Chapter 12

Systems of Equations and Inequalities

12.R Chapter Review

1. Solve the first equation for y, substitute into the second equation and solve:

$$\begin{cases} 2x - y = 5 & \rightarrow \quad y = 2x - 5 \\ 5x + 2y = 8 \end{cases}$$

$$5x + 2(2x - 5) = 8 \rightarrow 5x + 4x - 10 = 8$$

$$9x = 18 \rightarrow x = 2 \rightarrow y = 2(2) - 5 = 4 - 5 = -1$$

The solution is $x = 2,\ y = -1$.

2. Solve the second equation for y, substitute into the first equation and solve:

$$\begin{cases} 2x + 3y = 2 \\ 7x - y = 3 & \rightarrow \quad y = 7x - 3 \end{cases}$$

$$2x + 3(7x - 3) = 2 \rightarrow 2x + 21x - 9 = 2$$

$$23x = 11 \quad \rightarrow \quad x = \frac{11}{23} \rightarrow y = 7\left(\frac{11}{23}\right) - 3 = \frac{77}{23} - \frac{69}{23} = \frac{8}{23}$$

The solution is $x = \frac{11}{23},\ y = \frac{8}{23}$.

3. Solve the second equation for x, substitute into the first equation and solve:

$$\begin{cases} 3x - 4y = 4 \\ x - 3y = \frac{1}{2} & \rightarrow \quad x = 3y + \frac{1}{2} \end{cases}$$

$$3\left(3y + \frac{1}{2}\right) - 4y = 4 \rightarrow 9y + \frac{3}{2} - 4y = 4 \rightarrow 5y = \frac{5}{2} \rightarrow y = \frac{1}{2} \rightarrow x = 3\left(\frac{1}{2}\right) + \frac{1}{2} = 2$$

The solution is $x = 2,\ y = \frac{1}{2}$.

4. Solve the first equation for y, substitute into the second equation and solve:

$$\begin{cases} 2x + y = 0 & \rightarrow \quad y = -2x \\ 5x - 4y = -\frac{13}{2} \end{cases}$$

$$5x - 4(-2x) = -\frac{13}{2} \rightarrow 5x + 8x = -\frac{13}{2} \rightarrow 13x = -\frac{13}{2} \quad \rightarrow \quad x = -\frac{1}{2} \rightarrow y = -2\left(-\frac{1}{2}\right) = 1$$

The solution is $x = -\frac{1}{2},\ y = 1$.

5. Solve the first equation for x, substitute into the second equation and solve:

$$\begin{cases} x-2y-4=0 & \rightarrow \quad x=2y+4 \\ 3x+2y-4=0 \end{cases}$$

$$3(2y+4)+2y-4=0 \rightarrow 6y+12+2y-4=0$$

$$8y=-8 \rightarrow y=-1 \rightarrow x=2(-1)+4=2$$

The solution is $x=2,\ y=-1$.

6. Solve the first equation for x, substitute into the second equation and solve:

$$\begin{cases} x-3y+5=0 & \rightarrow \quad x=3y-5 \\ 2x+3y-5=0 \end{cases}$$

$$2(3y-5)+3y-5=0 \rightarrow 6y-10+3y-5=0$$

$$9y=15 \quad \rightarrow \quad y=\frac{5}{3} \rightarrow x=3\left(\frac{5}{3}\right)-5=0$$

The solution is $x=0,\ y=\frac{5}{3}$.

7. Substitute the first equation into the second equation and solve:

$$\begin{cases} y=2x-5 \\ x=3y+4 \end{cases}$$

$$x=3(2x-5)+4 \rightarrow x=6x-15+4$$

$$-5x=-11 \rightarrow x=\frac{11}{5} \rightarrow y=2\left(\frac{11}{5}\right)-5=-\frac{3}{5}$$

The solution is $x=\frac{11}{5},\ y=-\frac{3}{5}$.

8. Substitute the first equation into the second equation and solve:

$$\begin{cases} x=5y+2 \\ y=5x+2 \end{cases}$$

$$y=5(5y+2)+2 \rightarrow y=25y+10+2$$

$$-24y=12 \quad \rightarrow \quad y=-\frac{1}{2} \rightarrow x=5\left(-\frac{1}{2}\right)+2=-\frac{5}{2}+2=-\frac{1}{2}$$

The solution is $x=-\frac{1}{2},\ y=-\frac{1}{2}$.

9. Multiply each side of the first equation by 5 and each side of the second equation by 30 and add to eliminate y:

$$\begin{cases} x-\ y+4=0 & \xrightarrow{5} \quad 5x-5y+20=0 \\ \frac{1}{2}x+\frac{1}{6}y+\frac{2}{5}=0 & \xrightarrow{30} \quad \underline{15x+5y+\ 12=0} \\ & \qquad\quad 20x \qquad +32=0 \end{cases}$$

$$20x=-32 \rightarrow x=-\frac{8}{5} \qquad \text{Substitute and solve for y:} \quad -\frac{8}{5}-y+4=0 \quad \rightarrow \quad y=\frac{12}{5}$$

The solution of the system is $x=-\frac{8}{5},\ y=\frac{12}{5}$.

10. Solve the second equation for y, substitute into the first equation and solve:

$$\begin{cases} x + \frac{1}{4}y = 2 \\ y + 4x + 2 = 0 \quad \rightarrow \quad y = -4x - 2 \end{cases}$$

$$x + \frac{1}{4}(-4x - 2) = 2$$

$$x - x - \frac{1}{2} = 2 \rightarrow 0 = \frac{5}{2}$$

There is no solution to the system. The system of equations is inconsistent.

11. Rewrite each equation and add to eliminate y:

$$\begin{cases} x - 2y - 8 = 0 & \rightarrow & x - 2y = 8 \\ 2x + 2y - 10 = 0 & \rightarrow & \underline{2x + 2y = 10} \end{cases}$$

$$3x = 18$$

$$x = 6$$

Substitute and solve for y:

$$6 - 2y = 8 \rightarrow -2y = 2 \rightarrow y = -1$$

The solution of the system is $x = 6,\ y = -1$.

12. Rewrite each equation and add to eliminate y:

$$\begin{cases} x - 3y + \frac{7}{2} = 0 & \rightarrow & x - 3y = -\frac{7}{2} \\ \frac{1}{2}x + 3y - 5 = 0 & \rightarrow & \underline{\frac{1}{2}x + 3y = 5} \end{cases}$$

$$\frac{3}{2}x = \frac{3}{2}$$

$$x = 1$$

Substitute and solve for y:

$$\frac{1}{2}(1) + 3y = 5 \quad \rightarrow \quad 3y = \frac{9}{2} \quad \rightarrow \quad y = \frac{3}{2}$$

The solution of the system is $x = 1,\ y = \frac{3}{2}$.

13. Solve the first equation for y, substitute into the second equation and solve:

$$\begin{cases} y - 2x = 11 \quad \rightarrow \quad y = 2x + 11 \\ 2y - 3x = 18 \end{cases}$$

$$2(2x + 11) - 3x = 18$$

$$4x + 22 - 3x = 18 \rightarrow x = -4 \rightarrow y = 2(-4) + 11 = 3$$

The solution is $x = -4,\ y = 3$.

14. Multiply each side of the second equation by 2 and add to eliminate y:

$$\begin{cases} 3x-4y-12=0 \\ 5x+2y+6=0 \end{cases} \begin{array}{l} \longrightarrow \\ \xrightarrow{2} \end{array} \begin{array}{r} 3x-4y=12 \\ \underline{10x+4y=-12} \\ 13x \quad = 0 \\ x=0 \end{array}$$

Substitute and solve for y:

$$3(0)-4y=12 \quad \to \quad -4y=12 \quad \to \quad y=-3$$

The solution is $x=0,\ y=-3$.

15. Multiply each side of the first equation by 2 and each side of the second equation by 3 and add to eliminate y:

$$\begin{cases} 2x+3y-13=0 \\ 3x-2y \quad =0 \end{cases} \begin{array}{l} \xrightarrow{2} \\ \xrightarrow{3} \end{array} \begin{array}{r} 4x+6y-26=0 \\ \underline{9x-6y \quad =0} \\ 13x \quad -26=0 \\ 13x=26 \\ x=2 \end{array}$$

Substitute and solve for y:

$$\begin{aligned} 3(2)-2y&=0 \\ -2y&=-6 \\ y&=3 \end{aligned}$$

The solution of the system is $x=2,\ y=3$.

16. Multiply each side of the first equation by 5 and each side of the second equation by –4 and add to eliminate x:

$$\begin{cases} 4x+5y=21 \\ 5x+6y=42 \end{cases} \begin{array}{l} \xrightarrow{5} \\ \xrightarrow{-4} \end{array} \begin{array}{r} 20x+25y=105 \\ \underline{-20x-24y=-168} \\ y=-63 \end{array}$$

Substitute and solve for x:

$$\begin{aligned} 4x+5(-63)&=21 \\ 4x-315&=21 \\ 4x&=336 \\ x&=84 \end{aligned}$$

The solution of the system is $x=84,\ y=-63$.

17. Multiply each side of the second equation by –3 and add to eliminate x:

$$\begin{cases} 3x-2y=8 \\ x-\frac{2}{3}y=12 \end{cases} \begin{array}{l} \longrightarrow \\ \xrightarrow{-3} \end{array} \begin{array}{r} 3x-2y=8 \\ \underline{-3x+2y=-36} \\ 0=-28 \end{array}$$

The system has no solution, so the system is inconsistent.

18. Multiply each side of the first equation by –2 and add to eliminate x:

$$\begin{cases} 2x + 5y = 10 \\ 4x + 10y = 15 \end{cases} \qquad \begin{array}{l} \xrightarrow{-2} \\ \xrightarrow{} \end{array} \qquad \begin{array}{r} -4x - 10y = -20 \\ \underline{4x + 10y = 15} \\ 0 = -5 \end{array}$$

The system has no solution, so the system is inconsistent.

19. Multiply each side of the first equation by –2 and add to the second equation to eliminate x; and multiply each side of the first equation by –3 and add to the third equation to eliminate x:

$$\begin{cases} x + 2y - z = 6 \\ 2x - y + 3z = -13 \\ 3x - 2y + 3z = -16 \end{cases} \qquad \begin{array}{l} \xrightarrow{-2} \\ \xrightarrow{} \end{array} \qquad \begin{array}{r} -2x - 4y + 2z = -12 \\ \underline{2x - y + 3z = -13} \\ -5y + 5z = -25 \end{array} \xrightarrow{-1/5} \quad y - z = 5$$

$$\begin{array}{l} \xrightarrow{-3} \\ \xrightarrow{} \end{array} \qquad \begin{array}{r} -3x - 6y + 3z = -18 \\ \underline{3x - 2y + 3z = -16} \\ -8y + 6z = -34 \end{array}$$

Multiply each side of the first result by 8 and add to the second result to eliminate y:

$$\begin{array}{r} y - z = 5 \\ -8y + 6z = -34 \end{array} \qquad \begin{array}{l} \xrightarrow{8} \\ \xrightarrow{} \end{array} \qquad \begin{array}{r} 8y - 8z = 40 \\ \underline{-8y + 6z = -34} \\ -2z = 6 \\ z = -3 \end{array}$$

Substituting and solving for the other variables:

$$\begin{array}{r} y - (-3) = 5 \\ y = 2 \end{array} \qquad \begin{array}{r} x + 2(2) - (-3) = 6 \\ x + 4 + 3 = 6 \\ x = -1 \end{array}$$

The solution is $x = -1, \; y = 2, \; z = -3$.

20. Add the first equation and the second equation to eliminate z; and multiply each side of the first equation by 2 and add to the third equation to eliminate z:

$$\begin{cases} x + 5y - z = 2 \\ 2x + y + z = 7 \\ x - y + 2z = 11 \end{cases} \qquad \begin{array}{l} \xrightarrow{} \\ \xrightarrow{} \end{array} \qquad \begin{array}{r} x + 5y - z = 2 \\ \underline{2x + y + z = 7} \\ 3x + 6y \quad = 9 \end{array} \xrightarrow{-1/3} \quad -x - 2y = -3$$

$$\begin{array}{l} \xrightarrow{2} \\ \xrightarrow{} \end{array} \qquad \begin{array}{r} 2x + 10y - 2z = 4 \\ \underline{x - y + 2z = 11} \\ 3x + 9y \quad = 15 \end{array} \xrightarrow{1/3} \quad x + 3y = 5$$

Add the two results to eliminate x:

$$\begin{array}{r} -x - 2y = -3 \\ \underline{x + 3y = 5} \\ y = 2 \end{array}$$

Substituting and solving for the other variables:

$$\begin{array}{r} x + 3(2) = 5 \\ x + 6 = 5 \\ x = -1 \end{array} \qquad \begin{array}{r} 2(-1) + 2 + z = 7 \\ -2 + 2 + z = 7 \\ z = 7 \end{array}$$

The solution is $x = -1, \; y = 2, \; z = 7$.

