Chapter 1

Equations and Inequalities

1.1 Equations

1. $$\begin{aligned} 7x &= 21 \\ \frac{7x}{7} &= \frac{21}{7} \\ x &= 3 \end{aligned}$$

2. $$\begin{aligned} 6x &= -24 \\ \frac{6x}{6} &= \frac{-24}{6} \\ x &= -4 \end{aligned}$$

3. $$\begin{aligned} 3x + 15 &= 0 \\ 3x + 15 - 15 &= 0 - 15 \\ 3x &= -15 \\ \frac{3x}{3} &= \frac{-15}{3} \\ x &= -5 \end{aligned}$$

4. $$\begin{aligned} 6x + 18 &= 0 \\ 6x + 18 - 18 &= 0 - 18 \\ 6x &= -18 \\ \frac{6x}{6} &= \frac{-18}{6} \\ x &= -3 \end{aligned}$$

5. $$\begin{aligned} 2x - 3 &= 0 \\ 2x - 3 + 3 &= 0 + 3 \\ 2x &= 3 \\ \frac{2x}{2} &= \frac{3}{2} \\ x &= \frac{3}{2} \end{aligned}$$

6. $$\begin{aligned} 3x + 4 &= 0 \\ 3x + 4 - 4 &= 0 - 4 \\ 3x &= -4 \\ \frac{3x}{3} &= \frac{-4}{3} \\ x &= -\frac{4}{3} \end{aligned}$$

7. $$\begin{aligned} \frac{1}{3}x &= \frac{5}{12} \\ (3)\left(\frac{1}{3}x\right) &= \left(\frac{5}{12}\right)(3) \\ x &= \frac{5}{4} \end{aligned}$$

8. $$\begin{aligned} \frac{2}{3}x &= \frac{9}{2} \\ (6)\left(\frac{2}{3}x\right) &= \left(\frac{9}{2}\right)(6) \\ 4x &= 27 \\ \frac{4x}{4} &= \frac{27}{4} \\ x &= \frac{27}{4} \end{aligned}$$

9.
$$\begin{aligned} 3x+4&=x\\ 3x+4-4&=x-4\\ 3x&=x-4\\ 3x-x&=x-4-x\\ 2x&=-4\\ \frac{2x}{2}&=\frac{-4}{2}\\ x&=-2 \end{aligned}$$

10.
$$\begin{aligned} 2x+9&=5x\\ 2x+9-9&=5x-9\\ 2x&=5x-9\\ 2x-5x&=5x-9-5x\\ -3x&=-9\\ \frac{-3x}{-3}&=\frac{-9}{-3}\\ x&=3 \end{aligned}$$

11.
$$\begin{aligned} 2t-6&=3-t\\ 2t-6+6&=3-t+6\\ 2t&=9-t\\ 2t+t&=9-t+t\\ 3t&=9\\ \frac{3t}{3}&=\frac{9}{3}\\ t&=3 \end{aligned}$$

12.
$$\begin{aligned} 5y+6&=-18-y\\ 5y+6-6&=-18-y-6\\ 5y&=-y-24\\ 5y+y&=-y-24+y\\ 6y&=-24\\ \frac{6y}{6}&=\frac{-24}{6}\\ y&=-4 \end{aligned}$$

13.
$$\begin{aligned} 6-x&=2x+9\\ 6-x-6&=2x+9-6\\ -x&=2x+3\\ -x-2x&=2x+3-2x\\ -3x&=3\\ \frac{-3x}{-3}&=\frac{3}{-3}\rightarrow x=-1 \end{aligned}$$

14.
$$\begin{aligned} 3-2x&=2-x\\ 3-2x-3&=2-x-3\\ -2x&=-x-1\\ -2x+x&=-x-1+x\\ -x&=-1\\ \frac{-x}{-1}&=\frac{-1}{-1}\rightarrow x=1 \end{aligned}$$

15.
$$\begin{aligned} 3+2n&=4n+7\\ 3+2n-3&=4n+7-3\\ 2n&=4n+4\\ 2n-4n&=4n+4-4n\\ -2n&=4\\ \frac{-2n}{-2}&=\frac{4}{-2}\rightarrow n=-2 \end{aligned}$$

16.
$$\begin{aligned} 6-2m&=3m+1\\ 6-2m-6&=3m+1-6\\ -2m&=3m-5\\ -2m-3m&=3m-5-3m\\ -5m&=-5\\ \frac{-5m}{-5}&=\frac{-5}{-5}\rightarrow m=1 \end{aligned}$$

17.
$$\begin{aligned} 2(3+2x)&=3(x-4)\\ 6+4x&=3x-12\\ 6+4x-6&=3x-12-6\\ 4x&=3x-18\\ 4x-3x&=3x-18-3x\\ x&=-18 \end{aligned}$$

18.
$$\begin{aligned} 3(2-x)&=2x-1\\ 6-3x&=2x-1\\ 6-3x-6&=2x-1-6\\ -3x&=2x-7\\ -3x-2x&=2x-7-2x\\ -5x&=-7\\ \frac{-5x}{-5}&=\frac{-7}{-5}\rightarrow x=\frac{7}{5} \end{aligned}$$

19.
$$8x-(3x+2)=3x-10$$
$$8x-3x-2=3x-10$$
$$5x-2=3x-10$$
$$5x-2+2=3x-10+2$$
$$5x=3x-8$$
$$5x-3x=3x-8-3x$$
$$2x=-8$$
$$\frac{2x}{2}=\frac{-8}{2}\rightarrow x=-4$$

20.
$$7-(2x-1)=10$$
$$7-2x+1=10$$
$$8-2x=10$$
$$8-2x-8=10-8$$
$$-2x=-2$$
$$\frac{-2x}{-2}=\frac{-2}{-2}\rightarrow x=1$$

