e) The graph is unbounded.

(f) Find the vertices:

The intersection of $x + y = 4$ and the y-axis is (0, 4).

The intersection of $x + y = 4$ and the x-axis is (4, 0).

The two corner points are (0, 4), and (4, 0).

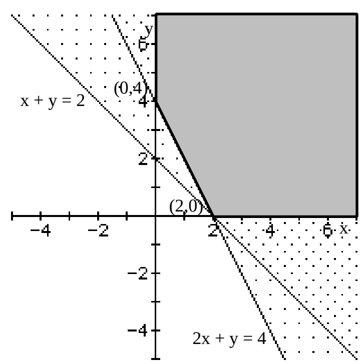
33. Graph the system of linear inequalities:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \geq 2 \\ 2x + y \geq 4 \end{cases}$$

(a) Graph $x \geq 0;\ y \geq 0$. Shaded region is the first quadrant.

(b) Graph the line $x + y = 2$. Use a solid line since the inequality uses ≥.

Choose a test point not on the line, such as (0, 0). Since 0 + 0 ≥ 2 is false, shade the opposite side of the line from (0, 0).



(c) Graph the line $2x + y = 4$. Use a solid line since the inequality uses ≥.

Choose a test point not on the line, such as (0, 0). Since $2(0) + 0 \geq 4$ is false, shade the opposite side of the line from (0, 0).

(d) The overlapping region is the solution.

(e) The graph is unbounded.

(f) Find the vertices:

The intersection of $x + y = 2$ and the x-axis is (2, 0).

The intersection of $2x + y = 4$ and the y-axis is (0, 4).

The two corner points are (2, 0), and (0, 4).

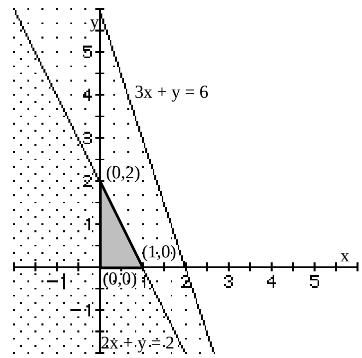
34. Graph the system of linear inequalities:

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ 3x + y \le 6 \\ 2x + y \le 2 \end{cases}$$

(a) Graph $x \ge 0$; $y \ge 0$. Shaded region is the first quadrant.

(b) Graph the line $3x + y = 6$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $3(0) + 0 \le 6$ is true, shade the side of the line containing (0, 0).

(c) Graph the line $2x + y = 2$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $2(0) + 0 \le 2$ is true, shade the side of the line containing (0, 0).



(d) The overlapping region is the solution. (e) The graph is bounded.

(f) Find the vertices:
The intersection of $x = 0$ and $y = 0$ is (0, 0).
The intersection of $2x + y = 2$ and the x-axis is (1, 0).
The intersection of $2x + y = 2$ and the y-axis is (0, 2).
The three corner points are (0, 0), (1, 0), and (0, 2).

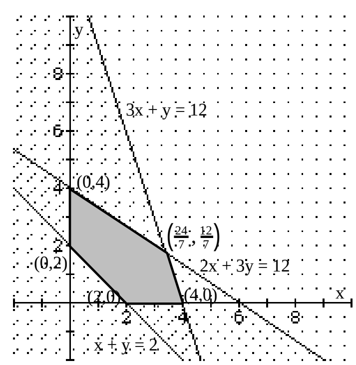
35. Graph the system of linear inequalities:

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ x + y \ge 2 \\ 2x + 3y \le 12 \\ 3x + y \le 12 \end{cases}$$

(a) Graph $x \ge 0$; $y \ge 0$. Shaded region is the first quadrant.

(b) Graph the line $x + y = 2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $0 + 0 \ge 2$ is false, shade the opposite side of the line from (0, 0).

(c) Graph the line $2x + 3y = 12$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $2(0) + 3(0) \le 12$ is true, shade the side of the line containing (0, 0).



(d) Graph the line $3x + y = 12$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $3(0) + 0 \le 12$ is true, shade the side of the line containing (0, 0).

(e) The overlapping region is the solution.

(f) The graph is bounded.