21. $A+C=\begin{bmatrix}1&0\\2&4\\-1&2\end{bmatrix}+\begin{bmatrix}3&-4\\1&5\\5&-2\end{bmatrix}=\begin{bmatrix}4&-4\\3&9\\4&0\end{bmatrix}$

22. $A-C=\begin{bmatrix}1&0\\2&4\\-1&2\end{bmatrix}-\begin{bmatrix}3&-4\\1&5\\5&-2\end{bmatrix}=\begin{bmatrix}-2&4\\1&-1\\-6&4\end{bmatrix}$

23. $6A=6\begin{bmatrix}1&0\\2&4\\-1&2\end{bmatrix}=\begin{bmatrix}6&0\\12&24\\-6&12\end{bmatrix}$

24. $-4B=-4\begin{bmatrix}4&-3&0\\1&1&-2\end{bmatrix}=\begin{bmatrix}-16&12&0\\-4&-4&8\end{bmatrix}$

25. $AB=\begin{bmatrix}1&0\\2&4\\-1&2\end{bmatrix}\begin{bmatrix}4&-3&0\\1&1&-2\end{bmatrix}=\begin{bmatrix}4&-3&0\\12&-2&-8\\-2&5&-4\end{bmatrix}$

26. $BA=\begin{bmatrix}4&-3&0\\1&1&-2\end{bmatrix}\begin{bmatrix}1&0\\2&4\\-1&2\end{bmatrix}=\begin{bmatrix}-2&-12\\5&0\end{bmatrix}$

27. $CB=\begin{bmatrix}3&-4\\1&5\\5&-2\end{bmatrix}\begin{bmatrix}4&-3&0\\1&1&-2\end{bmatrix}=\begin{bmatrix}8&-13&8\\9&2&-10\\18&-17&4\end{bmatrix}$

28. $BC=\begin{bmatrix}4&-3&0\\1&1&-2\end{bmatrix}\begin{bmatrix}3&-4\\1&5\\5&-2\end{bmatrix}=\begin{bmatrix}9&-31\\-6&5\end{bmatrix}$

29. Augment the matrix with the identity and use row operations to find the inverse:

$$A=\begin{bmatrix}4&6\\1&3\end{bmatrix}\rightarrow\left[\begin{array}{cc|cc}4&6&1&0\\1&3&0&1\end{array}\right]$$

$$\rightarrow\left[\begin{array}{cc|cc}1&3&0&1\\4&6&1&0\end{array}\right]\rightarrow\left[\begin{array}{cc|cc}1&3&0&1\\0&-6&1&-4\end{array}\right]\rightarrow\left[\begin{array}{cc|cc}1&3&0&1\\0&1&-\frac{1}{6}&\frac{2}{3}\end{array}\right]\rightarrow\left[\begin{array}{cc|cc}1&0&\frac{1}{2}&-1\\0&1&-\frac{1}{6}&\frac{2}{3}\end{array}\right]$$

Interchange r_1 and r_2 $\quad R_2=-4r_1+r_2$ $\quad R_2=-\frac{1}{6}r_2$ $\quad R_1=-3r_2+r_1$

$$A^{-1}=\begin{bmatrix}\frac{1}{2}&-1\\-\frac{1}{6}&\frac{2}{3}\end{bmatrix}$$

30. Augment the matrix with the identity and use row operations to find the inverse:

$$A=\begin{bmatrix}-3&2\\1&-2\end{bmatrix}\rightarrow\left[\begin{array}{cc|cc}-3&2&1&0\\1&-2&0&1\end{array}\right]$$

$$\rightarrow \left[\begin{array}{cc|cc} 1 & -2 & 0 & 1 \\ -3 & 2 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & -2 & 0 & 1 \\ 0 & -4 & 1 & 3 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & -2 & 0 & 1 \\ 0 & 1 & -\frac{1}{4} & -\frac{3}{4} \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{4} & -\frac{3}{4} \end{array}\right]$$

Interchange r_1 and r_2 $\quad R_2 = 3r_1 + r_2 \quad R_2 = -\frac{1}{4}r_2 \quad R_1 = 2r_2 + r_1$

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{4} & -\frac{3}{4} \end{bmatrix}$$

31. Augment the matrix with the identity and use row operations to find the inverse:

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 1 & 0 \\ 0 & -4 & -1 & -1 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & -4 & -1 & -1 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & -2 & 3 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & 0 & 7 & 3 & -4 & 1 \end{array}\right]$$

$R_2 = -r_1 + r_2 \quad R_2 = -r_2 \quad R_1 = -3r_2 + r_1$

$R_3 = -r_1 + r_3 \quad R_3 = 4r_2 + r_3$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & -2 & 3 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{3}{7} & -\frac{4}{7} & \frac{1}{7} \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{5}{7} & \frac{9}{7} & \frac{3}{7} \\ 0 & 1 & 0 & \frac{1}{7} & \frac{1}{7} & -\frac{2}{7} \\ 0 & 0 & 1 & \frac{3}{7} & -\frac{4}{7} & \frac{1}{7} \end{array}\right] \rightarrow A^{-1} = \begin{bmatrix} -\frac{5}{7} & \frac{9}{7} & \frac{3}{7} \\ \frac{1}{7} & \frac{1}{7} & -\frac{2}{7} \\ \frac{3}{7} & -\frac{4}{7} & \frac{1}{7} \end{bmatrix}$$

$R_3 = \frac{1}{7}r_3 \quad R_1 = 3r_3 + r_1$

$R_2 = -2r_3 + r_2$

32. Augment the matrix with the identity and use row operations to find the inverse:

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|ccc} 3 & 1 & 2 & 1 & 0 & 0 \\ 3 & 2 & -1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 3 & 2 & -1 & 0 & 1 & 0 \\ 3 & 1 & 2 & 1 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & -1 & -4 & 0 & 1 & -3 \\ 0 & -2 & -1 & 1 & 0 & -3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 4 & 0 & -1 & 3 \\ 0 & -2 & -1 & 1 & 0 & -3 \end{array}\right]$$

Interchange r_1 and r_2 $\quad R_2 = -3r_1 + r_2 \quad R_2 = -r_2$

$R_3 = -3r_1 + r_3$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 1 & -2 \\ 0 & 1 & 4 & 0 & -1 & 3 \\ 0 & 0 & 7 & 1 & -2 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 1 & -2 \\ 0 & 1 & 4 & 0 & -1 & 3 \\ 0 & 0 & 1 & \frac{1}{7} & -\frac{2}{7} & \frac{3}{7} \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{7} & \frac{1}{7} & -\frac{5}{7} \\ 0 & 1 & 0 & -\frac{4}{7} & \frac{1}{7} & \frac{9}{7} \\ 0 & 0 & 1 & \frac{1}{7} & -\frac{2}{7} & \frac{3}{7} \end{array}\right]$$

$R_1 = -r_2 + r_1 \quad R_3 = \frac{1}{7}r_3 \quad R_1 = 3r_3 + r_1$

$R_3 = 2r_2 + r_3 \quad R_2 = -4r_3 + r_2$

$$A^{-1} = \begin{bmatrix} \frac{3}{7} & \frac{1}{7} & -\frac{5}{7} \\ -\frac{4}{7} & \frac{1}{7} & \frac{9}{7} \\ \frac{1}{7} & -\frac{2}{7} & \frac{3}{7} \end{bmatrix}$$

33. Augment the matrix with the identity and use row operations to find the inverse:

$$A=\begin{bmatrix}4 & -8\\ -1 & 2\end{bmatrix} \to \left[\begin{array}{cc|cc}4 & -8 & 1 & 0\\ -1 & 2 & 0 & 1\end{array}\right]$$

$$\to \left[\begin{array}{cc|cc}-1 & 2 & 0 & 1\\ 4 & -8 & 1 & 0\end{array}\right] \to \left[\begin{array}{cc|cc}-1 & 2 & 0 & 1\\ 0 & 0 & 1 & 4\end{array}\right] \to \left[\begin{array}{cc|cc}1 & -2 & 0 & -1\\ 0 & 0 & 1 & 4\end{array}\right]$$

Interchange r_1 and r_2 $\quad R_2=4r_1+r_2 \quad R_1=-r_1$

There is no inverse because there is no way to obtain the identity on the left side. The matrix is singular.

34. Augment the matrix with the identity and use row operations to find the inverse:

$$A=\begin{bmatrix}-3 & 1\\ -6 & 2\end{bmatrix} \to \left[\begin{array}{cc|cc}-3 & 1 & 1 & 0\\ -6 & 2 & 0 & 1\end{array}\right]$$

$$\to \left[\begin{array}{cc|cc}1 & -\frac{1}{3} & -\frac{1}{3} & 0\\ -6 & 2 & 0 & 1\end{array}\right] \to \left[\begin{array}{cc|cc}1 & -\frac{1}{3} & -\frac{1}{3} & 0\\ 0 & 0 & -2 & 1\end{array}\right]$$

$R_1=-\frac{1}{3}r_1 \quad R_2=6r_1+r_2$

There is no inverse because there is no way to obtain the identity on the left side. The matrix is singular.

35. $\begin{cases}3x-2y=1\\ 10x+10y=5\end{cases}$ can be written as: $\left[\begin{array}{cc|c}3 & -2 & 1\\ 10 & 10 & 5\end{array}\right]$

$$\to \left[\begin{array}{cc|c}3 & -2 & 1\\ 1 & 16 & 2\end{array}\right] \to \left[\begin{array}{cc|c}1 & 16 & 2\\ 3 & -2 & 1\end{array}\right] \to \left[\begin{array}{cc|c}1 & 16 & 2\\ 0 & -50 & -5\end{array}\right] \to \left[\begin{array}{cc|c}1 & 16 & 2\\ 0 & 1 & \frac{1}{10}\end{array}\right] \to \left[\begin{array}{cc|c}1 & 0 & \frac{2}{5}\\ 0 & 1 & \frac{1}{10}\end{array}\right]$$

$R_2=-3r_1+r_2$ $\quad$ Interchange r_1 and r_2 $\quad R_2=-3r_1+r_2 \quad R_2=-\frac{1}{50}r_2 \quad R_1=-16r_2+r_1$

The solution is $x=\dfrac{2}{5},\ y=\dfrac{1}{10}$.

36. $\begin{cases}3x+2y=6\\ x-y=-\dfrac{1}{2}\end{cases}$ can be written as: $\left[\begin{array}{cc|c}3 & 2 & 6\\ 1 & -1 & -\frac{1}{2}\end{array}\right]$

$$\to \left[\begin{array}{cc|c}1 & -1 & -\frac{1}{2}\\ 3 & 2 & 6\end{array}\right] \to \left[\begin{array}{cc|c}1 & -1 & -\frac{1}{2}\\ 0 & 5 & \frac{15}{2}\end{array}\right] \to \left[\begin{array}{cc|c}1 & -1 & -\frac{1}{2}\\ 0 & 1 & \frac{3}{2}\end{array}\right] \to \left[\begin{array}{cc|c}1 & 0 & 1\\ 0 & 1 & \frac{3}{2}\end{array}\right]$$

Interchange r_1 and r_2 $\quad R_2=-3r_1+r_2 \quad R_2=\frac{1}{5}r_2 \quad R_1=r_2+r_1$

The solution is $x=1,\ y=\dfrac{3}{2}$.

37. $\begin{cases} 5x+6y-3z=6 \\ 4x-7y-2z=-3 \\ 3x+y-7z=1 \end{cases}$ can be written as: $\left[\begin{array}{ccc|c} 5 & 6 & -3 & 6 \\ 4 & -7 & -2 & -3 \\ 3 & 1 & -7 & 1 \end{array}\right]$

$$\to \left[\begin{array}{ccc|c} 1 & 13 & -1 & 9 \\ 4 & -7 & -2 & -3 \\ 3 & 1 & -7 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 13 & -1 & 9 \\ 0 & -59 & 2 & -39 \\ 0 & -38 & -4 & -26 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 13 & -1 & 9 \\ 0 & 1 & -\frac{2}{59} & \frac{39}{59} \\ 0 & -38 & -4 & -26 \end{array}\right]$$

$R_1 = -r_2 + r_1$ $\quad R_2 = -4r_1 + r_2$ $\quad R_2 = -\frac{1}{59} r_2$

$R_3 = -3r_1 + r_3$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & -\frac{33}{59} & \frac{24}{59} \\ 0 & 1 & -\frac{2}{59} & \frac{39}{59} \\ 0 & 0 & -\frac{312}{59} & -\frac{52}{59} \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -\frac{33}{59} & \frac{24}{59} \\ 0 & 1 & -\frac{2}{59} & \frac{39}{59} \\ 0 & 0 & 1 & \frac{1}{6} \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & \frac{1}{6} \end{array}\right]$$

$R_1 = -13r_2 + r_1$ $\quad R_3 = -\frac{59}{312} r_3$ $\quad R_1 = \frac{33}{59} r_3 + r_1$

$R_3 = 38r_2 + r_3$ $\quad R_2 = \frac{2}{59} r_3 + r_2$

The solution is $x = \frac{1}{2}, y = \frac{2}{3}, z = \frac{1}{6}$.

38. $\begin{cases} 2x+y+z=5 \\ 4x-y-3z=1 \\ 8x+y-z=5 \end{cases}$ can be written as: $\left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 4 & -1 & -3 & 1 \\ 8 & 1 & -1 & 5 \end{array}\right]$

$$\to \left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 0 & -3 & -5 & -9 \\ 0 & -3 & -5 & -15 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & 1 & \frac{5}{3} & 3 \\ 0 & -3 & -5 & -15 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{3} & 1 \\ 0 & 1 & \frac{5}{3} & 3 \\ 0 & 0 & 0 & -6 \end{array}\right]$$

$R_2 = -2r_1 + r_2$ $\quad R_1 = \frac{1}{2} r_1$ $\quad R_1 = -\frac{1}{2} r_2 + r_1$

$R_3 = -4r_1 + r_3$ $\quad R_2 = -\frac{1}{3} r_2$ $\quad R_3 = 3r_2 + r_3$

There is no solution; the system is inconsistent.

39. $\begin{cases} x - 2z = 1 \\ 2x+3y = -3 \\ 4x-3y-4z=3 \end{cases}$ can be written as: $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 2 & 3 & 0 & -3 \\ 4 & -3 & -4 & 3 \end{array}\right]$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 3 & 4 & -5 \\ 0 & -3 & 4 & -1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{4}{3} & -\frac{5}{3} \\ 0 & -3 & 4 & -1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{4}{3} & -\frac{5}{3} \\ 0 & 0 & 8 & -6 \end{array}\right]$$

$R_2 = -2r_1 + r_2$ $\quad R_2 = \frac{1}{3} r_2$ $\quad R_3 = 3r_2 + r_3$

$R_3 = -4r_1 + r_3$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{4}{3} & -\frac{5}{3} \\ 0 & 0 & 1 & -\frac{3}{4} \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & -\frac{2}{3} \\ 0 & 0 & 1 & -\frac{3}{4} \end{array}\right]$$

$R_3 = \frac{1}{8} r_3$ $\quad R_1 = 2r_3 + r_1$

$R_2 = -\frac{4}{3} r_3 + r_2$

The solution is $x = -\frac{1}{2}, y = -\frac{2}{3}, z = -\frac{3}{4}$.

