Chapter 14

Counting and Probability

14.2 Permutations and Combinations

1. $P(6, 2) = \frac{6!}{(6-2)!} = \frac{6!}{4!} = \frac{6\ 5\ 4!}{4!} = 30$

2. $P(7, 2) = \frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{7\ 6\ 5!}{5!} = 42$

3. $P(4, 4) = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4\ 3\ 2\ 1}{1} = 24$

4. $P(8, 8) = \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{8\ 7\ 6\ 5\ 4\ 3\ 2\ 1}{1} = 40320$

5. $P(7, 0) = \frac{7!}{(7-0)!} = \frac{7!}{7!} = 1$

6. $P(9, 0) = \frac{9!}{(9-0)!} = \frac{9!}{9!} = 1$

7. $P(8, 4) = \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{8\ 7\ 6\ 5\ 4!}{4!} = 1680$

8. $P(8, 3) = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\ 7\ 6\ 5!}{5!} = 336$

9. $C(8, 2) = \frac{8!}{(8-2)!\,2!} = \frac{8!}{6!\,2!} = \frac{8\ 7\ 6!}{6!\ 2\ 1} = 28$

10. $C(8, 6) = \frac{8!}{(8-6)!\,6!} = \frac{8!}{2!\,6!} = \frac{8\ 7\ 6!}{6!\ 2\ 1} = 28$

11. $C(7, 4) = \frac{7!}{(7-4)!\,4!} = \frac{7!}{3!\,4!} = \frac{7\ 6\ 5\ 4!}{4!\ 3\ 2\ 1} = 35$

12. $C(6, 2) = \frac{6!}{(6-2)!\,2!} = \frac{6!}{4!\,2!} = \frac{6\ 5\ 4!}{4!\ 2\ 1} = 15$

13. $C(15, 15) = \frac{15!}{(15-15)!\,15!} = \frac{15!}{0!\,15!} = \frac{15!}{15!\ 1} = 1$

14. $C(18, 1) = \frac{18!}{(18-1)!\,1!} = \frac{18!}{17!\,1!} = \frac{18\ 17!}{17!\ 1} = 18$

15. $C(26, 13) = \dfrac{26!}{(26-13)!13!} = \dfrac{26!}{13!13!} = 10{,}400{,}600$

16. $C(18, 9) = \dfrac{18!}{(18-9)!\,9!} = \dfrac{18!}{9!9!} = 48620$

17. {*abc, abd, abe, acb, acd, ace, adb, adc, ade, aeb, aec, aed, bac, bad, bae, bca, bcd, bce, bda, bdc, bde, bea, bec, bed, cab, cad, cae, cba, cbd, cbe, cda, cdb, cde, cea, ceb, ced, dab, dac, dae, dba, dbc, dbe, dca, dcb, dce, dea, deb, dec, eab, eac, ead, eba, ebc, ebd, eca, ecb, ecd, eda, edb, edc*}

$$P(5,3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5\ 4\ 3\ 2!}{2!} = 60$$

18. {*ab, ac, ad, ae, ba, bc, bd, be, ca, cb, cd, ce, da, db, dc, de, ea, eb, ec, ed*}

$$P(5,2) = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5\ 4\ 3!}{3!} = 20$$

19. {123, 124, 132, 134, 142, 143, 213, 214, 231, 234, 241, 243, 312, 314, 321, 324, 341, 342, 412, 413, 421, 423, 431, 432}

$$P(4,3) = \frac{4!}{(4-3)!} = \frac{4!}{1!} = \frac{4\ 3\ 2\ 1}{1} = 24$$

20. {123, 124, 125, 126, 132, 134, 135, 136, 142, 143, 145, 146, 152, 153, 154, 156, 162, 163, 164, 165, 213, 214, 215, 216, 231, 234, 235, 236, 241, 243, 245, 246, 251, 253, 254, 256, 261, 263, 264, 265, 312, 314, 315, 316, 321, 324, 325, 326, 341, 342, 345, 346, 351, 352, 354, 356, 361, 362, 364, 365, 412, 413, 415, 416, 421, 423, 425, 426, 431, 432, 435, 436, 451, 452, 453, 456, 461, 462, 463, 465, 512, 513, 514, 516, 521, 523, 524, 526, 531, 532, 534, 536, 541, 542, 543, 546, 561, 562, 563, 564, 612, 613, 614, 615, 621, 623, 624, 625, 631, 632, 634, 635, 641, 642, 643, 645, 651, 652, 653, 654}

$$P(6,3) = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6\ 5\ 4\ 3!}{3!} = 120$$

