# Chapter 1

# Equations and Inequalities

## 1.4 Radical Equations; Equations Quadratic in Form

1. $$\begin{aligned} \sqrt{2t-1} &= 1 \\ \left(\sqrt{2t-1}\right)^2 &= 1^2 \\ 2t-1 &= 1 \\ 2t &= 2 \\ t &= 1 \end{aligned}$$
Check: $\sqrt{2(1)-1} = \sqrt{1} = 1$
The solution is $t = 1$.

2. $$\begin{aligned} \sqrt{3t+4} &= 2 \\ \left(\sqrt{3t+4}\right)^2 &= 2^2 \\ 3t+4 &= 4 \\ 3t &= 0 \\ t &= 0 \end{aligned}$$
Check: $\sqrt{3(0)+4} = \sqrt{4} = 2$
The solution is $t = 0$.

3. $\sqrt{3t+4} = -6$

Since the principal square root is always a non-negative number, this equation has no real solution.

4. $\sqrt{5t+3} = -2$

Since the principal square root is always a non-negative number, this equation has no real solution.

5. $$\begin{aligned} \sqrt[3]{1-2x} - 3 &= 0 \\ \sqrt[3]{1-2x} &= 3 \\ \left(\sqrt[3]{1-2x}\right)^3 &= 3^3 \\ 1-2x &= 27 \\ -2x &= 26 \\ x &= -13 \end{aligned}$$
Check: $\sqrt[3]{1-2(-13)} - 3$
$= \sqrt[3]{27} - 3 = 0$
The solution is $x = -13$.

6. $$\begin{aligned} \sqrt[3]{1-2x} - 1 &= 0 \\ \sqrt[3]{1-2x} &= 1 \\ \left(\sqrt[3]{1-2x}\right)^3 &= 1^3 \\ 1-2x &= 1 \\ -2x &= 0 \\ x &= 0 \end{aligned}$$
Check: $\sqrt[3]{1-2(0)} - 1 = \sqrt[3]{1} - 1 = 0$
The solution is $x = 0$.

7. $x = 8\sqrt{x}$

$$(x)^2 = \left(8\sqrt{x}\right)^2$$
$$x^2 = 64x$$
$$x^2 - 64x = 0$$
$$x(x-64) = 0$$
$$x = 0 \text{ or } x = 64$$

Check

$x = 0: 0 = 8\sqrt{0}$

$0 = 0$

$x = 64: \quad 64 = 8\sqrt{64}$

$64 = (8)(8) = 64$

The solution set is $\{0, 64\}$.

8. $x = 3\sqrt{x}$

$$(x)^2 = \left(3\sqrt{x}\right)^2$$
$$x^2 = 9x$$
$$x^2 - 9x = 0$$
$$x(x-9) = 0$$
$$x = 0 \text{ or } x = 9$$

Check

$x = 0: 0 = 3\sqrt{0}$

$0 = 0$

$x = 9: 9 = 3\sqrt{9}$

$9 = (3)(3) = 9$

The solution set is $\{0, 9\}$.

9. $\sqrt{15-2x} = x$

$$\left(\sqrt{15-2x}\right)^2 = x^2$$
$$15 - 2x = x^2$$
$$x^2 + 2x - 15 = 0$$
$$(x+5)(x-3) = 0$$
$$x = -5 \text{ or } x = 3$$

Check $-5$: $\sqrt{15-2(-5)} = \sqrt{25}$
$= 5 \neq -5$

Check 3: $\sqrt{15-2(3)} = \sqrt{9} = 3 = 3$

The solution is $x = 3$.

10. $\sqrt{12-x} = x$

$$\left(\sqrt{12-x}\right)^2 = x^2$$
$$12 - x = x^2$$
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$
$$x = -4 \text{ or } x = 3$$

Check $-4$: $\sqrt{12-(-4)} = \sqrt{16}$
$= 4 \neq -4$

Check 3: $\sqrt{12-3} = \sqrt{9} = 3 = 3$

The solution is $x = 3$.

11. $x = 2\sqrt{x-1}$

$$x^2 = \left(2\sqrt{x-1}\right)^2$$
$$x^2 = 4(x-1)$$
$$x^2 = 4x - 4$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$
$$x = 2$$

Check: $2 = 2\sqrt{2-1} \rightarrow 2 = 2$

The solution is $x = 2$.

12. $x = 2\sqrt{-x-1}$

$$x^2 = \left(2\sqrt{-x-1}\right)^2$$
$$x^2 = 4(-x-1)$$
$$x^2 = -4x - 4$$
$$x^2 + 4x + 4 = 0$$
$$(x+2)^2 = 0$$
$$x = -2$$

Check: $-2 = 2\sqrt{-(-2)-1}$
$\rightarrow -2 \neq 2$

No real solution.

13.
$$\sqrt{x^2 - x - 4} = x + 2$$
$$\left(\sqrt{x^2 - x - 4}\right)^2 = (x+2)^2$$
$$x^2 - x - 4 = x^2 + 4x + 4$$
$$-8 = 5x \rightarrow -\frac{8}{5} = x$$

Check

$x = -\frac{8}{5}$ : $\sqrt{\left(-\frac{8}{5}\right)^2 - \left(-\frac{8}{5}\right) - 4} = \left(-\frac{8}{5}\right) + 2$

$$\sqrt{\frac{64}{25} + \frac{8}{5} - 4} = \frac{2}{5}$$
$$\sqrt{\frac{64 + 40 - 100}{25}} = \frac{2}{5}$$
$$\sqrt{\frac{4}{25}} = \frac{2}{5}$$

$\frac{2}{5} = \frac{2}{5}$ The solution is $x = -\frac{8}{5}$.

14.
$$\sqrt{3 - x + x^2} = x - 2$$
$$\left(\sqrt{3 - x + x^2}\right)^2 = (x-2)^2$$
$$3 - x + x^2 = x^2 - 4x + 4$$
$$5x = 1$$
$$x = \frac{1}{5}$$

Check

$x = \frac{1}{5}$ : $\sqrt{3 - \left(\frac{1}{5}\right) + \left(\frac{1}{5}\right)^2} = \left(\frac{1}{5}\right) - 2$

$$\sqrt{3 - \frac{1}{5} + \frac{1}{25}} = -\frac{9}{5}$$

Since the principal square root is always a non-negative number, x = 1/5 does not check, therefore this equation has no real solution.

15.
$$3 + \sqrt{3x+1} = x$$
$$\sqrt{3x+1} = x - 3$$
$$\left(\sqrt{3x+1}\right)^2 = (x-3)^2$$
$$3x + 1 = x^2 - 6x + 9$$
$$0 = x^2 - 9x + 8$$
$$(x-1)(x-8) = 0$$
$$x = 1 \text{ or } x = 8$$

Check 1: $3 + \sqrt{3(1)+1}$
$$= 3 + \sqrt{4} = 5 \neq 1$$

Check 8: $3 + \sqrt{3(8)+1}$
$$= 3 + \sqrt{25} = 8 = 8$$

The solution is $x = 8$.

16.
$$2 + \sqrt{12 - 2x} = x$$
$$\sqrt{12 - 2x} = x - 2$$
$$\left(\sqrt{12 - 2x}\right)^2 = (x-2)^2$$
$$12 - 2x = x^2 - 4x + 4$$
$$0 = x^2 - 2x - 8$$
$$(x+2)(x-4) = 0$$
$$x = -2 \text{ or } x = 4$$

Check $-2$: $2 + \sqrt{12 - 2(-2)}$
$$= 2 + \sqrt{16} = 6 \neq -2$$

Check 4: $2 + \sqrt{12 - 2(4)}$
$$= 2 + \sqrt{4} = 4 = 4$$

The solution is $x = 4$.

17. $\sqrt{2x+3}-\sqrt{x+1}=1$

$$\begin{aligned}
\sqrt{2x+3} &= 1+\sqrt{x+1}\\
\left(\sqrt{2x+3}\right)^2 &= \left(1+\sqrt{x+1}\right)^2\\
2x+3 &= 1+2\sqrt{x+1}+x+1\\
x+1 &= 2\sqrt{x+1}\\
(x+1)^2 &= \left(2\sqrt{x+1}\right)^2\\
x^2+2x+1 &= 4(x+1)\\
x^2+2x+1 &= 4x+4\\
x^2-2x-3 &= 0\\
(x+1)(x-3) &= 0\\
x=-1 &\text{ or } x=3
\end{aligned}$$

Check $-1$: $\sqrt{2(-1)+3}-\sqrt{-1+1}$
$=\sqrt{1}-\sqrt{0}=1-0=1=1$

Check 3: $\sqrt{2(3)+3}-\sqrt{3+1}$
$=\sqrt{9}-\sqrt{4}=3-2=1=1$

The solution is $x=-1$ or $x=3$.