40. $\begin{cases} x+2y-z=2 \\ 2x-2y+z=-1 \\ 6x+4y+3z=5 \end{cases}$ can be written as: $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & -2 & 1 & -1 \\ 6 & 4 & 3 & 5 \end{array}\right]$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -6 & 3 & -5 \\ 0 & -8 & 9 & -7 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -\frac{1}{2} & \frac{5}{6} \\ 0 & -8 & 9 & -7 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & -\frac{1}{2} & \frac{5}{6} \\ 0 & 0 & 5 & -\frac{1}{3} \end{array}\right]$$

$R_2=-2r_1+r_2$ $\quad R_2=-\frac{1}{6}r_2$ $\quad R_1=-2r_2+r_1$

$R_3=-6r_1+r_3$ $\quad R_3=8r_2+r_3$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & -\frac{1}{2} & \frac{5}{6} \\ 0 & 0 & 1 & -\frac{1}{15} \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & \frac{4}{5} \\ 0 & 0 & 1 & -\frac{1}{15} \end{array}\right]$$

$R_3=\frac{1}{5}r_3$ $\quad R_2=\frac{1}{2}r_3+r_2$

The solution is $x=\dfrac{1}{3}, y=\dfrac{4}{5}, z=-\dfrac{1}{15}$.

41. $\begin{cases} x-y+z=0 \\ x-y-5z=6 \\ 2x-2y+z=1 \end{cases}$ can be written as: $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 1 & -1 & -5 & 6 \\ 2 & -2 & 1 & 1 \end{array}\right]$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & -6 & 6 \\ 0 & 0 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \begin{cases} x=y+1 \\ z=-1 \end{cases}$$

$R_2=-r_1+r_2$ $\quad R_2=-\frac{1}{6}r_2$ $\quad R_1=-r_2+r_1$

$R_3=-2r_1+r_3$ $\quad R_3=r_2+r_3$

The solution is $x=y+1$, $z=-1$, y is any real number..

42. $\begin{cases} 4x-3y+5z=0 \\ 2x+4y-3z=0 \\ 6x+2y+z=0 \end{cases}$ can be written as: $\left[\begin{array}{ccc|c} 4 & -3 & 5 & 0 \\ 2 & 4 & -3 & 0 \\ 6 & 2 & 1 & 0 \end{array}\right]$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -\frac{3}{4} & \frac{5}{4} & 0 \\ 2 & 4 & -3 & 0 \\ 6 & 2 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -\frac{3}{4} & \frac{5}{4} & 0 \\ 0 & \frac{11}{2} & -\frac{11}{2} & 0 \\ 0 & \frac{13}{2} & -\frac{13}{2} & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -\frac{3}{4} & \frac{5}{4} & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1=\frac{1}{4}r_1$ $\quad R_2=-2r_1+r_2$ $\quad R_2=\frac{2}{11}r_2$ $\quad R_1=\frac{3}{4}r_2+r_1$

$R_3=-6r_1+r_3$ $\quad R_3=\frac{2}{13}r_3$ $\quad R_3=-r_2+r_3$

The solution is $x=-\dfrac{1}{2}z, y=z$, z is any real number..

43. $\begin{cases} x-y-z-t=1 \\ 2x+y+z+2t=3 \\ x-2y-2z-3t=0 \\ 3x-4y+z+5t=-3 \end{cases}$ can be written as: $\left[\begin{array}{cccc|c} 1 & -1 & -1 & -1 & 1 \\ 2 & 1 & 1 & 2 & 3 \\ 1 & -2 & -2 & -3 & 0 \\ 3 & -4 & 1 & 5 & -3 \end{array}\right]$

$$\rightarrow\left[\begin{array}{cccc|c}1&-1&-1&-1&1\\0&3&3&4&1\\0&-1&-1&-2&-1\\0&-1&4&8&-6\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&-1&-1&-1&1\\0&-1&-1&-2&-1\\0&3&3&4&1\\0&-1&4&8&-6\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&-1&-1&-1&1\\0&1&1&2&1\\0&3&3&4&1\\0&-1&4&8&-6\end{array}\right]$$

$R_2=-2r_1+r_2$ Interchange r_2 and r_3 $R_2=-r_2$
$R_3=-r_1+r_3$
$R_4=-3r_1+r_4$

$$\rightarrow\left[\begin{array}{cccc|c}1&0&0&1&2\\0&1&1&2&1\\0&0&0&-2&-2\\0&0&5&10&-5\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&0&0&1&2\\0&1&1&2&1\\0&0&0&1&1\\0&0&1&2&-1\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&0&0&1&2\\0&1&1&2&1\\0&0&1&2&-1\\0&0&0&1&1\end{array}\right]$$

$R_1=r_2+r_1$ $R_3=-\frac{1}{2}r_3$ Interchange r_3 and r_4
$R_3=-3r_2+r_3$ $R_4=\frac{1}{5}r_4$
$R_4=r_2+r_4$

$$\rightarrow\left[\begin{array}{cccc|c}1&0&0&1&2\\0&1&0&0&2\\0&0&1&2&-1\\0&0&0&1&1\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&0&0&0&1\\0&1&0&0&2\\0&0&1&0&-3\\0&0&0&1&1\end{array}\right]$$

$R_2=-r_3+r_2$ $R_1=-r_4+r_1$
$R_3=-2r_4+r_3$

The solution is $x=1,\ y=2,\ z=-3,\ t=1$.

44. $\begin{cases}x-3y+3z-t=4\\x+2y-z=-3\\x+3z+2t=3\\x+y+5z=6\end{cases}$ can be written as: $\left[\begin{array}{cccc|c}1&-3&3&-1&4\\1&2&-1&0&-3\\1&0&3&2&3\\1&1&5&0&6\end{array}\right]$

$$\rightarrow\left[\begin{array}{cccc|c}1&-3&3&-1&4\\0&5&-4&1&-7\\0&3&0&3&-1\\0&4&2&1&2\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&-3&3&-1&4\\0&1&-\frac{4}{5}&\frac{1}{5}&-\frac{7}{5}\\0&3&0&3&-1\\0&4&2&1&2\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&0&\frac{3}{5}&-\frac{2}{5}&-\frac{1}{5}\\0&1&-\frac{4}{5}&\frac{1}{5}&-\frac{7}{5}\\0&0&\frac{12}{5}&\frac{12}{5}&\frac{16}{5}\\0&0&\frac{26}{5}&\frac{1}{5}&\frac{38}{5}\end{array}\right]$$

$R_2=-r_1+r_2$ $R_2=\frac{1}{5}r_2$ $R_1=3r_2+r_1$
$R_3=-r_1+r_3$ $R_3=-3r_2+r_3$
$R_4=-r_1+r_4$ $R_4=-4r_2+r_4$

$$\rightarrow\left[\begin{array}{cccc|c}1&0&\frac{3}{5}&-\frac{2}{5}&-\frac{1}{5}\\0&1&-\frac{4}{5}&\frac{1}{5}&-\frac{7}{5}\\0&0&1&1&\frac{4}{3}\\0&0&\frac{26}{5}&\frac{1}{5}&\frac{38}{5}\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&0&0&-1&-1\\0&1&0&1&-\frac{1}{3}\\0&0&1&1&\frac{4}{3}\\0&0&0&-5&\frac{2}{3}\end{array}\right]\rightarrow\left[\begin{array}{cccc|c}1&0&0&-1&-1\\0&1&0&1&-\frac{1}{3}\\0&0&1&1&\frac{4}{3}\\0&0&0&1&-\frac{2}{15}\end{array}\right]$$

$R_3=\frac{5}{12}r_3$ $R_1=-\frac{3}{5}r_3+r_1$ $R_4=-\frac{1}{5}r_4$
$R_2=\frac{4}{5}r_3+r_2$
$R_4=-\frac{26}{5}r_3+r_4$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{17}{15} \\ 0 & 1 & 0 & 0 & -\frac{1}{5} \\ 0 & 0 & 1 & 0 & \frac{22}{15} \\ 0 & 0 & 0 & 1 & -\frac{2}{15} \end{array}\right]$$

$R_1 = r_4 + r_1$

$R_2 = -r_4 + r_2$

$R_3 = -r_4 + r_3$

The solution is $x = -\dfrac{17}{15}, y = -\dfrac{1}{5}, z = \dfrac{22}{15}, t = -\dfrac{2}{15}$.

45. Evaluating the determinant:

$$\begin{vmatrix} 3 & 4 \\ 1 & 3 \end{vmatrix} = 3(3) - 4(1) = 9 - 4 = 5$$

46. Evaluating the determinant:

$$\begin{vmatrix} -4 & 0 \\ 1 & 3 \end{vmatrix} = -4(3) - 1(0) = -12 - 0 = -12$$

47. Evaluating the determinant:

$$\begin{vmatrix} 1 & 4 & 0 \\ -1 & 2 & 6 \\ 4 & 1 & 3 \end{vmatrix} = 1\begin{vmatrix} 2 & 6 \\ 1 & 3 \end{vmatrix} - 4\begin{vmatrix} -1 & 6 \\ 4 & 3 \end{vmatrix} + 0\begin{vmatrix} -1 & 2 \\ 4 & 1 \end{vmatrix}$$

$$= 1[2(3) - 6(1)] - 4[-1(3) - 6(4)] + 0[-1(1) - 2(4)]$$

$$= 1(6 - 6) - 4(-3 - 24) + 0(-1 - 8)$$

$$= 1(0) - 4(-27) + 0(-9) = 0 + 108 + 0 = 108$$

48. Evaluating the determinant:

$$\begin{vmatrix} 2 & 3 & 10 \\ 0 & 1 & 5 \\ -1 & 2 & 3 \end{vmatrix} = 2\begin{vmatrix} 1 & 5 \\ 2 & 3 \end{vmatrix} - 3\begin{vmatrix} 0 & 5 \\ -1 & 3 \end{vmatrix} + 10\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix}$$

$$= 2[1(3) - 2(5)] - 3[0(3) - (-1)(5)] + 10[0(2) - (-1)(1)]$$

$$= 2(3 - 10) - 3(0 + 5) + 10(0 + 1) = 2(-7) - 3(5) + 10(1)$$

$$= -14 - 15 + 10 = -19$$

49. Evaluating the determinant:

$$\begin{vmatrix} 2 & 1 & -3 \\ 5 & 0 & 1 \\ 2 & 6 & 0 \end{vmatrix} = 2\begin{vmatrix} 0 & 1 \\ 6 & 0 \end{vmatrix} - 1\begin{vmatrix} 5 & 1 \\ 2 & 0 \end{vmatrix} + (-3)\begin{vmatrix} 5 & 0 \\ 2 & 6 \end{vmatrix}$$

$$= 2(0 - 6) - 1(0 - 2) + (-3)(30 - 0) = -12 + 2 - 90 = -100$$

50. Evaluating the determinant:

$$\begin{vmatrix} -2 & 1 & 0 \\ 1 & 2 & 3 \\ -1 & 4 & 2 \end{vmatrix} = -2\begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix}$$

$$= -2(4-12) - 1(2+3) + 0(4+2) = 16 - 5 + 0 = 11$$

51. Set up and evaluate the determinants to use Cramer's Rule:

$$\begin{cases} x - 2y = 4 \\ 3x + 2y = 4 \end{cases}$$

$$D = \begin{vmatrix} 1 & -2 \\ 3 & 2 \end{vmatrix} = 1(2) - 3(-2) = 2 + 6 = 8$$

$$D_x = \begin{vmatrix} 4 & -2 \\ 4 & 2 \end{vmatrix} = 4(2) - 4(-2) = 8 + 8 = 16$$

$$D_y = \begin{vmatrix} 1 & 4 \\ 3 & 4 \end{vmatrix} = 1(4) - 4(3) = 4 - 12 = -8$$

Find the solutions by Cramer's Rule: $x = \frac{D_x}{D} = \frac{16}{8} = 2 \qquad y = \frac{D_y}{D} = \frac{-8}{8} = -1$

52. Set up and evaluate the determinants to use Cramer's Rule:

$$\begin{cases} x - 3y = -5 \\ 2x + 3y = 5 \end{cases}$$

$$D = \begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix} = 1(3) - 2(-3) = 3 + 6 = 9$$

$$D_x = \begin{vmatrix} -5 & -3 \\ 5 & 3 \end{vmatrix} = -5(3) - 5(-3) = -15 + 15 = 0$$

$$D_y = \begin{vmatrix} 1 & -5 \\ 2 & 5 \end{vmatrix} = 1(5) - 2(-5) = 5 + 10 = 15$$

Find the solutions by Cramer's Rule: $x = \frac{D_x}{D} = \frac{0}{9} = 0 \qquad y = \frac{D_y}{D} = \frac{15}{9} = \frac{5}{3}$

53. Set up and evaluate the determinants to use Cramer's Rule:

$$\begin{cases} 2x + 3y = 13 \\ 3x - 2y = 0 \end{cases}$$

$$D = \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = -4 - 9 = -13$$

$$D_x = \begin{vmatrix} 13 & 3 \\ 0 & -2 \end{vmatrix} = -26 - 0 = -26$$

$$D_y = \begin{vmatrix} 2 & 13 \\ 3 & 0 \end{vmatrix} = 0 - 39 = -39$$

Find the solutions by Cramer's Rule: $x = \frac{D_x}{D} = \frac{-26}{-13} = 2 \qquad y = \frac{D_y}{D} = \frac{-39}{-13} = 3$

