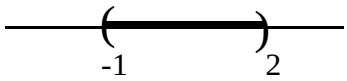
Equations and Inequalities

1.5 Solving Inequalities

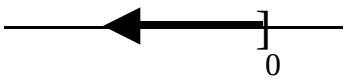
1. $[0, 2]$ $0 \le x \le 2$

2. $[2,\infty)$ $2 \le x < \infty$

3. $(-1,2)$ $-1 < x < 2$



4. $(-\infty,0]$ $-\infty < x \le 0$



5. $[0,3)$ $0 \le x < 3$

6. $(-1,1]$ $-1 < x \le 1$

(d) (a) $6 < 8$
(d) $-2 < 0$
(d) $9 < 15$
(d) $-6 > -10$

(d) (a) $5 > 4$
(d) $-3 > -4$
(d) $6 > 3$
(d) $-4 < -2$

9. (a) $7 > 0$
(b) $-1 > -8$
(c) $12 > -9$
(d) $-8 < 6$

10. (a) $0 > -2$
(b) $-8 > -10$
(c) $-9 > -15$
(d) $6 < 10$

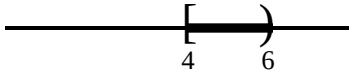
11. (a) $2x + 4 < 5$
(b) $2x - 4 < -3$
(c) $6x + 3 < 6$
(d) $-4x - 2 > -4$

12. (a) $4 - 2x > 8$
(b) $-4 - 2x > 0$
(c) $3 - 6x > 15$
(d) $-2 + 4x < -10$

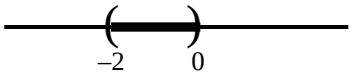
13. $[0, 4]$

14. $(-1, 5)$

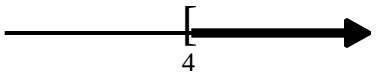
15. $[4, 6)$



16. $(-2, 0)$

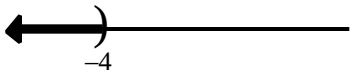


17. $[4, \infty)$

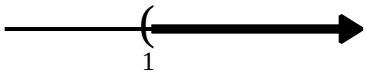


18. $(-\infty, 5]$

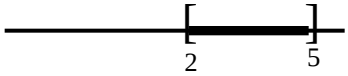
19. $(-\infty, -4)$



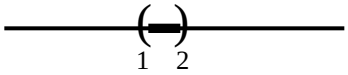
20. $(1, \infty)$



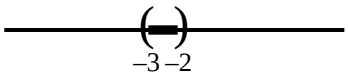
21. $2 \le x \le 5$



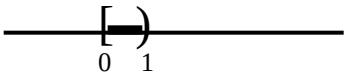
22. $1 < x < 2$



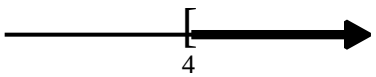
23. $-3 < x < -2$



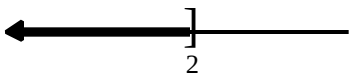
24. $0 \le x < 1$



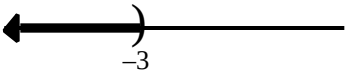
25. $x \ge 4$



26. $x \le 2$



27. $x < -3$



28. $x > -8$

29. If $x < 5$, then $x - 5 < 0$.

30. If $x < -4$, then $x + 4 < 0$.

31. If $x > -4$, then $x + 4 > 0$.

32. If $x > 6$, then $x - 6 > 0$.

33. If $x \ge -4$, then $3x \ge -12$.

34. If $x \le 3$, then $2x \le 6$.

35. If $x > 6$, then $-2x < -12$.

36. If $x > -2$, then $-4x < 8$.

37. If $x \ge 5$, then $-4x \le -20$.

38. If $x \le -4$, then $-3x \ge 12$.

39. If $2x > 6$, then $x > 3$.

40. If $3x \le 12$, then $x \le 4$.

41. If $\frac{-1}{2}x \le 3$, then $x \ge -6$.

42. If $-\frac{1}{4}x > 1$, then $x < -4$.

43.

$$x + 1 < 5$$
$$x + 1 - 1 < 5 - 1$$
$$x < 4$$
$$\{x \mid x < 4\} \text{ or } (-\infty, 4)$$

44.

$$x - 6 < 1$$
$$x - 6 + 6 < 1 + 6$$
$$x < 7$$
$$\{x \mid x < 7\} \text{ or } (-\infty, 7)$$

45.

