Chapter 12

Systems of Equations and Inequalities

12.9 Linear Programming

1. $z = x + y$

Vertex	Value of $z = x + y$
(0, 3)	z = 0 + 3 = 3
(0, 6)	z = 0 + 6 = 6
(5, 6)	z = 5 + 6 = 11
(5, 2)	z = 5 + 2 = 7
(4, 0)	z = 4 + 0 = 4

The maximum value is 11 at (5, 6), and the minimum value is 3 at (0, 3).

2. $z = 2x + 3y$

Vertex	Value of $z = 2x + 3y$
(0, 3)	z = 2(0) + 3(3) = 9
(0, 6)	z = 2(0) + 3(6) = 18
(5, 6)	z = 2(5) + 3(6) = 28
(5, 2)	z = 2(5) + 3(2) = 16
(4, 0)	z = 2(4) + 3(0) = 8

The maximum value is 28 at (5, 6), and the minimum value is 8 at (4, 0).

3. $z = x + 10y$

Vertex	Value of $z = x + 10y$
(0, 3)	z = 0 + 10(3) = 30
(0, 6)	z = 0 + 10(6) = 60
(5, 6)	z = 5 + 10(6) = 65
(5, 2)	z = 5 + 10(2) = 25
(4, 0)	z = 4 + 10(0) = 4

The maximum value is 65 at (5, 6), and the minimum value is 4 at (4, 0).

4. $z = 10x + y$

Vertex	Value of $z = 10x + y$
(0, 3)	z = 10(0) + 3 = 3
(0, 6)	z = 10(0) + 6 = 6
(5, 6)	z = 10(5) + 6 = 56
(5, 2)	z = 10(5) + 2 = 52
(4, 0)	z = 10(4) + 0 = 40

The maximum value is 56 at (5, 6), and the minimum value is 3 at (0, 3).

5. $z = 5x + 7y$

Vertex	Value of $z = 5x + 7y$
(0, 3)	z = 5(0) + 7(3) = 21
(0, 6)	z = 5(0) + 7(6) = 42
(5, 6)	z = 5(5) + 7(6) = 67
(5, 2)	z = 5(5) + 7(2) = 39
(4, 0)	z = 5(4) + 7(0) = 20

The maximum value is 67 at (5, 6), and the minimum value is 20 at (4, 0).

6. $z = 7x + 5y$

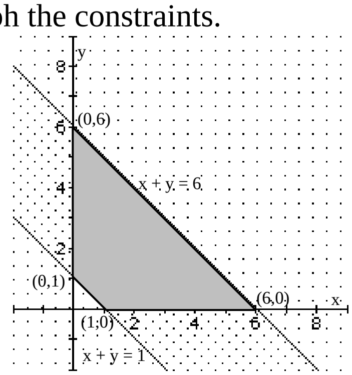
Vertex	Value of $z = 7x + 5y$
(0, 3)	z = 7(0) + 5(3) = 15
(0, 6)	z = 7(0) + 5(6) = 30
(5, 6)	z = 7(5) + 5(6) = 65
(5, 2)	z = 7(5) + 5(2) = 45
(4, 0)	z = 7(4) + 5(0) = 28

The maximum value is 65 at (5, 6), and the minimum value is 15 at (0, 3).

7. Maximize $z = 2x + y$

Subject to $x \geq 0$, $y \geq 0$, $x + y \leq 6$, $x + y \geq 1$

Graph the constraints.



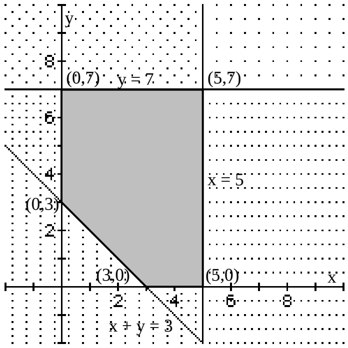
The corner points are (0, 1), (1, 0), (0, 6), (6, 0).

Evaluate the objective function:

Vertex	Value of $z = 2x + y$
(0, 1)	z = 2(0) + 1 = 1
(0, 6)	z = 2(0) + 6 = 6
(1, 0)	z = 2(1) + 0 = 2
(6, 0)	z = 2(6) + 0 = 12

The maximum value is 12 at (6, 0).

8. Maximize $z = x + 3y$
Subject to $x \geq 0$, $y \geq 0$, $x + y \geq 3$, $x \leq 5$, $y \leq 7$
Graph the constraints.

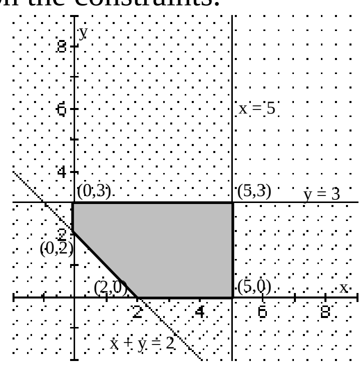


The corner points are (0, 3), (3, 0), (0, 7), (5, 0), (5, 7).
Evaluate the objective function:

Vertex	Value of $z = x + 3y$
(0, 3)	z = 0 + 3(3) = 9
(0, 7)	z = 0 + 3(7) = 21
(3, 0)	z = 3 + 3(0) = 3
(5, 0)	z = 5 + 3(0) = 5
(5, 7)	z = 5 + 3(7) = 26

The maximum value is 26 at (5, 7).

9. Minimize $z = 2x + 5y$
Subject to $x \geq 0$, $y \geq 0$, $x + y \geq 2$, $x \leq 5$, $y \leq 3$
Graph the constraints.

