Chapter 1

Equations and Inequalities

1.3 Quadratic Equations

1. $x^2 - 9x = 0$
$x(x-9) = 0$
$x = 0$ or $x = 9$
The solution set is {0, 9}.

2. $x^2 + 4x = 0$
$x(x+4) = 0$
$x = 0$ or $x = -4$
The solution set is {–4, 0}.

3. $x^2 - 25 = 0$
$(x+5)(x-5) = 0$
$x = -5$ or $x = 5$
The solution set is {–5, 5}.

4. $x^2 - 9 = 0$
$(x+3)(x-3) = 0$
$x = -3$ or $x = 3$
The solution set is {–3, 3}.

5. $z^2 + z - 6 = 0$
$(z+3)(z-2) = 0$
$z = -3$ or $z = 2$
The solution set is {–3, 2}.

6. $v^2 + 7v + 6 = 0$
$(v+6)(v+1) = 0$
$v = -6$ or $v = -1$
The solution set is {–6, -1}.

7. $2x^2 - 5x - 3 = 0$
$(2x+1)(x-3) = 0$
$x = \frac{-1}{2}$ or $x = 3$
The solution set is $\left\{\frac{-1}{2}, 3\right\}$.

8. $3x^2 + 5x + 2 = 0$
$(3x+2)(x+1) = 0$
$x = \frac{-2}{3}$ or $x = -1$
The solution set is $\left\{-\frac{2}{3}, -1\right\}$.

9. $3t^2 - 48 = 0$
$3(t^2 - 16) = 0$
$3(t+4)(t-4) = 0$
$t = -4$ or $t = 4$
The solution set is {–4, 4}.

10. $2y^2 - 50 = 0$
$2(y^2 - 25) = 0$
$2(y+5)(y-5) = 0$
$y = -5$ or $y = 5$
The solution set is {–5, 5}.

11. $x(x-8) + 12 = 0$
$x^2 - 8x + 12 = 0$
$(x-6)(x-2) = 0$
$x = 6$ or $x = 2$
The solution set is {2, 6}.

12. $x(x+4) = 12$
$x^2 + 4x = 12$
$x^2 + 4x - 12 = 0$
$(x+6)(x-2) = 0$
$x = -6$ or $x = 2$
The solution set is {–6, 2}.

13. $4x^2 + 9 = 12x$
$4x^2 - 12x + 9 = 0$
$(2x - 3)^2 = 0$
$x = \frac{3}{2}$
The solution set is $\left\{\frac{3}{2}\right\}$.

14. $25x^2 + 16 = 40x$
$25x^2 - 40x + 16 = 0$
$(5x - 4)^2 = 0$
$x = \frac{4}{5}$
The solution set is $\left\{\frac{4}{5}\right\}$.

15. $6(p^2 - 1) = 5p$
$6p^2 - 6 = 5p$
$6p^2 - 5p - 6 = 0$
$(3p + 2)(2p - 3) = 0$
$p = \frac{-2}{3}$ or $p = \frac{3}{2}$
The solution set is $\left\{\frac{-2}{3}, \frac{3}{2}\right\}$.

16. $2(2u^2 - 4u) + 3 = 0$
$4u^2 - 8u + 3 = 0$
$(2u - 1)(2u - 3) = 0$
$u = \frac{1}{2}$ or $u = \frac{3}{2}$
The solution set is $\left\{\frac{1}{2}, \frac{3}{2}\right\}$.

17. $6x - 5 = \frac{6}{x}$
$6x^2 - 5x = 6$
$6x^2 - 5x - 6 = 0$
$(3x + 2)(2x - 3) = 0$
$x = \frac{-2}{3}$ or $x = \frac{3}{2}$
Since neither of these values causes a denominator to equal zero, the solution set is $\left\{\frac{-2}{3}, \frac{3}{2}\right\}$.

18. $x + \frac{12}{x} = 7$
$x^2 + 12 = 7x$
$x^2 - 7x + 12 = 0$
$(x - 3)(x - 4) = 0$
$x = 3$ or $x = 4$
Since neither of these values causes a denominator to equal zero, the solution set is $\{3, 4\}$.

19. $$\frac{4(x-2)}{x-3}+\frac{3}{x}=\frac{-3}{x(x-3)}$$
$$x(x-3)\left(\frac{4(x-2)}{x-3}+\frac{3}{x}\right)=\left(\frac{-3}{x(x-3)}\right)x(x-3)$$
$$x(x-3)\left(\frac{4(x-2)}{x-3}\right)+x(x-3)\left(\frac{3}{x}\right)=-3$$
$$x(4(x-2))+(x-3)(3)=-3$$
$$4x^2-8x+3x-9=-3$$
$$4x^2-5x-6=0$$
$$(4x+3)(x-2)=0$$
$$x=-\frac{3}{4}\text{ or } x=2$$

Since neither of these values causes a denominator to equal zero, the solution set is $\left\{-\frac{3}{4},2\right\}$.

20. $$\frac{5}{x+4}=4+\frac{3}{x-2}$$
$$(x+4)(x-2)\left(\frac{5}{x+4}\right)=\left(4+\frac{3}{x-2}\right)(x+4)(x-2)$$
$$(x-2)(5)=4(x+4)(x-2)+(3)(x+4)$$
$$5x-10=4(x^2+2x-8)+3x+12$$
$$5x-10=4x^2+8x-32+3x+12$$
$$0=4x^2+6x-10$$
$$0=2(2x^2+3x-5)$$
$$0=2(2x+5)(x-1)$$
$$x=-\frac{5}{2}\text{ or } x=1$$

Since neither of these values causes a denominator to equal zero, the solution set is $\left\{-\frac{5}{2},1\right\}$.

21. $$x^2=25$$
$$x=\pm\sqrt{25}$$
$$x=\pm5$$
The solution set is $\{-5,5\}$.

22. $$x^2=36$$
$$x=\pm\sqrt{36}$$
$$x=\pm6$$
The solution set is $\{-6,6\}$.

23. $$(x-1)^2=4$$
$$x-1=\pm\sqrt{4}$$
$$x-1=\pm2$$
$$x-1=2\text{ or } x-1=-2$$
$$\rightarrow x=3\text{ or } x=-1$$
The solution set is $\{-1,3\}$.