21. {*abc, abd, abe, acd, ace, ade, bcd, bce, bde, cde*}

$$C(5,3) = \frac{5!}{(5-3)!3!} = \frac{5\ 4\ 3!}{2\ 1\ 3!} = 10$$

22. {*ab, ac, ad, ae, bc, bd, be, cd, ce, de*}

$$C(5,2) = \frac{5!}{(5-2)!2!} = \frac{5\ 4\ 3!}{2\ 1\ 3!} = 10$$

23. {123, 124, 134, 234} $C(4,3) = \dfrac{4!}{(4-3)!3!} = \dfrac{4\ 3!}{1!3!} = 4$

24. {123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456}

$$C(6,3) = \frac{6!}{(6-3)!\,3!} = \frac{6\ 5\ 4\ 3!}{3\ 2\ 1\ 3!} = 20$$

25. There are 5 choices of shirts and 3 choices of ties; there are (5)(3) = 15 combinations.

26. There are 3 choices of blouses and 5 choices of skirts; there are (3)(5) = 15 different outfits.

27. There are 4 choices for the first letter in the code and 4 choices for the second letter in the code; there are (4)(4) = 16 possible two-letter codes.

28. There are 5 choices for the first letter in the code and 5 choices for the second letter in the code; there are (5)(5) = 25 possible two-letter codes.

29. There are two choices for each of three positions; there are (2)(2)(2) = 8 possible three-digit numbers.

30. There are ten choices for each of three positions; there are (10)(10)(10) = 1000 possible three-digit numbers.

31. To line up the four people, there are 4 choices for the first position, 3 choices for the second position, 2 choices for the third position, and 1 choice for the fourth position. Thus there are (4)(3)(2)(1) = 24 possible ways four people can be lined up.

32. To stack the five boxes, there are 5 choices for the first position, 4 choices for the second position, 3 choices for the third position, 2 choices for the fourth position, and 1 choice for the fifth position. Thus, there are (5)(4)(3)(2)(1) = 120 possible ways five boxes can be stacked up.

33. Since no letter can be repeated, there are 5 choices for the first letter, 4 choices for the second letter, and 3 choices for the third letter. Thus, there are (5)(4)(3) = 60 possible three-letter codes.

34. Since no letter can be repeated, there are 6 choices for the first letter, 5 choices for the second letter, 4 choices for the third letter, and 3 choices for the fourth letter. Thus, there are (6)(5)(4)(3) = 360 possible three-letter codes.

35. There are 26 possible one-letter names. There are (26)(26) = 676 possible two-letter names. There are (26)(26)(26) = 17576 possible three-letter names. Thus, there are 26 + 676 + 17576 = 18,278 possible companies that can be listed on the New York Stock Exchange.

36. There are (26)(26)(26)(26) = 456,976 possible four-letter names. There are (26)(26)(26)(26)(26) = 11,881,376 possible five-letter names. Thus, there are 456,976 + 11,881,376 = 12,338,352 possible companies that can be listed on the NASDAQ.

37. A committee of 4 from a total of 7 students is given by:

$$C(7,4) = \frac{7!}{(7-4)!\,4!} = \frac{7!}{3!\,4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{3 \cdot 2 \cdot 1 \cdot 4!} = 35$$

35 committees are possible.

38. A committee of 3 from a total of 8 professors is given by:

$$C(8,3) = \frac{8!}{(8-3)!\,3!} = \frac{8!}{5!\,3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 1 \cdot 5!} = 56$$

56 committees are possible.

39. There are 2 possible answers for each question. Therefore, there are $2^{10} = 1024$ different possible arrangements of the answers.

40. There are 4 possible answers for each question. Therefore, there are $4^{5} = 1024$ different possible arrangements of the answers.

41. There are 9 choices for the first digit, and 10 choices for each of the other three digits. Thus, there are (9)(10)(10)(10) = 9000 possible four-digit numbers.

42. There are 8 choices for the first digit, and 10 choices for each of the other four digits. Thus, there are (8)(10)(10)(10)(10) = 80,000 possible five-digit numbers.

43. There are 5 choices for the first position, 4 choices for the second position, 3 choices for the third position, 2 choices for the fourth position, and 1 choice for the fifth position. Thus, there are (5)(4)(3)(2)(1) = 120 possible arrangements of the books.