54. Set up and evaluate the determinants to use Cramer's Rule:

$$\begin{cases} 3x - 4y = 12 \\ 5x + 2y = -6 \end{cases}$$

$$D = \begin{vmatrix} 3 & -4 \\ 5 & 2 \end{vmatrix} = 6 + 20 = 26$$

$$D_x = \begin{vmatrix} 12 & -4 \\ -6 & 2 \end{vmatrix} = 24 - 24 = 0$$

$$D_y = \begin{vmatrix} 3 & 12 \\ 5 & -6 \end{vmatrix} = -18 - 60 = -78$$

Find the solutions by Cramer's Rule: $x = \dfrac{D_x}{D} = \dfrac{0}{26} = 0 \qquad y = \dfrac{D_y}{D} = \dfrac{-78}{26} = -3$

55. Set up and evaluate the determinants to use Cramer's Rule:

$$\begin{cases} x + 2y - z = 6 \\ 2x - y + 3z = -13 \\ 3x - 2y + 3z = -16 \end{cases}$$

$$D = \begin{vmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 3 \end{vmatrix} = 1\begin{vmatrix} -1 & 3 \\ -2 & 3 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} + (-1)\begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix}$$

$$= 1(-3 + 6) - 2(6 - 9) - 1(-4 + 3) = 3 + 6 + 1 = 10$$

$$D_x = \begin{vmatrix} 6 & 2 & -1 \\ -13 & -1 & 3 \\ -16 & -2 & 3 \end{vmatrix} = 6\begin{vmatrix} -1 & 3 \\ -2 & 3 \end{vmatrix} - 2\begin{vmatrix} -13 & 3 \\ -16 & 3 \end{vmatrix} + (-1)\begin{vmatrix} -13 & -1 \\ -16 & -2 \end{vmatrix}$$

$$= 6(-3 + 6) - 2(-39 + 48) - 1(26 - 16) = 18 - 18 - 10 = -10$$

$$D_y = \begin{vmatrix} 1 & 6 & -1 \\ 2 & -13 & 3 \\ 3 & -16 & 3 \end{vmatrix} = 1\begin{vmatrix} -13 & 3 \\ -16 & 3 \end{vmatrix} - 6\begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} + (-1)\begin{vmatrix} 2 & -13 \\ 3 & -16 \end{vmatrix}$$

$$= 1(-39 + 48) - 6(6 - 9) - 1(-32 + 39) = 9 + 18 - 7 = 20$$

$$D_z = \begin{vmatrix} 1 & 2 & 6 \\ 2 & -1 & -13 \\ 3 & -2 & -16 \end{vmatrix} = 1\begin{vmatrix} -1 & -13 \\ -2 & -16 \end{vmatrix} - 2\begin{vmatrix} 2 & -13 \\ 3 & -16 \end{vmatrix} + 6\begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix}$$

$$= 1(16 - 26) - 2(-32 + 39) + 6(-4 + 3) = -10 - 14 - 6 = -30$$

Find the solutions by Cramer's Rule:

$$x = \frac{D_x}{D} = \frac{-10}{10} = -1 \qquad y = \frac{D_y}{D} = \frac{20}{10} = 2 \qquad z = \frac{D_z}{D} = \frac{-30}{10} = -3$$

56. Set up and evaluate the determinants to use Cramer's Rule:

$$\begin{cases} x - y + z = 8 \\ 2x + 3y - z = -2 \\ 3x - y - 9z = 9 \end{cases}$$

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & -1 \\ 3 & -1 & -9 \end{vmatrix} = 1\begin{vmatrix} 3 & -1 \\ -1 & -9 \end{vmatrix} - (-1)\begin{vmatrix} 2 & -1 \\ 3 & -9 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix}$$

$$= 1(-27 - 1) + 1(-18 + 3) + 1(-2 - 9) = -28 - 15 - 11 = -54$$

$$D_x = \begin{vmatrix} 8 & -1 & 1 \\ -2 & 3 & -1 \\ 9 & -1 & -9 \end{vmatrix} = 8\begin{vmatrix} 3 & -1 \\ -1 & -9 \end{vmatrix} - (-1)\begin{vmatrix} -2 & -1 \\ 9 & -9 \end{vmatrix} + 1\begin{vmatrix} -2 & 3 \\ 9 & -1 \end{vmatrix}$$

$$= 8(-27-1) + 1(18+9) + 1(2-27) = -224 + 27 - 25 = -222$$

$$D_y = \begin{vmatrix} 1 & 8 & 1 \\ 2 & -2 & -1 \\ 3 & 9 & -9 \end{vmatrix} = 1\begin{vmatrix} -2 & -1 \\ 9 & -9 \end{vmatrix} - 8\begin{vmatrix} 2 & -1 \\ 3 & -9 \end{vmatrix} + 1\begin{vmatrix} 2 & -2 \\ 3 & 9 \end{vmatrix}$$

$$= 1(18+9) - 8(-18+3) + 1(18+6) = 27 + 120 + 24 = 171$$

$$D_z = \begin{vmatrix} 1 & -1 & 8 \\ 2 & 3 & -2 \\ 3 & -1 & 9 \end{vmatrix} = 1\begin{vmatrix} 3 & -2 \\ -1 & 9 \end{vmatrix} - (-1)\begin{vmatrix} 2 & -2 \\ 3 & 9 \end{vmatrix} + 8\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix}$$

$$= 1(27-2) + 1(18+6) + 8(-2-9) = 25 + 24 - 88 = -39$$

Find the solutions by Cramer's Rule:

$$x = \frac{D_x}{D} = \frac{-222}{-54} = \frac{37}{9} \qquad y = \frac{D_y}{D} = \frac{171}{-54} = -\frac{19}{6} \qquad z = \frac{D_z}{D} = \frac{-39}{-54} = \frac{13}{18}$$

57. Find the partial fraction decomposition:

$$\frac{6}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4} \quad \text{(Multiply both sides by } x(x-4).\text{)}$$

$$6 = A(x-4) + Bx$$

Let $x = 4$: then $6 = A(4-4) + B(4) \rightarrow 4B = 6 \rightarrow B = \frac{3}{2}$

Let $x = 0$: then $6 = A(0-4) + B(0) \rightarrow -4A = 6 \rightarrow A = -\frac{3}{2}$

$$\frac{6}{x(x-4)} = \frac{\left(-\frac{3}{2}\right)}{x} + \frac{\left(\frac{3}{2}\right)}{x-4}$$

58. Find the partial fraction decomposition:

$$\frac{x}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3} \quad \text{(Multiply both sides by } (x+2)(x-3).\text{)}$$

$$x = A(x-3) + B(x+2)$$

Let $x = -2$: then $-2 = A(-2-3) + B(-2+2) \rightarrow -5A = -2 \rightarrow A = \frac{2}{5}$

Let $x = 3$: then $3 = A(3-3) + B(3+2) \rightarrow 5B = 3 \rightarrow B = \frac{3}{5}$

$$\frac{x}{(x+2)(x-3)} = \frac{\left(\frac{2}{5}\right)}{x+2} + \frac{\left(\frac{3}{5}\right)}{x-3}$$

59. Find the partial fraction decomposition:

$$\frac{x-4}{x^2(x-1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1} \text{ (Multiply both sides by } x^2(x-1).)$$

$$x-4=Ax(x-1)+B(x-1)+Cx^2$$

Let $x=1$: then $1-4=A(1)(1-1)+B(1-1)+C(1)^2 \rightarrow -3=C \rightarrow C=-3$

Let $x=0$: then $0-4=A(0)(0-1)+B(0-1)+C(0)^2 \rightarrow -4=-B \rightarrow B=4$

Let $x=2$: then $2-4=A(2)(2-1)+B(2-1)+C(2)^2 \rightarrow -2=2A+B+4C$

$$\rightarrow 2A=-2-4-4(-3) \rightarrow 2A=6 \rightarrow A=3$$

$$\frac{x-4}{x^2(x-1)}=\frac{3}{x}+\frac{4}{x^2}+\frac{-3}{x-1}$$

60. Find the partial fraction decomposition:

$$\frac{2x-6}{(x-2)^2(x-1)}=\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{x-1} \text{ (Multiply both sides by } (x-2)^2(x-1).)$$

$$2x-6=A(x-2)(x-1)+B(x-1)+C(x-2)^2$$

Let $x=1$: then $2(1)-6=A(1-2)(1-1)+B(1-1)+C(1-2)^2$

$$\rightarrow -4=C \rightarrow C=-4$$

Let $x=2$: then $2(2)-6=A(2-2)(2-1)+B(2-1)+C(2-2)^2 \rightarrow -2=B$

Let $x=0$: then $2(0)-6=A(0-2)(0-1)+B(0-1)+C(0-2)^2$

$$\rightarrow -6=2A-B+4C \rightarrow -6=2A-(-2)+4(-4)$$

$$\rightarrow 2A=8 \rightarrow A=4$$

$$\frac{2x-6}{(x-2)^2(x-1)}=\frac{4}{x-2}+\frac{-2}{(x-2)^2}+\frac{-4}{x-1}$$

61. Find the partial fraction decomposition:

$$\frac{x}{(x^2+9)(x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+9} \text{ (Multiply both sides by } (x+1)(x^2+9).)$$

$$x=A(x^2+9)+(Bx+C)(x+1)$$

Let $x=-1$: then $-1=A((-1)^2+9)+(B(-1)+C)(-1+1)$

$$\rightarrow -1=A(10)+(-B+C)(0) \rightarrow -1=10A \rightarrow A=-\frac{1}{10}$$

Let $x=1$: then $1=A(1^2+9)+(B(1)+C)(1+1) \rightarrow 1=10A+2B+2C$

$$\rightarrow 1=10\left(-\frac{1}{10}\right)+2B+2C \rightarrow 2=2B+2C \rightarrow B+C=1$$

Let $x=0$: then $0=A(0^2+9)+(B(0)+C)(0+1) \rightarrow 0=9A+C$

$$\rightarrow 0=9\left(-\frac{1}{10}\right)+C \rightarrow C=\frac{9}{10} \quad B=1-C \rightarrow B=1-\frac{9}{10} \rightarrow B=\frac{1}{10}$$

$$\frac{x}{(x^2+9)(x+1)}=\frac{-\left(\frac{1}{10}\right)}{x+1}+\frac{\left(\frac{1}{10}x+\frac{9}{10}\right)}{x^2+9}$$

62. Find the partial fraction decomposition:

$$\frac{3x}{(x-2)(x^2+1)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1} \quad \text{(Multiply both sides by } (x-2)(x^2+1).\text{)}$$

$$3x = A(x^2+1) + (Bx+C)(x-2)$$

Let $x = 2$: then $3(2) = A((2)^2+1) + (B(2)+C)(2-2)$

$$\to\ 6 = 5A \ \to\ A = \frac{6}{5}$$

Let $x = 0$: then $3(0) = A(0^2+1) + (B(0)+C)(0-2) \ \to\ 0 = A - 2C$

$$\to\ 0 = \frac{6}{5} - 2C \ \to\ 2C = \frac{6}{5} \ \to\ C = \frac{3}{5}$$

Let $x = 1$: then $3(1) = A(1^2+1) + (B(1)+C)(1-2) \ \to\ 3 = 2A - B - C$

$$\to\ 3 = 2\left(\frac{6}{5}\right) - B - \frac{3}{5} \ \to\ B = -\frac{6}{5}$$

$$\frac{3x}{(x-2)(x^2+1)} = \frac{\left(\frac{6}{5}\right)}{x-2} + \frac{\left(-\frac{6}{5}x + \frac{3}{5}\right)}{x^2+1}$$

63. Find the partial fraction decomposition:

$$\frac{x^3}{(x^2+4)^2} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2} \quad \text{(Multiply both sides by } (x^2+4)^2.\text{)}$$

$$x^3 = (Ax+B)(x^2+4) + Cx + D$$

$$x^3 = Ax^3 + Bx^2 + 4Ax + 4B + Cx + D$$

$$x^3 = Ax^3 + Bx^2 + (4A+C)x + 4B + D$$

$$A = 1$$

$$B = 0$$

$$4A + C = 0 \ \to\ 4(1) + C = 0 \ \to\ C = -4$$

$$4B + D = 0 \ \to\ 4(0) + D = 0 \ \to\ D = 0$$

$$\frac{x^3}{(x^2+4)^2} = \frac{x}{x^2+4} + \frac{-4x}{(x^2+4)^2}$$

64. Find the partial fraction decomposition:

$$\frac{x^3+1}{(x^2+16)^2} = \frac{Ax+B}{x^2+16} + \frac{Cx+D}{(x^2+16)^2} \quad \text{(Multiply both sides by } (x^2+16)^2.\text{)}$$

$$x^3 + 1 = (Ax+B)(x^2+16) + Cx + D$$

$$x^3 + 1 = Ax^3 + Bx^2 + 16Ax + 16B + Cx + D$$

$$x^3 + 1 = Ax^3 + Bx^2 + (16A + C)x + 16B + D$$

$$A = 1$$
$$B = 0$$
$$16A + C = 0 \rightarrow 16(1) + C = 0 \rightarrow C = -16$$
$$16B + D = 1 \rightarrow 16(0) + D = 1 \rightarrow D = 1$$
$$\frac{x^3+1}{(x^2+16)^2} = \frac{x}{x^2+16} + \frac{-16x+1}{(x^2+16)^2}$$

65. Find the partial fraction decomposition:

$$\frac{x^2}{(x^2+1)(x^2-1)} = \frac{x^2}{(x^2+1)(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

(Multiply both sides by $(x-1)(x+1)(x^2+1)$.)

$$x^2 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x-1)(x+1)$$

Let $x = 1$: then $1^2 = A(1+1)(1^2+1) + B(1-1)(1^2+1) + (C(1)+D)(1-1)(1+1)$

$$\rightarrow 1 = 4A \rightarrow A = \frac{1}{4}$$

Let $x = -1$: then

$$(-1)^2 = A(-1+1)((-1)^2+1) + B(-1-1)((-1)^2+1) + (C(-1) + D)(-1-1)(-1+1)$$

$$\rightarrow 1 = -4B \rightarrow B = -\frac{1}{4}$$

Let $x = 0$: then

$$0^2 = A(0+1)(0^2+1) + B(0-1)(0^2+1) + (C(0)+D)(0-1)(0+1)$$

$$\rightarrow 0 = A - B - D \rightarrow 0 = \frac{1}{4} - \left(-\frac{1}{4}\right) - D \rightarrow D = \frac{1}{2}$$