24. $$(x+2)^2=1$$
$$x+2=\pm\sqrt{1}$$
$$x+2=\pm1$$
$$x+2=1\text{ or } x+2=-1$$
$$\rightarrow x=-1\text{ or } x=-3$$
The solution set is $\{-3,-1\}$.

25. $$(2x+3)^2=9$$
$$2x+3=\pm\sqrt{9}$$
$$2x+3=\pm3$$
$$2x+3=3\text{ or } 2x+3=-3$$
$$\rightarrow x=0\text{ or } x=-3$$
The solution set is $\{-3,0\}$.

26. $$(3x-2)^2=4$$
$$3x-2=\pm\sqrt{4}$$
$$3x-2=\pm2$$
$$3x-2=2\text{ or } 3x-2=-2$$
$$\rightarrow x=\frac{4}{3}\text{ or } x=0$$
The solution set is $\left\{0,\frac{4}{3}\right\}$.

27. $\left(\frac{8}{2}\right)^2 = 4^2 = 16$

28. $\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

29. $\left(\frac{\left(\frac{1}{2}\right)}{2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

30. $\left(\frac{\left(-\frac{1}{3}\right)}{2}\right)^2 = \left(-\frac{1}{6}\right)^2 = \frac{1}{36}$

31. $\left(\frac{\left(-\frac{2}{3}\right)}{2}\right)^2 = \left(-\frac{1}{3}\right)^2 = \frac{1}{9}$

32. $\left(\frac{\left(-\frac{2}{5}\right)}{2}\right)^2 = \left(-\frac{1}{5}\right)^2 = \frac{1}{25}$

33.
$$\begin{aligned} x^2 + 4x &= 21 \\ x^2 + 4x + 4 &= 21 + 4 \\ (x+2)^2 &= 25 \\ x + 2 &= \pm\sqrt{25} \\ x + 2 &= \pm 5 \\ x &= -2 \pm 5 \rightarrow x = 3 \text{ or } x = -7 \end{aligned}$$
The solution set is $\{-7, 3\}$.

34.
$$\begin{aligned} x^2 - 6x &= 13 \\ x^2 - 6x + 9 &= 13 + 9 \\ (x-3)^2 &= 22 \\ x - 3 &= \pm\sqrt{22} \\ x &= 3 \pm \sqrt{22} \end{aligned}$$
The solution set is $\{3+\sqrt{22}, 3-\sqrt{22}\}$.

35.
$$\begin{aligned} x^2 - \frac{1}{2}x - \frac{3}{16} &= 0 \\ x^2 - \frac{1}{2}x &= \frac{3}{16} \\ x^2 - \frac{1}{2}x + \frac{1}{16} &= \frac{3}{16} + \frac{1}{16} \\ \left(x - \frac{1}{4}\right)^2 &= \frac{1}{4} \\ x - \frac{1}{4} &= \pm\sqrt{\frac{1}{4}} \\ x - \frac{1}{4} &= \pm\frac{1}{2} \\ x &= \frac{1}{4} \pm \frac{1}{2} \rightarrow x = \frac{3}{4} \\ \text{or } x &= -\frac{1}{4} \end{aligned}$$
The solution set is $\left\{-\frac{1}{4}, \frac{3}{4}\right\}$.

36.
$$\begin{aligned} x^2 + \frac{2}{3}x - \frac{1}{3} &= 0 \\ x^2 + \frac{2}{3}x &= \frac{1}{3} \\ x^2 + \frac{2}{3}x + \frac{1}{9} &= \frac{1}{3} + \frac{1}{9} \\ \left(x + \frac{1}{3}\right)^2 &= \frac{4}{9} \\ x + \frac{1}{3} &= \pm\sqrt{\frac{4}{9}} \\ x + \frac{1}{3} &= \pm\frac{2}{3} \\ x &= -\frac{1}{3} \pm \frac{2}{3} \rightarrow x = \frac{1}{3} \\ \text{or } x &= -1 \end{aligned}$$
The solution set is $\left\{-1, \frac{1}{3}\right\}$.

37.
$$3x^2 + x - \frac{1}{2} = 0$$
$$x^2 + \frac{1}{3}x - \frac{1}{6} = 0$$
$$x^2 + \frac{1}{3}x = \frac{1}{6}$$
$$x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{1}{6} + \frac{1}{36}$$
$$\left(x + \frac{1}{6}\right)^2 = \frac{7}{36}$$
$$x + \frac{1}{6} = \pm\sqrt{\frac{7}{36}}$$
$$x + \frac{1}{6} = \pm\frac{\sqrt{7}}{6}$$
$$x = -\frac{1}{6} \pm \frac{\sqrt{7}}{6}$$

The solution set is $\left\{-\frac{1}{6} + \frac{\sqrt{7}}{6}, -\frac{1}{6} - \frac{\sqrt{7}}{6}\right\}$.

38.
$$2x^2 - 3x - 1 = 0$$
$$x^2 - \frac{3}{2}x - \frac{1}{2} = 0$$
$$x^2 - \frac{3}{2}x = \frac{1}{2}$$
$$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{1}{2} + \frac{9}{16}$$
$$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$$
$$x - \frac{3}{4} = \pm\sqrt{\frac{17}{16}}$$
$$x - \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$$
$$x = \frac{3}{4} \pm \frac{\sqrt{17}}{4}$$

The solution set is $\left\{\frac{3}{4} + \frac{\sqrt{17}}{4}, \frac{3}{4} - \frac{\sqrt{17}}{4}\right\}$.