(g) Find the vertices:

The intersection of $x + y = 2$ and the y-axis is (0, 2).
The intersection of $x + y = 2$ and the x-axis is (2, 0).
The intersection of $2x + 3y = 12$ and the y-axis is (0, 4).
The intersection of $3x + y = 12$ and the x-axis is (4, 0).
To find the intersection of $2x + 3y = 12$ and $3x + y = 12$, solve the system:

$$\begin{cases} 2x + 3y = 12 \\ 3x + y = 12 \quad \rightarrow \quad y = 12 - 3x \end{cases}$$

Substitute and solve:

$$2x + 3(12 - 3x) = 12 \rightarrow 2x + 36 - 9x = 12$$

$$-7x = -24 \rightarrow x = \frac{24}{7}$$

$$y = 12 - 3\left(\frac{24}{7}\right) = 12 - \frac{72}{2} = \frac{12}{7}$$

The point of intersection is $\left(\frac{24}{7}, \frac{12}{7}\right)$.

The five corner points are (0, 2), (0, 4), (2, 0), (4, 0), and $\left(\frac{24}{7}, \frac{12}{7}\right)$.

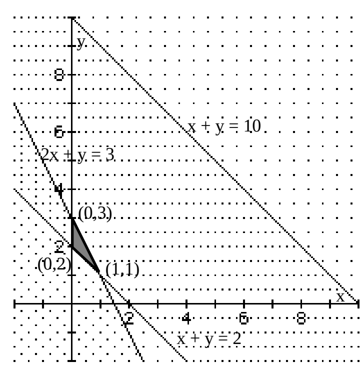
36. Graph the system of linear inequalities:

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ x + y \ge 2 \\ x + y \le 10 \\ 2x + y \le 3 \end{cases}$$

(a) Graph $x \ge 0; y \ge 0$. Shaded region is the first quadrant.

(b) Graph the line $x + y = 2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $0 + 0 \ge 2$ is false, shade the opposite side of the line from (0, 0).

(c) Graph the line $x + y = 10$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0 + 0 \le 10$ is true, shade the side of the line containing (0, 0).



(d) Graph the line $2x + y = 3$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $2(0) + 0 \le 3$ is true, shade the side of the line containing (0, 0).

(e) The overlapping region is the solution.

(f) The graph is bounded.

(g) Find the vertices:

The intersection of $x + y = 2$ and the y-axis is (0, 2).

The intersection of $2x+y=3$ and the y-axis is (0, 3).
To find the intersection of $2x+y=3$ and $x+y=2$, solve the system:

$$\begin{cases} 2x+y=3 \\ x+y=2 \quad \rightarrow \quad y=2-x \end{cases}$$

Substitute and solve:

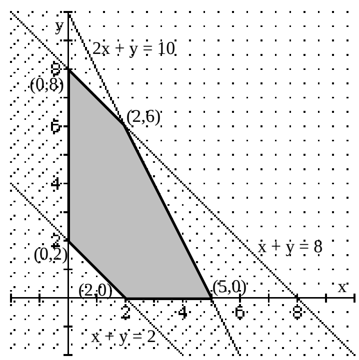
$$2x+2-x=3 \rightarrow x=1$$
$$y=2-1=1$$

The point of intersection is (1, 1).
The three corner points are (0, 2), (0, 3), and (1, 1).

37. Graph the system of linear inequalities:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x+y \geq 2 \\ x+y \leq 8 \\ 2x+y \leq 10 \end{cases}$$

(a) Graph $x \geq 0$; $y \geq 0$. Shaded region is the first quadrant.
(b) Graph the line $x+y=2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $0+0 \geq 2$ is false, shade the opposite side of the line from (0, 0).
(c) Graph the line $x+y=8$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0+0 \leq 8$ is true, shade the side of the line containing (0, 0).



(d) Graph the line $2x+y=10$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $2(0)+0 \leq 10$ is true, shade the side of the line containing (0, 0).
(e) The overlapping region is the solution.
(f) The graph is bounded.
(g) Find the vertices:
The intersection of $x+y=2$ and the y-axis is (0, 2).
The intersection of $x+y=2$ and the x-axis is (2, 0).
The intersection of $x+y=8$ and the y-axis is (0, 8).
The intersection of $2x+y=10$ and the x-axis is (5, 0).
To find the intersection of $x+y=8$ and $2x+y=10$, solve the system:

$$\begin{cases} x+y=8 \quad \rightarrow \quad y=8-x \\ 2x+y=10 \end{cases}$$

Substitute and solve:

$$2x+8-x=10 \rightarrow x=2$$
$$y=8-2=6$$

The point of intersection is (2, 6).
The five corner points are (0, 2), (0, 8), (2, 0), (5, 0), and (2, 6).