18. $\sqrt{3x+7}+\sqrt{x+2}=1$

$$\begin{aligned}
\sqrt{3x+7} &= 1-\sqrt{x+2}\\
\left(\sqrt{3x+7}\right)^2 &= \left(1-\sqrt{x+2}\right)^2\\
3x+7 &= 1-2\sqrt{x+2}+x+2\\
2x+4 &= -2\sqrt{x+2}\\
-x-2 &= \sqrt{x+2}\\
(-x-2)^2 &= \left(\sqrt{x+2}\right)^2\\
x^2+4x+4 &= x+2\\
x^2+3x+2 &= 0\\
(x+1)(x+2) &= 0\\
x=-1 &\text{ or } x=-2
\end{aligned}$$

Check $-1$: $\sqrt{3(-1)+7}+\sqrt{-1+2}$
$=\sqrt{4}+\sqrt{1}=2+1=3\neq 1$

Check $-2$: $\sqrt{3(-2)+7}+\sqrt{-2+2}$
$=\sqrt{1}+\sqrt{0}=1+0=1=1$

The solution is $x=-2$.

19. $\sqrt{3x+1}-\sqrt{x-1}=2$

$$\begin{aligned}
\sqrt{3x+1} &= 2+\sqrt{x-1}\\
\left(\sqrt{3x+1}\right)^2 &= \left(2+\sqrt{x-1}\right)^2\\
3x+1 &= 4+4\sqrt{x-1}+x-1\\
2x-2 &= 4\sqrt{x-1}\\
(2x-2)^2 &= \left(4\sqrt{x-1}\right)^2\\
4x^2-8x+4 &= 16(x-1)\\
x^2-2x+1 &= 4x-4\\
x^2-6x+5 &= 0\\
(x-1)(x-5) &= 0\\
x=1 &\text{ or } x=5
\end{aligned}$$

Check 1: $\sqrt{3(1)+1}-\sqrt{1-1}$
$=\sqrt{4}-\sqrt{0}=2-0=2=2$

Check 5: $\sqrt{3(5)+1}-\sqrt{5-1}$
$=\sqrt{16}-\sqrt{4}=4-2=2=2$

The solution is $x=1$ or $x=5$.

20. $\sqrt{3x-5}-\sqrt{x+7}=2$

$$\begin{aligned}
\sqrt{3x-5} &= 2+\sqrt{x+7}\\
\left(\sqrt{3x-5}\right)^2 &= \left(2+\sqrt{x+7}\right)^2\\
3x-5 &= 4+4\sqrt{x+7}+x+7\\
2x-16 &= 4\sqrt{x+7}\\
(2x-16)^2 &= \left(4\sqrt{x+7}\right)^2\\
4x^2-64x+256 &= 16(x+7)\\
4x^2-64x+256 &= 16x+112\\
4x^2-80x+144 &= 0\\
x^2-20x+36 &= 0\\
(x-2)(x-18) &= 0\\
x=2 &\text{ or } x=18
\end{aligned}$$

Check 2: $\sqrt{3(2)-5}-\sqrt{2+7}$
$=\sqrt{1}-\sqrt{9}=1-3=-2\neq 2$

Check 18: $\sqrt{3(18)-5}-\sqrt{18+7}$
$=\sqrt{49}-\sqrt{25}=7-5=2=2$

The solution is $x=18$.

21.
$$\sqrt{3-2\sqrt{x}} = \sqrt{x}$$
$$\left(\sqrt{3-2\sqrt{x}}\right)^2 = \left(\sqrt{x}\right)^2$$
$$3-2\sqrt{x} = x$$
$$-2\sqrt{x} = x-3$$
$$\left(-2\sqrt{x}\right)^2 = (x-3)^2$$
$$4x = x^2 - 6x + 9$$
$$0 = x^2 - 10x + 9$$
$$0 = (x-9)(x-1)$$
$$x = 9 \text{ or } x = 1$$

Check

$$x = 9: \sqrt{3-2\sqrt{9}} = \sqrt{9}$$
$$\sqrt{3-2(3)} = 3$$
$$\sqrt{-3} \neq 3$$

Check

$$x = 1: \sqrt{3-2\sqrt{1}} = \sqrt{1}$$
$$\sqrt{3-2(1)} = 1$$
$$\sqrt{1} = 1$$

The solution is $x = 1$.

22.
$$\sqrt{10+3\sqrt{x}} = \sqrt{x}$$
$$\left(\sqrt{10+3\sqrt{x}}\right)^2 = \left(\sqrt{x}\right)^2$$
$$10+3\sqrt{x} = x$$
$$3\sqrt{x} = x-10$$
$$\left(3\sqrt{x}\right)^2 = (x-10)^2$$
$$9x = x^2 - 20x + 100$$
$$0 = x^2 - 29x + 100$$
$$0 = (x-25)(x-4)$$
$$x = 25 \text{ or } x = 4$$

Check

$$x = 25: \sqrt{10+3\sqrt{25}} = \sqrt{25}$$
$$\sqrt{10+(3)(5)} = 5$$
$$\sqrt{25} = 5$$

Check

$$x = 4: \sqrt{10+3\sqrt{4}} = \sqrt{4}$$
$$\sqrt{10+(3)(2)} = 2$$
$$\sqrt{16} \neq 2$$

The solution is $x = 25$.

23.
$$(3x+1)^{1/2} = 4$$
$$\left((3x+1)^{1/2}\right)^2 = (4)^2$$
$$3x+1 = 16$$
$$3x = 15$$
$$x = 5$$

Check

$$x = 5: (3(5)+1)^{1/2} = 4$$
$$16^{1/2} = 4$$
$$4 = 4$$

The solution is $x = 5$.

24.
$$(3x-5)^{1/2} = 2$$
$$\left((3x-5)^{1/2}\right)^2 = (2)^2$$
$$3x-5 = 4$$
$$3x = 9$$
$$x = 3$$

Check

$$x = 3: (3(3)-5)^{1/2} = 2$$
$$4^{1/2} = 2$$
$$2 = 2$$

The solution is $x = 3$.

25. $(5x-2)^{1/3}=2$

$$\left((5x-2)^{1/3}\right)^3=(2)^3$$
$$5x-2=8$$
$$5x=10$$
$$x=2$$

Check

$x=2$: $(5(2)-2)^{1/3}=2$

$$8^{1/3}=2$$
$$2=2$$

The solution is $x=2$.

26. $(2x+1)^{1/3}=-1$

$$\left((2x+1)^{1/3}\right)^3=(-1)^3$$
$$2x+1=-1$$
$$2x=-2$$
$$x=-1$$

Check

$x=-1$: $(2(-1)+1)^{1/3}=-1$

$$(-1)^{1/3}=-1$$
$$-1=-1$$

The solution is $x=-1$.

27. $(x^2+9)^{1/2}=5$

$$\left((x^2+9)^{1/2}\right)^2=(5)^2$$
$$x^2+9=25$$
$$x^2=16$$
$$x=-4 \text{ or } x=4$$

Check

$x=-4$: $((-4)^2+9)^{1/2}=5$

$$25^{1/2}=5$$
$$5=5$$

$x=4$: $((4)^2+9)^{1/2}=5$

$$25^{1/2}=5$$
$$5=5$$

The solution set is $\{-4,4\}$.

28. $(x^2-16)^{1/2}=9$

$$\left((x^2-16)^{1/2}\right)^2=(9)^2$$
$$x^2-16=81$$
$$x^2=97$$
$$x=-\sqrt{97} \text{ or } x=\sqrt{97}$$

Check

$x=-\sqrt{97}$: $\left(\left(-\sqrt{97}\right)^2-16\right)^{1/2}=9$

$$(97-16)^{1/2}=9$$
$$81^{1/2}=9$$
$$9=9$$

$x=\sqrt{97}$: $\left(\left(\sqrt{97}\right)^2-16\right)^{1/2}=9$

$$(97-16)^{1/2}=9$$
$$81^{1/2}=9$$
$$9=9$$

The solution set is $\{-\sqrt{97},\sqrt{97}\}$.

29.

$$x^{3/2} - 3x^{1/2} = 0$$
$$x^{3/2} = 3x^{1/2}$$
$$\left(x^{3/2}\right)^2 = \left(3x^{1/2}\right)^2$$
$$x^3 = 9x$$
$$x^3 - 9x = 0$$
$$x\left(x^2 - 9\right) = 0$$
$$x = 0 \text{ or } x = -3 \text{ or } x = 3$$

Check

$x = 0 : 0^{3/2} - 3\left(0^{1/2}\right) = 0$

$0 = 0$

$x = -3 : (-3)^{3/2} - 3\left((-3)^{1/2}\right) = 0$

$\left(\sqrt{-3}\right)^3 - 3\left(\sqrt{-3}\right) \neq 0$

$x = 3: \quad (3)^{3/2} - 3\left((3)^{1/2}\right) = 0$

$(3)^{3/2} - (3)^{3/2} = 0$

$0 = 0$

The solution set is $\{0,3\}$.