39.
$$x^2 - 4x + 2 = 0$$
$a = 1$, $b = -4$, $c = 2$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{16 - 8}}{2} = \frac{4 \pm \sqrt{8}}{2}$$
$$= \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$
$$\left\{2 - \sqrt{2},\ 2 + \sqrt{2}\right\}$$

40.
$$x^2 + 4x + 2 = 0$$
$a = 1$, $b = 4$, $c = 2$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{16 - 8}}{2} = \frac{-4 \pm \sqrt{8}}{2}$$
$$= \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$
$$\left\{-2 - \sqrt{2}, -2 + \sqrt{2}\right\}$$

41.
$$x^2 - 4x - 1 = 0$$
$a = 1$, $b = -4$, $c = -1$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{16 + 4}}{2} = \frac{4 \pm \sqrt{20}}{2}$$
$$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$
$$\left\{2 - \sqrt{5},\ 2 + \sqrt{5}\right\}$$

42.
$$x^2 + 6x + 1 = 0$$
$a = 1$, $b = 6$, $c = 1$
$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{36 - 4}}{2} = \frac{-6 \pm \sqrt{32}}{2}$$
$$= \frac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2}$$
$$\left\{-3 - 2\sqrt{2}, -3 + 2\sqrt{2}\right\}$$

43. $2x^2 - 5x + 3 = 0$

$a = 2, \quad b = -5, \quad c = 3$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25 - 24}}{4} = \frac{5 \pm 1}{4}$$

$$\left\{1, \frac{3}{2}\right\}$$

44. $2x^2 + 5x + 3 = 0$

$a = 2, \quad b = 5, \quad c = 3$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25 - 24}}{4} = \frac{-5 \pm 1}{4}$$

$$\left\{-1, -\frac{3}{2}\right\}$$

45. $4y^2 - y + 2 = 0$

$a = 4, \quad b = -1, \quad c = 2$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(2)}}{2(4)}$$

$$= \frac{1 \pm \sqrt{1 - 32}}{8} = \frac{1 \pm \sqrt{-31}}{8}$$

No real solution.

46. $4t^2 + t + 1 = 0$

$a = 4, \quad b = 1, \quad c = 1$

$$t = \frac{-1 \pm \sqrt{1^2 - 4(4)(1)}}{2(4)}$$

$$= \frac{-1 \pm \sqrt{1 - 16}}{8} = \frac{-1 \pm \sqrt{-15}}{8}$$

No real solution.

47. $4x^2 = 1 - 2x$

$4x^2 + 2x - 1 = 0$

$a = 4, \quad b = 2, \quad c = -1$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-2 \pm \sqrt{4 + 16}}{8} = \frac{-2 \pm \sqrt{20}}{8}$$

$$= \frac{-2 \pm 2\sqrt{5}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

$$\left\{\frac{-1 - \sqrt{5}}{4}, \frac{-1 + \sqrt{5}}{4}\right\}$$

48. $2x^2 = 1 - 2x$

$2x^2 + 2x - 1 = 0$

$a = 2, \quad b = 2, \quad c = -1$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{4 + 8}}{4} = \frac{-2 \pm \sqrt{12}}{4}$$

$$= \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

$$\left\{\frac{-1 - \sqrt{3}}{2}, \frac{-1 + \sqrt{3}}{2}\right\}$$

49. $4x^2 = 9x$
$4x^2 - 9x = 0$
$a = 4, \quad b = -9, \quad c = 0$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(4)(0)}}{2(4)}$$
$$= \frac{9 \pm \sqrt{81}}{8}$$
$$= \frac{9 \pm 9}{8}$$
$$\rightarrow x = \frac{9+9}{8} \text{ or } x = \frac{9-9}{8}$$
$$x = \frac{18}{8} = \frac{9}{4} \text{ or } x = 0$$
$$\left\{0, \frac{9}{4}\right\}$$

50. $5x = 4x^2$
$0 = 4x^2 - 5x$
$a = 4, \quad b = -5, \quad c = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(0)}}{2(4)}$$
$$= \frac{5 \pm \sqrt{25}}{8}$$
$$= \frac{5 \pm 5}{8}$$
$$\rightarrow x = \frac{5+5}{8} \text{ or } x = \frac{5-5}{8}$$
$$x = \frac{10}{8} = \frac{5}{4} \text{ or } x = 0$$
$$\left\{0, \frac{5}{4}\right\}$$

51. $9t^2 - 6t + 1 = 0$
$a = 9, \quad b = -6, \quad c = 1$

$$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)}$$
$$= \frac{6 \pm \sqrt{36 - 36}}{18}$$
$$= \frac{6 \pm 0}{18} = \frac{1}{3}$$
$$\left\{\frac{1}{3}\right\}$$

52. $4u^2 - 6u + 9 = 0$
$a = 4, \quad b = -6, \quad c = 9$

$$u = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$$
$$= \frac{6 \pm \sqrt{36 - 144}}{8}$$
$$= \frac{6 \pm \sqrt{-108}}{8}$$

No real solution.

53.
$$\frac{3}{4}x^2-\frac{1}{4}x-\frac{1}{2}=0$$
$$4\left(\frac{3}{4}x^2-\frac{1}{4}x-\frac{1}{2}\right)=(0)(4)$$
$$3x^2-x-2=0$$
$$a=3,\quad b=-1,\quad c=-2$$
$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(-2)}}{2(3)}$$
$$=\frac{1\pm\sqrt{1+24}}{6}$$
$$=\frac{1\pm\sqrt{25}}{6}=\frac{1\pm 5}{6}$$
$$\rightarrow x=\frac{1+5}{6}\text{ or } x=\frac{1-5}{6}$$
$$x=\frac{6}{6}=1\text{ or } x=\frac{-4}{6}=-\frac{2}{3}$$
$$\left\{-\frac{2}{3},1\right\}$$

54.
$$\frac{2}{3}x^2-x-3=0$$
$$3\left(\frac{2}{3}x^2-x-3\right)=(0)(3)$$
$$2x^2-3x-9=0$$
$$a=2,\quad b=-3,\quad c=-9$$
$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-9)}}{2(2)}$$
$$=\frac{3\pm\sqrt{9+72}}{4}$$
$$=\frac{3\pm\sqrt{81}}{4}=\frac{3\pm 9}{4}\rightarrow x=\frac{3+9}{4}\text{ or } x=\frac{3-9}{4}$$
$$x=\frac{12}{4}=3\text{ or } x=\frac{-6}{4}=-\frac{3}{2}$$
$$\left\{-\frac{3}{2},3\right\}$$

55.
$$4-\frac{1}{x}-\frac{2}{x^2}=0$$
$$\left(x^2\right)\left(4-\frac{1}{x}-\frac{2}{x^2}\right)=(0)\left(x^2\right)$$
$$4x^2-x-2=0$$
$$a=4,\quad b=-1,\quad c=-2$$
$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(4)(-2)}}{2(4)}$$
$$=\frac{1\pm\sqrt{1+32}}{8}$$
$$=\frac{1\pm\sqrt{33}}{8}$$
Since neither of these values causes a denominator to equal zero, the solution set is $\left\{\frac{1+\sqrt{33}}{8},\frac{1-\sqrt{33}}{8}\right\}$.