44. (a) There are 26 choices for each of the first two positions, and 10 choices for each of the next four positions. Thus, there are (26)(26)(10)(10)(10)(10) = 6,760,000 possible license plates.

(b) There are 26 choices for each of the first two positions, 10 choices for the first digit, 9 choices for the second digit, 8 choices for the third digit, and 7 choices for the fourth digit. Thus, there are (26)(26)(10)(9)(8)(7) = 3,407,040 possible license plates.

(c) There are 26 choices for the first letter, 25 choices for the second letter, 10 choices for the first digit, 9 choices for the second digit, 8 choices for the third digit, and 7 choices for the fourth digit. Thus, there are (26)(25)(10)(9)(8)(7) = 3,276,000 possible license plates.

45. There are 8 choices for the DOW stocks, 15 choices for the NASDAQ stocks, and 4 choices for the global stocks. Thus, there are (8)(15)(4) = 480 different portfolios.

46. There are 50 choices for the first number, 50 choices for the second number, and 50 choices for the third number. Thus, there are (50)(50)(50) = 125,000 different lock combinations.

47. The first person can have any of 365 days, the second person can have any of the remaining 364 days. Thus, there are (365)(364) = 132,860 possible ways two people can have different birthdays.

48. The first person can have any of 365 days, the second person can have any of the remaining 364 days, the third person can have any of the remaining 363 days, the fourth person can have any of the remaining 362 days, and the fifth person can have any of the remaining 361 days. Thus, there are (365)(364)(363)(362)(361) = $6.302555019 \times 10^{12}$ possible ways five people can have different birthdays.

49. Choosing 2 boys from the 4 boys can be done C(4,2) ways, and choosing 3 girls from the 8 girls can be done in C(8,3) ways. Thus, there are a total of:

$$C(4,2)\cdot C(8,3) = \frac{4!}{(4-2)!\,2!}\cdot\frac{8!}{(8-3)!\,3!} = \frac{4!}{2!\,2!}\cdot\frac{8!}{5!\,3!}$$
$$= \frac{4\cdot 3!}{2\cdot 1\cdot 2\cdot 1}\cdot\frac{8\cdot 7\cdot 6\cdot 5!}{5!\,3!} = 336$$

50. The committee is made up of 2 of 4 administrators, 3 of 8 faculty, and 5 of 20 students. The number of possible committees is:

$$C(4,2)\cdot C(8,3)\cdot C(20,5) = \frac{4!}{(4-2)!\,2!}\cdot\frac{8!}{(8-3)!\,3!}\cdot\frac{20!}{(20-5)!\,5!}$$
$$= \frac{4!}{2!\,2!}\cdot\frac{8!}{5!\,3!}\cdot\frac{20!}{15!\,5!} = \frac{4!}{2\cdot 1\cdot 2\cdot 1}\cdot\frac{8\cdot 7\cdot 6\cdot 5!}{5\cdot 4!\cdot 3!}\cdot\frac{20\cdot 19\cdot 18\cdot 17\cdot 16\cdot 15!}{15!\,5!}$$
$$= 5{,}209{,}344 \text{ possible committees}$$

51. This is a permutation with repetition. There are $\frac{9!}{2!\,2!} = 90{,}720$ different words.

52. This is a permutation with repetition. There are $\frac{11!}{2!\,2!\,2!} = 4{,}989{,}600$ different words.

53. (a) $C(7,2)\cdot C(3,1) = 21\cdot 3 = 63$
(b) $C(7,3) = 35$
(c) $C(3,3) = 1$

54. (a) $C(15,5)\cdot C(10,0) = \frac{15!}{10!\,5!}\cdot\frac{10!}{0!\,10!} = \frac{15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10!}{10!\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = 3003$

(b) $C(15,3)\cdot C(10,2) = \frac{15!}{12!\,3!}\cdot\frac{10!}{8!\,2!} = \frac{15\cdot 14\cdot 13\cdot 12!}{12!\cdot 3\cdot 2\cdot 1}\cdot\frac{10\cdot 9\cdot 8!}{8!\cdot 2\cdot 1} = 20475$

(c) $C(15,4)\cdot C(10,1) + C(15,5)\cdot C(10,0) = \frac{15\cdot 14\cdot 13\cdot 12\cdot 11!}{11!\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{10\cdot 9!}{9!\cdot 1} + 3003 = 16653$