21.
$$\frac{3}{2}x+2=\frac{1}{2}-\frac{1}{2}x$$
$$\frac{3}{2}x+2-2=\frac{1}{2}-\frac{1}{2}x-2$$
$$\frac{3}{2}x=-\frac{3}{2}-\frac{1}{2}x$$
$$\frac{3}{2}x+\frac{1}{2}x=-\frac{3}{2}-\frac{1}{2}x+\frac{1}{2}x$$
$$2x=-\frac{3}{2}$$
$$\left(\frac{1}{2}\right)(2x)=\left(-\frac{3}{2}\right)\left(\frac{1}{2}\right)$$
$$x=-\frac{3}{4}$$

22.
$$\frac{1}{3}x=2-\frac{2}{3}x$$
$$\frac{1}{3}x+\frac{2}{3}x=2-\frac{2}{3}x+\frac{2}{3}x$$
$$x=2$$

23.
$$\frac{1}{2}x-5=\frac{3}{4}x$$
$$\frac{1}{2}x-5+5=\frac{3}{4}x+5$$
$$\frac{1}{2}x=\frac{3}{4}x+5$$
$$\frac{1}{2}x-\frac{3}{4}x=\frac{3}{4}x+5-\frac{3}{4}x$$
$$\frac{2}{4}x-\frac{3}{4}x=5$$
$$-\frac{1}{4}x=5$$
$$(-4)\left(-\frac{1}{4}x\right)=(5)(-4)$$
$$x=-20$$

24.
$$1-\frac{1}{2}x=6$$
$$1-\frac{1}{2}x-1=6-1$$
$$-\frac{1}{2}x=5$$
$$(-2)\left(-\frac{1}{2}x\right)=(5)(-2)$$
$$x=-10$$

25.
$$\begin{aligned} \tfrac{2}{3}p &= \tfrac{1}{2}p + \tfrac{1}{3} \\ 6\left(\tfrac{2}{3}p\right) &= 6\left(\tfrac{1}{2}p + \tfrac{1}{3}\right) \\ 4p &= 3p + 2 \\ 4p - 3p &= 3p + 2 - 3p \\ p &= 2 \end{aligned}$$

26.
$$\begin{aligned} \tfrac{1}{2} - \tfrac{1}{3}p &= \tfrac{4}{3} \\ 6\left(\tfrac{1}{2} - \tfrac{1}{3}p\right) &= 6\left(\tfrac{4}{3}\right) \\ 3 - 2p &= 8 \\ 3 - 2p - 3 &= 8 - 3 \\ -2p &= 5 \\ \frac{-2p}{-2} &= \frac{5}{-2} \rightarrow p = -\frac{5}{2} \end{aligned}$$

27.
$$\begin{aligned} 0.9t &= 0.4 + 0.1t \\ 0.9t - 0.1t &= 0.4 + 0.1t - 0.1t \\ 0.8t &= 0.4 \\ \frac{0.8t}{0.8} &= \frac{0.4}{0.8} \rightarrow t = 0.5 \end{aligned}$$

28.
$$\begin{aligned} 0.9t &= 1 + t \\ 0.9t - t &= 1 + t - t \\ -0.1t &= 1 \\ \frac{-0.1t}{-0.1} &= \frac{1}{-0.1} \rightarrow t = -10 \end{aligned}$$

29.
$$\begin{aligned} \frac{x+1}{3} + \frac{x+2}{7} &= 2 \\ (21)\left(\frac{x+1}{3} + \frac{x+2}{7}\right) &= (2)(21) \\ (21)\left(\frac{x+1}{3}\right) + (21)\left(\frac{x+2}{7}\right) &= 42 \\ 7(x+1) + (3)(x+2) &= 42 \\ 7x + 7 + 3x + 6 &= 42 \\ 10x + 13 &= 42 \\ 10x + 13 - 13 &= 42 - 13 \\ 10x &= 29 \\ \frac{10x}{10} &= \frac{29}{10} \\ x &= 2.9 \end{aligned}$$

30.
$$\begin{aligned} \frac{2x+1}{3} + 16 &= 3x \\ 3\left(\frac{2x+1}{3} + 16\right) &= 3(3x) \\ 2x + 1 + 48 &= 9x \\ 2x + 49 &= 9x \\ 2x + 49 - 2x &= 9x - 2x \\ 49 &= 7x \\ \frac{49}{7} &= \frac{7x}{7} \\ x &= 7 \end{aligned}$$

31.
$$\frac{2}{y}+\frac{4}{y}=3$$
$$y\left(\frac{2}{y}+\frac{4}{y}\right)=y(3)$$
$$2+4=3y$$
$$6=3y$$
$$\frac{6}{3}=\frac{3y}{3}$$
$$y=2$$
and since y = 2 does not cause a denominator to equal zero, the solution set is {2}.

32.
$$\frac{4}{y}-5=\frac{5}{2y}$$
$$2y\left(\frac{4}{y}-5\right)=2y\left(\frac{5}{2y}\right)$$
$$8-10y=5$$
$$8-10y-8=5-8$$
$$-10y=-3$$
$$\frac{-10y}{-10}=\frac{-3}{-10}$$
$$y=\frac{3}{10}$$
and since $y=\frac{3}{10}$ does not cause a denominator to equal zero, the solution set is $\left\{\frac{3}{10}\right\}$.