Let $x = 2$: then

$$2^2 = A(2+1)(2^2+1) + B(2-1)(2^2+1) + (C(2)+D)(2-1)(2+1)$$

$$\rightarrow 4 = 15A + 5B + 6C + 3D \rightarrow 4 = 15\left(\frac{1}{4}\right) + 5\left(-\frac{1}{4}\right) + 6C + 3\left(\frac{1}{2}\right)$$

$$\rightarrow 6C = 4 - \frac{15}{4} + \frac{5}{4} - \frac{3}{2} \rightarrow 6C = 0 \rightarrow C = 0$$

$$\frac{x^2}{(x^2+1)(x^2-1)} = \frac{x^2}{(x^2+1)(x-1)(x+1)} = \frac{\left(\frac{1}{4}\right)}{x-1} + \frac{-\left(\frac{1}{4}\right)}{x+1} + \frac{\left(\frac{1}{2}\right)}{x^2+1}$$

66. Find the partial fraction decomposition:

$$\frac{4}{(x^2+4)(x^2-1)} = \frac{4}{(x^2+4)(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+4}$$

(Multiply both sides by $(x-1)(x+1)(x^2+4)$.)

$$4 = A(x+1)(x^2+4) + B(x-1)(x^2+4) + (Cx+D)(x-1)(x+1)$$

Let $x=1$: then $4=A(1+1)(1^2+4)+B(1-1)(1^2+4)+(C(1)+D)(1-1)(1+1)$

$$\to\ 4=10A\ \to\ A=\frac{2}{5}$$

Let $x=-1$: then

$$4=A(-1+1)((-1)^2+4)+B(-1-1)((-1)^2+4)+(C(-1)+D)(-1-1)(-1+1)$$

$$\to\ 4=-10B\ \to\ B=-\frac{2}{5}$$

Let $x=0$: then

$$4=A(0+1)(0^2+4)+B(0-1)(0^2+4)+(C(0)+D)(0-1)(0+1)$$

$$\to\ 4=4A-4B-D\ \to\ 4=\frac{8}{5}-\left(-\frac{8}{5}\right)-D\ \to\ D=-\frac{4}{5}$$

Let $x=2$: then

$$4=A(2+1)(2^2+4)+B(2-1)(2^2+4)+(C(2)+D)(2-1)(2+1)$$

$$\to\ 4=24A+8B+6C+3D\ \to\ 4=24\left(\frac{2}{5}\right)+8\left(-\frac{2}{5}\right)+6C+3\left(-\frac{4}{5}\right)$$

$$\to\ 6C=4-\frac{48}{5}+\frac{16}{5}+\frac{12}{5}\ \to\ 6C=0\ \to\ C=0$$

$$\frac{4}{(x^2+4)(x^2-1)}=\frac{4}{(x^2+4)(x-1)(x+1)}=\frac{\left(\frac{2}{5}\right)}{x-1}+\frac{-\left(\frac{2}{5}\right)}{x+1}+\frac{-\left(\frac{4}{5}\right)}{x^2+4}$$

67. Solve the first equation for y, substitute into the second equation and solve:

$$\begin{cases}2x+y+3=0 & \to\ y=-2x-3\\ x^2+y^2=5\end{cases}$$

$$x^2+(-2x-3)^2=5\to x^2+4x^2+12x+9=5$$

$$5x^2+12x+4=0\to(5x+2)(x+2)=0$$

$$x=-\frac{2}{5}\quad \text{or } x=-2$$

$$y=-\frac{11}{5}\qquad y=1$$

Solutions: $\left(-\frac{2}{5},-\frac{11}{5}\right),(-2,1)$.

68. Add the equations to eliminate y, and solve:

$$\begin{cases}x^2+y^2=\ 16\\ \underline{2x-y^2=-8}\end{cases}$$

$$x^2+2x=\ 8$$

$$x^2+2x-8=0\to(x+4)(x-2)=0$$

$$x=-4 \text{ or } x=2$$

If $x = -4$: $(-4)^2 + y^2 = 16 \rightarrow y^2 = 0 \rightarrow y = 0$

If $x = 2$: $(2)^2 + y^2 = 16 \rightarrow y^2 = 12 \rightarrow y = \pm\sqrt{12} = \pm 2\sqrt{3}$

Solutions: $(-4, 0), \left(2, 2\sqrt{3}\right), \left(2, -2\sqrt{3}\right)$.

69. Multiply each side of the second equation by 2 and add the equations to eliminate xy:

$$\begin{cases} 2xy + y^2 = 10 \\ -xy + 3y^2 = 2 \end{cases} \quad \begin{array}{r} \longrightarrow \quad 2xy + y^2 = 10 \\ \xrightarrow{2} \quad \underline{-2xy + 6y^2 = 4} \end{array}$$

$$7y^2 = 14 \rightarrow y^2 = 2 \rightarrow y = \pm\sqrt{2}$$

If $y = \sqrt{2}$: $2x\left(\sqrt{2}\right) + \left(\sqrt{2}\right)^2 = 10 \rightarrow 2\sqrt{2}x = 8 \rightarrow x = \dfrac{8}{2\sqrt{2}} = 2\sqrt{2}$

If $y = -\sqrt{2}$: $2x\left(-\sqrt{2}\right) + \left(-\sqrt{2}\right)^2 = 10 \rightarrow -2\sqrt{2}x = 8 \rightarrow x = \dfrac{8}{-2\sqrt{2}} = -2\sqrt{2}$

Solutions: $\left(2\sqrt{2}, \sqrt{2}\right), \left(-2\sqrt{2}, -\sqrt{2}\right)$

70. Multiply each side of the first equation by –2 and add the equations to eliminate y:

$$\begin{cases} 3x^2 - y^2 = 1 \\ 7x^2 - 2y^2 = 5 \end{cases} \quad \begin{array}{r} \xrightarrow{-2} \quad -6x^2 + 2y^2 = -2 \\ \longrightarrow \quad \underline{7x^2 - 2y^2 = 5} \\ x^2 \qquad = 3 \end{array}$$

$$x = \pm\sqrt{3}$$

If $x = \sqrt{3}$: $3\left(\sqrt{3}\right)^2 - y^2 = 1 \rightarrow -y^2 = -8 \rightarrow y = \pm\sqrt{8} = \pm 2\sqrt{2}$

If $x = -\sqrt{3}$: $3\left(-\sqrt{3}\right)^2 - y^2 = 1 \rightarrow -y^2 = -8 \rightarrow y = \pm\sqrt{8} = \pm 2\sqrt{2}$

Solutions: $\left(\sqrt{3}, 2\sqrt{2}\right), \left(\sqrt{3}, -2\sqrt{2}\right), \left(-\sqrt{3}, 2\sqrt{2}\right), \left(-\sqrt{3}, -2\sqrt{2}\right)$

71. Substitute into the second equation into the first equation and solve:

$$\begin{cases} x^2 + y^2 = 6y \\ x^2 = 3y \end{cases}$$

$3y + y^2 = 6y \rightarrow y^2 - 3y = 0 \rightarrow y(y-3) = 0 \rightarrow y = 0$ or $y = 3$

If $y = 0$: $x^2 = 3(0) \rightarrow x^2 = 0 \rightarrow x = 0$

If $y = 3$: $x^2 = 3(3) \rightarrow x^2 = 9 \rightarrow x = \pm 3$

Solutions: (0, 0), (–3, 3), (3, 3)

72. Multiply each side of the second equation by –1 and add the equations to eliminate y:

$$\begin{cases} 2x^2 + y^2 = 9 \\ x^2 + y^2 = 9 \end{cases} \quad \begin{array}{r} \longrightarrow \quad 2x^2 + y^2 = 9 \\ \xrightarrow{-1} \quad \underline{-x^2 - y^2 = -9} \\ x^2 \qquad = 0 \end{array}$$

$$x = 0$$

If $x = 0$: $0^2 + y^2 = 9 \rightarrow y^2 = 9 \rightarrow y = \pm 3$

Solutions: (0, 3), (0, –3)

73. Factor the second equation, solve for x, substitute into the first equation and solve:

$$\begin{cases} 3x^2+4xy+5y^2=8 \\ x^2+3xy+2y^2=0 \end{cases} \rightarrow (x+2y)(x+y)=0 \rightarrow x=-2y \text{ or } x=-y$$

Substitute $x=-2y$ and solve:

$$\begin{aligned} 3x^2+4xy+5y^2&=8 \\ 3(-2y)^2+4(-2y)y+5y^2&=8 \\ 12y^2-8y^2+5y^2&=8 \\ 9y^2&=8 \\ y^2&=\frac{8}{9} \\ y&=\pm\frac{2\sqrt{2}}{3} \end{aligned}$$

Substitute $x=-y$ and solve:

$$\begin{aligned} 3x^2+4xy+5y^2&=8 \\ 3(-y)^2+4(-y)y+5y^2&=8 \\ 3y^2-4y^2+5y^2&=8 \\ 4y^2&=8 \\ y^2&=2 \\ y&=\pm\sqrt{2} \end{aligned}$$

If $y=\frac{2\sqrt{2}}{3}$: $\quad x=-2\left(\frac{2\sqrt{2}}{3}\right)=\frac{-4\sqrt{2}}{3}$

If $y=\frac{-2\sqrt{2}}{3}$: $\quad x=-2\left(\frac{-2\sqrt{2}}{3}\right)=\frac{4\sqrt{2}}{3}$

If $y=\sqrt{2}$: $\quad x=-\sqrt{2}$

If $y=-\sqrt{2}$: $\quad x=\sqrt{2}$

Solutions: $\left(\frac{-4\sqrt{2}}{3},\frac{2\sqrt{2}}{3}\right),\left(\frac{4\sqrt{2}}{3},\frac{-2\sqrt{2}}{3}\right),\left(-\sqrt{2},\sqrt{2}\right),\left(\sqrt{2},-\sqrt{2}\right)$

74. Multiply each side of the first equation by 2 and each side of the second equation by 3 and add to eliminate the constant:

$$\begin{cases} 3x^2+2xy-2y^2=6 & \xrightarrow{\cdot 2} & 6x^2+4xy-4y^2=12 \\ xy-2y^2=-4 & \xrightarrow{\cdot 3} & 3xy-6y^2=-12 \end{cases}$$

$$\begin{aligned} 6x^2+7xy-10y^2&=0 \\ (6x-5y)(x+2y)&=0 \\ x=\frac{5}{6}y \text{ or } x&=-2y \end{aligned}$$

Substitute $x=\frac{5}{6}y$ and solve:

$$\begin{aligned} \frac{5}{6}y\ \ y-2y^2&=-4 \\ -\frac{7}{6}y^2=-4 \rightarrow y^2&=\frac{24}{7} \\ y&=\pm\frac{2\sqrt{42}}{7} \end{aligned}$$

Substitute $x=-2y$ and solve:

$$\begin{aligned} -2y\ \ y-2y^2&=-4 \\ -4y^2&=-4 \\ y^2&=1 \\ y&=\pm1 \end{aligned}$$

If $y=\frac{2\sqrt{42}}{7}$: $\quad x=\frac{5}{6}\ \frac{2\sqrt{42}}{7}=\frac{5\sqrt{42}}{21}$

If $y=-\frac{2\sqrt{42}}{7}$: $\quad x=\frac{5}{6}\ \frac{-2\sqrt{42}}{7}=-\frac{5\sqrt{42}}{21}$

If $y=1$: $\quad x=-2$

If $y=-1$: $\quad x=2$

Solutions: $\left(\frac{5\sqrt{42}}{21},\frac{2\sqrt{42}}{7}\right),\left(-\frac{5\sqrt{42}}{21},-\frac{2\sqrt{42}}{7}\right),(-2,1),(2,-1)$

75. Multiply each side of the second equation by –y and add the equations to eliminate y:

$$\begin{cases} x^2-3x+y^2+y=-2 & \longrightarrow & x^2-3x+y^2+y=-2 \\ \dfrac{x^2-x}{y}+y+1=0 & \longrightarrow & \underline{-x^2+x-y^2-y=0} \\ & & -2x=-2 \\ & & x=1 \end{cases}$$

If $x=1$: $1^2-3(1)+y^2+y=-2 \;\rightarrow\; y^2+y=0 \;\rightarrow\; y(y+1)=0$
$\rightarrow\; y=0$ or $y=-1$

Note that $y\neq 0$ because that would cause division by zero in the original equation.
Solution: (1, –1)

76. Multiply each side of the second equation by –x and add the equations to eliminate x:

$$\begin{cases} x^2+x+y^2=y+2 & \longrightarrow & x^2+x+y^2=y+2 \\ x+1=\dfrac{2-y}{x} & \longrightarrow & \underline{-x^2-x=y-2} \end{cases}$$

$$y^2=2y \rightarrow y^2-2y=0 \rightarrow y(y-2)=0$$
$$y=0 \text{ or } y=2$$

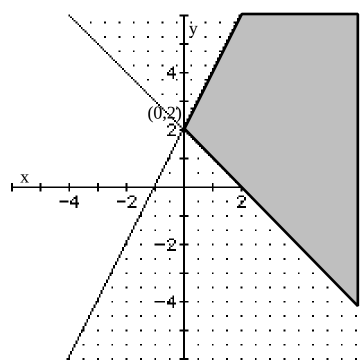
If y = 0: $x^2+x+0^2=0+2 \;\rightarrow\; x^2+x-2=0 \;\rightarrow\; (x-1)(x+2)=0$
$\rightarrow\; x=1$ or $x=-2$

If y = 2: $x^2+x+2^2=2+2 \;\rightarrow\; x^2+x=0 \;\rightarrow\; x(x+1)=0$
$\rightarrow\; x=0$ or $x=-1$

Note that $x\neq 0$ because that would cause division by zero in the original equation.
Solutions: (1, 0), (–2, 0), (–1, 2)

77. Graph the system of linear inequalities:

$$\begin{cases} -2x+y\le 2 \\ x+y\ge 2 \end{cases}$$



(a) Graph the line $-2x+y=2$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0).
Since $-2(0)+0\le 2$ is true, shade the side of the line containing (0, 0).