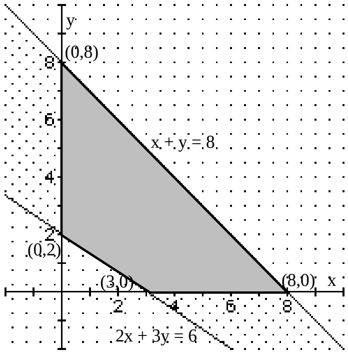


The corner points are (0, 2), (2, 0), (0, 3), (5, 0), (5, 3).
Evaluate the objective function:

Vertex	Value of $z = 2x + 5y$
(0, 2)	z = 2(0) + 5(2) = 10
(0, 3)	z = 2(0) + 5(3) = 15
(2, 0)	z = 2(2) + 5(0) = 4
(5, 0)	z = 2(5) + 5(0) = 10
(5, 3)	z = 2(5) + 5(3) = 25

The minimum value is 4 at (2, 0).

10. Minimize $z = 3x + 4y$
Subject to $x \ge 0, \ y \ge 0, \ 2x + 3y \ge 6, \ x + y \le 8$
Graph the constraints.

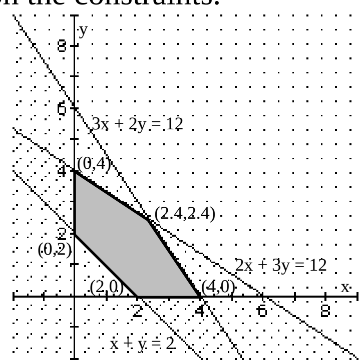


The corner points are (0, 2), (3, 0), (0, 8), (8, 0).
Evaluate the objective function:

Vertex	Value of $z = 3x + 4y$
(0, 2)	z = 3(0) + 4(2) = 8
(0, 8)	z = 3(0) + 4(8) = 32
(3, 0)	z = 3(3) + 4(0) = 9
(8, 0)	z = 3(8) + 4(0) = 24

The minimum value is 8 at (0, 2).

11. Maximize $z = 3x + 5y$
Subject to $x \ge 0, \ y \ge 0, \ x + y \ge 2, \ 2x + 3y \le 12, \ 3x + 2y \le 12$
Graph the constraints.



To find the intersection of $2x + 3y = 12$ and $3x + 2y = 12$, solve the system:

$$\begin{cases} 2x + 3y = 12 \\ 3x + 2y = 12 \quad \rightarrow \quad y = 6 - \frac{3}{2}x \end{cases}$$

Substitute and solve:

$$2x + 3\left(6 - \frac{3}{2}x\right) = 12 \rightarrow 2x + 18 - \frac{9}{2}x = 12 \rightarrow -\frac{5}{2}x = -6$$

$$x = \frac{12}{5} \qquad y = 6 - \frac{3}{2}\left(\frac{12}{5}\right) = 6 - \frac{18}{5} = \frac{12}{5}$$

The point of intersection is $(2.4, 2.4)$.

The corner points are (0, 2), (2, 0), (0, 4), (4, 0), (2.4, 2.4).

Evaluate the objective function:

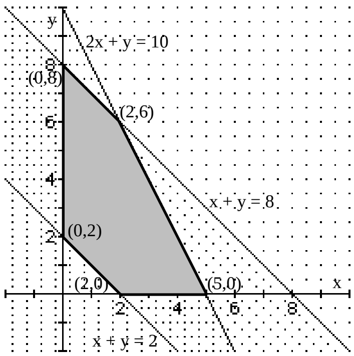
Vertex	Value of $z = 3x + 5y$
(0, 2)	z = 3(0) + 5(2) = 10
(0, 4)	z = 3(0) + 5(4) = 20
(2, 0)	z = 3(2) + 5(0) = 6
(4, 0)	z = 3(4) + 5(0) = 12
(2.4,2.4)	z = 3(2.4) + 5(2.4) = 19.2

The maximum value is 20 at (0, 4).

12. Maximize $z = 5x + 3y$

Subject to $x \geq 0$, $y \geq 0$, $x + y \geq 2$, $x + y \leq 8$, $2x + y \leq 10$

Graph the constraints.



To find the intersection of $x + y = 8$ and $2x + y = 10$, solve the system:

$$\begin{cases} x + y = 8 & \rightarrow \quad y = 8 - x \\ 2x + y = 10 \end{cases}$$

Substitute and solve:

$$2x + 8 - x = 10$$

$$x = 2 \qquad y = 8 - 2 = 6$$

The point of intersection is (2, 6).

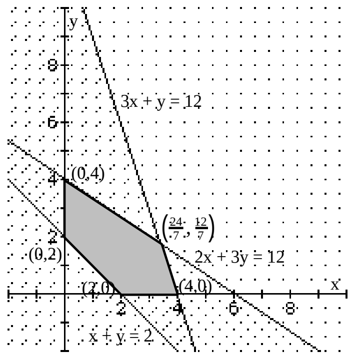
The corner points are (0, 2), (2, 0), (0, 8), (5, 0), (2, 6).

Evaluate the objective function:

Vertex	Value of $z = 5x + 3y$
(0, 2)	z = 5(0) + 3(2) = 6
(0, 8)	z = 5(0) + 3(8) = 24
(2, 0)	z = 5(2) + 3(0) = 10
(5, 0)	z = 5(5) + 3(0) = 25
(2, 6)	z = 5(2) + 3(6) = 28

The maximum value is 28 at (2, 6).

13. Minimize $z = 5x + 4y$
Subject to $x \ge 0$, $y \ge 0$, $x + y \ge 2$, $2x + 3y \le 12$, $3x + y \le 12$
Graph the constraints.



To find the intersection of $2x + 3y = 12$ and $3x + y = 12$, solve the system:

$$\begin{cases} 2x + 3y = 12 \\ 3x + y = 12 \quad \rightarrow \quad y = 12 - 3x \end{cases}$$

Substitute and solve:

$$2x + 3(12 - 3x) = 12 \rightarrow 2x + 36 - 9x = 12 \rightarrow -7x = -24 \rightarrow x = \frac{24}{7}$$

$$y = 12 - 3\left(\frac{24}{7}\right) = 12 - \frac{72}{7} = \frac{12}{7}$$

The point of intersection is $\left(\frac{24}{7}, \frac{12}{7}\right)$.