30.

$$x^{3/4} - 9x^{1/4} = 0$$
$$x^{3/4} = 9x^{1/4}$$
$$\left(x^{3/4}\right)^4 = \left(9x^{1/4}\right)^4$$
$$x^3 = 6561x$$
$$x^3 - 6561x = 0$$
$$x\left(x^2 - 6561\right) = 0$$
$$x = 0 \text{ or } x = -81 \text{ or } x = 81$$

Check

$x = 0 : 0^{3/4} - 9(0)^{1/4} = 0$

$0 = 0$

$x = -81 : (-81)^{3/4} - 9\left((-81)^{1/4}\right) = 0$

$\left(\sqrt[4]{-81}\right)^3 - 9\left(\sqrt[4]{-81}\right) \neq 0$

$x = 81 : (81)^{3/4} - 9\left((81)^{1/4}\right) = 0$

$(81)^{3/4} - 9\left((81)^{1/4}\right) = 0$

$27 - 27 = 0$

$0 = 0$

The solution set is $\{0,81\}$.

31.

$$x^4 - 5x^2 + 4 = 0$$
$$\left(x^2 - 4\right)\left(x^2 - 1\right) = 0$$
$$x^2 - 4 = 0 \text{ or } x^2 - 1 = 0$$
$$x = \pm 2 \text{ or } x = \pm 3$$

The solution set is $\{-3,-2,2,3\}$.

32.

$$x^4 - 10x^2 + 25 = 0$$
$$\left(x^2 - 5\right)\left(x^2 - 5\right) = 0$$
$$x^2 - 5 = 0$$
$$x = \pm\sqrt{5}$$

The solution set is $\{-\sqrt{5}, \sqrt{5}\}$.

33.

$$3x^4 - 2x^2 - 1 = 0$$
$$\left(3x^2 + 1\right)\left(x^2 - 1\right) = 0$$
$$3x^2 + 1 = 0 \text{ or } x^2 - 1 = 0$$

$3x^2 = -1$ which is impossible

or

$x = \pm 1$

The solution set is $\{-1,1\}$.

34.

$$2x^4 - 5x^2 - 12 = 0$$
$$\left(2x^2 + 3\right)\left(x^2 - 4\right) = 0$$
$$2x^2 + 3 = 0 \text{ or } x^2 - 4 = 0$$

$2x^2 = -3$ which is impossible

or

$x = \pm 2$

The solution set is $\{-2,2\}$.

35.
$x^6 + 7x^3 - 8 = 0$
$(x^3 + 8)(x^3 - 1) = 0$
$x^3 + 8 = 0$ or $x^3 - 1 = 0$
$x^3 = -8 \rightarrow x = -2$
or
$x^3 = 1 \rightarrow x = 1$

The solution set is $\{-2, 1\}$.

36.
$x^6 - 7x^3 - 8 = 0$
$(x^3 - 8)(x^3 + 1) = 0$
$x^3 - 8 = 0$ or $x^3 + 1 = 0$
$x^3 = 8 \rightarrow x = 2$
or
$x^3 = -1 \rightarrow x = -1$

The solution set is $\{-1, 2\}$.

37.
$(x + 2)^2 + 7(x + 2) + 12 = 0$
$let\ p = x + 2 \rightarrow p^2 = (x + 2)^2$
$p^2 + 7p + 12 = 0$
$(p + 3)(p + 4) = 0$
$p + 3 = 0$ or $p + 4 = 0$
$p = -3 \rightarrow x + 2 = -3 \rightarrow x = -5$
or
$p = -4 \rightarrow x + 2 = -4 \rightarrow x = -6$
The solution set is $\{-6, -5\}$.

38.
$(2x + 5)^2 - (2x + 5) - 6 = 0$
$let\ p = 2x + 5 \rightarrow p^2 = (2x + 5)^2$
$p^2 - p - 6 = 0$
$(p - 3)(p + 2) = 0$
$p - 3 = 0$ or $p + 2 = 0$
$p = 3 \rightarrow 2x + 5 = 3 \rightarrow x = -1$
or
$p = -2 \rightarrow 2x + 5 = -2 \rightarrow x = -\frac{7}{2}$
The solution set is $\left\{-\frac{7}{2}, -1\right\}$.

39.
$(3x + 4)^2 - 6(3x + 4) + 9 = 0$
$let\ p = 3x + 4 \rightarrow p^2 = (3x + 4)^2$
$p^2 - 6p + 9 = 0$
$(p - 3)(p - 3) = 0$
$p - 3 = 0$
$p = 3 \rightarrow 3x + 4 = 3 \rightarrow x = -\frac{1}{3}$
The solution set is $\left\{-\frac{1}{3}\right\}$.

40.
$(2 - x)^2 + (2 - x) - 20 = 0$
$let\ p = 2 - x \rightarrow p^2 = (2 - x)^2$
$p^2 + p - 20 = 0$
$(p + 5)(p - 4) = 0$
$p + 5 = 0$ or $p - 4 = 0$
$p = -5 \rightarrow 2 - x = -5 \rightarrow x = 7$
or
$p = 4 \rightarrow 2 - x = 4 \rightarrow x = -2$
The solution set is $\{-2, 7\}$.

41. $2(s+1)^2 - 5(s+1) = 3$

$let\ p = s+1 \rightarrow p^2 = (s+1)^2$

$2p^2 - 5p = 3$

$2p^2 - 5p - 3 = 0$

$(2p+1)(p-3) = 0$

$2p+1 = 0$ or $p-3 = 0$

$p = -\frac{1}{2} \rightarrow s+1 = -\frac{1}{2} \rightarrow s = -\frac{3}{2}$

or

$p = 3 \rightarrow s+1 = 3 \rightarrow s = 2$

The solution set is $\left\{-\frac{3}{2}, 2\right\}$.

42. $3(1-y)^2 + 5(1-y) + 2 = 0$

$let\ p = 1-y \rightarrow p^2 = (1-y)^2$

$3p^2 + 5p + 2 = 0$

$(3p+2)(p+1) = 0$

$3p+2 = 0$ or $p+1 = 0$

$p = -\frac{2}{3} \rightarrow 1-y = -\frac{2}{3} \rightarrow y = \frac{5}{3}$

or

$p = -1 \rightarrow 1-y = -1 \rightarrow y = 2$

The solution set is $\left\{\frac{5}{3}, 2\right\}$.

43. $x - 4x\sqrt{x} = 0$

$x = 4x\sqrt{x}$

$(x)^2 = \left(4x\sqrt{x}\right)^2$

$x = 16x^2x$

$x = 16x^3$

$0 = 16x^3 - x$

$0 = x\left(16x^2 - 1\right)$

$x = 0$

or $16x^2 - 1 = 0 \rightarrow x = \pm\frac{1}{4}$

Check

$x = 0: 0 - 4(0)\sqrt{0} = 0$

$0 = 0$

$x = -\frac{1}{4}: \left(-\frac{1}{4}\right) - 4\left(-\frac{1}{4}\right)\sqrt{-\frac{1}{4}} \neq 0$

$x = \frac{1}{4}: \left(\frac{1}{4}\right) - 4\left(\frac{1}{4}\right)\sqrt{\frac{1}{4}} = 0$

$\left(\frac{1}{4}\right) - 1\left(\frac{1}{2}\right) = 0$

$-\frac{1}{4} \neq 0$

The solution set is $\{0\}$.

44. $x + 8\sqrt{x} = 0$

$8\sqrt{x} = -x$

$\left(8\sqrt{x}\right)^2 = (-x)^2$

$64x = x^2$

$0 = x^2 - 64x$

$0 = x(x-64)$

$x = 0$ or $x = 64$

Check

$x = 0: 0 + 8\sqrt{0} = 0$

$0 = 0$

$x = 64: 64 + 8\sqrt{64} = 0$

$64 + 64 \neq 0$

The solution set is $\{0\}$.

45.
$$x+\sqrt{x}=20$$
$$let\ p=\sqrt{x}\ \rightarrow\ p^2=x$$
$$p^2+p=20$$
$$p^2+p-20=0$$
$$(p+5)(p-4)=0$$
$$p+5=0\ \text{ or }\ p-4=0$$
$$p=-5\rightarrow\sqrt{x}=-5\rightarrow x=25$$
or
$$p=4\rightarrow\sqrt{x}=4\rightarrow x=16$$

Check
$$x=25: 25+\sqrt{25}=20$$
$$25+5\neq 20$$

$$x=16: 16+\sqrt{16}=20$$
$$16+4=20$$

The solution set is $\{16\}$.