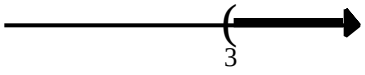
$$1 - 2x \le 3$$
$$-2x \le 2$$
$$x \ge -1$$
$$\{x \mid x \ge -1\} \text{ or } [-1, +\infty)$$

46.

$$2 - 3x \le 5$$
$$-3x \le 3$$
$$x \ge -1$$
$$\{x \mid x \ge -1\} \text{ or } [-1, +\infty)$$

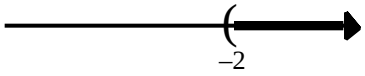
47.

$$3x - 7 > 2$$
$$3x > 9$$
$$x > 3$$
$$\{x \mid x > 3\} \text{ or } (3, +\infty)$$



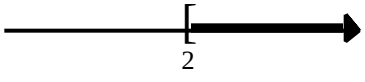
48.

$$2x + 5 > 1$$
$$2x > -4$$
$$x > -2$$
$$\{x \mid x > -2\} \text{ or } (-2, +\infty)$$



49.

$$3x - 1 \ge 3 + x$$
$$2x \ge 4$$
$$x \ge 2$$
$$\{x \mid x \ge 2\} \text{ or } [2, +\infty)$$



50.

$$2x - 2 \ge 3 + x$$
$$x \ge 5$$
$$\{x \mid x \ge 5\} \text{ or } [5, +\infty)$$

51.

$$-2(x + 3) < 8$$
$$-2x - 6 < 8$$
$$-2x < 14$$
$$x > -7$$
$$\{x \mid x > -7\} \text{ or } (-7, +\infty)$$

52.

$$-3(1 - x) < 12$$
$$-3 + 3x < 12$$
$$3x < 15$$
$$x < 5$$
$$\{x \mid x < 5\} \text{ or } (-\infty, 5)$$

53.

$$\begin{aligned} 4-3(1-x) &\le 3 \\ 4-3+3x &\le 3 \\ 3x+1 &\le 3 \\ 3x &\le 2 \\ x &\le \frac{2}{3} \end{aligned}$$

$\left\{x \mid x \le \frac{2}{3}\right\}$ or $\left(-\infty, \frac{2}{3}\right]$

54.

$$\begin{aligned} 8-4(2-x) &\le -2x \\ 8-8+4x &\le -2x \\ 4x &\le -2x \\ 6x &\le 0 \\ x &\le 0 \end{aligned}$$

$\{x \mid x \le 0\}$ or $(-\infty, 0]$

55.

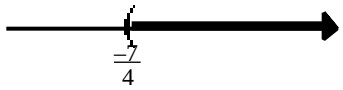
$$\begin{aligned} \tfrac{1}{2}(x-4) &> x+8 \\ \tfrac{1}{2}x-2 &> x+8 \\ \tfrac{-1}{2}x &> 10 \\ x &< -20 \end{aligned}$$

$\{x \mid x < -20\}$ or $(-\infty, -20)$

56.

$$\begin{aligned} 3x+4 &> \tfrac{1}{3}(x-2) \\ 3x+4 &> \tfrac{1}{3}x-\tfrac{2}{3} \\ 9x+12 &> x-2 \\ 8x &> -14 \\ x &> -\frac{7}{4} \end{aligned}$$

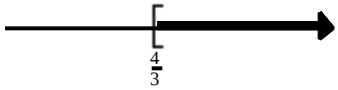
$\left\{x \mid x > -\frac{7}{4}\right\}$ or $\left(-\frac{7}{4}, \infty\right)$



57.

$$\begin{aligned} \frac{x}{2} &\ge 1-\frac{x}{4} \\ 2x &\ge 4-x \\ 3x &\ge 4 \\ x &\ge \frac{4}{3} \end{aligned}$$

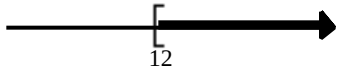
$\left\{x \mid x \ge \frac{4}{3}\right\}$ or $\left[\frac{4}{3}, +\infty\right)$



58.

$$\begin{aligned} \frac{x}{3} &\ge 2+\frac{x}{6} \\ 2x &\ge 12+x \\ x &\ge 12 \end{aligned}$$

$\{x \mid x \ge 12\}$ or $[12, +\infty)$



59.

$$\begin{aligned} 0 &\le 2x-6 \le 4 \\ 6 &\le 2x \le 10 \\ 3 &\le x \le 5 \end{aligned}$$

$\{x \mid 3 \le x \le 5\}$ or $[3, 5]$

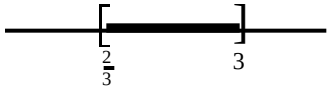
60.

$$\begin{aligned} 4 &\le 2x+2 \le 10 \\ 2 &\le 2x \le 8 \\ 1 &\le x \le 4 \end{aligned}$$

$\{x \mid 1 \le x \le 4\}$ or $[1, 4]$

61.

$$-5 \le 4 - 3x \le 2$$
$$-9 \le -3x \le -2$$
$$3 \ge x \ge \frac{2}{3}$$
$$\left\{x \middle| \frac{2}{3} \le x \le 3\right\} \text{ or } \left[\frac{2}{3}, 3\right]$$



62.