(b) Graph the line $x+y=2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0).
Since $0+0\ge 2$ is false, shade the opposite side of the line from (0, 0).

(c) The overlapping region is the solution.

(d) The graph is unbounded.

(e) Find the vertices:
To find the intersection of $x+y=2$ and $-2x+y=2$, solve the system:

$$\begin{cases} x+y=2 & \rightarrow & x=2-y \\ -2x+y=2 \end{cases}$$

Substitute and solve:

$$-2(2-y)+y=2 \rightarrow -4+2y+y=2 \rightarrow 3y=6 \rightarrow y=2$$

$$x=2-2=0$$

The point of intersection is (0, 2).

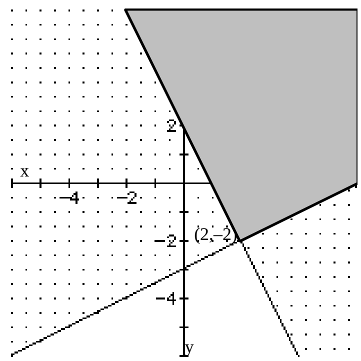
The corner point is (0, 2).

78. Graph the system of linear inequalities:

$$\begin{cases} x-2y \le 6 \\ 2x+y \ge 2 \end{cases}$$

(a) Graph the line $x-2y=6$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0-2(0)\le 6$ is true, shade the side of the line containing (0, 0).

(b) Graph the line $2x+y=2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $2(0)+0 \ge 2$ is false, shade the opposite side of the line from (0, 0).



(c) The overlapping region is the solution.

(d) The graph is unbounded.

(e) Find the vertices:
To find the intersection of $x-2y=6$ and $2x+y=2$, solve the system:

$$\begin{cases} x-2y=6 & \rightarrow \quad x=2y+6 \\ 2x+y=2 \end{cases}$$

Substitute and solve:

$$2(2y+6)+y=2 \rightarrow 4y+12+y=2 \rightarrow 5y=-10 \rightarrow y=-2$$

$$x=2(-2)+6=2$$

The point of intersection is (2, –2).

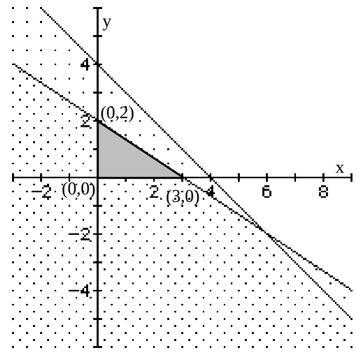
The corner point is (2, –2).

79. Graph the system of linear inequalities:

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ x + y \le 4 \\ 2x + 3y \le 6 \end{cases}$$

(a) Graph $x \ge 0$; $y \ge 0$. Shaded region is the first quadrant.

(b) Graph the line $x + y = 4$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $0 + 0 \le 4$ is true, shade the side of the line containing (0, 0).



(c) Graph the line $2x + 3y = 6$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $2(0) + 3(0) \le 6$ is true, shade the side of the line containing (0, 0).

(d) The overlapping region is the solution.

(e) The graph is bounded.

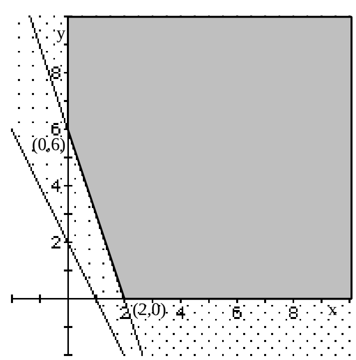
(f) Find the vertices:
The x-axis and y-axis intersect at (0, 0).
The intersection of $2x + 3y = 6$ and the y-axis is (0, 2).
The intersection of $2x + 3y = 6$ and the x-axis is (3, 0).
The three corner points are (0, 0), (0, 2), and (3, 0).

80. Graph the system of linear inequalities:

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ 3x + y \ge 6 \\ 2x + y \ge 2 \end{cases}$$

(a) Graph $x \ge 0$; $y \ge 0$. Shaded region is the first quadrant.

(b) Graph the line $3x + y = 6$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since 3(0) + 0 ≥ 6 is false, shade the opposite side of the line from (0, 0).



(c) Graph the line $2x + y = 2$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since 2(0) + 0 ≥ 2 is false, shade the opposite side of the line from (0, 0).

(d) The overlapping region is the solution.

(e) The graph is unbounded.

(f) Find the vertices:
The intersection of $3x + y = 6$ and the y-axis is (0, 6).
The intersection of $3x + y = 6$ and the x-axis is (2, 0).
The two corner points are (0, 6), and (2, 0).

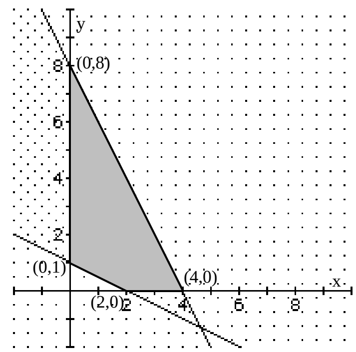
81. Graph the system of linear inequalities:

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ 2x + y \le 8 \\ x + 2y \ge 2 \end{cases}$$

(a) Graph $x \ge 0$; $y \ge 0$. Shaded region is the first quadrant.

(b) Graph the line $2x + y = 8$. Use a solid line since the inequality uses ≤.

Choose a test point not on the line, such as (0, 0). Since 2(0) + 0 ≤ 8 is true, shade the side of the line containing (0, 0).



(c) Graph the line $x + 2y = 2$. Use a solid line since the inequality uses ≥.

Choose a test point not on the line, such as (0, 0). Since 0 + 2(0) ≥ 2 is false, shade the opposite side of the line from (0, 0).

(d) The overlapping region is the solution.

(e) The graph is bounded.

(f) Find the vertices:

The intersection of $x + 2y = 2$ and the y-axis is (0, 1).

The intersection of $x + 2y = 2$ and the x-axis is (2, 0).

The intersection of $2x + y = 8$ and the y-axis is (0, 8).

The intersection of $2x + y = 8$ and the x-axis is (4, 0).

The four corner points are (0, 1), (0, 8), (2, 0), and (4, 0).

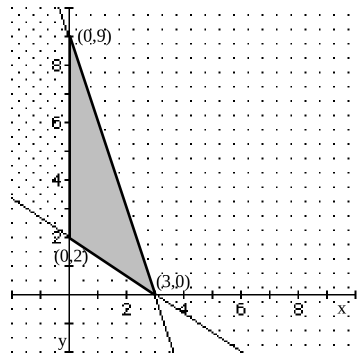
82. Graph the system of linear inequalities:

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ 3x + y \le 9 \\ 2x + 3y \ge 6 \end{cases}$$

(a) Graph $x \ge 0$; $y \ge 0$. Shaded region is the first quadrant.

(b) Graph the line $3x + y = 9$. Use a solid line since the inequality uses ≤.
Choose a test point not on the line, such as (0, 0). Since $3(0) + 0 \le 9$ is true, shade the side of the line containing (0, 0).

(c) Graph the line $2x + 3y = 6$. Use a solid line since the inequality uses ≥.
Choose a test point not on the line, such as (0, 0). Since $2(0) + 3(0) \ge 6$ is false, shade the opposite side of the line from (0, 0).



(d) The overlapping region is the solution.

(e) The graph is bounded.

(f) Find the vertices:
The intersection of $2x + 3y = 6$ and the y-axis is (0, 2).
The intersection of $2x + 3y = 6$ and the x-axis is (3, 0).
The intersection of $3x + y = 9$ and the y-axis is (0, 9).
The intersection of $3x + y = 9$ and the x-axis is (3, 0).
The three corner points are (0, 2), (0, 9), and (3, 0).

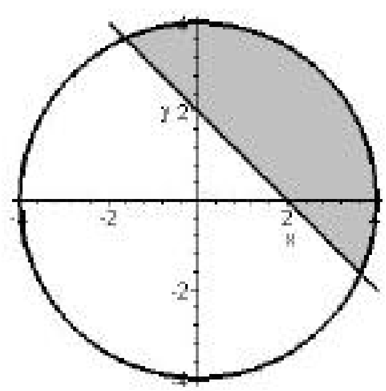
83. Graph the system of inequalities:

$$\begin{cases} x^2 + y^2 \le 16 \\ x + y \ge 2 \end{cases}$$

(a) Graph the circle $x^2 + y^2 = 16$. Use a solid line since the inequality uses ≤. Choose a test point not on the circle, such as (0, 0). Since $0^2 + 0^2 \le 16$ is true, shade the side of the circle containing (0, 0).

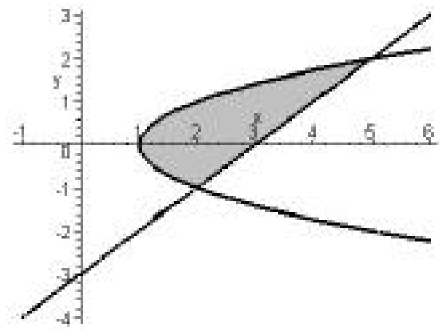
(b) Graph the line $x + y = 2$. Use a solid line since the inequality uses ≥. Choose a test point not on the line, such as (0, 0). Since $0 + 0 \ge 2$ is false, shade the opposite side of the line from (0, 0).

(c) The overlapping region is the solution.



84. Graph the system of inequalities:

$$\begin{cases} y^2 \le x - 1 \\ x - y \le 3 \end{cases}$$



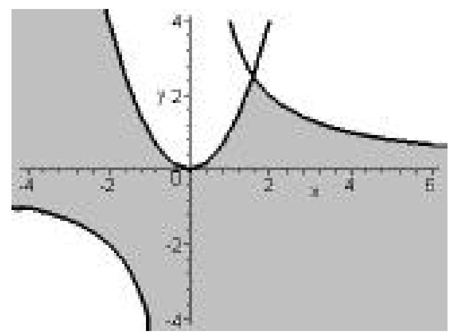
(a) Graph the parabola $y^2 = x - 1$. Use a solid line since the inequality uses $\le$. Choose a test point not on the parabola, such as (0, 0). Since $0^2 \le 0 - 1$ is false, shade the opposite side of the parabola from (0, 0).

(b) Graph the line $x - y = 3$. Use a solid line since the inequality uses $\le$. Choose a test point not on the line, such as (0, 0). Since $0 - 0 \le 3$ is true, shade the same side of the line as (0, 0).

(c) The overlapping region is the solution.

85. Graph the system of inequalities:

$$\begin{cases} y \le x^2 \\ xy \le 4 \end{cases}$$



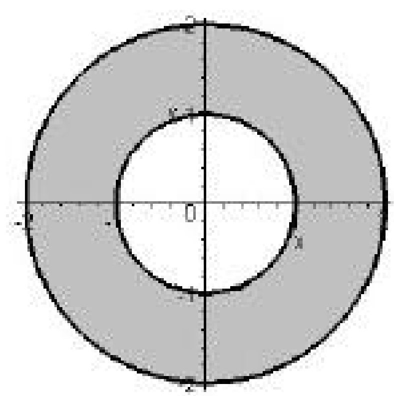
(a) Graph the parabola $y = x^2$. Use a solid line since the inequality uses $\le$. Choose a test point not on the parabola, such as (1, 2). Since $2 \le 1^2$ is false, shade the opposite side of the parabola from (1, 2).

(b) Graph the hyperbola $xy = 4$. Use a solid line since the inequality uses $\le$. Choose a test point not on the hyperbola, such as (1, 2). Since $1 \cdot 2 \le 4$ is true, shade the same side of the hyperbola as (1, 2).

(c) The overlapping region is the solution.

86. Graph the system of inequalities:

$$\begin{cases} x^2 + y^2 \ge 1 \\ x^2 + y^2 \le 4 \end{cases}$$

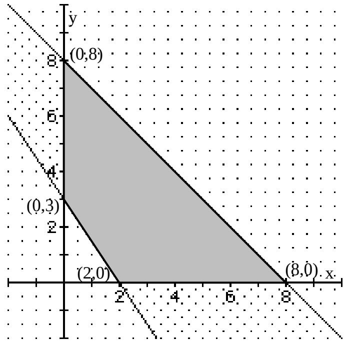


(a) Graph the circle $x^2 + y^2 = 1$. Use a solid line since the inequality uses $\ge$. Choose a test point not on the circle, such as (0, 0). Since $0^2 + 0^2 \ge 1$ is false, shade the opposite side of the circle from (0, 0).

(b) Graph the circle $x^2 + y^2 = 4$. Use a solid line since the inequality uses $\le$. Choose a test point not on the circle, such as (0, 0). Since $0^2 + 0^2 \le 4$ is true, shade the same side of the circle as (0, 0).

(c) The overlapping region is the solution.

87. Maximize $z = 3x + 4y$ Subject to $x \ge 0$, $y \ge 0$, $3x + 2y \ge 6$, $x + y \le 8$
Graph the constraints.

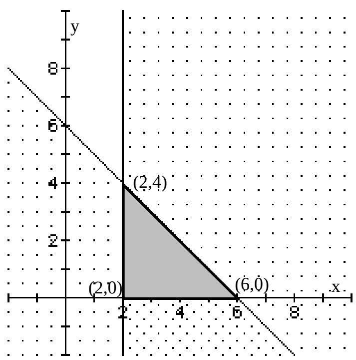


The corner points are (0, 3), (2, 0), (0, 8), (8, 0).
Evaluate the objective function:

Vertex	Value of $z = 3x + 4y$
(0, 3)	z = 3(0) + 4(3) = 12
(0, 8)	z = 3(0) + 4(8) = 32
(2, 0)	z = 3(2) + 4(0) = 6
(8, 0)	z = 3(8) + 4(0) = 24

The maximum value is 32 at (0, 8).