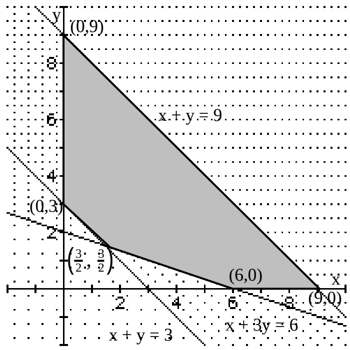
The corner points are (0, 2), (2, 0), (0, 4), (4, 0), $\left(\frac{24}{7}, \frac{12}{7}\right)$.

Evaluate the objective function:

Vertex	Value of $z = 5x + 4y$
(0, 2)	z = 5(0) + 4(2) = 8
(0, 4)	z = 5(0) + 4(4) = 16
(2, 0)	z = 5(2) + 4(0) = 10
(4, 0)	z = 5(4) + 4(0) = 20
$\left(\frac{24}{7}, \frac{12}{7}\right)$	$z = 5\left(\frac{24}{7}\right) + 4\left(\frac{12}{7}\right) = \frac{120}{7} + \frac{48}{7} = \frac{168}{7} = 24$

The minimum value is 8 at (0, 2).

14. Minimize $z = 2x + 3y$
Subject to $x \geq 0$, $y \geq 0$, $x + y \geq 3$, $x + y \leq 9$, $x + 3y \geq 6$
Graph the constraints.



To find the intersection of $x + y = 3$ and $x + 3y = 6$, solve the system:

$$\begin{cases} x + y = 3 & \rightarrow \quad y = 3 - x \\ x + 3y = 6 \end{cases}$$

Substitute and solve:

$$x + 3(3 - x) = 6 \rightarrow x + 9 - 3x = 6 \rightarrow -2x = -3$$

$$x = 1.5 \qquad y = 3 - 1.5 = 1.5$$

The point of intersection is $(1.5, 1.5)$.

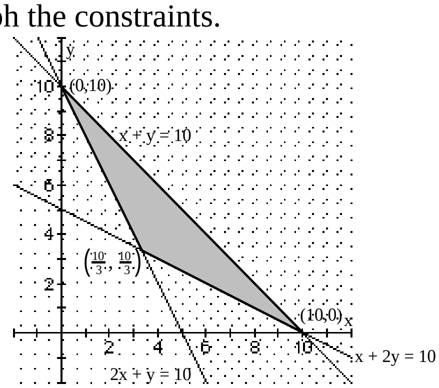
The corner points are (0, 3), (6, 0), (0, 9), (9, 0), $(1.5, 1.5)$.

Evaluate the objective function:

Vertex	Value of $z = 2x + 3y$
(0, 3)	z = 2(0) + 3(3) = 9
(0, 9)	z = 2(0) + 3(9) = 27
(6, 0)	z = 2(6) + 3(0) = 12
(9, 0)	z = 2(9) + 3(0) = 18
$(1.5, 1.5)$	z = $2(1.5) + 3(1.5) = 3 + 4.5 = 7.5$

The minimum value is 7.5 at $(1.5, 1.5)$.

15. Maximize $z = 5x + 2y$
Subject to $x \geq 0$, $y \geq 0$, $x + y \leq 10$, $2x + y \geq 10$, $x + 2y \geq 10$
Graph the constraints.



To find the intersection of $2x + y = 10$ and $x + 2y = 10$, solve the system:

$$\begin{cases} 2x + y = 10 & \rightarrow \quad y = 10 - 2x \\ x + 2y = 10 \end{cases}$$

Substitute and solve:

$$x+2(10-2x)=10 \rightarrow x+20-4x=10 \rightarrow -3x=-10 \rightarrow x=\frac{10}{3}$$

$$y=10-2\left(\frac{10}{3}\right)=10-\frac{20}{3}=\frac{10}{3}$$

The point of intersection is $\left(\frac{10}{3},\frac{10}{3}\right)$.

The corner points are (0, 10), (10, 0), $\left(\frac{10}{3},\frac{10}{3}\right)$.

Evaluate the objective function:

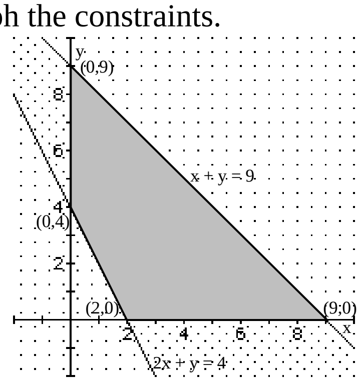
Vertex	Value of $z=5x+2y$
(0, 10)	z = 5(0) + 2(10) = 20
(10, 0)	z = 5(10) + 2(0) = 50
$\left(\frac{10}{3},\frac{10}{3}\right)$	$z=5\left(\frac{10}{3}\right)+2\left(\frac{10}{3}\right)=\frac{50}{3}+\frac{20}{3}=\frac{70}{3}=23\frac{1}{3}$

The maximum value is 50 at (10, 0).

16. Maximize $z=2x+4y$

Subject to $x\geq 0$, $y\geq 0$, $x+y\leq 9$, $2x+y\geq 4$

Graph the constraints.



The corner points are (0, 9), (9, 0), (0, 4), (2, 0).

Evaluate the objective function:

Vertex	Value of $z=2x+4y$
(0, 9)	z = 2(0) + 4(9) = 36
(9, 0)	z = 2(9) + 4(0) = 18
(0, 4)	z = 2(0) + 4(4) = 16
(2, 0)	z = 2(2) + 4(0) = 4

The maximum value is 36 at (0, 9).