46.
$$x+\sqrt{x}=6$$
$$let\ p=\sqrt{x}\ \rightarrow\ p^2=x$$
$$p^2+p=6$$
$$p^2+p-6=0$$
$$(p+3)(p-2)=0$$
$$p+3=0\ \text{ or }\ p-2=0$$
$$p=-3\rightarrow\sqrt{x}=-3\rightarrow x=9$$
or
$$p=2\rightarrow\sqrt{x}=2\rightarrow x=4$$

Check
$$x=9: 9+\sqrt{9}=6$$
$$9+3\neq 6$$

$$x=4: 4+\sqrt{4}=6$$
$$4+2=6$$

The solution set is $\{4\}$.

47.
$$t^{1/2}-2t^{1/4}+1=0$$
$$let\ p=t^{1/4}\rightarrow p^2=t^{1/2}$$
$$p^2-2p+1=0$$
$$(p-1)(p-1)=0$$
$$p-1=0$$
$$p=1\rightarrow t^{1/4}=1\rightarrow t=1$$

Check
$$t=1: 1^{1/2}-2(1)^{1/4}+1=0$$
$$1-2+1=0$$
$$0=0$$
The solution set is $\{1\}$.

48.
$$z^{1/2}-4t^{1/4}+4=0$$
$$let\ p=z^{1/4}\rightarrow p^2=z^{1/2}$$
$$p^2-4p+4=0$$
$$(p-2)(p-2)=0$$
$$p-2=0$$
$$p=2\rightarrow z^{1/4}=2\rightarrow z=16$$

Check
$$z=16: 16^{1/2}-4(16)^{1/4}+4=0$$
$$4-8+4=0$$
$$0=0$$
The solution set is $\{16\}$.

49. $4x^{1/2} - 9x^{1/4} + 4 = 0$

$let\ \ p = x^{1/4} \to p^2 = x^{1/2}$

$$4p^2 - 9p + 4 = 0 \to p = \frac{9 \pm \sqrt{81-64}}{8} = \frac{9 \pm \sqrt{17}}{8}$$

$$\to x^{1/4} = \frac{9 \pm \sqrt{17}}{8} \to x = \left(\frac{9 \pm \sqrt{17}}{8}\right)^4$$

Check

$$x = \left(\frac{9+\sqrt{17}}{8}\right)^4 : \quad 4\left(\left(\frac{9+\sqrt{17}}{8}\right)^4\right)^{1/2} - 9\left(\left(\frac{9+\sqrt{17}}{8}\right)^4\right)^{1/4} + 4 = 0$$

$$4\left(\frac{9+\sqrt{17}}{8}\right)^2 - 9\left(\frac{9+\sqrt{17}}{8}\right) + 4 = 0$$

$$4\frac{\left(9+\sqrt{17}\right)^2}{64} - 9\left(\frac{9+\sqrt{17}}{8}\right) + 4 = 0$$

$$64\left(4\frac{\left(9+\sqrt{17}\right)^2}{64} - 9\left(\frac{9+\sqrt{17}}{8}\right) + 4\right) = (0)(64)$$

$$4\left(9+\sqrt{17}\right)^2 - 72\left(9+\sqrt{17}\right) + 256 = 0$$

$$4\left(81 + 18\sqrt{17} + 17\right) - 72\left(9+\sqrt{17}\right) + 256 = 0$$

$$324 + 72\sqrt{17} + 68 - 648 - 72\sqrt{17} + 256 = 0$$

$$0 = 0$$

$$x = \left(\frac{9-\sqrt{17}}{8}\right)^4 : \quad 4\left(\left(\frac{9-\sqrt{17}}{8}\right)^4\right)^{1/2} - 9\left(\left(\frac{9-\sqrt{17}}{8}\right)^4\right)^{1/4} + 4 = 0$$

$$4\left(\frac{9-\sqrt{17}}{8}\right)^2 - 9\left(\frac{9-\sqrt{17}}{8}\right) + 4 = 0 \to 4\left(81 - 18\sqrt{17} + 17\right) - 72\left(9-\sqrt{17}\right) + 256 = 0$$

$$324 - 72\sqrt{17} + 68 - 648 + 72\sqrt{17} + 256 = 0$$

$$0 = 0$$

$$4\frac{\left(9-\sqrt{17}\right)^2}{64} - 9\left(\frac{9-\sqrt{17}}{8}\right) + 4 = 0\Bigg\} \to 64\left(4\frac{\left(9-\sqrt{17}\right)^2}{64} - 9\left(\frac{9-\sqrt{17}}{8}\right) + 4\right) = (0)(64)$$

$$\to 4\left(9-\sqrt{17}\right)^2 - 72\left(9-\sqrt{17}\right) + 256 = 0 \quad \to 0 = 0 \quad \text{the solution set is } \left\{\frac{9 \pm \sqrt{17}}{8}\right\}$$

50. $x^{1/2} - 3x^{1/4} + 2 = 0$

$let\ p = x^{1/4} \rightarrow p^2 = x^{1/2}$

$p^2 - 3p + 2 = 0$

$(p-2)(p-1) = 0$

$p = 2 \rightarrow x^{1/4} = 2 \rightarrow x = 16$

or $p = 1 \rightarrow x^{1/4} = 1 \rightarrow x = 1$

Check

$x = 16:\ 16^{1/2} - 3(16)^{1/4} + 2 = 0$

$4 - 6 + 2 = 0$

$0 = 0$

$x = 1:\ 1^{1/2} - 3(1)^{1/4} + 2 = 0$

$1 - 3 + 2 = 0$

$0 = 0$

The solution set is $\{1, 16\}$.

51. $\sqrt[4]{5x^2 - 6} = x$

$\left(\sqrt[4]{5x^2 - 6}\right)^4 = x^4$

$5x^2 - 6 = x^4$

$0 = x^4 - 5x^2 + 6$

$let\ p = x^2 \rightarrow p^2 = x^4$

$0 = p^2 - 5p + 6$

$(p-3)(p-2) = 0$

$p = 3 \rightarrow x^2 = 3 \rightarrow x = \pm\sqrt{3}$

or $p = 2 \rightarrow x^2 = 2 \rightarrow x = \pm\sqrt{2}$

Check

$x = -\sqrt{3}:\ \sqrt[4]{5\left(-\sqrt{3}\right)^2 - 6} = -\sqrt{3}$

$\sqrt[4]{15 - 6} = -\sqrt{3}$

$\sqrt[4]{9} \neq -\sqrt{3}$

$x = \sqrt{3}:\ \sqrt[4]{5\left(\sqrt{3}\right)^2 - 6} = \sqrt{3}$

$\sqrt[4]{15 - 6} = \sqrt{3}$

$\sqrt[4]{9} = \sqrt{3}$

$\sqrt{3} = \sqrt{3}$

$x = -\sqrt{2}:\ \sqrt[4]{5\left(-\sqrt{2}\right)^2 - 6} = -\sqrt{2}$

$\sqrt[4]{10 - 6} = -\sqrt{2}$

$\sqrt[4]{4} \neq -\sqrt{2}$

$x = \sqrt{2}:\ \sqrt[4]{5\left(\sqrt{2}\right)^2 - 6} = \sqrt{2}$

$\sqrt[4]{10 - 6} = \sqrt{2}$

$\sqrt[4]{4} = \sqrt{2}$

$\sqrt{2} = \sqrt{2}$

The solution set is $\left\{\sqrt{2}, \sqrt{3}\right\}$.

52. $\sqrt[4]{4-5x^2} = x$

$\left(\sqrt[4]{4-5x^2}\right)^4 = x^4$

$4-5x^2 = x^4$

$0 = x^4 + 5x^2 - 4$

$let\ p = x^2 \rightarrow p^2 = x^4$

$0 = p^2 + 5p - 4$

$p = \frac{-5 \pm \sqrt{25+16}}{2} = \frac{-5 \pm \sqrt{41}}{2}$

$\rightarrow x^2 = \frac{-5 \pm \sqrt{41}}{2} \rightarrow x = \pm\sqrt{\frac{-5 \pm \sqrt{41}}{2}}$

but since $-5-\sqrt{41} < 0$,

$x = \pm\sqrt{\frac{-5-\sqrt{41}}{2}}$ is undefined

Check:

$x = \pm\sqrt{\frac{-5+\sqrt{41}}{2}}$ :

$\sqrt[4]{4-5\left(\pm\sqrt{\frac{-5+\sqrt{41}}{2}}\right)^2} = \pm\sqrt{\frac{-5+\sqrt{41}}{2}}$

$\sqrt[4]{4-5\left(\frac{-5+\sqrt{41}}{2}\right)} = \pm\sqrt{\frac{-5+\sqrt{41}}{2}}$