33.
$$\frac{1}{2}+\frac{2}{x}=\frac{3}{4}$$
$$(4x)\left(\frac{1}{2}+\frac{2}{x}\right)=\left(\frac{3}{4}\right)(4x)$$
$$(4x)\left(\frac{1}{2}\right)+(4x)\left(\frac{2}{x}\right)=3x$$
$$2x+8=3x$$
$$2x+8-8=3x-8$$
$$2x=3x-8$$
$$2x-3x=3x-8-3x$$
$$-x=-8$$
$$\frac{-x}{-1}=\frac{-8}{-1}$$
$$x=8$$
and since x = 8 does not cause any denominator to equal zero, x = 8 solves the original equation.

34.
$$\frac{3}{x}-\frac{1}{3}=\frac{1}{6}$$
$$(6x)\left(\frac{3}{x}-\frac{1}{3}\right)=\left(\frac{1}{6}\right)(6x)$$
$$(6x)\left(\frac{3}{x}\right)-(6x)\left(\frac{1}{3}\right)=x$$
$$18-2x=x$$
$$18-2x+2x=x+2x$$
$$18=3x$$
$$\frac{18}{3}=\frac{3x}{3}$$
$$6=x$$
and since x = 6 does not cause any denominator to equal zero, x = 6 solves the original equation.

35.
$$\begin{aligned}(x+7)(x-1)&=(x+1)^2\\ x^2+6x-7&=x^2+2x+1\\ x^2+6x-7-x^2&=x^2+2x+1-x^2\\ 6x-7&=2x+1\\ 6x-7+7&=2x+1+7\\ 6x&=2x+8\\ 6x-2x&=2x+8-2x\\ 4x&=8\\ \frac{4x}{4}&=\frac{8}{4}\\ x&=2\end{aligned}$$

36.
$$\begin{aligned}(x+2)(x-3)&=(x+3)^2\\ x^2-x-6&=x^2+6x+9\\ x^2-x-6-x^2&=x^2+6x+9-x^2\\ -x-6&=6x+9\\ -x-6+6&=6x+9+6\\ -x&=6x+15\\ -x-6x&=6x+15-6x\\ -7x&=15\\ \frac{-7x}{-7}&=\frac{15}{-7}\to x=-\frac{15}{7}\end{aligned}$$

37.
$$\begin{aligned}x(2x-3)&=(2x+1)(x-4)\\ 2x^2-3x&=2x^2-7x-4\\ 2x^2-3x-2x^2&=2x^2-7x-4-2x^2\\ -3x&=-7x-4\\ -3x+7x&=-7x-4+7x\\ 4x&=-4\\ \frac{4x}{4}&=\frac{-4}{4}\to x=-1\end{aligned}$$

38.
$$\begin{aligned}x(1+2x)&=(2x-1)(x-2)\\ x+2x^2&=2x^2-5x+2\\ 2x^2+x-2x^2&=2x^2-5x+2-2x^2\\ x&=-5x+2\\ x+5x&=-5x+2+5x\\ 6x&=2\\ \frac{6x}{6}&=\frac{2}{6}\to x=\frac{1}{3}\end{aligned}$$

39.
$$\begin{aligned}z(z^2+1)&=3+z^3\\ z^3+z&=3+z^3\\ z^3+z-z^3&=3+z^3-z^3\\ z&=3\end{aligned}$$

40.
$$\begin{aligned}w(4-w^2)&=8-w^3\\ 4w-w^3&=8-w^3\\ 4w-w^3+w^3&=8-w^3+w^3\\ 4w&=8\\ \frac{4w}{4}&=\frac{8}{4}\to w=2\end{aligned}$$

41.
$$\begin{aligned}\frac{x}{x-2}+3&=\frac{2}{x-2}\\ (x-2)\left(\frac{x}{x-2}+3\right)&=\left(\frac{2}{x-2}\right)(x-2)\\ (x-2)\left(\frac{x}{x-2}\right)+(x-2)(3)&=2\\ x+3x-6&=2\\ 4x-6&=2\\ 4x-6+6&=2+6\\ 4x&=8\\ \frac{4x}{4}&=\frac{8}{4}\to x=2\end{aligned}$$

but x = 2 causes a denominator to equal zero, so we must discard this answer. Therefore the original equation has no real solution.

42.
$$\begin{aligned}\frac{2x}{x+3}&=\frac{-6}{x+3}-2\\ (x+3)\left(\frac{2x}{x+3}\right)&=\left(\frac{-6}{x+3}-2\right)(x+3)\\ 2x&=\left(\frac{-6}{x+3}\right)(x+3)-(2)(x+3)\\ 2x&=-6-2x-6\\ 2x&=-12-2x\\ 2x+2x&=-12-2x+2x\\ 4x&=-12\\ \frac{4x}{4}&=\frac{-12}{4}\to x=-3\end{aligned}$$

but x = - 3 causes a denominator to equal zero, so we must discard this answer. Therefore the original equation has no real solution.

43.
$$x^2 = 9x$$
$$x^2 - 9x = 0$$
$$x(x-9) = 0$$
$$x = 0 \text{ or } x = 9$$
The solution set is {0, 9}.

44.
$$4x^3 = x^2$$
$$4x^3 - x^2 = x^2 - x^2$$
$$4x^3 - x^2 = 0$$
$$x^2(4x-1) = 0$$
$$x^2 = 0 \text{ or}$$
$$4x - 1 = 0 \rightarrow x = 0$$
$$or \quad x = \frac{1}{4}$$
The solution set is $\left\{0, \frac{1}{4}\right\}$

45.
$$t^3 - 9t^2 = 0$$
$$t^2(t-9) = 0$$
$$t^2 = 0$$
$$\text{or } t - 9 = 0 \rightarrow t = 0$$
$$or \quad t = 9$$
The solution set is $\{0, 9\}$

46.
$$4z^3 - 8z^2 = 0$$
$$4z^2(z-2) = 0$$
$$4z^2 = 0$$
$$\text{or } z - 2 = 0 \rightarrow z = 0$$
$$or \quad z = 2$$
The solution set is $\{0, 2\}$