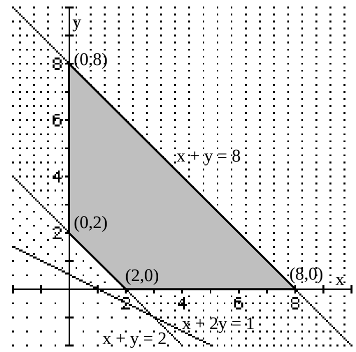
38. Graph the system of linear inequalities:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \geq 2 \\ x + y \leq 8 \\ x + 2y \geq 1 \end{cases}$$

(a) Graph $x \geq 0;\ y \geq 0$. Shaded region is the first quadrant.

(b) Graph the line $x + y = 2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $0 + 0 \geq 2$ is false, shade the opposite side of the line from (0, 0).

(c) Graph the line $x + y = 8$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0 + 0 \leq 8$ is true, shade the side of the line containing (0, 0).



(d) Graph the line $x + 2y = 1$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $0 + 2(0) \geq 1$ is false, shade the opposite side of the line from (0, 0).

(e) The overlapping region is the solution.

(f) The graph is bounded.

(g) Find the vertices:
The intersection of $x + y = 2$ and the y-axis is (0, 2).
The intersection of $x + y = 2$ and the x-axis is (2, 0).
The intersection of $x + y = 8$ and the y-axis is (0, 8).
The intersection of $x + y = 8$ and the x-axis is (8, 0).
The four corner points are (0, 2), (0, 8), (2, 0), and (8, 0).

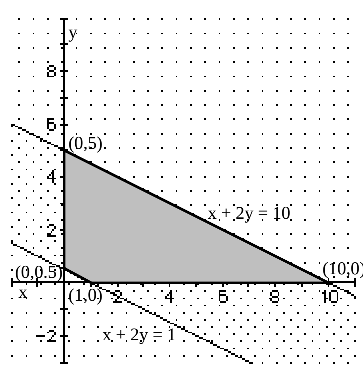
39. Graph the system of linear inequalities:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + 2y \geq 1 \\ x + 2y \leq 10 \end{cases}$$

(a) Graph $x \geq 0;\ y \geq 0$. Shaded region is the first quadrant.

(b) Graph the line $x + 2y = 1$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $0 + 2(0) \geq 1$ is false, shade the opposite side of the line from (0, 0).

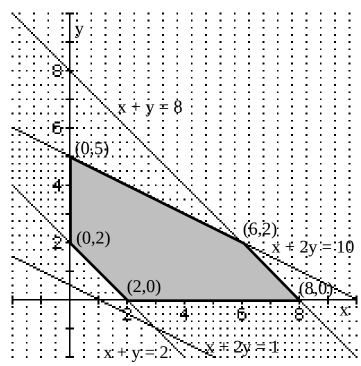
(c) Graph the line $x + 2y = 10$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0 + 2(0) \leq 10$ is true, shade the side of the line containing (0, 0).



(d) The overlapping region is the solution.
(e) The graph is bounded.
(f) Find the vertices:
The intersection of $x+2y=1$ and the y-axis is (0, 0.5).
The intersection of $x+2y=1$ and the x-axis is (1, 0).
The intersection of $x+2y=10$ and the y-axis is (0, 5).
The intersection of $x+2y=10$ and the x-axis is (10, 0).
The four corner points are (0, 0.5), (0, 5), (1, 0), and (10, 0).