$\sqrt[4]{\frac{8-5\left(-5+\sqrt{41}\right)}{2}} = \pm\sqrt{\frac{-5+\sqrt{41}}{2}}$

$\sqrt[4]{\frac{33+\sqrt{41}}{2}} = \pm\sqrt{\frac{-5+\sqrt{41}}{2}}$

which is only true when $x = \sqrt{\frac{-5+\sqrt{41}}{2}}$

53. $x^2 + 3x + \sqrt{x^2+3x} = 6$

$let\ p = \sqrt{x^2+3x} \rightarrow p^2 = x^2 + 3x$

$p^2 + p = 6$

$p^2 + p - 6 = 0$

$(p+3)(p-2) = 0$

$p = -3$ or $p = 2$

$\rightarrow \sqrt{x^2+3x} = -3$ which is impossible since the principal square root is always a non-negative number.

or

$\sqrt{x^2+3x} = 2 \rightarrow x^2 + 3x = 4$

$\rightarrow x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4$ or $x = 1$

Check

$x = -4$ : $(-4)^2 + 3(-4) + \sqrt{(-4)^2 + 3(-4)} = 6$

$16 - 12 + \sqrt{16-12} = 6$

$16 - 12 + \sqrt{4} = 6$

$6 = 6$

$x = 1$ : $(1)^2 + 3(1) + \sqrt{(1)^2 + 3(1)} = 6$

$1 + 3 + \sqrt{1+3} = 6$

$4 + \sqrt{4} = 6$

$6 = 6$

The solution set is $\{-4, 1\}$.

54. $x^2-3x-\sqrt{x^2-3x}=2$

$let\ p=\sqrt{x^2-3x}\rightarrow p^2=x^2-3x$

$p^2-p=2$

$p^2-p-2=0$

$(p+1)(p-2)=0$

$p=-1$ or $p=2$

$p=-1\rightarrow\sqrt{x^2-3x}=-1$ which is impossible since the principal square root is always a non-negative number.

or

$p=2\rightarrow\sqrt{x^2-3x}=2\rightarrow x^2-3x=4$

$\rightarrow x^2-3x-4=0$

$(x-4)(x+1)=0$

$x=4$ or $x=-1$

Check

$x=4:\ (4)^2-3(4)-\sqrt{(4)^2-3(4)}=2$

$16-12-\sqrt{4}=2$

$4-2=2$

$2=2$

$x=-1:\ (-1)^2-3(-1)-\sqrt{(-1)^2-3(-1)}=2$

$1+3-\sqrt{4}=2$

$4-2=2$

The solution set is $\{-1,4\}$.

55. $\dfrac{1}{(x+1)^2}=\dfrac{1}{x+1}+2$

$let\ p=\dfrac{1}{x+1}\rightarrow p^2=\left(\dfrac{1}{x+1}\right)^2$

$p^2=p+2$

$p^2-p-2=0$

$(p+1)(p-2)=0$

$p=-1$ or $p=2$

$p=-1\rightarrow\dfrac{1}{x+1}=-1\rightarrow 1=-x-1\rightarrow x=-2$

or

$\dfrac{1}{x+1}=2\rightarrow 1=2x+2\rightarrow x=-\dfrac{1}{2}$

Check

$x=-2:\ \dfrac{1}{(-2+1)^2}=\dfrac{1}{-2+1}+2$

$1=-1+2$

$1=1$

$x=-\dfrac{1}{2}:\ \dfrac{1}{\left(-\dfrac{1}{2}+1\right)^2}=\dfrac{1}{-\dfrac{1}{2}+1}+2$

$4=2+2$

$4=4$

The solution set is $\left\{-2,-\dfrac{1}{2}\right\}$.

56. $\dfrac{1}{(x-1)^2}+\dfrac{1}{x-1}=12$

$let\ p=\dfrac{1}{x-1} \rightarrow p^2=\left(\dfrac{1}{x-1}\right)^2$

$p^2+p=12$

$p^2+p-12=0$

$(p+4)(p-3)=0$

$p=-4$ or $p=3$

$p=-4 \rightarrow \dfrac{1}{x-1}=-4$

$\rightarrow 1=-4x+4 \rightarrow 4x=3$

$\rightarrow x=\dfrac{3}{4}$

or

$\dfrac{1}{x-1}=3 \rightarrow 1=3x-3 \rightarrow x=\dfrac{4}{3}$

Check

$x=\dfrac{3}{4}:\ \dfrac{1}{\left(\frac{3}{4}-1\right)^2}+\dfrac{1}{\frac{3}{4}-1}=12$

$16-4=12$

$12=12$

$x=\dfrac{4}{3}:\ \dfrac{1}{\left(\frac{4}{3}-1\right)^2}+\dfrac{1}{\frac{4}{3}-1}=12$

$9+3=12$

$12=12$

The solution set is $\left\{\dfrac{3}{4},\dfrac{4}{3}\right\}$.

57. $3x^{-2}-7x^{-1}-6=0$

$let\ p=x^{-1} \rightarrow p^2=x^{-2}$

$3p^2-7p-6=0$

$(3p+2)(p-3)=0$

$p=-\dfrac{2}{3}$ or $p=3$

$p=-\dfrac{2}{3} \rightarrow x^{-1}=-\dfrac{2}{3} \rightarrow \left(x^{-1}\right)^{-1}$

$=\left(-\dfrac{2}{3}\right)^{-1} \rightarrow x=-\dfrac{3}{2}$

or

$p=3 \rightarrow x^{-1}=3 \rightarrow \left(x^{-1}\right)^{-1}=(3)^{-1} \rightarrow x=\dfrac{1}{3}$

Check

$x=-\dfrac{3}{2}:\ 3\left(-\dfrac{3}{2}\right)^{-2}-7\left(-\dfrac{3}{2}\right)^{-1}-6=0$

$3\left(\dfrac{4}{9}\right)-7\left(-\dfrac{2}{3}\right)-6=0$

$\dfrac{4}{3}+\dfrac{14}{3}-6=0$

$6-6=0$

$0=0$

$x=\dfrac{1}{3}:\ 3\left(\dfrac{1}{3}\right)^{-2}-7\left(\dfrac{1}{3}\right)^{-1}-6=0$

$3(9)-7(3)-6=0$

$27-21-6=0$

$6-6=0$

$0=0$

The solution set is $\left\{-\dfrac{3}{2},\dfrac{1}{3}\right\}$.

58. $2x^{-2}-3x^{-1}-4=0$

*let* $p=x^{-1} \rightarrow p^2=x^{-2}$

$2p^2-3p-4=0$

$p=\dfrac{3\pm\sqrt{9+32}}{4}=\dfrac{3\pm\sqrt{41}}{4}$

$p=\dfrac{3+\sqrt{41}}{4} \rightarrow x^{-1}=\dfrac{3+\sqrt{41}}{4}$

$\rightarrow \left(x^{-1}\right)^{-1}=\left(\dfrac{3+\sqrt{41}}{4}\right)^{-1} \rightarrow x=\dfrac{4}{3+\sqrt{41}}$

or

$p=\dfrac{3-\sqrt{41}}{4} \rightarrow x^{-1}=\dfrac{3-\sqrt{41}}{4}$

$\rightarrow \left(x^{-1}\right)^{-1}=\left(\dfrac{3-\sqrt{41}}{4}\right)^{-1} \rightarrow x=\dfrac{4}{3-\sqrt{41}}$

Check

$x=\dfrac{4}{3+\sqrt{41}}$ :

$2\left(\dfrac{4}{3+\sqrt{41}}\right)^{-2}-3\left(\dfrac{4}{3+\sqrt{41}}\right)^{-1}-4=0$

$2\dfrac{\left(3+\sqrt{41}\right)^2}{16}-3\left(\dfrac{3+\sqrt{41}}{4}\right)-4=0$

$(16)\left(2\dfrac{\left(9+6\sqrt{41}+41\right)}{16}-3\left(\dfrac{3+\sqrt{41}}{4}\right)-4\right)=(0)(16)$

$2\left(9+6\sqrt{41}+41\right)-12\left(3+\sqrt{41}\right)-64=0$

$18+12\sqrt{41}+82-36-12\sqrt{41}-64=0$

$0=0$

$x=\dfrac{4}{3-\sqrt{41}}$ :

$$\left.\begin{array}{l}
2\left(\dfrac{4}{3-\sqrt{41}}\right)^{-2}-3\left(\dfrac{4}{3-\sqrt{41}}\right)^{-1}-4=0 \\
2\dfrac{\left(3-\sqrt{41}\right)^2}{16}-3\left(\dfrac{3-\sqrt{41}}{4}\right)-4=0 \\
(16)\left(2\dfrac{\left(9-6\sqrt{41}+41\right)}{16}-3\left(\dfrac{3-\sqrt{41}}{4}\right)-4\right)=(0)(16)
\end{array}\right\} \rightarrow$$