47.
$$\frac{2x}{x^2-4} = \frac{4}{x^2-4} - \frac{3}{x+2}$$
$$\frac{2x}{(x+2)(x-2)} = \frac{4}{(x+2)(x-2)} - \frac{3}{x+2}$$
$$(x+2)(x-2)\left(\frac{2x}{(x+2)(x-2)}\right) = \left(\frac{4}{(x+2)(x-2)} - \frac{3}{x+2}\right)(x+2)(x-2)$$
$$2x = \left(\frac{4}{(x+2)(x-2)}\right)(x+2)(x-2) - \left(\frac{3}{x+2}\right)(x+2)(x-2)$$
$$2x = 4 - (3)(x-2)$$
$$2x = 4 - 3x + 6$$
$$2x = 10 - 3x$$
$$2x + 3x = 10 - 3x + 3x$$
$$5x = 10$$
$$\frac{5x}{5} = \frac{10}{5}$$
$$x = 2$$
but $x = 2$ causes a denominator to equal zero, so we must discard this answer.
Therefore the original equation has no real solution.

48. $$\frac{x}{x^2-9}+\frac{4}{x+3}=\frac{3}{x^2-9}$$

$$\frac{x}{(x+3)(x-3)}+\frac{4}{x+3}=\frac{3}{x^2-9}$$

$$(x+3)(x-3)\left(\frac{x}{(x+3)(x-3)}+\frac{4}{x+3}\right)=\left(\frac{3}{x^2-9}\right)(x+3)(x-3)$$

$$(x+3)(x-3)\left(\frac{x}{(x+3)(x-3)}\right)+(x+3)(x-3)\left(\frac{4}{x+3}\right)=\left(\frac{3}{(x+3)(x-3)}\right)(x+3)(x-3)$$

$$x+(x-3)4=3$$

$$x+4x-12=3$$

$$5x-12=3$$

$$5x-12+12=3+12$$

$$5x=15$$

$$\frac{5x}{5}=\frac{15}{5}$$

$$x=3$$

but $x = 3$ causes a denominator to equal zero, so we must discard this answer. Therefore the original equation has no real solution.

49. $$\frac{x}{x+2}=\frac{3}{2}$$

$$(2)(x+2)\left(\frac{x}{x+2}\right)=\left(\frac{3}{2}\right)(2)(x+2)$$

$$(2)x=3(x+2)$$

$$2x=3x+6$$

$$2x-3x=3x+6-3x$$

$$-x=6$$

$$\frac{-x}{-1}=\frac{6}{-1}\rightarrow x=-6$$

and since x = - 6 does not cause any denominator to equal zero, x = - 6 solves the original equation.

50. $$\frac{3x}{x-1}=2$$

$$(x-1)\left(\frac{3x}{x-1}\right)=(2)(x-1)$$

$$3x=2x-2$$

$$3x-2x=2x-2-2x$$

$$x=-2$$

and since $x = -2$ does not cause a denominator to equal zero, $x = -2$ solves the original equation.

51. $$\frac{5}{2x-3}=\frac{3}{x+5}$$

$$\begin{aligned}
(2x-3)(x+5)\left(\frac{5}{2x-3}\right) &= \left(\frac{3}{x+5}\right)(2x-3)(x+5)\\
(x+5)(5) &= (3)(2x-3)\\
5x+25 &= 6x-9\\
5x+25-6x &= 6x-9-6x\\
25-x &= -9\\
25-x-25 &= -9-25\\
-x &= -34\\
\frac{-x}{-1} &= \frac{-34}{-1}\\
x &= 34
\end{aligned}$$

and since x = 34 does not cause any denominator to equal zero, x = 34 solves the original equation.

52. $$\frac{-4}{x+4}=\frac{-3}{x+6}$$

$$\begin{aligned}
(x+6)(x+4)\left(\frac{-4}{x+4}\right) &= \left(\frac{-3}{x+6}\right)(x+6)(x+4)\\
(x+6)(-4) &= (-3)(x+4)\\
-4x-24 &= -3x-12\\
-4x-24+4x &= -3x-12+4x\\
-24 &= -12+x\\
-24+12 &= -12+x+12\\
-12 &= x
\end{aligned}$$

and since x = - 12 does not cause any denominator to equal zero, x = - 12 solves the original equation.

53. $$\frac{6t+7}{4t-1}=\frac{3t+8}{2t-4}$$

$$\begin{aligned}
(4t-1)(2t-4)\left(\frac{6t+7}{4t-1}\right) &= \left(\frac{3t+8}{2t-4}\right)(4t-1)(2t-4)\\
(2t-4)(6t+7) &= (3t+8)(4t-1)\\
12t^2+14t-24t-28 &= 12t^2-3t+32t-8\\
12t^2+14t-24t-28-12t^2 &= 12t^2-3t+32t-8-12t^2\\
14t-24t-28 &= -3t+32t-8
\end{aligned}$$

$$
\begin{aligned}
-10t-28 &= 29t-8\\
-10t-28-29t &= 29t-8-29t\\
28-39t-28 &= -8-28\\
-39t &= -36\\
\frac{-39t}{-39} &= \frac{-36}{-39}\\
t &= \frac{12}{13}
\end{aligned}
$$

and since $t=\frac{12}{13}$ does not cause any denominator to equal zero, $t=\frac{12}{13}$ solves the original equation.