17. Let x = the number of downhill skis produced.

Let y = the number of cross-country skis produced.

The total profit is: $P=70x+50y$. Profit is to be maximized; thus, this is the objective function.

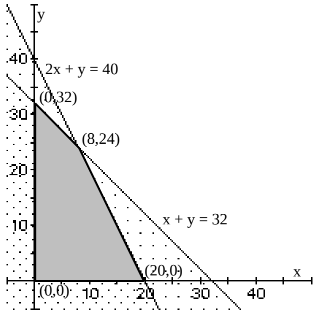
The constraints are:

$x\geq 0$, $y\geq 0$ A positive number of skis must be produced.

$2x+y\leq 40$ Only 40 hours of manufacturing time is available.

$x+y\leq 32$ Only 32 hours of finishing time is available.

Graph the constraints.



To find the intersection of $x + y = 32$ and $2x + y = 40$, solve the system:

$$\begin{cases} x + y = 32 \quad \rightarrow \quad y = 32 - x \\ 2x + y = 40 \end{cases}$$

Substitute and solve:

$$2x + 32 - x = 40 \rightarrow x = 8 \rightarrow y = 32 - 8 = 24$$

The point of intersection is (8, 24).

The corner points are (0, 0), (0, 32), (20, 0), (8, 24).

Evaluate the objective function:

Vertex	Value of $P = 70x + 50y$
(0, 0)	P = 70(0) + 50(0) = 0
(0, 32)	P = 70(0) + 50(32) = 1600
(20, 0)	P = 70(20) + 50(0) = 1400
(8, 24)	P = 70(8) + 50(24) = 1760

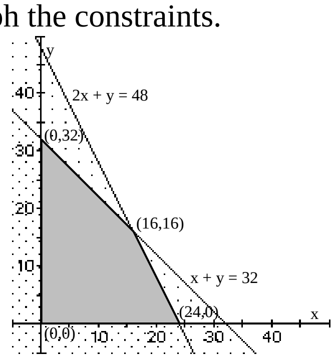
The maximum profit is $1760, when 8 downhill skis and 24 cross-country skis are produced.

With the increase of the manufacturing time to 48 hours, we do the following:

The constraints are:

$x \geq 0, \quad y \geq 0$	A positive number of skis must be produced.
$2x + y \leq 48$	Only 48 hours of manufacturing time is available.
$x + y \leq 32$	Only 32 hours of finishing time is available.

Graph the constraints.



To find the intersection of $x + y = 32$ and $2x + y = 48$, solve the system:

$$\begin{cases} x + y = 32 \quad \rightarrow \quad y = 32 - x \\ 2x + y = 48 \end{cases}$$

Substitute and solve:

$$2x + 32 - x = 48 \rightarrow x = 16 \rightarrow y = 32 - 16 = 16$$

The point of intersection is (16, 16).

The corner points are (0, 0), (0, 32), (24, 0), (16, 16).

Evaluate the objective function:

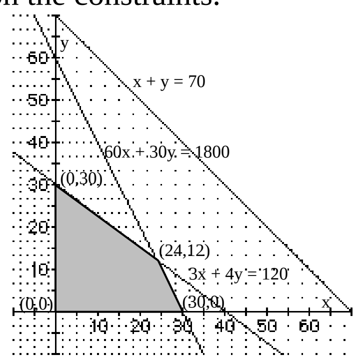
Vertex	Value of $P = 70x + 50y$
(0, 0)	P = 70(0) + 50(0) = 0
(0, 32)	P = 70(0) + 50(32) = 1600
(24, 0)	P = 70(24) + 50(0) = 1680
(16, 16)	P = 70(16) + 50(16) = 1920

The maximum profit is $1920, when 16 downhill skis and 16 cross-country skis are produced.

18. Let x = the number of acres of soybeans planted.
Let y = the number of acres of wheat planted.
The total profit is: $P = 180x + 100y$. Profit is to be maximized; thus, this is the objective function.
The constraints are:

$x \ge 0,\ \ y \ge 0$ A non-negative number of acres must be planted.
$x + y \le 70$ Acres available to plant.
$60x + 30y \le 1800$ Money available for preparation.
$3x + 4y \le 120$ Total workdays available.

Graph the constraints.



To find the intersection of $60x + 30y = 1800$ and $3x + 4y = 120$, solve the system:

$$\begin{cases} 60x + 30y = 1800 \quad \rightarrow \quad 2x + y = 60 \quad \rightarrow \quad y = 60 - 2x \\ 3x + 4y = 120 \end{cases}$$

Substitute and solve:

$$3x + 4(60 - 2x) = 120 \rightarrow 3x + 240 - 8x = 120 \rightarrow -5x = -120 \rightarrow x = 24$$

$$y = 60 - 2(24) = 12$$

The point of intersection is (24, 12).

The corner points are (0, 0), (0, 30), (30, 0), (24, 12).
Evaluate the objective function:

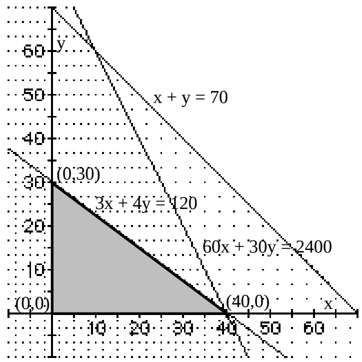
Vertex	Value of $P = 180x + 100y$
(0, 0)	P = 180(0) + 100(0) = 0
(0, 30)	P = 180(0) + 100(30) = 3000
(30, 0)	P = 180(30) + 100(0) = 5400
(24, 12)	P = 180(24) + 100(12) = 5520

The maximum profit is $5520, when 24 acres of soybeans and 12 acres of wheat are planted.

