Chapter 14

Counting and Probability

14.R Chapter Review

1. $A \cup B = \{1, 3, 5, 7\} \cup \{3, 5, 6, 7, 8\} = \{1, 3, 5, 6, 7, 8\}$

2. $B \cup C = \{3, 5, 6, 7, 8\} \cup \{2, 3, 7, 8, 9\} = \{2, 3, 5, 6, 7, 8, 9\}$

3. $A \cap C = \{1, 3, 5, 7\} \cap \{2, 3, 7, 8, 9\} = \{3, 7\}$

4. $A \cap B = \{1, 3, 5, 7\} \cap \{3, 5, 6, 7, 8\} = \{3, 5, 7\}$

5. $\overline{A} \cup \overline{B} = \overline{\{1, 3, 5, 7\}} \cup \overline{\{3, 5, 6, 7, 8\}} = \{2, 4, 6, 8, 9\} \cup \{1, 2, 4, 9\} = \{1, 2, 4, 6, 8, 9\}$

6. $\overline{B} \cap \overline{C} = \overline{\{3, 5, 6, 7, 8\}} \cap \overline{\{2, 3, 7, 8. 9\}} = \{1, 2, 4, 9\} \cap \{1, 4, 5, 6\} = \{1, 4\}$

7. $\overline{B \cap C} = \overline{\{3, 5, 6, 7, 8\} \cap \{2, 3, 7, 8, 9\}} = \overline{\{3, 7, 8\}} = \{1, 2, 4, 5, 6, 9\}$

8. $\overline{A \cup B} = \overline{\{1, 3, 5, 7\} \cup \{3, 5, 6, 7, 8\}} = \overline{\{1, 3, 5, 6, 7, 8\}} = \{2, 4, 9\}$

9. $n(A) = 8,\ n(B) = 12,\ n(A \cap B) = 3$
$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 8 + 12 - 3 = 17$$

10. $n(A) = 12,\ n(A \cup B) = 30,\ n(A \cap B) = 6$
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 30 &= 12 + n(B) - 6 \\ n(B) &= 30 - 12 + 6 = 24 \end{aligned}$$

11. From the figure:
$$n(A) = 20 + 2 + 6 + 1 = 29$$

12. From the figure:
$$n(A \text{ or } B) = 20 + 2 + 6 + 1 + 5 + 0 = 34$$

13. From the figure:
$$n(A \text{ and } C) = n(A \cap C) = 1 + 6 = 7$$

14. From the figure:
$$n(\text{not in } B) = 20 + 1 + 4 + 20 = 45$$

15. From the figure:
$n(\text{neither in } A \text{ nor in } C) = n(\overline{A \cup C}) = 20 + 5 = 25$

16. From the figure:
$n(\text{in } B \text{ but not in } C) = 2 + 5 = 7$

17. $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

18. $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

19. $P(8,3) = \dfrac{8!}{(8-3)!} = \dfrac{8!}{5!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 336$

20. $P(7,3) = \dfrac{7!}{(7-3)!} = \dfrac{7!}{4!} = \dfrac{7 \cdot 6 \cdot 5 \cdot 4!}{4!} = 210$

21. $C(8,3) = \dfrac{8!}{(8-3)!\,3!} = \dfrac{8!}{5!\,3!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1} = 56$

22. $C(7,3) = \dfrac{7!}{(7-3)!\,3!} = \dfrac{7!}{4!\,3!} = \dfrac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2 \cdot 1} = 35$

23. There are 2 choices of material, 3 choices of color, and 10 choices of size. The complete assortment would have: $2 \cdot 3 \cdot 10 = 60$ suits.

24. This is a permutation of 5 items taken 5 at a time. There are
$P(5,5) = \dfrac{5!}{(5-5)!} = \dfrac{5!}{0!} = 5! = 120$ possible wirings.

25. There are two possible outcomes for each game or
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7 = 128$ outcomes for 7 games.

26. There are two possible outcomes for each game or
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6 = 64$ outcomes for 6 games.

27. Since order is significant, this is a permutation.
$P(9,4) = \dfrac{9!}{(9-4)!} = \dfrac{9!}{5!} = \dfrac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 3024$ ways to seat 4 people in 9 seats.

28. Since order is significant, this is a permutation.
$P(4,4) = \dfrac{4!}{(4-4)!} = \dfrac{4!}{0!} = \dfrac{4 \cdot 3 \cdot 2 \cdot 1}{1} = 24$ arrangements of the letters in ROSE.

29. Choose 4 runners - order is not significant:
$C(8,4) = \dfrac{8!}{(8-4)!\,4!} = \dfrac{8!}{4!\,4!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 4!} = 70$ ways a squad can be chosen.

30. Choose 3 problems - order is not significant:
$C(10,3) = \dfrac{10!}{(10-3)!\,3!} = \dfrac{10!}{7!\,3!} = \dfrac{10 \cdot 9 \cdot 8 \cdot 7!}{3 \cdot 2 \cdot 1 \cdot 7!} = 120$ different tests are possible.

31. Choose 14 teams 2 at a time:

$$C(14,2) = \frac{14!}{(14-2)!\,2!} = \frac{14!}{12!\,2!} = \frac{14\ 13\ 12!}{12!\ 2\ 1} = 91 \text{ ways to pair 14 teams.}$$

32. (a) Since order is important, this is a permutation:

$$P(5,5)\,P(5,5) = \frac{5!}{(5-5)!}\ \frac{5!}{(5-5!} = 5!\ 5! = 120\ 120 = 14400 \text{ different arrangements.}$$

(b) There would be $5\ 5\ 4\ 4\ 3\ 3\ 2\ 2\ 1\ 1 = 14400$ different arrangements.