56.
$$4+\frac{1}{x}-\frac{1}{x^2}=0$$
$$\left(x^2\right)\left(4+\frac{1}{x}-\frac{1}{x^2}\right)=(0)\left(x^2\right)$$
$$4x^2+x-1=0$$
$$a=4,\quad b=1,\quad c=-1$$
$$x=\frac{-1\pm\sqrt{1^2-4(4)(-1)}}{2(4)}$$
$$=\frac{-1\pm\sqrt{1+16}}{8}$$
$$=\frac{-1\pm\sqrt{17}}{8}$$
Since neither of these values causes a denominator to equal zero, the solution set is $\left\{\frac{-1+\sqrt{17}}{8},\frac{-1-\sqrt{17}}{8}\right\}$.

57.
$$3x = 1 - \frac{1}{x}$$
$$x(3x) = \left(1 - \frac{1}{x}\right)(x)$$
$$3x^2 = x - 1$$
$$3x^2 - x + 1 = 0$$
$$a = 3, \quad b = -1 \quad c = 1$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(1)}}{2(4)}$$
$$= \frac{1 \pm \sqrt{1-12}}{8}$$
$$= \frac{1 \pm \sqrt{-11}}{8}$$

No real solutions.

58.
$$x = 1 - \frac{4}{x}$$
$$x(x) = \left(1 - \frac{4}{x}\right)(x)$$
$$x^2 = x - 4$$
$$x^2 - x + 4 = 0$$
$$a = 1, \quad b = -1 \quad c = 4$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(4)}}{2(1)}$$
$$= \frac{1 \pm \sqrt{1-16}}{2}$$
$$= \frac{1 \pm \sqrt{-15}}{2}$$

No real solutions.

59.
$$x^2 - 4.1x + 2.2 = 0$$
$$a = 1, \quad b = -4.1, \quad c = 2.2$$
$$x = \frac{-(-4.1) \pm \sqrt{(-4.1)^2 - 4(1)(2.2)}}{2(1)}$$
$$= \frac{4.1 \pm \sqrt{16.81 - 8.8}}{2} = \frac{4.1 \pm \sqrt{8.01}}{2}$$
$$x \approx 3.47, x \approx 0.64$$
$$\{3.47, 0.64\}$$

60.
$$x^2 + 3.9x + 1.8 = 0$$
$$a = 1, \quad b = 3.9, \quad c = 1.8$$
$$x = \frac{-(3.9) \pm \sqrt{(3.9)^2 - 4(1)(1.8)}}{2(1)}$$
$$= \frac{-3.9 \pm \sqrt{15.21 - 7.2}}{2} = \frac{-3.9 \pm \sqrt{8.01}}{2}$$
$$x \approx -0.54, x \approx -3.37$$
$$\{-0.54, -3.37\}$$

61.
$$x^2 + \sqrt{3}x - 3 = 0$$
$$a = 1, \quad b = \sqrt{3}, \quad c = -3$$
$$x = \frac{-(\sqrt{3}) \pm \sqrt{(\sqrt{3})^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{-\sqrt{3} \pm \sqrt{3+12}}{2} = \frac{-\sqrt{3} \pm \sqrt{15}}{2}$$
$$x \approx 1.07, x \approx -2.80$$
$$\{1.07, -2.80\}$$

62.
$$x^2 + \sqrt{2}x - 2 = 0$$
$$a = 1, \quad b = \sqrt{2}, \quad c = -2$$
$$x = \frac{-(\sqrt{2}) \pm \sqrt{(\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{-\sqrt{2} \pm \sqrt{2+8}}{2} = \frac{-\sqrt{2} \pm \sqrt{10}}{2}$$
$$x \approx 0.87, x \approx -2.29$$
$$\{0.87, -2.29\}$$

63. $\pi x^2 - x - \pi = 0$
$a = \pi, \quad b = -1, \quad c = -\pi$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(\pi)(-\pi)}}{2(\pi)}$$
$$= \frac{1 \pm \sqrt{1 + 4\pi^2}}{2\pi}$$
$x \approx 1.17, x \approx -0.83$
$\{1.17, -0.83\}$

64. $\pi x^2 + \pi x - 2 = 0$
$a = \pi, \quad b = \pi, \quad c = -2$
$$x = \frac{-(\pi) \pm \sqrt{(\pi)^2 - 4(\pi)(-2)}}{2(\pi)}$$
$$= \frac{-\pi \pm \sqrt{\pi^2 + 8\pi}}{2\pi}$$
$x \approx 0.44, x \approx -1.44$
$\{0.44, -1.44\}$

65. $3x^2 + 8\pi x + \sqrt{29} = 0$
$a = 3, \quad b = 8\pi, \quad c = \sqrt{29}$
$$x = \frac{-(8\pi) \pm \sqrt{(8\pi)^2 - 4(3)\sqrt{29}}}{2(3)}$$
$$= \frac{-8\pi \pm \sqrt{64\pi^2 - 12\sqrt{29}}}{6}$$
$x \approx -0.22, x \approx -8.16$
$\{-0.22, -8.16\}$

66. $\pi x^2 - 15\sqrt{2}x + 20 = 0$
$a = \pi, \quad b = -15\sqrt{2}, \quad c = 20$
$$x = \frac{-(-15\sqrt{2}) \pm \sqrt{(-15\sqrt{2})^2 - 4(\pi)20}}{2(\pi)}$$
$$= \frac{15\sqrt{2} \pm \sqrt{450 - 80\pi}}{2\pi}$$
$x \approx 6.66, x \approx 0.10$
$\{6.66, 0.10\}$

67. $x^2 - 5 = 0$
$x^2 = 5$
$x = \pm\sqrt{5}$
$\{\sqrt{5}, -\sqrt{5}\}$

68. $x^2 - 6 = 0$
$x^2 = 6$
$x = \pm\sqrt{6}$
$\{\sqrt{6}, -\sqrt{6}\}$

69. $16x^2 - 8x + 1 = 0$
$(4x - 1)(4x - 1) = 0$
$4x - 1 = 0 \rightarrow x = \frac{1}{4}$
$\left\{\frac{1}{4}\right\}$