With the increase of the preparation costs to $2400, we do the following:
The constraints are:

$x \ge 0,\ \ y \ge 0$	A non-negative number of acres must be planted.
$x + y \le 70$	Acres available to plant.
$60x + 30y \le 2400$	Money available for preparation.
$3x + 4y \le 120$	Total workdays available.

Graph the constraints.



The corner points are (0, 0), (0, 30), (40, 0).
Evaluate the objective function:

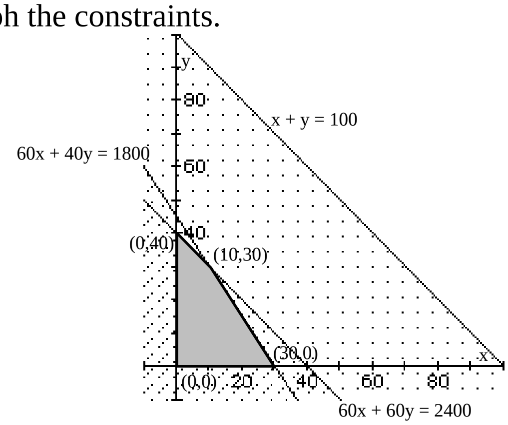
Vertex	Value of $P = 180x + 100y$
(0, 0)	P = 180(0) + 100(0) = 0
(0, 30)	P = 180(0) + 100(30) = 3000
(40, 0)	P = 180(40) + 100(0) = 7200

The maximum profit is $7200, when 40 acres of soybeans and 0 acres of wheat are planted.

19. Let x = the number of acres of corn planted.
Let y = the number of acres of soybeans planted.
The total profit is: $P = 250x + 200y$. Profit is to be maximized; thus, this is the objective function.
The constraints are:

$x \ge 0,\ \ y \ge 0$	A non-negative number of acres must be planted.
$x + y \le 100$	Acres available to plant.
$60x + 40y \le 1800$	Money available for cultivation costs.
$60x + 60y \le 2400$	Money available for labor costs.

Graph the constraints.



To find the intersection of $60x + 40y = 1800$ and $60x + 60y = 2400$, solve the system:

$$\begin{cases} 60x + 40y = 1800 \quad \to \quad 60x = 1800 - 40y \\ 60x + 60y = 2400 \end{cases}$$

Substitute and solve:

$$1800 - 40y + 60y = 2400 \to 20y = 600 \to y = 30$$

$$60x = 1800 - 40(30) \to 60x = 600 \to x = 10$$

The point of intersection is (10, 30).

The corner points are (0, 0), (0, 40), (30, 0), (10, 30).

Evaluate the objective function:

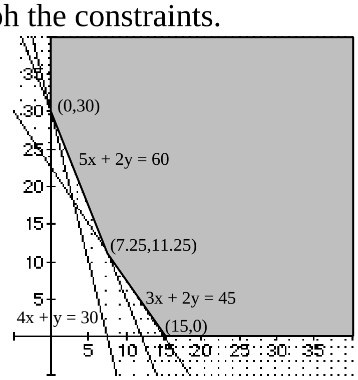
Vertex	Value of $P = 250x + 200y$
(0, 0)	P = 250(0) + 200(0) = 0
(0, 40)	P = 250(0) + 200(40) = 8000
(30, 0)	P = 250(30) + 200(0) = 7500
(10, 30)	P = 250(10) + 200(30) = 8500

The maximum profit is $8500, when 10 acres of corn and 30 acres of soybeans are planted.

20. Let x = the number of ounces of Supplement A to be taken.
Let y = the number of ounces of Supplement B to be taken.
The total cost is: $C = 1.5x + y$. Cost is to be minimized; thus, this is the objective function.
The constraints are:

$x \geq 0,\ y \geq 0$ A non-negative number of ounces must be taken.
$5x + 2y \geq 60$ Carbohydrates needed in diet.
$3x + 2y \geq 45$ Protein needed in diet.
$4x + y \geq 30$ Fat needed in diet.

Graph the constraints.



To find the intersection of $5x + 2y = 60$ and $3x + 2y = 45$, solve the system:

$$\begin{cases} 5x + 2y = 60 \quad \to \quad 2y = 60 - 5x \\ 3x + 2y = 45 \end{cases}$$

Substitute and solve:

$$3x + 60 - 5x = 45 \to -2x = -15 \to x = 7.5$$

$$2y = 60 - 5(7.5) = 22.5 \quad \to \quad y = 11.25$$

The point of intersection is (7.5, 11.25).

The corner points are (0, 30), (15, 0), (7.5, 11.25).

Evaluate the objective function:

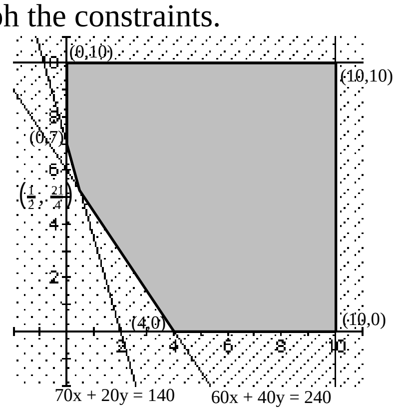
Vertex	Value of $C = 1.5x + y$
(0, 30)	C = 1.5(0) + 30 = 30
(15, 0)	C = 1.5(15) + 0 = 22.5
(7.5, 11.25)	C = 1.5(7.5) + 11.25 = 22.5

The minimum cost is $22.50, when 15 ounces of Supplement A and 0 ounces of Supplement B are used in the diet or when 7.5 ounces of Supplement A and 11.25 ounces of Supplement B are used in the diet.