40. Graph the system of linear inequalities:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x+2y \geq 1 \\ x+2y \leq 10 \\ x+y \geq 2 \\ x+y \leq 8 \end{cases}$$



(a) Graph $x \geq 0; y \geq 0$. Shaded region is the first quadrant.
(b) Graph the line $x+2y=1$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $0+2(0) \geq 1$ is false, shade the opposite side of the line from (0, 0).
(c) Graph the line $x+2y=10$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0+2(0) \leq 10$ is true, shade the side of the line containing (0, 0).
(d) Graph the line $x+y=2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $0+0 \geq 2$ is false, shade the opposite side of the line from (0, 0).
(e) Graph the line $x+y=8$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0+0 \leq 8$ is true, shade the side of the line containing (0, 0).
(f) The overlapping region is the solution.
(g) The graph is bounded.
(h) Find the vertices:
The intersection of $x+y=2$ and the y-axis is (0, 2).
The intersection of $x+y=2$ and the x-axis is (2, 0).
The intersection of $x+2y=10$ and the y-axis is (0, 5).
The intersection of $x+y=8$ and the x-axis is (8, 0).
To find the intersection of $x+y=8$ and $x+2y=10$, solve the system:

$$\begin{cases} x+y=8 \quad \rightarrow \quad y=8-x \\ x+2y=10 \end{cases}$$

Substitute and solve:

$$x+2(8-x)=10 \rightarrow x+16-2x=10 \rightarrow -x=-6 \rightarrow x=6 \rightarrow y=8-6=2$$

The point of intersection is (6, 2).
The five corner points are (0, 2), (0, 5), (2, 0), (8, 0), and (6, 2).

41. The system of linear inequalities is:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 4 \\ x + y \leq 6 \end{cases}$$

42. The system of linear inequalities is:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 6 \\ y \leq 5 \\ x + y \geq 2 \end{cases}$$

43. The system of linear inequalities is:

$$\begin{cases} x \geq 0 \\ y \geq 15 \\ x \leq 20 \\ x + y \leq 50 \\ x - y \leq 0 \end{cases}$$

44. The system of linear inequalities is:

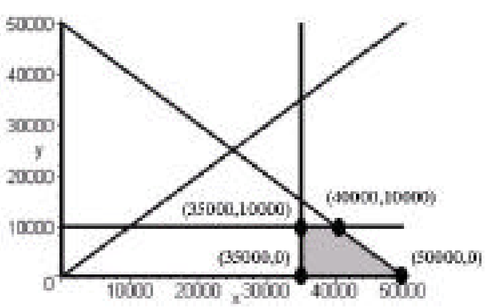
$$\begin{cases} x \geq 0 \\ y \leq 6 \\ x \leq 5 \\ 3x + 4y \geq 12 \\ 2x - y \leq 8 \end{cases}$$

45. (a) Let x = the amount invested in Treasury bills.
Let y = the amount invested in corporate bonds.

The constraints are:

$x \geq 0, y \geq 0$	A non-negative amount must be invested.
$x + y \leq 50000$	Total investment cannot exceed \$50,000.
$y \leq 10000$	Amount invested in corporate bonds must not exceed \$10,000.
$x \geq 35000$	Amount invested in Treasury bills must be at least \$35,000.
$x > y$	Amount invested in Treasury bills must be greater than the amount invested in corporate bonds.

(b) Graph the system $\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \leq 50000 \\ y \leq 10000 \\ x \geq 35000 \\ x > y \end{cases}$



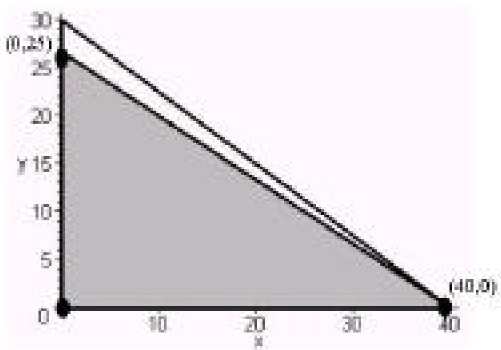
The corner points are (35000, 0), (35000, 10000), (40000, 10000), (50000, 0).

46. (a) Let x = the # of standard model trucks.
Let y = the # of deluxe model trucks.

The constraints are:

$x \ge 0, y \ge 0$	A non-negative number of trucks must be manufactured.
$2x + 3y \le 80$	Total painting hours worked cannot exceed 80.
$3x + 4y \le 120$	Total detailing hours worked cannot exceed 120.

(b) Graph the system.



The corner points are (0, 0), (0, 25), (40, 0).

47. (a) Let x = the # of packages of the economy blend.
Let y = the # of packages of the superior blend.