70. $9x^2 - 6x + 1 = 0$
$(3x - 1)(3x - 1) = 0$
$3x - 1 = 0 \rightarrow x = \frac{1}{3}$
$\left\{\frac{1}{3}\right\}$

71. $10x^2 - 19x - 15 = 0$
$(5x + 3)(2x - 5) = 0$
$5x + 3 = 0$ or $2x - 5 = 0$
$\rightarrow x = -\frac{3}{5}$ or $x = \frac{5}{2}$
$\left\{-\frac{3}{5}, \frac{5}{2}\right\}$

72. $6x^2 + 7x - 20 = 0$
$(3x - 4)(2x + 5) = 0$
$3x - 4 = 0$ or $2x + 5 = 0$
$\rightarrow x = \frac{4}{3}$ or $x = -\frac{5}{2}$
$\left\{-\frac{5}{2}, \frac{4}{3}\right\}$

73.

$$2 + z = 6z^2$$
$$0 = 6z^2 - z - 2$$
$$0 = (3z - 2)(2z + 1)$$
$$3z - 2 = 0 \text{ or } 2z + 1 = 0$$
$$\rightarrow z = \frac{2}{3} \text{ or } x = -\frac{1}{2}$$
$$\left\{-\frac{1}{2}, \frac{2}{3}\right\}$$

74.

$$2 = y + 6y^2$$
$$0 = 6y^2 + y - 2$$
$$0 = (3y + 2)(2y - 1)$$
$$3y + 2 = 0 \text{ or } 2y - 1 = 0$$
$$\rightarrow y = -\frac{2}{3} \text{ or } y = \frac{1}{2}$$
$$\left\{-\frac{2}{3}, \frac{1}{2}\right\}$$

75.

$$x^2 + \sqrt{2}x = \frac{1}{2}$$
$$x^2 + \sqrt{2}x - \frac{1}{2} = 0$$
$$2\left(x^2 + \sqrt{2}x - \frac{1}{2}\right) = (0)(2)$$
$$2x^2 + 2\sqrt{2}x - 1 = 0$$
$$a = 2, \quad b = 2\sqrt{2}, \quad c = -1$$
$$x = \frac{-(2\sqrt{2}) \pm \sqrt{(2\sqrt{2})^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-2\sqrt{2} \pm \sqrt{8+8}}{4} = \frac{-2\sqrt{2} \pm \sqrt{16}}{4}$$
$$= \frac{-2\sqrt{2} \pm 4}{4} = \frac{-\sqrt{2} \pm 2}{2}$$
$$\left\{\frac{-\sqrt{2}+2}{2}, \frac{-\sqrt{2}-2}{2}\right\}$$

76.

$$\frac{1}{2}x^2 = \sqrt{2}x + 1$$
$$\frac{1}{2}x^2 - \sqrt{2}x - 1 = 0$$
$$2\left(\frac{1}{2}x^2 - \sqrt{2}x - 1\right) = (0)(2)$$
$$x^2 - 2\sqrt{2}x - 2 = 0$$
$$a = 1, \quad b = -2\sqrt{2}, \quad c = -2$$
$$x = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{2\sqrt{2} \pm \sqrt{8+8}}{2} = \frac{2\sqrt{2} \pm \sqrt{16}}{2}$$
$$= \frac{2\sqrt{2} \pm 4}{2} = \frac{\sqrt{2} \pm 2}{1}$$
$$\left\{\sqrt{2}+2, \sqrt{2}-2\right\}$$

77.

$$x^2 + x = 4$$
$$x^2 + x - 4 = 0$$
$$a = 1, \quad b = 1, \quad c = -4$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$
$$\left\{\frac{-1+\sqrt{17}}{2}, \frac{-1-\sqrt{17}}{2}\right\}$$

78.

$$x^2 + x = 1$$
$$x^2 + x - 1 = 0$$
$$a = 1, \quad b = 1, \quad c = -1$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$
$$\left\{\frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2}\right\}$$

79. $2x^2 - 6x + 7 = 0$
$a = 2, \quad b = -6, \quad c = 7$
$b^2 - 4ac = (-6)^2 - 4(2)(7)$
$= 36 - 56 = -20$
Since the discriminant < 0, we have no real solutions

80. $x^2 + 4x + 7 = 0$
$a = 1, \quad b = 4, \quad c = 7$
$b^2 - 4ac = (4)^2 - 4(1)(7)$
$= 16 - 28 = -12$
since the discriminant < 0, we have no real solutions

81. $9x^2 - 30x + 25 = 0$
$a = 9, \quad b = -30, \quad c = 25$
$b^2 - 4ac = (-30)^2 - 4(9)(25)$
$= 900 - 900 = 0$
since the discriminant = 0, we have one repeated real solution

82. $25x^2 - 20x + 4 = 0$
$a = 25, \quad b = -20, \quad c = 4$
$b^2 - 4ac = (-20)^2 - 4(25)(4)$
$= 400 - 400 = 0$
since the discriminant = 0, we have one repeated real solution

83. $3x^2 + 5x - 8 = 0$
$a = 3, \quad b = 5, \quad c = -8$
$b^2 - 4ac = (5)^2 - 4(3)(-8)$
$= 25 + 96 = 121$
since the discriminant > 0, we have two unequal real solutions

84. $2x^2 - 3x - 7 = 0$
$a = 2, \quad b = -3, \quad c = -7$
$b^2 - 4ac = (-3)^2 - 4(2)(-7)$
$= 9 + 56 = 65$
since the discriminant > 0, we have two unequal real solutions

85. Let w represent the width of window.
Then $l = w + 2$ represents the length of the window.
Since the area is 143 square feet, we have:
$w(w + 2) = 143$
$w^2 + 2w - 143 = 0 \rightarrow (w + 13)(w - 11) = 0 \rightarrow w = -13$ which is not practical
or $w = 11$
The width of the rectangular window is 11 feet and the length is 13 feet.

86. Let w represent the width of window.
Then $l = w + 1$ represents the length of the window.
Since the area is 306 square centimeters, we have:
$w(w + 1) = 306$
$w^2 + w - 306 = 0 \rightarrow (w + 18)(w - 17) = 0 \rightarrow w = -18$ which is not practical
or $w = 17$
The width of the rectangular window is 17 centimeters and the length is 18 centimeters.