21. Let x = the number of hours that machine 1 is operated.
Let y = the number of hours that machine 2 is operated.
The total cost is: $C = 50x + 30y$. Cost is to be minimized; thus, this is the objective function.
The constraints are:

$x \ge 0$, $y \ge 0$ A positive number of hours must be used.
$x \le 10$ 10 hours available on machine 1.
$y \le 10$ 10 hours available on machine 2.
$60x + 40y \ge 240$ At least 240 8-inch plyers must be produced.
$70x + 20y \ge 140$ At least 140 6-inch plyers must be produced.

Graph the constraints.



To find the intersection of $60x + 40y = 240$ and $70x + 20y = 140$, solve the system:

$$\begin{cases} 60x + 40y = 240 \\ 70x + 20y = 140 \end{cases} \rightarrow \quad 20y = 140 - 70x$$

Substitute and solve:

$$60x + 2(140 - 70x) = 240 \rightarrow 60x + 280 - 140x = 240$$
$$-80x = -40 \rightarrow x = 0.5$$
$$20y = 140 - 70(0.5) \rightarrow 20y = 105 \rightarrow y = 5.25$$

The point of intersection is $(0.5, 5.25)$.

The corner points are (0, 7), (0, 10), (4, 0), (10, 0), (10, 10), $(0.5, 5.25)$.

Evaluate the objective function:

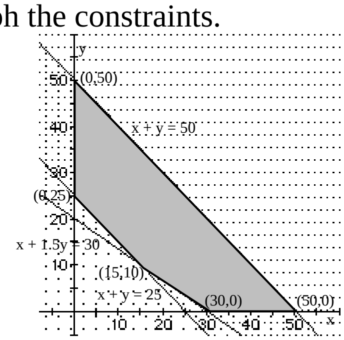
Vertex	Value of $C = 50x + 30y$
(0, 7)	C = 50(0) + 30(7) = 210
(0, 10)	C = 50(0) + 30(10) = 300
(4, 0)	C = 50(4) + 30(0) = 200
(10, 0)	C = 50(10) + 30(0) = 500
(10, 10)	C = 50(10) + 30(10) = 800
$(0.5, 5.25)$	C = $50(0.5) + 30(5.25) = 182.50$

The minimum cost is \$182.50, when machine 1 is used for 0.5 hours and machine 2 is used for 5.25 hours.

22. Let x = the number of newer trees.
Let y = the number of older trees.
The total cost is: $C = 15x + 20y$. Cost is to be minimized; thus, this is the objective function.
The constraints are:

$x \geq 0, \quad y \geq 0$	A non-negative number of trees must be pruned.
$x + y \geq 25$	Must prune at least 25 trees.
$x + y \leq 50$	There are only 50 trees in the orchard.
$x + 1.5y \geq 30$	Contract is for at least 30 hours.

Graph the constraints.



To find the intersection of $x + y = 25$ and $x + 1.5y = 30$, solve the system:

$$\begin{cases} x + y = 25 \\ x + 1.5y = 30 \quad \rightarrow \quad x = 30 - 1.5y \end{cases}$$

Substitute and solve:

$$30 - 1.5y + y = 25$$

$$-0.5y = -5 \rightarrow y = 10 \rightarrow x = 30 - 1.5(10) = 15$$

The point of intersection is (15, 10).

The corner points are (0, 25), (0, 50), (30, 0), (50, 0), (15, 10).
Evaluate the objective function:

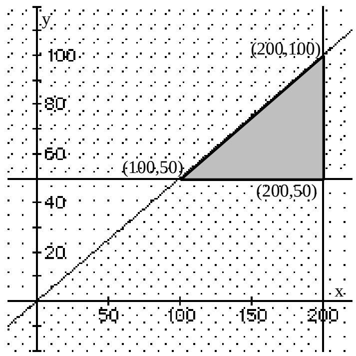
Vertex	Value of $C = 15x + 20y$
(0, 25)	C = 15(0) + 20(25) = 500
(0, 50)	C = 15(0) + 20(50) = 1000
(30, 0)	C = 15(30) + 20(0) = 450
(50, 0)	C = 15(50) + 20(0) = 750
(15, 10)	C = 15(15) + 20(10) = 425

The minimum cost is \$425, when 15 newer trees and 10 older trees are pruned.

23. Let x = the number of pounds of ground beef.
Let y = the number of pounds of ground pork.
The total cost is: $C = 0.75x + 0.45y$. Cost is to be minimized; thus, this is the objective function.
The constraints are:

$x \geq 0$, $y \geq 0$ A positive number of pounds must be used.
$x \leq 200$ Only 200 pounds of ground beef are available.
$y \geq 50$ At least 50 pounds of ground pork must be used.
$0.75x + 0.60y \geq 0.70(x + y) \rightarrow 0.05x \geq 0.10y$ Leanness condition to be met.

Graph the constraints.



The corner points are (100, 50), (200, 50), (200, 100).
Evaluate the objective function:

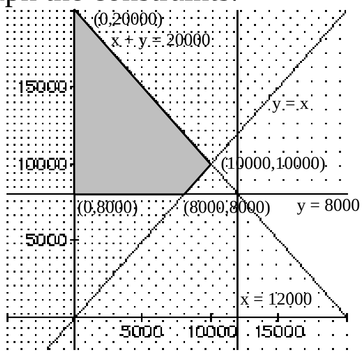
Vertex	Value of $C = 0.75x + 0.45y$
(100, 50)	C = 0.75(100) + 0.45(50) = 97.50
(200, 50)	C = 0.75(200) + 0.45(50) = 172.50
(200,100)	C = 0.75(200) + 0.45(100) = 195.00

The minimum cost is \$97.50, when 100 pounds of ground beef and 50 pounds of ground pork are used.