The constraints are:

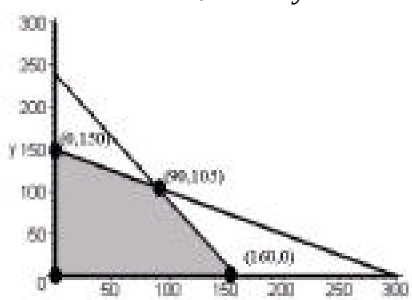
$x \ge 0, y \ge 0$	A non-negative # of packages must be produced.
$4x + 8y \le 75\ 16$	Total amount of grade A coffee cannot exceed 75 pounds. (Note: 75 pounds = (75)(16) ounces.)
$12x + 8y \le 120\ 16$	Total amount of grade B coffee cannot exceed 120 pounds. (Note: 120 pounds = (120)(16) ounces.)

We can simplify the equations

$$4x + 8y \le 75\ 16 \rightarrow x + 2y \le 75\ 4 \rightarrow x + 2y \le 300$$

$$12x + 8y \le 120\ 16 \rightarrow 3x + 2y \le 120\ 4 \rightarrow 3x + 2y \le 480$$

(b) Graph the system. $\begin{cases} x \ge 0 \\ y \ge 0 \\ x + 2y \le 300 \\ 3x + 2y \le 480 \end{cases}$



The corner points are (0, 0), (0, 150), (90, 105), (160, 0).

48. (a) Let x = the # of lower priced packages.
Let y = the # of quality packages.

The constraints are:

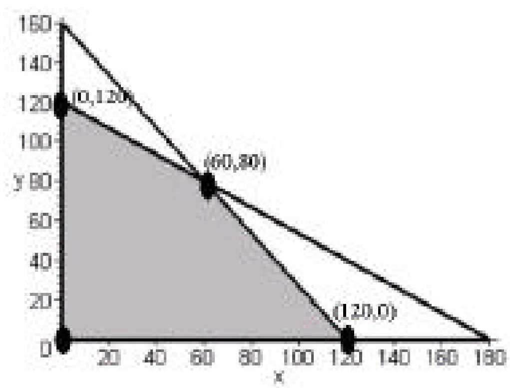
$x \geq 0, y \geq 0$	A non-negative # of packages must be produced.
$8x + 6y \leq 120\ \ 16$	Total amount of peanuts cannot exceed 120 pounds. (Note: 120 pounds = (120)(16) ounces.)
$4x + 6y \leq 90\ \ 16$	Total amount of cashews cannot exceed 90 pounds. (Note: 90 pounds = (90)(16) ounces.)

We can simplify the equations

$8x + 6y \leq 120\ \ 16 \rightarrow 4x + 3y \leq 120\ \ 4 \rightarrow 4x + 3y \leq 480$

$4x + 6y \leq 90\ \ 16 \rightarrow 2x + 3y \leq 90\ \ 4 \rightarrow 2x + 3y \leq 360$

(b) Graph the system. $\begin{cases} x \geq 0 \\ y \geq 0 \\ 4x + 3y \leq 480 \\ 2x + 3y \leq 360 \end{cases}$



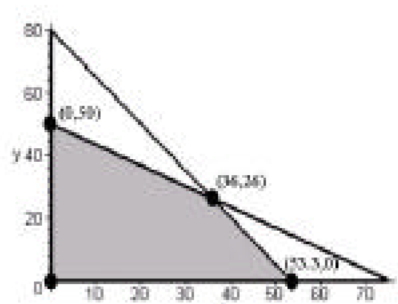
The corner points are (0, 0), (0, 120), (60, 80), (120, 0).

49. (a) Let x = the # of microwaves.
Let y = the # of printers.

The constraints are:

$x \geq 0, y \geq 0$	A non-negative # of items must be shipped.
$30x + 20y \leq 1600$	Total cargo weight cannot exceed 1600 pounds.
$2x + 3y \leq 150$	Total cargo volume cannot exceed 150 cubic feet.

(b) Graph the system. $\begin{cases} x \geq 0 \\ y \geq 0 \\ 30x + 20y \leq 1600 \\ 2x + 3y \leq 150 \end{cases}$



The corner points are (0, 0), (0, 50), (36, 26), (53.3, 0).