88. Maximize $z = 2x + 4y$ Subject to $x \ge 0$, $y \ge 0$, $x + y \le 6$, $x \ge 2$
Graph the constraints.

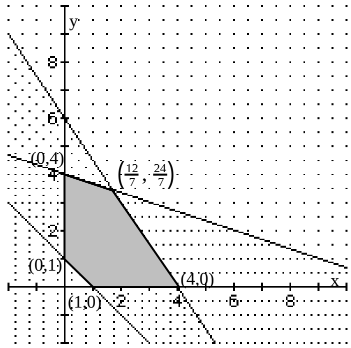


The corner points are (2, 4), (2, 0), (6, 0).
Evaluate the objective function:

Vertex	Value of $z = 2x + 4y$
(2, 4)	z = 2(2) + 4(4) = 20
(2, 0)	z = 2(2) + 4(0) = 4
(6, 0)	z = 2(6) + 4(0) = 12

The maximum value is 20 at (2, 4).

89. Minimize $z = 3x + 5y$
Subject to $x \ge 0$, $y \ge 0$, $x + y \ge 1$, $3x + 2y \le 12$, $x + 3y \le 12$
Graph the constraints.



To find the intersection of $3x + 2y = 12$ and $x + 3y = 12$, solve the system:

$$\begin{cases} 3x + 2y = 12 \\ x + 3y = 12 \quad \to \quad x = 12 - 3y \end{cases}$$

Substitute and solve:

$$3(12 - 3y) + 2y = 12 \to 36 - 9y + 2y = 12 \to -7y = -24 \to y = \frac{24}{7}$$

$$x = 12 - 3\left(\frac{24}{7}\right) = 12 - \frac{72}{7} = \frac{12}{7}$$

The point of intersection is $\left(\frac{12}{7}, \frac{24}{7}\right)$.

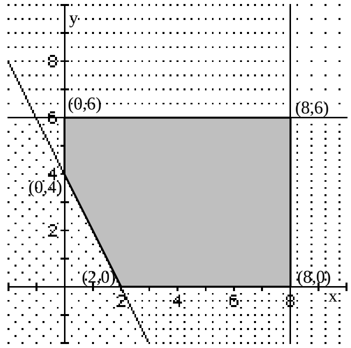
The corner points are (0, 1), (1, 0), (0, 4), (4, 0), $\left(\frac{12}{7}, \frac{24}{7}\right)$.

Evaluate the objective function:

Vertex	Value of $z = 3x + 5y$
(0, 1)	z = 3(0) + 5(1) = 5
(0, 4)	z = 3(0) + 5(4) = 20
(1, 0)	z = 3(1) + 5(0) = 3
(4, 0)	z = 3(4) + 5(0) = 12
$\left(\frac{12}{7}, \frac{24}{7}\right)$	$z = 3\left(\frac{12}{7}\right) + 5\left(\frac{24}{7}\right) = \frac{36}{7} + \frac{120}{7} = \frac{156}{7} \approx 22.3$

The minimum value is 3 at (1, 0).

90. Minimize $z = 3x + y$
Subject to $x \ge 0,\ \ y \ge 0,\ \ x \le 8,\ \ y \le 6,\ \ 2x + y \ge 4$
Graph the constraints.

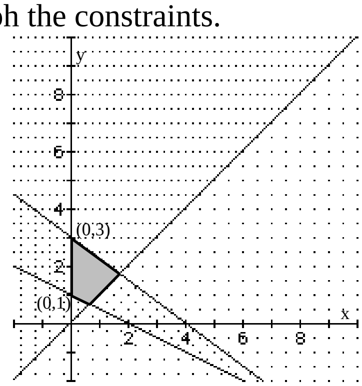


The corner points are (0, 4), (2, 0), (0, 6), (8, 0), (8, 6).
Evaluate the objective function:

Vertex	Value of $z = 3x + y$
(0, 6)	z = 3(0) + 6 = 6
(0, 4)	z = 3(0) + 4 = 4
(2, 0)	z = 3(2) + 0 = 6
(8, 0)	z = 3(8) + 0 = 24
(8, 6)	z = 3(8) + 6 = 30

The minimum value is 4 at (0, 4).

91. Maximize $z = 5x + 4y$ Subject to $x \ge 0,\ \ y \ge 0,\ \ x + 2y \ge 2,\ \ 3x + 4y \le 12,\ \ y \ge x$
Graph the constraints.



To find the intersection of $x + 2y = 2$ and $y = x$, substitute and solve:

$$x + 2x = 2 \rightarrow 3x = 2 \rightarrow x = \frac{2}{3} \rightarrow y = \frac{2}{3}$$

The point of intersection is $\left(\frac{2}{3}, \frac{2}{3}\right)$.

To find the intersection of $y = x$ and $3x + 4y = 12$, substitute and solve:

$$3x + 4x = 12 \rightarrow 7x = 12 \rightarrow x = \frac{12}{7} \rightarrow y = \frac{12}{7}$$

The point of intersection is $\left(\frac{12}{7}, \frac{12}{7}\right)$.

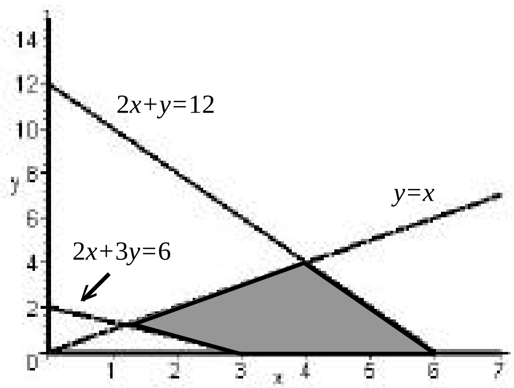
The corner points are (0, 1), (0, 3), $\left(\frac{2}{3}, \frac{2}{3}\right)$, $\left(\frac{12}{7}, \frac{12}{7}\right)$.

Evaluate the objective function:

Vertex	Value of $z = 5x + 4y$
(0, 1)	z = 5(0) + 4(1) = 4
(0, 3)	z = 5(0) + 4(3) = 12
$\left(\frac{2}{3}, \frac{2}{3}\right)$	$z = 5\left(\frac{2}{3}\right) + 4\left(\frac{2}{3}\right) = \frac{18}{3} = 6$
$\left(\frac{12}{7}, \frac{12}{7}\right)$	$z = 5\left(\frac{12}{7}\right) + 4\left(\frac{12}{7}\right) = \frac{108}{7} \approx 15.43$

The maximum value is $\frac{108}{7}$ at $\left(\frac{12}{7}, \frac{12}{7}\right)$.

92. Maximize $z = 4x + 5y$ Subject to $x \ge 0,\ y \ge 0,\ 2x + 3y \ge 6,\ x \ge y,\ 2x + y \le 12$
Graph the constraints.



The corner points are (1.2, 1.2), (3,0),4, 4), (6, 0).
Evaluate the objective function:

Vertex	Value of $z = 4x + 5y$
(1.2,1.2)	z = 4(1.2) + 5(1.2) = 10.8
(3, 0)	z = 4(4) + 5(0) = 16
(4, 4)	z = 4(4) + 5(4) = 36
(6, 0)	z = 4(6) + 5(0) = 24

The maximum value is 36 at (4, 4).

93. Multiply each side of the first equation by –2 and eliminate x:

$$\begin{cases} 2x + 5y = 5 & \longrightarrow & -4x - 10y = -10 \\ 4x + 10y = A & \longrightarrow & \underline{\ \ 4x + 10y = \ \ A} \\ & & 0 = A - 10 \end{cases}$$

If there are to be infinitely many solutions, the sum in elimination should be 0 = 0.
Therefore, $A - 10 = 0$ or $A = 10$.

94. Multiply each side of the first equation by –2 and eliminate x:

$$\begin{cases} 2x + 5y = 5 & \longrightarrow & -4x - 10y = -10 \\ 4x + 10y = A & \longrightarrow & \underline{\ \ 4x + 10y = \ \ A} \\ & & 0 = A - 10 \end{cases}$$

If the system is to be inconsistent, the sum in elimination should be 0 = any number except 0. Therefore, $A - 10 \ne 0$ or $A \ne 10$.

95. when $x=1,\ y=(1)^2+b(1)+c=2 \to 1+b+c=2 \to b+c=1$
when $x=-1,\ y=(-1)^2+b(-1)+c=3 \to 1-b+c=3 \to -b+c=2$

so we have the system $\begin{cases} b+c=1 \\ -b+c=2 \end{cases}$

subtracting the first equation from the second equation yields

$$\begin{array}{r} b+c=1 \\ \underline{-(-b+c=2)} \\ 2b=-1 \end{array} \qquad b=-0.5$$

Back-substituting we get: $-0.5+c=1 \to c=1.5$
Therefore, $y=x^2-0.5x+1.5$, which satisfies the given conditions.

96. when $x=1,\ y=(1)^2+b(1)+c=3 \to 1+b+c=3 \to b+c=2$
when $x=3,\ y=(3)^2+b(3)+c=3 \to 9+3b+c=5 \to 3b+c=-4$
so we have the system $\begin{cases} b+c=2 \\ 3b+c=-4 \end{cases}$

subtracting these equations yields

$$\begin{array}{r} b+c=2 \\ \underline{-(3b+c=-4)} \\ -2b=6 \end{array}$$

$$b=\frac{6}{-2}=-3$$

back substituting into the equation $b+c=2$, we get $-3+c=2 \to c=5$
Therefore, $y=x^2-3x+5$, which satisfies the given conditions.

97. $y=ax^2+bx+c$

At (0, 1) the equation becomes:

$1=a(0)^2+b(0)+c$
$c=1$

At (1, 0) the equation becomes:

$0=a(1)^2+b(1)+c$
$0=a+b+c$
$a+b+c=0$

At (–2, 1) the equation becomes:

$1=a(-2)^2+b(-2)+c=4a-2b+c \to 4a-2b+c=1$

The system of equations is:

$$\begin{cases} a+b+c=0 \\ 4a-2b+c=1 \\ c=1 \end{cases}$$

Substitute $c = 1$ into the first and second equations and simplify:

$$\begin{cases} a + b + 1 = 0 & \rightarrow & a + b = -1 & \rightarrow & a = -b - 1 \\ 4a - 2b + 1 = 1 & \rightarrow & 4a - 2b = 0 \end{cases}$$

Solve the first equation for a, substitute into the second equation and solve:

$$4(-b - 1) - 2b = 0$$
$$-4b - 4 - 2b = 0$$
$$-6b = 4 \rightarrow b = -\frac{2}{3} \rightarrow a = \frac{2}{3} - 1 = -\frac{1}{3}$$

The quadratic function is $y = -\frac{1}{3}x^2 - \frac{2}{3}x + 1$.

98. $x^2 + y^2 + Dx + Ey + F = 0$

At (0, 1) the equation becomes:

$$0^2 + 1^2 + D(0) + E(1) + F = 0$$
$$E + F = -1$$

At (1, 0) the equation becomes:

$$1^2 + 0^2 + D(1) + E(0) + F = 0$$
$$D + F = -1$$

At (–2, 1) the equation becomes:

$$(-2)^2 + 1^2 + D(-2) + E(1) + F = 0$$
$$-2D + E + F = -5$$

The system of equations is:

$$\begin{cases} E + F = -1 \\ D + F = -1 \\ -2D + E + F = -5 \end{cases}$$

Substitute $E + F = -1$ into the third equation and solve for D:

$$-2D + (-1) = -5 \rightarrow -2D = -4 \rightarrow D = 2$$

Substitute and solve:

$$2 + F = -1 \rightarrow F = -3$$
$$E + (-3) = -1 \rightarrow E = 2$$

The equation of the circle is $x^2 + y^2 + 2x + 2y - 3 = 0$.

99. Let x = the number of pounds of coffee that costs \$3.00 per pound.
Let y = the number of pounds of coffee that costs \$6.00 per pound.
Then $x + y = 100$ represents the total amount of coffee in the blend.
The value of the blend will be represented by the equation: $3x + 6y = 3.90(100)$.

Solve the system of equations:

$$\begin{cases} x + y = 100 & \rightarrow & y = 100 - x \\ 3x + 6y = 390 \end{cases}$$

Solve by substitution:

$$3x+6(100-x)=390 \rightarrow 3x+600-6x=390$$
$$-3x=-210 \rightarrow x=70$$
$$y=100-70=30$$

The blend is made up of 70 pounds of the \$3 per pound coffee and 30 pounds of the \$6 per pound coffee.

100. Let x = the number of acres of corn.
Let y = the number of acres of soybeans.
Then $x+y=1000$ represents the total acreage on the farm.
The total cost will be represented by the equation: $65x+45y=54325$.
Solve the system of equations:

$$\begin{cases} x+y=1000 \rightarrow y=1000-x \\ 65x+45y=54325 \end{cases}$$

Solve by substitution:

$$65x+45(1000-x)=54325 \rightarrow 65x+45000-45x=54325$$
$$20x=9325 \rightarrow x=466.25 \rightarrow y=1000-466.25 \rightarrow y=533.75$$

Corn should be planted on 466.25 acres and soybeans should be planted on 533.75 acres.

101. Let x = the number of small boxes.
Let y = the number of medium boxes.
Let z = the number of large boxes.
Oatmeal raisin equation: $x+2y+2z=15$
Chocolate chip equation: $x+y+2z=10$
Shortbread equation: $y+3z=11$
Multiply each side of the second equation by –1 and add to the first equation to eliminate x:

$$\begin{cases} x+2y+2z=15 \\ x+y+2z=10 \\ y+3z=11 \end{cases} \quad \begin{array}{r} x+2y+2z=15 \\ \underline{-x-y-2z=-10} \\ y=5 \end{array}$$

Substituting and solving for the other variables:

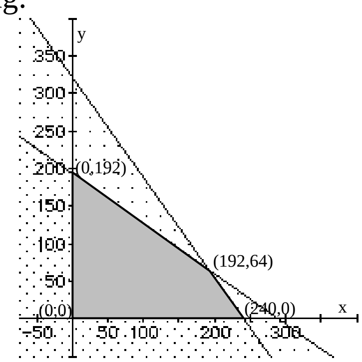
$$5+3z=11 \qquad x+5+2(2)=10$$
$$3z=6 \qquad x+9=10$$
$$z=2 \qquad x=1$$

1 small box, 5 medium boxes, and 2 large boxes of cookies should be purchased.