33. There are $8\ 10\ 10\ 10\ 10\ 10\ 2 = 1{,}600{,}000$ possible phone numbers.

34. There are $5\ 3\ 4 = 60$ different types of homes that can be built.

35. There are $24\ 9\ 10\ 10\ 10 = 216{,}000$ possible license plates.

36. There are $2^8 = 256$ different numbers.

37. Since there are repeated letters:

$$\frac{7!}{2!\,2!} = \frac{7\ 6\ 5\ 4\ 3\ 2\ 1}{2\ 1\ 2\ 1} = 1260 \text{ different words can be formed.}$$

38. Since there are repeated colors:

$$\frac{10!}{4!\ 3!\ 2!\ 1!} = \frac{10\ 9\ 8\ 7\ 6\ 5\ 4\ 3\ 2\ 1}{4\ 3\ 2\ 1\ 3\ 2\ 1\ 2\ 1\ 1} = 12600 \text{ different vertical arrangements.}$$

39. (a) $C(9,4)\ C(9,3)\,C(9,2) = 126\ 84\ 36 = 381{,}024$ committees can be formed.

(b) $C(9,4)\ C(5,3)\,C(2,2) = 126\ 10\ 1 = 1260$ committees can be formed.

40. (a) $C(5,1)\ C(8,3) = \frac{5!}{(5-1)!1!}\ \frac{8!}{(8-3)!3!} = 5\ 56 = 280$ committees containing exactly 1 man.

(b) $C(5,2)\,C(8,2) = 10\ 28 = 280$ committees containing exactly 2 women.

(c) $C(5,1)\ C(8,3) + C(5,2)\,C(8,2) + C(5,3)\,C(8,1) = 280 + 280 + 10\ 8 = 640$ committees containing at least 1 man.

41. (a) $365\ 364\ 363\ 362\ \ldots\ 348 = 8.634628387 \times 10^{45}$

(b) $P(\text{no one has same birthday}) = \frac{365\ 364\ 363\ 362\ \ldots\ 348}{365^{18}} = 0.6531 = 65.31\%$

(c) $P(\text{at least 2 have same birthday}) = 1 - P(\text{no one has same birthday})$

$= 1 - 0.6531 = 0.3469 = 34.69\%$

42. (a) $P(\text{heart disease}) = 0.321$

(b) $P(\text{not heart disease}) = 1 - P(\text{heart disease}) = 1 - 0.321 = 0.679$

43. (a) $P(\text{unemployed}) = 0.054 = 5.4\%$

(b) $P(\text{not unemployed}) = 1 - P(\text{unemployed}) = 1 - 0.054 = 0.946 = 94.6\%$

44. $P(40\text{ watt}) = \dfrac{n(40\text{ watt})}{n(\text{bulbs})} = \dfrac{3}{20}$

$P(\text{not 75 watt}) = 1 - P(75\text{ watt}) = 1 - \dfrac{n(75\text{ watt})}{n(\text{bulbs})} = 1 - \dfrac{11}{20} = \dfrac{9}{20}$

45. $P(\$1\text{ bill}) = \dfrac{n(\$1\text{ bill})}{n(S)} = \dfrac{4}{9}$

46. $P(\text{ROSE}) = \dfrac{1}{4}\ \dfrac{1}{3}\ \dfrac{1}{2}\ \dfrac{1}{1} = \dfrac{1}{24}$

47. Let S be all possible selections, let D be a card that is divisible by 5, and let PN be a 1 or a prime number.

$n(S) = 100$

$n(D) = 20$ (There are 20 numbers divisible by 5 between 1 and 100.)

$n(PN) = 26$ (There are 25 prime numbers less than or equal to 100.)

$P(D) = \dfrac{n(D)}{n(S)} = \dfrac{20}{100} = \dfrac{1}{5} = 0.2$

$P(PN) = \dfrac{n(PN)}{n(S)} = \dfrac{26}{100} = \dfrac{13}{50} = 0.26$

48. (a) $P(3\text{ Merlot}) = \dfrac{C(5,3)}{C(12,3)} = \dfrac{10}{220} = \dfrac{1}{22} \approx 0.0455$

(b) $P(2\text{ Merlot, 1 Cabernet}) = \dfrac{C(5,2)\,C(7,1)}{C(12,3)} = \dfrac{10\cdot 7}{220} = \dfrac{70}{220} = \dfrac{7}{22} \approx 0.3182$

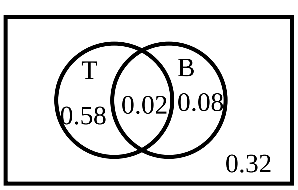
(c) $P(3\text{ Cabernet}) = \dfrac{C(7,3)}{C(12,3)} = \dfrac{35}{220} = \dfrac{7}{44} \approx 0.1591$

49. (a) $P(5\text{ heads}) = \dfrac{n(5\text{ heads})}{n(S)} = \dfrac{C(10,5)}{2^{10}} = \dfrac{\frac{10!}{5!5!}}{1024} = \dfrac{252}{1024} = 0.2461$

(b) $P(\text{all heads}) = \dfrac{n(\text{all heads})}{n(S)} = \dfrac{1}{2^{10}} = \dfrac{1}{1024} = 0.00098$

50. (a) $P(T \cup B) = P(T) + P(B) - P(T \cap B) = 0.6 + 0.1 - 0.02 = 0.68$

(b)



$P(T\text{ and not }B) = 0.58$

(c) $P(\overline{T \cup B}) = 1 - P(T \cup B) = 1 - 0.68 = 0.32$