24. Let x = the amount invested in junk bonds.
Let y = the amount invested in Treasury bills.
The total income is: $I = 0.09x + 0.07y$. Income is to be maximized; thus, this is the objective function.
The constraints are:

$x \geq 0$, $y \geq 0$ A non-negative amount must be invested.
$x + y \leq 20000$ Total investment cannot exceed \$20,000.
$x \leq 12000$ Amount invested in junk bonds must not exceed \$12,000.
$y \geq 8000$ Amount invested in Treasury bills must be at least \$8,000.
(a) $y \geq x$ Amount invested in Treasury bills must be equal to or greater than the amount invested in junk bonds.

Graph the constraints.



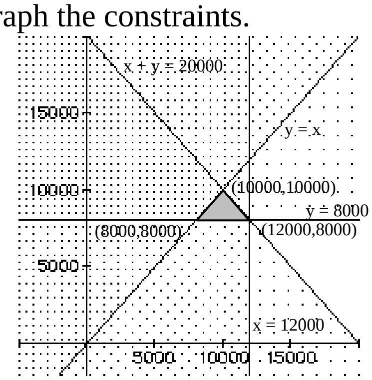
The corner points are (0, 20000), (0, 8000), (8000, 8000), (10000, 10000).
Evaluate the objective function:

Vertex	Value of $I = 0.09x + 0.07y$
(0, 20000)	I = 0.09(0) + 0.07(20000) = 1400
(0, 8000)	I = 0.09(0) + 0.07(8000) = 560
(8000, 8000)	I = 0.09(8000) + 0.07(8000) = 1280
(10000, 10000)	I = 0.09(10000) + 0.07(10000) = 1600

The maximum income is $1600, when $10,000 is invested in junk bonds and $10,000 is invested in Treasury bills.

(b) $y \le x$ Amount invested in Treasury bills must not exceed the amount invested in junk bonds.

Graph the constraints.



The corner points are (12000, 8000), (8000, 8000), (10000, 10000).
Evaluate the objective function:

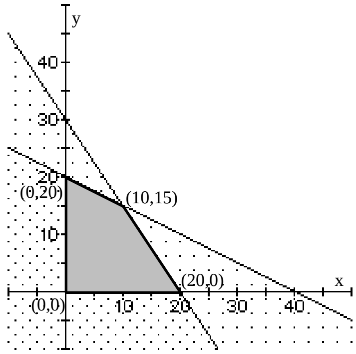
Vertex	Value of $I = 0.09x + 0.07y$
(12000, 8000)	I = 0.09(12000) + 0.07(8000) = 1640
(8000, 8000)	I = 0.09(8000) + 0.07(8000) = 1280
(10000, 10000)	I = 0.09(10000) + 0.07(10000) = 1600

The maximum income is $1640, when $12,000 is invested in junk bonds and $8,000 is invested in Treasury bills.

25. Let x = the number of racing skates manufactured.
Let y = the number of figure skates manufactured.
The total profit is: $P = 10x + 12y$. Profit is to be maximized; thus, this is the objective function.
The constraints are:

$x \geq 0, \ y \geq 0$	A positive number of skates must be manufactured.
$6x + 4y \leq 120$	Only 120 hours are available for fabrication.
$x + 2y \leq 40$	Only 40 hours are available for finishing.

Graph the constraints.



To find the intersection of $6x + 4y = 120$ and $x + 2y = 40$, solve the system:

$$\begin{cases} 6x + 4y = 120 \\ x + 2y = 40 \end{cases} \rightarrow \quad x = 40 - 2y$$

Substitute and solve:

$$6(40 - 2y) + 4y = 120 \rightarrow 240 - 12y + 4y = 120$$

$$-8y = -120 \rightarrow y = 15 \rightarrow x = 40 - 2(15) = 10$$

The point of intersection is (10, 15).

The corner points are (0, 0), (0, 20), (20, 0), (10, 15).
Evaluate the objective function:

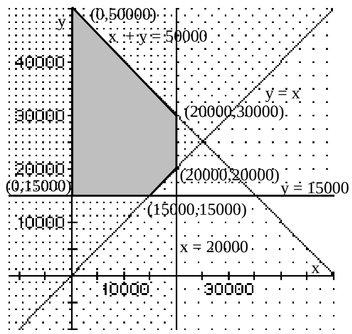
Vertex	Value of $P = 10x + 12y$
(0, 0)	P = 10(0) + 12(0) = 0
(0, 20)	P = 10(0) + 12(20) = 240
(20, 0)	P = 10(20) + 12(0) = 200
(10, 15)	P = 10(10) + 12(15) = 280

The maximum profit is $280, when 10 racing skates and 15 figure skates are produced.

26. Let x = the amount placed in the AAA bond.
Let y = the amount placed in a CD.
The total return is: $R = 0.08x + 0.04y$. Return is to be maximized; thus, this is the objective function.
The constraints are:

$x \geq 0, \ y \geq 0$	A positive amount must be invested in each.
$x + y \leq 50000$	Total investment cannot exceed $50,000.
$x \leq 20000$	Investment in the AAA bond cannot exceed $20,000.
$y \geq 15000$	Investment in the CD must be at least $15,000.
$y \geq x$	Investment in the CD must exceed or equal the investment in the bond.

Graph the constraints.



The corner points are (0, 50000), (0, 15000), (15000,15000), (20000,20000), (20000, 30000).

Evaluate the objective function:

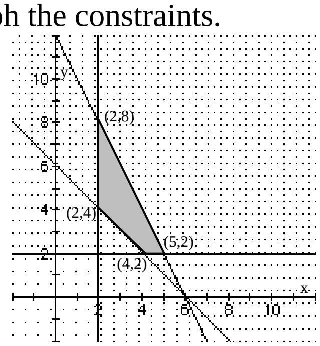
Vertex	Value of $R = 0.08x + 0.04y$
(0, 50000)	R = 0.08(0) + 0.04(50000) = 2000
(0, 15000)	R = 0.08(0) + 0.04(15000) = 600
(15000, 15000)	R = 0.08(15000) + 0.04(15000) = 1800
(20000, 20000)	R = 0.08(20000) + 0.04(20000) = 2400
(20000, 30000)	R = 0.08(20000) + 0.04(30000) = 2800

The maximum return is \$2800, when \$20,000 is invested in a AAA bond and \$30,000 is invested in a CD.