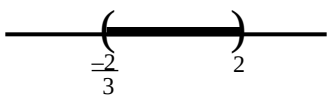
$$-3 \le 3 - 2x \le 9$$
$$-6 \le -2x \le 6$$
$$3 \ge x \ge -3$$
$$\{x \mid -3 \le x \le 3\} \text{ or } [-3, 3]$$

63.

$$-3 < \frac{2x-1}{4} < 0$$
$$-12 < 2x - 1 < 0$$
$$-11 < 2x < 1$$
$$\frac{-11}{2} < x < \frac{1}{2}$$
$$\left\{x \middle| \frac{-11}{2} < x < \frac{1}{2}\right\} \text{ or } \left(\frac{-11}{2}, \frac{1}{2}\right)$$

64.

$$0 < \frac{3x+2}{2} < 4$$
$$0 < 3x + 2 < 8$$
$$-2 < 3x < 6$$
$$-\frac{2}{3} < x < 2$$
$$\left\{x \middle| -\frac{2}{3} < x < 2\right\} \text{ or } \left(-\frac{2}{3}, 2\right)$$



65.

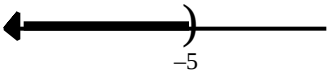
$$1 < 1 - \tfrac{1}{2}x < 4$$
$$0 < -\tfrac{1}{2}x < 3$$
$$0 > x > -6$$
$$\{x \mid -6 < x < 0\} \text{ or } (-6, 0)$$

66.

$$0 < 1 - \tfrac{1}{3}x < 1$$
$$-1 < -\tfrac{1}{3}x < 0$$
$$3 > x > 0$$
$$\{x \mid 0 < x < 3\} \text{ or } (0, 3)$$

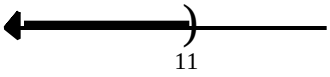
67.

$$(x+2)(x-3) > (x-1)(x+1)$$
$$x^2 - x - 6 > x^2 - 1$$
$$-x - 6 > -1$$
$$-x > 5$$
$$x < -5$$
$$\{x \mid x < -5\} \text{ or } (-\infty, -5)$$



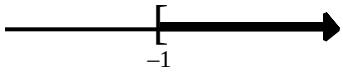
68.

$$(x-1)(x+1) > (x-3)(x+4)$$
$$x^2 - 1 > x^2 + x - 12$$
$$-1 > x - 12$$
$$-x > -11$$
$$x < 11$$
$$\{x \mid x < 11\} \text{ or } (-\infty, 11)$$



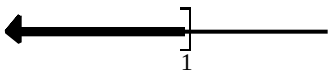
69.

$$x(4x+3) \le (2x+1)^2$$
$$4x^2 + 3x \le 4x^2 + 4x + 1$$
$$3x \le 4x + 1$$
$$-x \le 1$$
$$x \ge -1$$
$$\{x \mid x \ge -1\} \text{ or } [-1, +\infty)$$



70.

$$x(9x-5) \le (3x-1)^2$$
$$9x^2 - 5x \le 9x^2 - 6x + 1$$
$$-5x \le -6x + 1$$
$$x \le 1$$
$$\{x \mid x \le 1\} \text{ or } (-\infty, 1)$$



71.

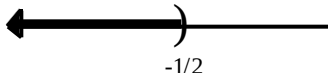
$$\frac{1}{2} \le \frac{x+1}{3} < \frac{3}{4}$$
$$6 \le 4x + 4 < 9$$
$$2 \le 4x < 5$$
$$\frac{1}{2} \le x < \frac{5}{4}$$
$$\left\{x \mid \frac{1}{2} \le x < \frac{5}{4}\right\} \text{ or } \left[\frac{1}{2}, \frac{5}{4}\right)$$

72.

$$\frac{1}{3} < \frac{x+1}{2} \le \frac{2}{3}$$
$$2 < 3x + 3 \le 4$$
$$-1 < 3x \le 1$$
$$-\frac{1}{3} < x \le \frac{1}{3}$$
$$\left\{x \mid -\frac{1}{3} < x \le \frac{1}{3}\right\} \text{ or } \left(-\frac{1}{3}, \frac{1}{3}\right]$$