87. Let l represent the length of the rectangle.
Let w represent the width of the rectangle.
The perimeter is 26 meters and the area is 40 square meters.

$$2l+2w=26 \rightarrow l+w=13 \rightarrow w=13-l$$

$$lw=40$$

$$l(13-l)=40 \rightarrow 13l-l^2=40 \rightarrow l^2-13l+40=0 \rightarrow (l-8)(l-5)=0$$

$$l=8 \text{ or } l=5$$

$$w=5 \quad w=8$$

The dimensions are 5 meters by 8 meters.

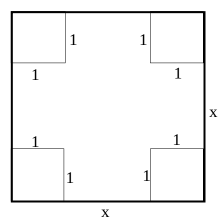
88. Let r represent the radius of the circle.
Since the field is a square with area 1250 square feet, the length of a side of the square is $\sqrt{1250}=25\sqrt{2}$ feet. The length of the diagonal is $2r$.
Use the Pythagorean Theorem to solve for r:

$$(2r)^2=\left(25\sqrt{2}\right)^2+\left(25\sqrt{2}\right)^2$$

$$4r^2=1250+1250 \rightarrow 4r^2=2500 \rightarrow r^2=625 \rightarrow r=25$$

The shortest radius setting for the sprinkler is 25 feet.

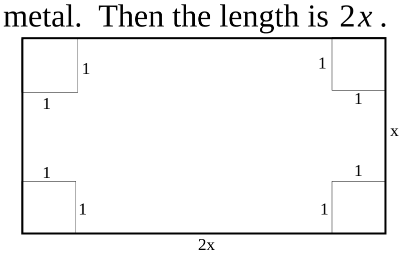
89. Let x represent the length of the side of the sheet metal.



$$(x-2)(x-2)(1)=4$$
$$x^2-4x+4=4$$
$$x^2-4x=0$$
$$x(x-4)=0$$
$$x=0 \text{ or } x=4$$

Since the side cannot be 0 feet long, the length of a side of the sheet metal is 4 feet.

90. Let x represent the width of the side of the sheet metal. Then the length is $2x$.



$$(x-2)(2x-2)(1)=4$$
$$2x^2-6x+4=4$$
$$2x^2-6x=0$$
$$2x(x-3)=0$$
$$x=0 \text{ or } x=3$$

Since the side cannot be 0 feet long, the width of the sheet metal is 3 feet and the length is 6 feet. The dimensions of the box are 1 foot by 4 feet by 1 foot.

91. (a) $s=96+80t-16t^2$. The ball strikes the ground when the height = 0.

So we solve $0=96+80t-16t^2$.

$$0=-16t^2+80t+96 \rightarrow \frac{0}{-16}=\frac{-16t^2+80t+96}{-16}$$

$0 = t^2 - 5t - 6 \rightarrow 0 = (t-6)(t+1) \rightarrow t = 6$ or $t = -1$, we discard the negative value since t represents elapsed time. Therefore, the ball hits the ground after 6 seconds.

(b) $s = 96 + 80t - 16t^2$. The ball passes the top of the building when the height = 96.

So we solve $96 = 96 + 80t - 16t^2$.

$96 = 96 + 80t - 16t^2 \rightarrow 0 = 80t - 16t^2 \rightarrow 0 = 16t(5-t)$

$t = 0$ or $t = 5$ we know that the ball starts (t = 0) at a height of 96 feet. Therefore, the ball hits the ground after 6 seconds.

92. $A = 2\pi r^2 + 2\pi r h$. Since $A = 188.5$ square inches and $h = 7$ inches,

$2\pi r^2 + 2\pi r(7) = 188.5 \rightarrow 2\pi r^2 + 14\pi r - 188.5 = 0$

$$r = \frac{-14\pi \pm \sqrt{(14\pi)^2 - 4(2\pi)(-188.5)}}{2(2\pi)} = \frac{-14\pi \pm \sqrt{6671.9642}}{4\pi}.$$

$r \approx 3$ or $r \approx -10$, which is not practical

The radius of the coffee can is approximately 3 inches.

93. Let x = number of boxes in excess of 150.

The total number of boxes ordered = 150 + x

The price per box = 200 - x

The customer's total bill = (# boxes ordered)(price per box)

$= (150 + x)(200 - x)$

So we need to solve the equation $(150 + x)(200 - x) = 30{,}625$.

$30{,}000 + 50x - x^2 = 30{,}625$

$0 = x^2 - 50x + 625 \rightarrow 0 = (x-25)(x-25) \rightarrow x = 25$

So the customer ordered a total of 150 + 25 = 175 boxes.

94. Let x be the width and $2x$ be the length of the patio. The height is $\frac{1}{3}$ foot.

$V = l\ w\ h = x(2x)\ \frac{1}{3} = 216$

$\frac{2}{3}x^2 = 216 \rightarrow x^2 = 324 \rightarrow x = \pm 18$

The dimensions of the patio are 18 feet by 36 feet.

95. Let x represent the width of the border measured in feet.

The total area is $A_T = (6 + 2x)(10 + 2x)$.

The area of the garden is $A_G = 6\ \ 10 = 60$.

The area of the border is $A_B = A_T - A_G = (6 + 2x)(10 + 2x) - 60$.

Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is

$0.25A_B = 0.25((6 + 2x)(10 + 2x) - 60)$

Solving the volume equation:

$$0.25\big((6+2x)(10+2x)-60\big)=27$$
$$60+32x+4x^2-60=108$$
$$4x^2+32x-108=0$$
$$x^2+8x-27=0$$
$$x=\frac{-8\pm\sqrt{8^2-4(1)(-27)}}{2(1)}=\frac{-8\pm\sqrt{172}}{2}$$
$$=\frac{-8\pm13.11}{2}=2.56 \text{ or } -10.56 \text{ which is not practical}$$

The width of the border is approximately 2.56 feet.

96. (a) $s=-4.9t^2+20t\cdot$

we solve

$$15=-4.9t^2+20t$$
$$4.9t^2-20t+15=0$$
$$t=\frac{-(-20)\pm\sqrt{(-20)^2-4(4.9)(15)}}{2(4.9)}=\frac{20\pm\sqrt{106}}{9.8}\approx\frac{20\pm10.296}{9.8}=3.09 \text{ or } 0.99$$

Therefore, after 0.99 seconds the object passes a height of 15 meters on its way up, and after 3.09 seconds the object passes a height of 15 meters on its way down.