$2\left(9-6\sqrt{41}+41\right)-12\left(3-\sqrt{41}\right)-64=0$

$18-12\sqrt{41}+82-36+12\sqrt{41}-64=0$

$0=0$

The solution set is $\left\{\dfrac{4}{3-\sqrt{41}},\dfrac{4}{3+\sqrt{41}}\right\}$

59. $2x^{2/3} - 5x^{1/3} - 3 = 0$

$let\ p = x^{1/3} \rightarrow p = x^{2/3}$

$2p^2 - 5p - 3 = 0$

$(2p+1)(p-3) = 0$

$p = -\frac{1}{2}$ or $p = 3$

$p = -\frac{1}{2} \rightarrow x^{1/3} = -\frac{1}{2}$

$\rightarrow \left(x^{1/3}\right)^3 = \left(-\frac{1}{2}\right)^3 \rightarrow x = -\frac{1}{8}$

or

$p = 3 \rightarrow x^{1/3} = 3 \rightarrow \left(x^{1/3}\right)^3 = (3)^3$

$\rightarrow x = 27$

Check

$x = -\frac{1}{8}:\ 2\left(-\frac{1}{8}\right)^{2/3} - 5\left(-\frac{1}{8}\right)^{1/3} - 3 = 0$

$2\left(\frac{1}{4}\right) - 5\left(-\frac{1}{2}\right) - 3 = 0$

$\frac{1}{2} + \frac{5}{2} - 3 = 0$

$3 - 3 = 0$

$0 = 0$

$x = 27:\ 2(27)^{2/3} - 5(27)^{1/3} - 3 = 0$

$2(9) - 5(3) - 3 = 0$

$18 - 15 - 3 = 0$

$3 - 3 = 0$

$0 = 0$

The solution set is $\left\{-\frac{1}{8}, 27\right\}$.

60. $3x^{4/3} + 5x^{2/3} - 2 = 0$

$$\left.\begin{array}{l} let\ p = x^{2/3} \rightarrow p = x^{4/3} \\ 3p^2 + 5p^2 - 2 = 0 \\ (3p-1)(p+2) = 0 \\ p = \frac{1}{3} \text{ or } p = -2 \end{array}\right\} \rightarrow$$

$p = \frac{1}{3} \rightarrow x^{2/3} = \frac{1}{3} \rightarrow \left(x^{2/3}\right)^{3/2} = \left(\frac{1}{3}\right)^{3/2}$

or

$p = -2 \rightarrow x^{2/3} = -2 \rightarrow \left(x^{2/3}\right)^{3/2} = (-2)^{3/2}$

which is impossible

Check

$$\left.\begin{array}{l} x = \left(\frac{1}{3}\right)^{3/2}: \\ 3\left(\left(\frac{1}{3}\right)^{3/2}\right)^{4/3} + 5\left(\left(\frac{1}{3}\right)^{3/2}\right)^{2/3} - 2 = 0 \\ 3\left(\frac{1}{3}\right)^2 + 5\left(\frac{1}{3}\right) - 2 = 0 \end{array}\right\} \rightarrow$$

$\frac{3}{9} + \frac{5}{3} - 2 = 0$

$\frac{1}{3} + \frac{5}{3} - 2 = 0$

$2 - 2 = 0$

$0 = 0$

The solution set is $\left\{\left(\frac{1}{3}\right)^{3/2}\right\}$.

61. $\left(\frac{v}{v+1}\right)^2+\frac{2v}{v+1}=8$

$let\ p=\frac{v}{v+1}\rightarrow p^2=\left(\frac{v}{v+1}\right)^2$

$\left(\frac{v}{v+1}\right)^2+\frac{2v}{v+1}=8$

$\rightarrow\left(\frac{v}{v+1}\right)^2+2\left(\frac{v}{v+1}\right)=8$

$p^2+2p=8$

$p^2+2p-8=0$

$(p+4)(p-2)=0$

$p=-4$ or $p=2$

$p=-4\rightarrow\frac{v}{v+1}=-4$

$\rightarrow v=-4v-4\rightarrow v=-\frac{4}{5}$

or

$p=2\rightarrow\frac{v}{v+1}=2$

$\rightarrow v=2v+2\rightarrow v=-2$

Check

$v=-\frac{4}{5}:\left(\frac{-\frac{4}{5}}{-\frac{4}{5}+1}\right)^2+\frac{2\left(-\frac{4}{5}\right)}{\left(-\frac{4}{5}\right)+1}=8$

$\frac{\left(\frac{16}{25}\right)}{\left(\frac{1}{25}\right)}+\frac{\left(-\frac{8}{5}\right)}{\left(\frac{1}{5}\right)}=8$

$16-8=8$

$8=8$

$v=-2:\left(\frac{-2}{-2+1}\right)^2+\frac{2(-2)}{(-2)+1}=8$

$4+4=8$

$8=8$

The solution set is $\left\{-\frac{4}{5},-2\right\}$.

62. $\left(\frac{y}{y-1}\right)^2=6\left(\frac{y}{y-1}\right)+7$

$let\ p=\frac{y}{y-1}\rightarrow p^2=\left(\frac{y}{y-1}\right)^2\rightarrow$

$p^2=6p+7$

$p=-1\rightarrow\frac{y}{y-1}=-1\rightarrow y=-y+1\rightarrow y=\frac{1}{2}$

or

$p=7\rightarrow\frac{y}{y-1}=7\rightarrow y=7y-7\rightarrow y=\frac{7}{6}$

Check

$y=\frac{1}{2}:\left(\frac{\frac{1}{2}}{\frac{1}{2}-1}\right)^2=6\left(\frac{\frac{1}{2}}{\frac{1}{2}-1}\right)+7$

$\frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{4}\right)}=6\left(\frac{\left(\frac{1}{2}\right)}{\left(-\frac{1}{2}\right)}\right)+7$

$1=6(-1)+7$

$1=1$

$y=\frac{7}{6}:\left(\frac{\frac{7}{6}}{\frac{7}{6}-1}\right)^2=6\left(\frac{\frac{7}{6}}{\frac{7}{6}-1}\right)+7$

$\frac{\left(\frac{49}{36}\right)}{\left(\frac{1}{36}\right)}=6\left(\frac{\left(\frac{7}{6}\right)}{\left(\frac{1}{6}\right)}\right)+7$

$49=42+7$

$49=49$

The solution set is $\left\{\frac{1}{2},\frac{7}{6}\right\}$.

63. $x - 4x^{1/2} + 2 = 0$

$let\ p = x^{1/2} \rightarrow p^2 = x^2$

$p^2 - 4p + 2 = 0$

$p = \dfrac{4 \pm \sqrt{16-8}}{2} = \dfrac{4 \pm \sqrt{8}}{2}$

$p = \dfrac{4+\sqrt{8}}{2} \rightarrow x^{1/2} = \dfrac{4+\sqrt{8}}{2}$

$\rightarrow \left(x^{1/2}\right)^2 = \left(\dfrac{4+\sqrt{8}}{2}\right)^2 \rightarrow x = \left(\dfrac{4+\sqrt{8}}{2}\right)^2$

or

$p = \dfrac{4-\sqrt{8}}{2} \rightarrow x^{1/2} = \dfrac{4-\sqrt{8}}{2}$

$\rightarrow \left(x^{1/2}\right)^2 = \left(\dfrac{4-\sqrt{8}}{2}\right)^2 \rightarrow x = \left(\dfrac{4-\sqrt{8}}{2}\right)^2$

Check

$x = \left(\dfrac{4+\sqrt{8}}{2}\right)^2 : \left(\dfrac{4+\sqrt{8}}{2}\right)^2 - 4\left(\dfrac{4+\sqrt{8}}{2}\right) + 2 = 0$

$\dfrac{16+8\sqrt{8}+8}{4} - 4\left(\dfrac{4+\sqrt{8}}{2}\right) + 2 = 0$

$4 + 2\sqrt{8} + 2 - 2\left(4+\sqrt{8}\right) + 2 = 0$

$4 + 2\sqrt{8} + 2 - 8 - 2\sqrt{8} + 2 = 0$

$0 = 0$

$x = \left(\dfrac{4-\sqrt{8}}{2}\right)^2 : \left(\dfrac{4-\sqrt{8}}{2}\right)^2 - 4\left(\dfrac{4-\sqrt{8}}{2}\right) + 2 = 0$

$\dfrac{16-8\sqrt{8}+8}{4} - 4\left(\dfrac{4-\sqrt{8}}{2}\right) + 2 = 0$

$4 - 2\sqrt{8} + 2 - 2\left(4-\sqrt{8}\right) + 2 = 0$

$4 - 2\sqrt{8} + 2 - 8 + 2\sqrt{8} + 2 = 0$

$0 = 0$

The solution set is

$\left\{\left(\dfrac{4+\sqrt{8}}{2}\right)^2, \left(\dfrac{4-\sqrt{8}}{2}\right)^2\right\} \rightarrow \{11.66, 0.34\}$.