54. $\dfrac{8w+5}{10w-7}=\dfrac{4w-3}{5w+7}$

$$
\begin{aligned}
(10w-7)(5w+7)\left(\frac{8w+5}{10w-7}\right) &= \left(\frac{4w-3}{5w+7}\right)(10w-7)(5w+7)\\
(5w+7)(8w+5) &= (4w-3)(10w-7)\\
40w^2+25w+56w+35 &= 40w^2-28w-30w+21\\
40w^2+25w+56w+35-40w^2 &= 40w^2-28w-30w+21-40w^2\\
25w+56w+35 &= -28w-30w+21\\
81w+35 &= -58w+21\\
81w+35+58w &= -58w+21+58w\\
139w+35 &= 21\\
139w+35-35 &= 21-35\\
139w &= -14\\
\frac{139w}{139} &= \frac{-14}{139}\\
w &= -\frac{14}{139}
\end{aligned}
$$

and since w = - 14/139 does not cause any denominator to equal zero, w = - 14/139 solves the original equation.

55. $$\frac{4}{x-2} = \frac{-3}{x+5} + \frac{7}{(x+5)(x-2)}$$

$$(x+5)(x-2)\left(\frac{4}{x-2}\right) = \left(\frac{-3}{x+5} + \frac{7}{(x+5)(x-2)}\right)(x+5)(x-2)$$

$$(x+5)(4) = \left(\frac{-3}{x+5}\right)(x+5)(x-2) + \left(\frac{7}{(x+5)(x-2)}\right)(x+5)(x-2)$$

$$4x + 20 = (-3)(x-2) + 7$$

$$4x + 20 = -3x + 6 + 7$$

$$4x + 20 = -3x + 13$$

$$4x + 20 + 3x = -3x - 8 + 3x$$

$$7x + 20 = -8$$

$$7x + 20 - 20 = -8 - 20$$

$$7x = -28$$

$$\frac{7x}{7} = \frac{-28}{7}$$

$$x = -4$$

and since x = - 4 does not cause any denominator to equal zero, x = - 4 solves the original equation.

56. $$\frac{-4}{2x+3} + \frac{1}{x-1} = \frac{1}{(2x+3)(x-1)}$$

$$(2x+3)(x-1)\left(\frac{-4}{2x+3} + \frac{1}{x-1}\right) = \left(\frac{1}{(2x+3)(x-1)}\right)(2x+3)(x-1)$$

$$(2x+3)(x-1)\left(\frac{-4}{2x+3}\right) + (2x+3)(x-1)\left(\frac{1}{x-1}\right) = 1$$

$$(x-1)(-4) + (2x+3)(1) = 1$$

$$-4x + 4 + 2x + 3 = 1$$

$$-2x + 7 = 1$$

$$-2x + 7 + 2x = 1 + 2x$$

$$7 = 1 + 2x$$

$$7 - 1 = 1 + 2x - 1$$

$$6 = 2x$$

$$\frac{6}{2} = \frac{2x}{2}$$

$$3 = x$$

and since x = 3 does not cause any denominator to equal zero, x = 3 solves the original equation.

57. $$\frac{2}{y+3}+\frac{3}{y-4}=\frac{5}{y+6}$$

$$(y+3)(y-4)(y+6)\left(\frac{2}{y+3}+\frac{3}{y-4}\right)=\left(\frac{5}{y+6}\right)(y+3)(y-4)(y+6)$$

$$(y+3)(y-4)(y+6)\left(\frac{2}{y+3}\right)+(y+3)(y-4)(y+6)\left(\frac{3}{y-4}\right)=(5)(y+3)(y-4)$$

$$(y-4)(y+6)(2)+(y+3)(y+6)(3)=(5)(y+3)(y-4)$$

$$(y^2+6y-4y-24)(2)+(y^2+6y+3y+18)(3)=(5)(y^2-4y+3y-12)$$

$$(y^2+2y-24)(2)+(y^2+9y+18)(3)=(5)(y^2-y-12)$$

$$2y^2+4y-48+3y^2+27y+54=5y^2-5y-60$$

$$5y^2+31y+6=5y^2-5y-60$$

$$5y^2+31y+6-5y^2=5y^2-5y-60-5y^2$$

$$31y+6=-5y-60$$

$$31y+6+5y=-5y-60+5y$$

$$36y+6=-60$$

$$36y+6-6=-60-6$$

$$36y=-66$$

$$\frac{36y}{36}=\frac{-66}{36}$$

$$y=-\frac{11}{6}$$

and since $y = - 11/6$ does not cause any denominator to equal zero, $y = - 11/6$ solves the original equation.

58. $$\frac{5}{5z-11}+\frac{4}{2z-3}=\frac{-3}{5-z}$$

$$(5z-11)(2z-3)(5-z)\left(\frac{5}{5z-11}+\frac{4}{2z-3}\right)=\left(\frac{-3}{5-z}\right)(5z-11)(2z-3)(5-z)$$

$$(5z-11)(2z-3)(5-z)\left(\frac{5}{5z-11}\right)+(5z-11)(2z-3)(5-z)\left(\frac{4}{2z-3}\right)=(-3)(5z-11)(2z-3)$$

$$(2z-3)(5-z)(5)+(5z-11)(5-z)(4)=(-3)(5z-11)(2z-3)$$

$$\left(10z-2z^2-15+3z\right)(5)+\left(25z-5z^2-55+11z\right)(4)=(-3)\left(10z^2-15z-22z+33\right)$$
$$\left(-2z^2+13z-15\right)(5)+\left(-5z^2+36z-55\right)(4)=(-3)\left(10z^2-37z+33\right)$$
$$-10z^2+85z-75-20z^2+144z-220=-30z^2+111z-99$$
$$-30z^2+229z-295=-30z^2+111z-99$$
$$-30z^2+229z-295+30z^2=-30z^2+111z-99+30z^2$$
$$229z-295=111z-99$$
$$229z-295-229z=111z-99-229z$$
$$-295=-118z-99$$
$$-295+99=-118z-99+99$$
$$-196=-118z$$
$$\frac{-196}{-118}=\frac{-118z}{-118}$$
$$\frac{98}{59}=z$$

and since $z=\frac{98}{59}$ does not cause any denominator to equal zero, $z=\frac{98}{59}$ solves the original equation.