(b) $s=-4.9t^2+20t\cdot$

we solve

$$0=-4.9t^2+20t \rightarrow 0=t(-4.9t+20) \rightarrow t=0 \text{ or } t=\frac{20}{4.9}\approx4.08$$

We know that the object starts (t = 0) at ground level. Therefore, the object hits the ground after 4.08 seconds.

(c) $s=-4.9t^2+20t\cdot$

we solve

$$100=-4.9t^2+20t$$
$$4.9t^2-20t+100=0$$
$$t=\frac{-(-20)\pm\sqrt{(-20)^2-4(4.9)(100)}}{2(4.9)}=\frac{20\pm\sqrt{-1560}}{9.8}$$

The equation has no real solution. Therefore the object never attains a height of 100 meters.

97. Let x represent the number of centimeters the length and width should be reduced.
$12 - x$ = the new length, $7 - x$ = the new width.
The new volume is 90% of the old volume.

$$(12 - x)(7 - x)(3) = 0.9(12)(7)(3)$$

$$3x^2 - 57x + 252 = 226.8 \rightarrow 3x^2 - 57x + 25.2 = 0 \rightarrow x^2 - 19x + 8.4 = 0$$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(8.4)}}{2(1)} = \frac{19 \pm \sqrt{327.4}}{2}$$

$$= \frac{19 \pm 18.09}{2} = 0.45 \text{ or } 18.55$$

Since 18.55 exceeds the dimensions, it is discarded.
The dimensions of the new chocolate bar are: 11.55 cm by 6.55 cm by 3 cm.

98. Let x represent the number of centimeters the length and width should be reduced.
$12 - x$ = the new length, $7 - x$ = the new width.
The new volume is 80% of the old volume.

$$(12 - x)(7 - x)(3) = 0.8(12)(7)(3)$$

$$3x^2 - 57x + 252 = 201.6$$

$$3x^2 - 57x + 50.4 = 0$$

$$x^2 - 19x + 16.8 = 0$$

$$x = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(16.8)}}{2(1)} = \frac{19 \pm \sqrt{293.8}}{2}$$

$$= \frac{19 \pm 17.1406}{2} = 0.93 \text{ or } 18.07$$

Since 18.07 exceeds the dimensions, it is discarded.
The dimensions of the new chocolate bar are: 11.07 cm by 6.07 cm by 3 cm.

99. Let x represent the width of the border measured in feet.
The radius of the pool is 5 feet.
Then $x + 5$ represents the radius of the circle, including both the pool and the border.
The total area of the pool and border is $A_T = \pi(x + 5)^2$.
The area of the pool is $A_P = \pi(5)^2 = 25\pi$.
The area of the border is $A_B = A_T - A_P = \pi(x + 5)^2 - 25\pi$.
Since the concrete is 3 inches or 0.25 feet thick, the volume of the concrete in the border is

$$0.25 A_B = 0.25\left(\pi(x + 5)^2 - 25\pi\right)$$

Solving the volume equation:

$$0.25\left(\pi(x + 5)^2 - 25\pi\right) = 27 \rightarrow \pi\left(x^2 + 10x + 25 - 25\right) = 108 \rightarrow \pi x^2 + 10\pi x - 108 = 0$$

$$x = \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-108)}}{2(\pi)}$$

$$= \frac{-31.42 \pm \sqrt{2344.1285}}{6.28} = \frac{-31.42 \pm 48.42}{6.28} = 2.71 \text{ or } -12.71$$

The width of the border is approximately 2.71 feet.

100. Let x represent the width of the border measured in feet.
The radius of the pool is 5 feet.
Then $x+5$ represents the radius of the circle, including both the pool and the border.
The total area of the pool and border is $A_T = \pi(x+5)^2$.
The area of the pool is $A_P = \pi(5)^2 = 25\pi$.
The area of the border is $A_B = A_T - A_P = \pi(x+5)^2 - 25\pi$.
Since the concrete is 4 inches or $\frac{1}{3}$ foot thick, the volume of the concrete in the border is

$$\tfrac{1}{3}A_B = \tfrac{1}{3}\left(\pi(x+5)^2 - 25\pi\right)$$

Solving the volume equation:

$$\begin{aligned}
\tfrac{1}{3}\left(\pi(x+5)^2 - 25\pi\right) &= 27\\
\pi\left(x^2+10x+25-25\right) &= 81\\
\pi x^2 + 10\pi x - 81 &= 0\\
x &= \frac{-10\pi \pm \sqrt{(10\pi)^2 - 4(\pi)(-81)}}{2(\pi)}\\
&= \frac{-31.42 \pm \sqrt{2004.8365}}{6.28}\\
&= \frac{-31.42 \pm 44.78}{6.28} = 2.13 \text{ or } -12.13, \text{which is not practical}
\end{aligned}$$

The width of the border is approximately 2.13 feet.

101. Let r represent the speed of the current.

	Rate	Time	Distance
Upstream	$15-r$	$\frac{10}{15-r}$	10
Downstream	$15+r$	$\frac{10}{15+r}$	10

Since the total time is 1.5 hours, we have:

$$\begin{aligned}
\frac{10}{15-r} + \frac{10}{15+r} &= 1.5\\
10(15+r) + 10(15-r) &= 1.5(15-r)(15+r)\\
150 + 10r + 150 - 10r &= 1.5(225-r^2)\\
300 &= 1.5(225-r^2)\\
200 &= 225 - r^2\\
r^2 - 25 &= 0\\
(r-5)(r+5) &= 0\\
r = 5 \text{ or } r &= -5
\end{aligned}$$

The speed of the current is 5 miles per hour.

102. (a) $s = 1280 - 32t - 16t^2$. The object strikes the ground when the height = 0.

So we solve $0 = 1280 - 32t - 16t^2$.

$$0 = -16t^2 - 32t + 1280$$
$$\frac{0}{-16} = \frac{-16t^2 - 32t + 1280}{-16}$$
$$0 = t^2 + 2t - 80$$
$$0 = (t-8)(t+10)$$

$t = 8$ or $t = -10$ we discard the negative value since t represents elapsed time. Therefore, the object hits the ground after 8 seconds.

(b) we compute the height when $t = 4$.