64. $x^{2/3} + 4x^{1/3} + 2 = 0$

*let* $p = x^{1/3}$

$p^2 + 4p + 2 = 0$

$p = \dfrac{-4 \pm \sqrt{16-8}}{2} = \dfrac{-4 \pm \sqrt{8}}{2}$

$p = \dfrac{-4+\sqrt{8}}{2} \rightarrow x^{1/3} = \dfrac{-4+\sqrt{8}}{2}$

$\rightarrow \left(x^{1/3}\right)^3 = \left(\dfrac{-4+\sqrt{8}}{2}\right)^3 \rightarrow x = \left(\dfrac{-4+\sqrt{8}}{2}\right)^3$

or

$p = \dfrac{-4-\sqrt{8}}{2} \rightarrow x^{1/3} = \dfrac{-4-\sqrt{8}}{2}$

$\rightarrow \left(x^{1/3}\right)^3 = \left(\dfrac{-4-\sqrt{8}}{2}\right)^3 \rightarrow x = \left(\dfrac{-4-\sqrt{8}}{2}\right)^3$

Check

$x = \left(\dfrac{-4+\sqrt{8}}{2}\right)^3$ :

$\left(\left(\dfrac{-4+\sqrt{8}}{2}\right)^3\right)^{2/3} + 4\left(\left(\dfrac{-4+\sqrt{8}}{2}\right)^3\right)^{1/3} + 2 = 0$

$\left(\dfrac{-4+\sqrt{8}}{2}\right)^2 + 4\left(\dfrac{-4+\sqrt{8}}{2}\right) + 2 = 0$

$\left(\dfrac{16-8\sqrt{8}+8}{4}\right) + 2\left(-4+\sqrt{8}\right) + 2 = 0$

$4 - 2\sqrt{8} + 2 + 2\left(-4+\sqrt{8}\right) + 2 = 0$

$4 - 2\sqrt{8} + 2 - 8 + 2\sqrt{8} + 2 = 0$

$0 = 0$

$x = \left(\dfrac{-4-\sqrt{8}}{2}\right)^3$ :

$\left(\left(\dfrac{-4-\sqrt{8}}{2}\right)^3\right)^{2/3} + 4\left(\left(\dfrac{-4-\sqrt{8}}{2}\right)^3\right)^{1/3} + 2 = 0$

$\left(\dfrac{-4-\sqrt{8}}{2}\right)^2 + 4\left(\dfrac{-4-\sqrt{8}}{2}\right) + 2 = 0$

$\left(\dfrac{16+8\sqrt{8}+8}{4}\right) + 2\left(-4-\sqrt{8}\right) + 2 = 0$

$4 + 2\sqrt{8} + 2 + 2\left(-4-\sqrt{8}\right) + 2 = 0$

$4 + 2\sqrt{8} + 2 - 8 - 2\sqrt{8} + 2 = 0$

$0 = 0$

The solution set is $\left\{\left(\dfrac{-4+\sqrt{8}}{2}\right)^3, \left(\dfrac{-4-\sqrt{8}}{2}\right)^3\right\}$

$\rightarrow \{-0.20, -39.80\}$

65.

$$x^4+\sqrt{3}x^2-3=0$$

$$let\ \ p=x^2 \rightarrow p^2=x^4$$

$$p^2+\sqrt{3}p-3=0$$

$$p=\frac{-\sqrt{3}\pm\sqrt{3+12}}{2}=\frac{-\sqrt{3}\pm\sqrt{15}}{2}$$

$$p=\frac{-\sqrt{3}+\sqrt{15}}{2}$$

$$\rightarrow x^2=\frac{-\sqrt{3}+\sqrt{15}}{2} \rightarrow x=\pm\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}}$$

or

$$p=\frac{-\sqrt{3}-\sqrt{15}}{2}$$

$$\rightarrow x^2=\frac{-\sqrt{3}-\sqrt{15}}{2} \rightarrow$$

$$x=\pm\sqrt{\frac{-\sqrt{3}-\sqrt{15}}{2}}$$

which is impossible since $\frac{-\sqrt{3}-\sqrt{15}}{2}<0$

.

Check

$x=\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}}$ :

$$\left(\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}}\right)^4+\sqrt{3}\left(\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}}\right)^2-3=0$$

$$\left(\frac{-\sqrt{3}+\sqrt{15}}{2}\right)^2+\sqrt{3}\left(\frac{-\sqrt{3}+\sqrt{15}}{2}\right)-3=0$$

$$\left(\frac{3-2\sqrt{3}\sqrt{15}+15}{4}\right)+\left(\frac{\sqrt{3}\left(-\sqrt{3}\right)+\sqrt{3}\sqrt{15}}{2}\right)-3=0$$

$$\left(\frac{18-2\sqrt{45}}{4}\right)+\left(\frac{-3+\sqrt{45}}{2}\right)-3=0$$

$$\left(\frac{9-\sqrt{45}}{2}\right)+\left(\frac{-3+\sqrt{45}}{2}\right)-3=0$$

$$\left(\frac{9-\sqrt{45}-3+\sqrt{45}}{2}\right)-3=0$$

$$3-3=0$$

$$0=0$$

$x=-\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}}$ :

$$\left(-\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}}\right)^4+\sqrt{3}\left(-\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}}\right)^2-3=0$$

$$\left(\frac{-\sqrt{3}+\sqrt{15}}{2}\right)^2+\sqrt{3}\left(\frac{-\sqrt{3}+\sqrt{15}}{2}\right)-3=0$$

$$\left(\frac{3-2\sqrt{3}\sqrt{15}+15}{4}\right)+\left(\frac{\sqrt{3}\left(-\sqrt{3}\right)+\sqrt{3}\sqrt{15}}{2}\right)-3=0$$

$$\left(\frac{18-2\sqrt{45}}{4}\right)+\left(\frac{-3+\sqrt{45}}{2}\right)-3=0$$

$$\left(\frac{9-\sqrt{45}}{2}\right)+\left(\frac{-3+\sqrt{45}}{2}\right)-3=0$$

$$\left(\frac{9-\sqrt{45}-3+\sqrt{45}}{2}\right)-3=0$$

$$3-3=0$$

$$0=0$$

The solution set is

$$\left\{\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}},-\sqrt{\frac{-\sqrt{3}+\sqrt{15}}{2}}\right\}$$

$$\rightarrow\{1.04,-1.04\}$$

66. $x^4+\sqrt{2}x^2-2=0$

$let\ p=x^2 \ \rightarrow\ p^2=x^4$

$p^2+\sqrt{2}p-2=0$

$$p=\frac{-\sqrt{2}\pm\sqrt{2+8}}{2}=\frac{-\sqrt{2}\pm\sqrt{10}}{2}$$

$$p=\frac{-\sqrt{2}+\sqrt{10}}{2}$$

$$\rightarrow x^2=\frac{-\sqrt{2}+\sqrt{10}}{2} \rightarrow x=\pm\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}$$

or

$$p=\frac{-\sqrt{2}-\sqrt{10}}{2} \rightarrow x^2=\frac{-\sqrt{2}-\sqrt{10}}{2}$$

$$\rightarrow x=\pm\sqrt{\frac{-\sqrt{2}-\sqrt{10}}{2}}$$

which is impossible since $\frac{-\sqrt{2}-\sqrt{10}}{2}<0$

Check

$x=\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}$ :

$$\left(\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}\right)^4+\sqrt{2}\left(\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}\right)^2-2=0$$

$$\left(\frac{-\sqrt{2}+\sqrt{10}}{2}\right)^2+\sqrt{2}\left(\frac{-\sqrt{2}+\sqrt{10}}{2}\right)-2=0$$

$$\left(\frac{2-2\sqrt{2}\sqrt{10}+10}{4}\right)+\left(\frac{\sqrt{2}\left(-\sqrt{2}\right)+\sqrt{2}\sqrt{10}}{2}\right)-2=0$$

$$\left(\frac{12-2\sqrt{20}}{4}\right)+\left(\frac{-2+\sqrt{20}}{2}\right)-2=0$$

$$\left(\frac{6-\sqrt{20}}{2}\right)+\left(\frac{-2+\sqrt{20}}{2}\right)-2=0$$

$$\left(\frac{6-\sqrt{20}-2+\sqrt{20}}{2}\right)-2=0$$

$2-2=0$

$0=0$

$x=-\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}$ :

$$\left(-\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}\right)^4+\sqrt{2}\left(-\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}\right)^2-2=0$$

$$\left(\frac{-\sqrt{2}+\sqrt{10}}{2}\right)^2+\sqrt{2}\left(\frac{-\sqrt{2}+\sqrt{10}}{2}\right)-2=0$$

$$\left(\frac{2-2\sqrt{2}\sqrt{10}+10}{4}\right)+\left(\frac{\sqrt{2}\left(-\sqrt{2}\right)+\sqrt{2}\sqrt{10}}{2}\right)-2=0$$

$$\left(\frac{12-2\sqrt{20}}{4}\right)+\left(\frac{-2+\sqrt{20}}{2}\right)-2=0$$

$$\left(\frac{6-\sqrt{20}}{2}\right)+\left(\frac{-2+\sqrt{20}}{2}\right)-2=0$$

$$\left(\frac{6-\sqrt{20}-2+\sqrt{20}}{2}\right)-2=0$$

$2-2=0$

$0=0$

The solution set is

$$\left\{\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}},-\sqrt{\frac{-\sqrt{2}+\sqrt{10}}{2}}\right\}$$