27. Let x = the number of metal fasteners.
Let y = the number of plastic fasteners.
The total cost is: $C = 9x + 4y$. Cost is to be minimized; thus, this is the objective function.
The constraints are:

$x \geq 2,\ \ y \geq 2$	At least 2 of each fastener must be made.
$x + y \geq 6$	At least 6 fasteners are needed.
$4x + 2y \leq 24$	Only 24 hours are available.

Graph the constraints.



The corner points are (2, 4), (2, 8), (4, 2), (5, 2).

Evaluate the objective function:

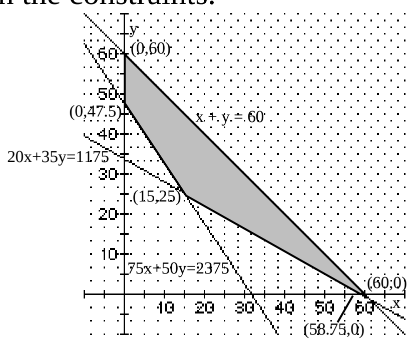
Vertex	Value of $C = 9x + 4y$
(2, 4)	C = 9(2) + 4(4) = 34
(2, 8)	C = 9(2) + 4(8) = 50
(4, 2)	C = 9(4) + 4(2) = 44
(5, 2)	C = 9(5) + 4(2) = 53

The minimum cost is \$34, when 2 metal fasteners and 4 plastic fasteners are ordered.

28. Let x = the amount of "Gourmet Dog".
Let y = the amount of "Chow Hound".
The total cost is: $C = 0.40x + 0.32y$. Cost is to be minimized; thus, this is the objective function.
The constraints are:

$x \geq 0,\quad y \geq 0$	A non-negative number of cans must be purchased.
$20x + 35y \geq 1175$	At least 1175 units of vitamins per month.
$75x + 50y \geq 2375$	At least 2375 calories per month.
$x + y \leq 60$	Storage space for 60 cans.

Graph the constraints.



The corner points are (0, 47.5), (0, 60), (60, 0), (58.75, 0), (15, 25).
Evaluate the objective function:

Vertex	Value of $C = 0.40x + 0.32y$
(0, 47.5)	C = 0.40(0) + 0.32(47.5) = 15.20
(0, 60)	C = 0.40(0) + 0.32(60) = 19.20
(60, 0)	C = 0.40(60) + 0.32(0) = 24.00
(58.75, 0)	C = 0.40(58.75) + 0.32(0) = 23.50
(15, 25)	C = 0.40(15) + 0.32(25) = 14.00

The minimum cost is \$14, when 15 cans of "Gourmet Dog" and 25 cans of "Chow Hound" are purchased.

29. Let x = the number of first-class seats.
Let y = the number of coach seats.
The constraints are:

$8 \leq x \leq 16$	Restriction on first-class seats.
$80 \leq y \leq 120$	Restriction on coach seats.

(a) $\dfrac{x}{y} \leq \dfrac{1}{12}$ Ratio of seats.

If $y = 120$, then $\dfrac{x}{120} \leq \dfrac{1}{12} \rightarrow 12x \leq 120 \rightarrow x \leq 10$

The maximum revenue will be obtained with 120 coach seats and 10 first-class seats. (Note that the first-class seats meet their constraint.)

(b) $\dfrac{x}{y} \leq \dfrac{1}{8}$ Ratio of seats.

If $y = 120$, then $\dfrac{x}{120} \leq \dfrac{1}{8} \rightarrow 8x \leq 120 \rightarrow x \leq 15$

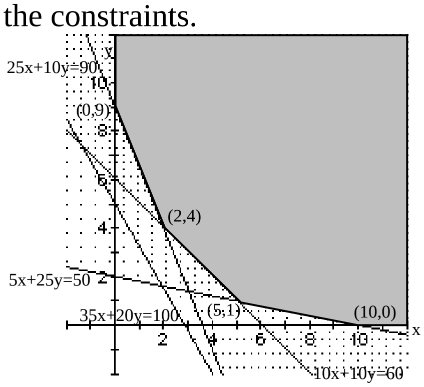
The maximum revenue will be obtained with 120 coach seats and 15 first-class seats. (Note that the first-class seats meet their constraint.)

30. Let x = the number of ounces of Supplement A.
Let y = the number of ounces of Supplement B.
The total cost is: $C = 0.06x + 0.08y$. Cost is to be minimized; thus, this is the objective function.

The constraints are:

$x \geq 0, \quad y \geq 0$	A non-negative number of ounces must be purchased.
$5x + 25y \geq 50$	At least 50 units of vitamin I.
$25x + 10y \geq 90$	At least 90 units of vitamin II.
$10x + 10y \geq 60$	At least 60 units of vitamin III.
$35x + 20y \geq 100$	At least 100 units of vitamin IV.

Graph the constraints.



The corner points are (0, 9), (2, 4), (5, 1), (10, 0).

Evaluate the objective function:

Vertex	Value of $C = 0.06x + 0.08y$
(0, 9)	C = 0.06(0) + 0.08(9) = 0.72
(2, 4)	C = 0.06(2) + 0.08(4) = 0.44
(5, 1)	C = 0.06(5) + 0.08(1) = 0.38
(10, 0)	C = 0.06(10) + 0.08(0) = 0.60

The minimum cost is \$0.38, when 5 ounces of Supplement A, and 1 ounce of Supplement B are used.