$s = 96 + 80(4) - 16(4)^2 = 160$.

The object reaches a height of 160 feet after 4 seconds.

103. given a quadratic equation $ax^2 + bx + c = 0$

the solutions are given by

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } x = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

adding these two values we get

$$\frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-b+\sqrt{b^2-4ac} - b - \sqrt{b^2-4ac}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}$$

104. given a quadratic equation $ax^2 + bx + c = 0$

the solutions are given by

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } x = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

multiplying these two values we get

$$\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) = \frac{\left(-b+\sqrt{b^2-4ac}\right)\left(-b-\sqrt{b^2-4ac}\right)}{4a^2}$$
$$= \frac{b^2 + b\sqrt{b^2-4ac} - b\sqrt{b^2-4ac} - \left(b^2-4ac\right)}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

105. the quadratic equation $kx^2 + x + k = 0$ will have a repeated solution provided the discriminant $= 0$ that is, we need $b^2 - 4ac = 0$ in the given quadratic equation
$b^2 - 4ac = (1)^2 - 4(k)(k) = 1 - 4k^2$
so we solve

$$1 - 4k^2 = 0 \rightarrow 1 = 4k^2 \rightarrow \frac{1}{4} = k^2 \rightarrow \pm\sqrt{\frac{1}{4}} = k \rightarrow \pm\frac{1}{2} = k$$

106. the quadratic equation $x^2 - kx + 4 = 0$ will have a repeated solution provided the discriminant $= 0$ that is, we need $b^2 - 4ac = 0$ in the given quadratic equation
$b^2 - 4ac = (-k)^2 - 4(1)(4) = k^2 - 16$
so we solve

$$k^2 - 16 = 0 \rightarrow k^2 = 16 \rightarrow k = \pm 4$$

107. the quadratic equation $ax^2 + bx + c = 0$ has solutions given by

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

the quadratic equation $ax^2 - bx + c = 0$ has solutions given by

$$x_3 = \frac{-(-b) + \sqrt{(-b)^2 - 4ac}}{2a} = \frac{b + \sqrt{b^2 - 4ac}}{2a} = -x_2$$

and

$$x_4 = \frac{-(-b) - \sqrt{(-b)^2 - 4ac}}{2a} = \frac{b - \sqrt{b^2 - 4ac}}{2a} = -x_1$$

so we have the negatives of the first pair of solutions.

108. The quadratic equation $ax^2 + bx + c = 0$ has solutions given by

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

The quadratic equation $cx^2 + bx + a = 0$ has solutions given by

$$x_3 = \frac{-b + \sqrt{b^2 - 4ca}}{2c} \quad \text{and} \quad x_4 = \frac{-b - \sqrt{b^2 - 4ca}}{2c}$$

Now notice that

$$(x_1)(x_4)=\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ca}}{2c}\right)$$

$$=\frac{b^2+b\sqrt{b^2-4ca}-b\sqrt{b^2-4ac}-\left(b^2-4ac\right)}{4ac}$$

$$=\frac{b^2-b^2+4ac}{4ac}=\frac{4ac}{4ac}=1$$

therefore, x_1 and x_4 are reciprocals of each other. A similar result holds for x_2 and x_3.

109. We need to solve the equation $\frac{1}{2}n(n+1)=666$

$$2\left(\frac{1}{2}n\right)(n+1)=(666)(2)$$

$$n(n+1)=1332$$

$$n^2+n=1332 \rightarrow n^2+n-1332=0 \rightarrow (n+37)(n-36)=0$$

$$n=-37 \text{ or } n=36$$

since *n* must be a positive integer (it represents how many numbers we add together), we discard the negative value. Therefore, we conclude that 1 + 2 + 3 +……..+ 36 = 666.

110. (a) we need to solve the equation $\frac{1}{2}n(n-3)=65$

$$2\left(\frac{1}{2}n\right)(n-3)=(65)(2)$$

$$n(n-3)=130 \rightarrow n^2-3n=130 \rightarrow n^2-3n-130=0 \rightarrow (n+10)(n-13)=0$$

$$n=-10 \text{ or } n=13$$

since *n* must be a positive integer (it represents the number of sides of the polygon), we discard the negative value. Therefore, we conclude that the polygon has 13 sides and 65 diagonals.

(b) we need to solve the equation $\frac{1}{2}n(n-3)=80$

$$2\left(\frac{1}{2}n\right)(n-3)=(80)(2)$$

$$n(n-3)=160 \rightarrow n^2-3n=160 \rightarrow n^2-3n-160=0$$

the solutions to this equation are $n=\frac{3\pm\sqrt{9+640}}{2}=\frac{3\pm\sqrt{649}}{2}$ but neither of these answers is a whole number. Therefore, there cannot exist a polygon having 80 diagonals.

111. Let t_1 and t_2 represent the times for the two segments of the trip.

	Rate	Time	Distance
Chicago to Atlanta	45	t_1	$45t_1$
Atlanta to Miami	55	t_2	$55t_2$

Since Atlanta is halfway between Chicago and Miami, the distances are equal.

$$45t_1 = 55t_2 \quad \rightarrow \quad t_1 = \frac{55}{45}t_2 = \frac{11}{9}t_2$$

Computing the average speed:

$$\text{Avg Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{45t_1 + 55t_2}{t_1 + t_2} = \frac{45\left(\frac{11}{9}t_2\right) + 55t_2}{\frac{11}{9}t_2 + t_2}$$

$$= \frac{55t_2 + 55t_2}{\frac{11t_2 + 9t_2}{9}} = \frac{110t_2}{\frac{20t_2}{9}} = \frac{990t_2}{20t_2} = \frac{99}{2} = 49.5 \text{ miles per hour}$$

The average speed for the trip from Chicago to Miami is 49.5 miles per hour.

112. The time traveled with the tail wind was:

$$919 = 550t$$

$$t = \frac{919}{550} \approx 1.671 \text{ hours}$$

Since they were 20 minutes early, the time in still air would have been:

$$1.671 \text{ hours } + 20 \text{ minutes } = 1.671 + 0.333 \approx 2 \text{ hours}$$

Thus the rate in still air is:

$$919 = r(2)$$

$$r \approx 460 \text{ nautical miles / hour}$$

The tail wind was 550 – 460 = 90 nautical miles per hour.

113 – 116. Answers will vary.