59.
$$\frac{x}{x^2-1}-\frac{x+3}{x^2-x}=\frac{-3}{x^2+x}$$
$$\frac{x}{(x+1)(x-1)}-\frac{x+3}{x(x-1)}=\frac{-3}{x(x+1)}$$
$$(x+1)(x-1)(x)\left(\frac{x}{(x+1)(x-1)}-\frac{x+3}{x(x-1)}\right)=\left(\frac{-3}{x(x+1)}\right)(x+1)(x-1)(x)$$
$$(x+1)(x-1)(x)\left(\frac{x}{(x+1)(x-1)}\right)-(x+1)(x-1)(x)\left(\frac{x+3}{x(x-1)}\right)=(-3)(x-1)$$
$$(x)(x)-(x+1)(x+3)=-3x+3$$
$$x^2-\left(x^2+3x+x+3\right)=-3x+3$$
$$x^2-\left(x^2+4x+3\right)=-3x+3$$
$$x^2-x^2-4x-3=-3x+3$$
$$-4x-3=-3x+3$$
$$-4x-3+4x=-3x+3+4x$$
$$-3=3+x$$
$$-3-3=3+x-3$$
$$-6=x$$

and since $x = -6$ does not cause any denominator to equal zero, $x = -6$ solves the original equation.

60. $$\frac{x+1}{x^2+2x}-\frac{x+4}{x^2+x}=\frac{-3}{x^2+3x+2}$$

$$\frac{x+1}{x(x+2)}-\frac{x+4}{x(x+1)}=\frac{-3}{(x+2)(x+1)}$$

$$x(x+2)(x+1)\left(\frac{x+1}{x(x+2)}-\frac{x+4}{x(x+1)}\right)=\left(\frac{-3}{(x+2)(x+1)}\right)x(x+2)(x+1)$$

$$x(x+2)(x+1)\left(\frac{x+1}{x(x+2)}\right)-x(x+2)(x+1)\left(\frac{x+4}{x(x+1)}\right)=(-3)x$$

$$(x+1)(x+1)-(x+2)(x+4)=-3x$$

$$x^2+2x+1-(x^2+4x+2x+8)=-3x$$

$$x^2+2x+1-(x^2+6x+8)=-3x$$

$$x^2+2x+1-x^2-6x-8=-3x$$

$$2x+1-6x-8=-3x$$

$$-4x-7=-3x$$

$$-4x-7+4x=-3x+4x$$

$$-7=x$$

and since $x = -7$ does not cause any denominator to equal zero, $x = -7$ solves the original equation.

61. $$3.2x+\frac{21.3}{65.871}=19.23$$

$$3.2x+\frac{21.3}{65.871}-\frac{21.3}{65.871}=19.23-\frac{21.3}{65.871}$$

$$3.2x=19.23-\frac{21.3}{65.871}$$

$$\left(\frac{1}{3.2}\right)(3.2x)=\left(19.23-\frac{21.3}{65.871}\right)\left(\frac{1}{3.2}\right)$$

$$x=\left(19.23-\frac{21.3}{65.871}\right)\left(\frac{1}{3.2}\right)$$

$$x\approx 5.91$$

62. $$6.2x-\frac{19.1}{83.72}=0.195$$

$$6.2x-\frac{19.1}{83.72}+\frac{19.1}{83.72}=0.195+\frac{19.1}{83.72}$$

$$6.2x=0.195+\frac{19.1}{83.72}$$

$$\left(\frac{1}{6.2}\right)(6.2x)=\left(0.195+\frac{19.1}{83.72}\right)\left(\frac{1}{6.2}\right)$$

$$x=\left(0.195+\frac{19.1}{83.72}\right)\left(\frac{1}{6.2}\right)$$

$$x\approx 0.07$$

63. $$14.72-21.58x=\frac{18}{2.11}x+2.4$$

$$14.72-21.58x-\frac{18}{2.11}x=\frac{18}{2.11}x+2.4-\frac{18}{2.11}x$$
$$14.72-21.58x-\frac{18}{2.11}x=2.4$$
$$14.72-21.58x-\frac{18}{2.11}x-14.72=2.4-14.72$$
$$-21.58x-\frac{18}{2.11}x=2.4-14.72$$
$$(x)\left(-21.58-\frac{18}{2.11}\right)=2.4-14.72$$
$$\left(\frac{1}{-21.58-\frac{18}{2.11}}\right)(x)\left(-21.58-\frac{18}{2.11}\right)=(2.4-14.72)\left(\frac{1}{-21.58-\frac{18}{2.11}}\right)$$
$$x=(2.4-14.72)\left(\frac{1}{-21.58-\frac{18}{2.11}}\right)$$
$$x\approx 0.41$$

64. $$18.63x-\frac{21.2}{2.6}=\frac{14}{2.32}x-20$$

$$18.63x-\frac{21.2}{2.6}-\frac{14}{2.32}x=\frac{14}{2.32}x-20-\frac{14}{2.32}x$$
$$18.63x-\frac{21.2}{2.6}-\frac{14}{2.32}x=-20$$
$$18.63x-\frac{21.2}{2.6}-\frac{14}{2.32}x+\frac{21.2}{2.6}=-20+\frac{21.2}{2.6}$$
$$18.63x-\frac{14}{2.32}x=-20+\frac{21.2}{2.6}$$
$$(x)\left(18.63-\frac{14}{2.32}\right)=-20+\frac{21.2}{2.6}$$
$$\left(\frac{1}{18.63-\frac{14}{2.32}}\right)(x)\left(18.63-\frac{14}{2.32}\right)=\left(-20+\frac{21.2}{2.6}\right)\left(\frac{1}{18.63-\frac{14}{2.32}}\right)$$

$$x = \left(-20 + \frac{21.2}{2.6}\right)\left(\frac{1}{18.63 - \frac{14}{2.32}}\right)$$

$$x \approx 0.94$$

65.

$$x^2 - 7x + 12 = 0$$
$$(x-4)(x-3) = 0$$
$$x - 4 = 0$$
$$\text{or } x - 3 = 0 \rightarrow x = 4$$
$$\text{or } x = 3$$

Therefore the solution set is $\{3,4\}$

66.