102. (a) Let x = the number of lower priced packages.
Let y = the number of quality packages.
Peanut inequality: $8x + 6y \le 120(16) \rightarrow 8x + 6y \le 1920$
Cashew inequality: $4x + 6y \le 72(16) \rightarrow 4x + 6y \le 1152$
The system of inequalities is:

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ 8x + 6y \le 1920 \\ 4x + 6y \le 1152 \end{cases}$$

(b) Graphing:



To find the intersection of $8x + 6y = 1920$ and $4x + 6y = 1152$, solve the system:

$$\begin{cases} 8x + 6y = 1920 \\ 4x + 6y = 1152 \end{cases} \rightarrow 6y = 1152 - 4x$$

Substitute and solve:

$$8x + 1152 - 4x = 1920$$

$$4x = 768 \rightarrow x = 192 \rightarrow 6y = 1152 - 4(192) = 384 \rightarrow y = 64$$

The corner points are (0, 0), (0, 192), (240, 0), and (192, 64).

103. Let x = the length of the lot.
Let y = the width of the lot.
Perimeter equation: $2x + 2y = 68$
Diagonal equation: $x^2 + y^2 = 26^2$
Solve the system of equations:

$$\begin{cases} 2x + 2y = 68 \rightarrow y = 34 - x \\ x^2 + y^2 = 676 \end{cases}$$

Solve by substitution:

$$x^2 + (34 - x)^2 = 676 \rightarrow x^2 + 1156 - 68x + x^2 = 676$$

$$2x^2 - 68x + 480 = 0 \rightarrow x^2 - 34x + 240 = 0$$

$$(x - 24)(x - 10) = 0 \rightarrow x = 24 \text{ or } x = 10$$

$$y = 10 \text{ or } y = 24$$

The dimensions of the lot are 24 feet by 10 feet.

104. Let x = the height of the window.
Let y = the width of the window.
Area equation: $xy = 4$
Diagonal equation: $x^2 + y^2 = \left(2\sqrt{2}\right)^2$
Solve the system of equations:

$$\begin{cases} xy = 4 & \rightarrow \quad y = \dfrac{4}{x} \\ x^2 + y^2 = 8 \end{cases}$$

Solve by substitution:

$$x^2 + \left(\frac{4}{x}\right)^2 = 8 \rightarrow x^2 + \frac{16}{x^2} = 8 \rightarrow x^4 + 16 = 8x^2$$

$$x^4 - 8x^2 + 16 = 0 \rightarrow \left(x^2 - 4\right)^2 = 0 \rightarrow x^2 = 4$$

$$x = 2 \text{ or } x = -2$$

$$y = 2 \qquad y = -2$$

The dimensions of the window are 2 feet by 2 feet. (Dimensions must be positive.)

105. Let x = the length of one leg.
Let y = the length of the other leg.
Perimeter equation: $x + y + 6 = 14$
Pythagorean equation: $x^2 + y^2 = 6^2$
Solve the system of equations:

$$\begin{cases} x + y = 8 & \rightarrow \quad y = 8 - x \\ x^2 + y^2 = 36 \end{cases}$$

Solve by substitution:

$$x^2 + (8 - x)^2 = 36 \rightarrow x^2 + 64 - 16x + x^2 = 36$$

$$2x^2 - 16x + 28 = 0 \rightarrow x^2 - 8x + 14 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 56}}{2} = \frac{8 \pm \sqrt{8}}{2} = \frac{8 \pm 2\sqrt{2}}{2} = 4 \pm \sqrt{2}$$

If $x = 4 - \sqrt{2}$, then $y = 8 - 4 + \sqrt{2} = 4 + \sqrt{2}$

If $x = 4 + \sqrt{2}$, then $y = 8 - 4 - \sqrt{2} = 4 - \sqrt{2}$

The legs are $4 + \sqrt{2}$ and $4 - \sqrt{2}$.

106. Let x = the length of the two equal sides in the isosceles triangle.
Let y = the length of the base.
The perimeter of the triangle: $x + x + y = 18$
Since the altitude to the base y is 6, the Pythagorean theorem will produce another equation: $\left(\frac{y}{2}\right)^2 + 6^2 = x^2$

Solve the system of equations:

$$\begin{cases} 2x + y = 18 & \rightarrow \quad y = 18 - 2x \\ \dfrac{y^2}{4} + 36 = x^2 \end{cases}$$

Solve the first equation for y, substitute into the second equation and solve:

$$\frac{(18-2x)^2}{4}+36=x^2 \rightarrow \frac{324-72x+4x^2}{4}+36=x^2$$

$$81-18x+x^2+36=x^2 \rightarrow -18x=-117 \rightarrow x=6.5$$

$$y=18-2(6.5)=5$$

The base of the triangle is 5 inches.

107. Let x = the length of the side of the smaller square.
Then $2x$ = the length of the side of the larger square.
The needed fencing is $4x+8x=12x$.
Solve the area equation:

$$x^2+(2x)^2=5000 \rightarrow 5x^2=5000 \rightarrow x^2=1000 \rightarrow x=10\sqrt{10}$$

$12x=120\sqrt{10}\approx 379.5$ feet of fence are needed.

108. Let x = the amount Katy receives.
Let y = the amount Mike receives.
Let z = the amount Danny receives.
Let w = the amount that Colleen receives.
Conditions:

$$x+y+z+w=45$$

$$y=2x$$

$$w=x$$

$$z=\frac{1}{2}x$$

Solve by substitution:

$$x+y+z+w=45$$

$$x+2x+x+\frac{1}{2}x=45 \rightarrow \frac{9}{2}x=45 \rightarrow x=10$$

Katy receives \$10, Mike receives \$20, Danny receives \$5, and Colleen receives \$10.

109. Let x = the speed of the boat in still water.
Let y = the speed of the river current.
Let d = the distance from Chiritza to the Flotel Orellana (100 kilometers)

	Rate	Time	Distance
trip downstream	$x+y$	$\frac{5}{2}$	100
trip downstream	$x-y$	3	100

The system of equations is:

$$\begin{cases}(x+y)\left(\frac{5}{2}\right)=100 & \rightarrow \quad 5x+5y=200\\ (x-y)(3)=d & \rightarrow \quad 3x-3y=100\end{cases}$$

$$\begin{aligned} 5x+5y=200 &\xrightarrow{3} \quad 15x+15y=600 \\ 3x-3y=100 &\xrightarrow{5} \quad \underline{+\ 15x-15y=500} \\ & \qquad\quad 30x=1100 \end{aligned} \qquad x=\frac{1100}{30}=\frac{110}{3}$$

$$\rightarrow 3\left(\frac{110}{3}\right)-3y=100 \rightarrow 110-3y=100 \rightarrow 10=3y \rightarrow y=\frac{10}{3}$$

The speed of the boat = $\frac{110}{3} \approx 36.67$ km/hr ; the speed of the current = $\frac{10}{3} \approx 3.33$ km/hr.

110. Let x = the speed of the plane in still air. (475 miles per hour)
Let y = the speed of the jet stream.
Let d = the distance from Chicago to Ft. Lauderdale.
The jet stream flows from Chicago to Ft. Lauderdale because the time is shorter in that direction.

	Rate	Time	Distance
Chicago to Ft. Lauderdale	$x+y$	$\frac{5}{2}$	d
Ft. Lauderdale to Chicago	$x-y$	$\frac{17}{6}$	d

The system of equations is:

$$\begin{cases} (x+y)\left(\frac{5}{2}\right)=d \quad \rightarrow \quad 5x+5y=2d \\ (x-y)\left(\frac{17}{6}\right)=d \quad \rightarrow \quad 17x-17y=6d \end{cases}$$

Substitute 475 for x and solve:

$$\begin{cases} 5(475)+5y=2d \quad \rightarrow \quad 2375+5y=2d \\ 17(475)-17y=6d \quad \rightarrow \quad 8075-17y=6d \end{cases}$$

$$8075-17y=3(2375+5y)$$

$$8075-17y=7125+15y \rightarrow 950=32y \rightarrow y \approx 29.69$$

The speed of the jet stream is approximately 29.69 miles per hour.

111. Let x = the number of hours for Bruce to do the job alone.
Let y = the number of hours for Bryce to do the job alone.
Let z = the number of hours for Marty to do the job alone.
Then $\frac{1}{x}$ represents the fraction of the job that Bruce does in one hour.
$\frac{1}{y}$ represents the fraction of the job that Bryce does in one hour.
$\frac{1}{z}$ represents the fraction of the job that Marty does in one hour.
The equation representing Bruce and Bryce working together is:

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{\left(\frac{4}{3}\right)}=\frac{3}{4}=0.75$$

The equation representing Bryce and Marty working together is:

$$\frac{1}{y}+\frac{1}{z}=\frac{1}{\left(\frac{8}{5}\right)}=\frac{5}{8}=0.675$$

The equation representing Bruce and Marty working together is:

$$\frac{1}{x}+\frac{1}{z}=\frac{1}{\left(\frac{8}{3}\right)}=0.375$$

Solve the system of equations:

$$\begin{cases} x^{-1}+y^{-1}=0.75 \\ y^{-1}+z^{-1}=0.675 \\ x^{-1}+z^{-1}=0.375 \end{cases}$$

Let $u=x^{-1},\ v=y^{-1},\ w=z^{-1}$

$$\begin{cases} u+v=0.75 \quad\rightarrow\quad u=0.75-v \\ v+w=0.675 \quad\rightarrow\quad w=0.675-v \\ u+w=0.375 \end{cases}$$

Substitute into the third equation and solve:

$$0.75-v+0.675-v=0.375 \rightarrow -2v=-1 \rightarrow v=0.5$$

$$u=0.75-0.5=0.25$$

$$w=0.675-0.5=0.125$$

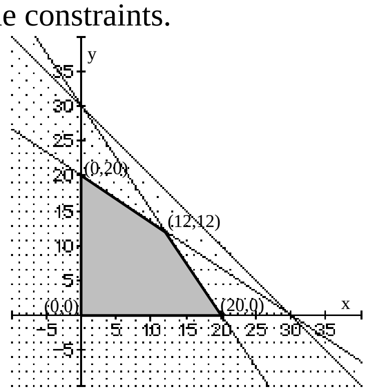
Solve for x, y, and z: $x=4,\ \ y=2,\ \ z=8$ (reciprocals)

Bruce can do the job in 4 hours, Bryce in 2 hours, and Marty in 8 hours.

112. Let x = the number of dancing girls produced.
Let y = the number of mermaids produced.
The total profit is: $P=25x+30y$. Profit is to be maximized; thus, this is the objective function.
The constraints are:

$x\ge 0,\ \ y\ge 0$	A non-negative number of figurines must be produced.
$3x+3y\le 90$	90 hours are available for molding.
$6x+4y\le 120$	120 hours are available for painting.
$2x+3y\le 60$	60 hours are available for glazing.

Graph the constraints.



To find the intersection of $6x+4y=120$ and $2x+3y=60$, solve the system:

$$\begin{cases} 6x+4y=120 \\ 2x+3y=60 \quad\rightarrow\quad 2x=60-3y \end{cases}$$

Substitute and solve:

$$3(2x)+4y=120 \rightarrow 3(60-3y)+4y=120$$

$$180-9y+4y=120 \rightarrow -5y=-60 \rightarrow y=12$$

$$2x=60-3(12)=24 \rightarrow x=12$$

The point of intersection is (12, 12).

The corner points are (0, 0), (0, 20), (20, 0), (12, 12).

Evaluate the objective function:

Vertex	Value of $P = 25x + 30y$
(0, 0)	P = 25(0) + 30(0) = 0
(0, 20)	P = 25(0) + 30(20) = 600
(20, 0)	P = 25(20) + 30(0) = 500
(12, 12)	P = 25(12) + 30(12) = 660

The maximum profit is $660, when 12 dancing girl and 12 mermaid figurines are produced each day.

To determine the excess, evaluate each constraint at x = 12 and y = 12:

Molding: $3x+3y=3(12)+3(12)=36+36=72$

Painting: $6x+4y=6(12)+4(12)=72+48=120$

Glazing: $2x+3y=2(12)+3(12)=24+36=60$

Painting and glazing are at their capacity. Molding has 18 more hours available, since only 72 of the 90 hours are used.

113. Let x = the number of gasoline engines produced each week.

Let y = the number of diesel engines produced each week.

The total cost is: $C = 450x + 550y$. Cost is to be minimized; thus, this is the objective function.

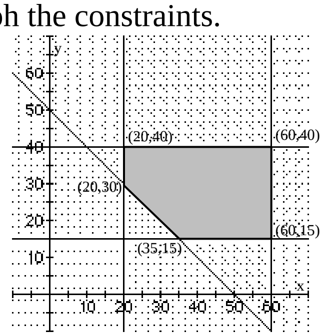
The constraints are:

$20 \le x \le 60$ number of gasoline engines needed and capacity each week.

$15 \le y \le 40$ number of diesel engines needed and capacity each week.

$x + y \ge 50$ number of engines produced to prevent layoffs.

Graph the constraints.



The corner points are (20, 30), (20, 40), (35, 15), (60, 15), (60, 40).

Evaluate the objective function:

Vertex	Value of $C = 450x + 550y$
(20, 30)	C = 450(20) + 550(30) = 25,500
(20, 40)	C = 450(20) + 550(40) = 31,000
(35, 15)	C = 450(35) + 550(15) = 24,000
(60, 15)	C = 450(60) + 550(15) = 35,250
(60, 40)	C = 450(60) + 550(40) = 49,000

The minimum cost is \$24,000, when 35 gasoline engines and 15 diesel engines are produced.
The excess capacity is 15 gasoline engines, since only 20 gasoline engines had to be delivered.