$\rightarrow\{0.94,-0.94\}$

67. $\pi(1+t)^2 = \pi + 1 + t$

$let\ p = 1+t \rightarrow p^2 = (1+t)^2$

$\pi p^2 = \pi + p \rightarrow \pi p^2 - p - \pi = 0$

$$p = \frac{1 \pm \sqrt{1+4\pi^2}}{2\pi}$$

$$\rightarrow 1 + t = \frac{1 \pm \sqrt{1+4\pi^2}}{2\pi}$$

$$\rightarrow t = -1 + \frac{1 \pm \sqrt{1+4\pi^2}}{2\pi}$$

$$t = -1 + \frac{1 \pm \sqrt{1+4\pi^2}}{2\pi}$$

Check

$$t = -1 + \frac{1+\sqrt{1+4\pi^2}}{2\pi}:$$

$$\pi\left(\frac{1+\sqrt{1+4\pi^2}}{2\pi}\right)^2 = \pi + \frac{1+\sqrt{1+4\pi^2}}{2\pi}$$

$$\pi\left(\frac{1+2\sqrt{1+4\pi^2}+1+4\pi^2}{4\pi^2}\right) = \pi + \frac{1+\sqrt{1+4\pi^2}}{2\pi}$$

$$\frac{2+2\sqrt{1+4\pi^2}+4\pi^2}{4\pi} = \frac{2\pi^2+1+\sqrt{1+4\pi^2}}{2\pi}$$

$$\frac{1+\sqrt{1+4\pi^2}+2\pi^2}{2\pi} = \frac{2\pi^2+1+\sqrt{1+4\pi^2}}{2\pi}$$

$0 = 0$

$$t = -1 + \frac{1-\sqrt{1+4\pi^2}}{2\pi}:$$

$$\pi\left(\frac{1-\sqrt{1+4\pi^2}}{2\pi}\right)^2 = \pi + \frac{1-\sqrt{1+4\pi^2}}{2\pi}$$

$$\pi\left(\frac{1-2\sqrt{1+4\pi^2}+1+4\pi^2}{4\pi^2}\right) = \pi + \frac{1-\sqrt{1+4\pi^2}}{2\pi}$$

$$\frac{2-2\sqrt{1+4\pi^2}+4\pi^2}{4\pi} = \frac{2\pi^2+1-\sqrt{1+4\pi^2}}{2\pi}$$

$$\frac{1-\sqrt{1+4\pi^2}+2\pi^2}{2\pi} = \frac{2\pi^2+1-\sqrt{1+4\pi^2}}{2\pi}$$

$0 = 0$

The solution set is

$$\left\{-1+\frac{1+\sqrt{1+4\pi^2}}{2\pi}, -1+\frac{1-\sqrt{1+4\pi^2}}{2\pi}\right\}$$

$$\rightarrow \{0.17, -1.85\}$$

68. $\pi(1+r)^2 = 2 + \pi(1+r)$

$let\ p = 1+r \rightarrow p^2 = (1+r)^2$

$$\left.\begin{aligned} &\pi p^2 = 2 + \pi p \rightarrow \pi p^2 - \pi p - 2 = 0 \\ &p = \frac{\pi \pm \sqrt{\pi^2+8\pi}}{2\pi} \end{aligned}\right\} \rightarrow$$

$$p = \frac{\pi \pm \sqrt{\pi^2+8\pi}}{2\pi} \rightarrow 1 + r = \frac{\pi \pm \sqrt{\pi^2+8\pi}}{2\pi}$$

$$r = -1 + \frac{\pi \pm \sqrt{\pi^2+8\pi}}{2\pi}$$

Check

$r=-1+\dfrac{\pi+\sqrt{\pi^2+8\pi}}{2\pi}$:

$\pi\left(\dfrac{\pi+\sqrt{\pi^2+8\pi}}{2\pi}\right)^2=2+\pi\left(\dfrac{\pi+\sqrt{\pi^2+8\pi}}{2\pi}\right)$

$\pi\left(\dfrac{\pi^2+2\pi\sqrt{\pi^2+8\pi}+\pi^2+8\pi}{4\pi^2}\right)$

$=2+\pi\left(\dfrac{\pi+\sqrt{\pi^2+8\pi}}{2\pi}\right)$

$\dfrac{2\pi^2+2\pi\sqrt{\pi^2+8\pi}+8\pi}{4\pi}=2+\dfrac{\pi+\sqrt{\pi^2+8\pi}}{2}$

$\dfrac{\pi+\sqrt{\pi^2+8\pi}+4}{2}=\dfrac{4+\pi+\sqrt{\pi^2+8\pi}}{2}$

$0=0$

$r=-1+\dfrac{\pi-\sqrt{\pi^2+8\pi}}{2\pi}$:

$\pi\left(\dfrac{\pi-\sqrt{\pi^2+8\pi}}{2\pi}\right)^2=2+\pi\left(\dfrac{\pi-\sqrt{\pi^2+8\pi}}{2\pi}\right)$

$\pi\left(\dfrac{\pi^2-2\pi\sqrt{\pi^2+8\pi}+\pi^2+8\pi}{4\pi^2}\right)=2+\pi\left(\dfrac{\pi-\sqrt{\pi^2+8\pi}}{2\pi}\right)$

$\dfrac{2\pi^2-2\pi\sqrt{\pi^2+8\pi}+8\pi}{4\pi}=2+\dfrac{\pi-\sqrt{\pi^2+8\pi}}{2}$

$\dfrac{\pi-\sqrt{\pi^2+8\pi}+4}{2}=\dfrac{4+\pi-\sqrt{\pi^2+8\pi}}{2}$

$0=0$

The solution set is

$\left\{-1+\dfrac{\pi+\sqrt{\pi^2+8\pi}}{2\pi},-1+\dfrac{\pi-\sqrt{\pi^2+8\pi}}{2\pi}\right\}$

$\to\{0.44,-1.44\}$

69. $k^2-k=12\to k^2-k-12=0$

$(k-4)(k+3)=0$

$k=4$ or $k=-3$

$\to\dfrac{x+3}{x-3}=4$

$\to x+3=4x-12\to x=15$

or

$\dfrac{x+3}{x-3}=-3$

$\to x+3=-3x+9\to x=\dfrac{3}{2}$

and since neither of these x values causes a denominator to equal zero, the solution set is $\left\{\dfrac{3}{2},15\right\}$.

70. $k^2-3k=28\to k^2-3k-28=0$

$(k+4)(k-7)=0$

$k=-4$ or $k=7$

$\to\dfrac{x+3}{x-4}=-4$

$\to x+3=-4x+16\to x=\dfrac{13}{5}$

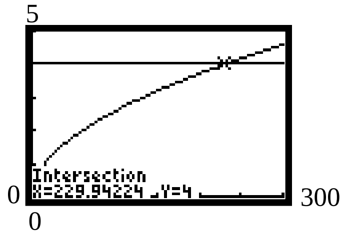
or

$\dfrac{x+3}{x-4}=7$

$\to x+3=7x-28\to x=\dfrac{31}{6}$

and since neither of these x values causes a denominator to equal zero, the solution set is $\left\{\dfrac{13}{5},\dfrac{31}{6}\right\}$.

71. Graph the equations and to find the x-coordinate of the points of intersection:




The distance to the water's surface is approximately 229.94 feet.

72. Answers will vary, one example is $\sqrt{x+1} = -1$

73. Answers will vary, one example is $x - \sqrt{x} - 2 = 0$

74. The step that leads to the possibility of extraneous solutions is the step in which each side of the equation is raised to whatever power is needed to remove the radical. In particular, raising both side of an equation to an even power has the potential of introducing a negative value that does not occur in the original equation.

This possibility does not arise when solving linear or quadratic equations because these equation can be solved by using basic arithmetic and / or factoring, which will not introduce extra negative values.