$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x - 3 = 0$$
$$\text{or } x + 2 = 0 \rightarrow x = 3$$
$$\text{or } x = -2.$$

Therefore the solution set is $\{-2,3\}$

67.

$$2x^2 + 5x - 3 = 0$$
$$(2x-1)(x+3) = 0$$
$$2x - 1 = 0$$
$$\text{or } x + 3 = 0 \rightarrow x = \frac{1}{2}$$
$$\text{or } x = -3$$

Therefore the solution set is $\left\{-3, \frac{1}{2}\right\}$.

68.

$$3x^2 + 5x - 2 = 0$$
$$(3x-1)(x+2) = 0$$
$$3x - 1 = 0$$
$$\text{or } x + 2 = 0 \rightarrow x = \frac{1}{3}$$
$$\text{or } x = -2$$

Therefore the solution set is $\left\{-2, \frac{1}{3}\right\}$.

69.

$$x^3 = 9x$$
$$x^3 - 9x = 0$$
$$x\left(x^2 - 9\right) = 0$$
$$x(x+3)(x-3) = 0$$
$$x = 0$$
$$\text{or } x + 3 = 0$$
$$\text{or } x - 3 = 0 \rightarrow x = 0$$
$$\text{or } x = -3 \text{ or } x = 3$$

Therefore the solution set is $\{-3,0,3\}$.

70.

$$x^4 = x^2$$
$$x^4 - x^2 = 0$$
$$x^2\left(x^2 - 1\right) = 0$$
$$x^2(x+1)(x-1) = 0$$
$$x^2 = 0$$
$$\text{or } x + 1 = 0$$
$$\text{or } x - 1 = 0 \rightarrow x = 0$$
$$\text{or } x = -1 \text{ or } x = 1$$

Therefore the solution set is $\{-1,0,1\}$.

71.

$$x^3 + x^2 - 20x = 0$$
$$x\left(x^2 + x - 20\right) = 0$$
$$x(x+5)(x-4) = 0$$
$$x = 0$$
$$\text{or } x + 5 = 0$$
$$\text{or } x - 4 = 0 \rightarrow x = 0$$
$$\text{or } x = -5 \text{ or } x = 4$$

Therefore the solution set is $\{-5,0,4\}$.

72.

$$x^3 + 6x^2 - 7x = 0$$
$$x\left(x^2 + 6x - 7\right) = 0$$
$$x(x+7)(x-1) = 0$$
$$x = 0 \text{ or } x + 7 = 0$$
$$\text{or } x - 1 = 0 \rightarrow x = 0$$
$$\text{or } x = -7 \text{ or } x = 1$$

Therefore the solution set is $\{-7,0,1\}$.

73. $x^3 + x^2 - x - 1 = 0$

We can factor by grouping to get

$$x^2(x+1) - (x+1) = 0$$
$$(x+1)(x^2-1) = 0$$
$$(x+1)(x+1)(x-1) = 0$$
$$x + 1 = 0$$
$$\text{or } x - 1 = 0 \rightarrow x = -1 \text{ or } x = 1$$

Therefore the solution set is $\{-1, 1\}$.

74. $x^3 + 4x^2 - x - 4 = 0$

We can factor by grouping to get

$$x^2(x+4) - (x+4) = 0$$
$$(x+4)(x^2-1) = 0$$
$$(x+4)(x+1)(x-1) = 0$$
$$x + 4 = 0$$
$$\text{or } x + 1 = 0$$
$$\text{or } x - 1 = 0 \rightarrow x = -4$$
$$\text{or } x = -1 \text{ or } x = 1$$

Therefore the solution set is $\{-4, -1, 1\}$.

75. $x^3 - 3x^2 - 4x + 12 = 0$

We can factor by grouping to get

$$x^2(x-3) - 4(x-3) = 0$$
$$(x-3)(x^2-4) = 0$$
$$(x-3)(x+2)(x-2) = 0$$
$$x - 3 = 0 \quad \text{or } x + 2 = 0$$
$$\text{or } x - 2 = 0 \rightarrow x = 3$$
$$\text{or } x = -2 \quad \text{or } x = 2$$

Therefore the solution set is $\{-2, 2, 3\}$.

76. $x^3 - 3x^2 - x + 3 = 0$

We can factor by grouping to get

$$x^2(x-3) - (x-3) = 0$$
$$(x-3)(x^2-1) = 0$$
$$(x-3)(x+1)(x-1) = 0$$
$$x - 3 = 0 \quad \text{or } x + 1 = 0$$
$$\text{or } x - 1 = 0 \rightarrow x = 3$$
$$\text{or } x = -1 \quad \text{or } x = 1$$

Therefore the solution set is $\{-1, 1, 3\}$.

77. $ax - b = c, \quad a \neq 0$

$$ax - b + b = c + b$$
$$ax = c + b$$
$$\frac{ax}{a} = \frac{c+b}{a}$$
$$x = \frac{c+b}{a}$$

78. $1 - ax = b, \quad a \neq 0$

$$1 - ax - 1 = b - 1$$
$$-ax = b - 1$$
$$\frac{-ax}{-a} = \frac{b-1}{-a}$$
$$x = \frac{b-1}{-a} = \frac{1-b}{a}$$

79. $\frac{x}{a} + \frac{x}{b} = c, \quad a \neq 0,\ b \neq 0,\ a \neq -b$

$$ab\left(\frac{x}{a} + \frac{x}{b}\right) = ab\ c$$
$$bx + ax = abc$$
$$x(a+b) = abc$$
$$\frac{x(a+b)}{a+b} = \frac{abc}{a+b}$$
$$x = \frac{abc}{a+b}$$

80. $\frac{a}{x} + \frac{b}{x} = c, \quad c \neq 0$

$$x\left(\frac{a}{x} + \frac{b}{x}\right) = x\ c$$
$$a + b = cx$$
$$\frac{a+b}{c} = \frac{cx}{c}$$
$$x = \frac{a+b}{c}$$

such that $a \neq -b$

81. $$\frac{1}{x-a}+\frac{1}{x+a}=\frac{2}{x-1}$$

$$(x-a)(x+a)(x-1)\left(\frac{1}{x-a}+\frac{1}{x+a}\right)=\left(\frac{2}{x-1}\right)(x-a)(x+a)(x-1)$$
$$(x+a)(x-1)(1)+(x-a)(x-1)(1)=(2)(x-a)(x+a)$$
$$x^2-x+ax-a+x^2-x-ax+a=2x^2-2a^2$$
$$2x^2-2x=2x^2-2a^2$$
$$-2x=-2a^2$$
$$\frac{-2x}{-2}=\frac{-2a^2}{-2}$$
$$x=a^2$$

such that $x \neq \pm a, x \neq 1$

82. $$\frac{b+c}{x+a}=\frac{b-c}{x-a}, c \neq 0, a \neq 0$$

$$(b+c)(x-a)=(b-c)(x+a)$$
$$bx-ba+cx-ca=bx+ba-cx-ca$$
$$-ba+cx-ca=ba-cx-ca$$
$$-ba+cx=ba-cx$$
$$2cx=2ba$$
$$\frac{2cx}{2c}=\frac{2ba}{2c}$$
$$x=\frac{ba}{c}$$

such that $x \neq \pm a$

83. $$x+2a=16+ax-6a$$

$$x=4 \rightarrow$$
$$4+2a=16+a(4)-6a$$
$$4+2a=16+4a-6a$$
$$4+2a=16-2a$$
$$4a=12$$
$$a=3$$

84. $$x+2b=x-4+2bx$$

$$x=2 \rightarrow$$
$$2+2b=2-4+2b(2)$$
$$2+2b=2-4+4b$$
$$2+2b=-2+4b$$
$$4=6b$$
$$\frac{2}{3}=b$$

85. Solving for R:

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$
$$RR_1R_2\left(\frac{1}{R}\right)=RR_1R_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$
$$R_1R_2=RR_2+RR_1$$
$$R_1R_2=R(R_2+R_1)$$
$$\frac{R_1R_2}{R_2+R_1}=\frac{R(R_2+R_1)}{R_2+R_1}$$
$$\frac{R_1R_2}{R_2+R_1}=R$$

86. Solving for r:

$$A = P(1+rt)$$
$$A = P + Prt$$
$$A - P = Prt$$
$$\frac{A-P}{Pt} = \frac{Prt}{Pt}$$
$$r = \frac{A-P}{Pt}$$

87. Solving for R:

$$F = \frac{mv^2}{R}$$
$$RF = R\left(\frac{mv^2}{R}\right)$$
$$RF = mv^2$$
$$\frac{RF}{F} = \frac{mv^2}{F} \rightarrow R = \frac{mv^2}{F}$$

88. Solving for T:

$$PV = nRT$$
$$\frac{PV}{nR} = \frac{nRT}{nR}$$
$$T = \frac{PV}{nR}$$

89. Solving for r:

$$S = \frac{a}{1-r}$$
$$S(1-r) = \left(\frac{a}{1-r}\right)(1-r)$$
$$S - Sr = a$$
$$S - Sr - S = a - S$$
$$-Sr = a - S$$
$$\frac{-Sr}{-S} = \frac{a-S}{-S} \rightarrow r = \frac{S-a}{S}$$

90. Solving for t:

$$v = -gt + v_0$$
$$v - v_0 = -gt$$
$$\frac{v-v_0}{-g} = \frac{-gt}{-g} \rightarrow t = \frac{v-v_0}{-g} = \frac{v_0 - v}{g}$$

91. Step 7 is only allowed if $x \neq 2$.
But step 1 states that $x = 2$, so we have a contradiction.

92. $x^2 = 9$ is not equivalent to $x = 3$ since $x^2 = 9$ also has $x = -3$ as a solution.
$x = \sqrt{9}$ is equivalent to $x = 3$ since the equations have equivalent solutions
$(x-1)(x-2) = (x-1)^2$ is not equivalent to $x - 2 = x - 1$ since the first equation has solution set {1,2}, but the second equation has no solution.

93. In order to solve $\frac{5}{x+3} + 3 = \frac{8+x}{x+3}$, we multiply each term by the expression "$x+3$" to get $(x+3)\left(\frac{5}{x+3} + 3\right) = \left(\frac{8+x}{x+3}\right)(x+3)$.
Now, provided $x \neq -3$, we can cancel the denominators to get

$$5 + (x+3)(3) = 8 + x$$
$$5 + 3x + 9 = 8 + x \rightarrow 2x = -6 \rightarrow x = -3$$

However, we already stated that $x \neq -3$. So we have a contradiction.

94. Answers will vary. One example is $3x + 1 = 3x + 6$.