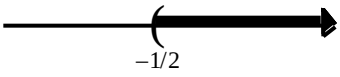
73.

$$(4x+2)^{-1} < 0$$
$$\frac{1}{4x+2} < 0 \rightarrow 4x + 2 < 0$$
$$x < -\frac{1}{2}$$
$$\left\{x \mid x < -\frac{1}{2}\right\} \text{ or } \left(-\infty, -\frac{1}{2}\right)$$



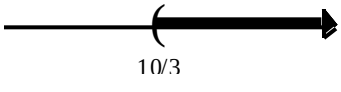
74.

$$(2x+1)^{-1} > 0$$
$$\frac{1}{2x+1} > 0 \rightarrow 2x + 1 > 0$$
$$x > -\frac{1}{2}$$
$$\left\{x \mid x > -\frac{1}{2}\right\} \text{ or } \left(-\frac{1}{2}, +\infty\right)$$



75.

$$0 < \frac{2}{x} < \frac{3}{5} \rightarrow 0 < \frac{2}{x} \quad \text{and} \quad \frac{2}{x} < \frac{3}{5}$$
$$0 < \frac{2}{x} \rightarrow x > 0 \quad \text{therefore} \quad \frac{2}{x} < \frac{3}{5} \rightarrow 10 < 3x \rightarrow \frac{10}{3} < x$$
$$\left\{x \mid \frac{10}{3} < x\right\} \text{ or } \left(\frac{10}{3}, +\infty\right)$$

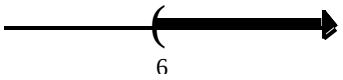


76.

$$0 < \frac{4}{x} < \frac{2}{3} \rightarrow 0 < \frac{4}{x} \quad \text{and} \quad \frac{4}{x} < \frac{2}{3}$$

$$0 < \frac{4}{x} \rightarrow x > 0 \quad \text{therefore} \quad \frac{4}{x} < \frac{2}{3} \rightarrow 12 < 2x \rightarrow 6 < x$$

$\{x \mid 6 < x\}$ or $(6, +\infty)$

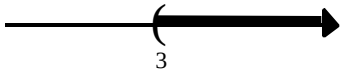


77.

$$0 < (2x-4)^{-1} < \frac{1}{2} \rightarrow 0 < \frac{1}{2x-4} \quad \text{and} \quad \frac{1}{2x-4} < \frac{1}{2}$$

$$0 < \frac{1}{2x-4} \rightarrow 2x - 4 > 0 \quad \text{therefore} \quad \frac{1}{2x-4} < \frac{1}{2} \rightarrow 2 < 2x - 4 \rightarrow 3 < x$$

$\{x \mid 3 < x\}$ or $(3, +\infty)$

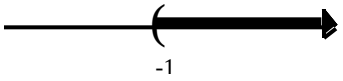


78.

$$0 < (3x+6)^{-1} < \frac{1}{3} \rightarrow 0 < \frac{1}{3x+6} < \frac{1}{3} \rightarrow 0 < \frac{1}{3x+6} \quad \text{and} \quad \frac{1}{3x+6} < \frac{1}{3}$$

$$0 < \frac{1}{3x+6} \rightarrow 3x + 6 > 0 \quad \text{therefore} \quad \frac{1}{3x+6} < \frac{1}{3} \rightarrow 3 < 3x + 6 \rightarrow -1 < x$$

$\{x \mid -1 < x\}$ or $(-1, +\infty)$



79. If $-1 < x < 1$, then $3 < x + 4 < 5 \rightarrow a = 3, b = 5$

80. If $-3 < x < 2$, then $-9 < -3x - 6 < -4 \rightarrow a = -9, b = -4$

81. If $2 < x < 3$, then $-12 < -4x < -8 \rightarrow a = -12, b = -8$

82. If $-4 < x < 0$, then $-2 < \frac{1}{2}x < 0 \rightarrow a = -2, b = 0$

83. If $0 < x < 4$, then $0 < 2x < 8 \rightarrow 3 < 2x + 3 < 11 \rightarrow a = 0, b = 11$

84.
If $-3 < x < 3$, then $6 > -2x > -6 \rightarrow 7 > 1 - 2x > -5$
$\rightarrow -5 < 1 - 2x < 7 \rightarrow a = -5, b = 7$

85.
If $-3 < x < 0$, then $1 < x + 4 < 4$
$\rightarrow 1 > \frac{1}{x+4} > \frac{1}{4} \rightarrow \frac{1}{4} < \frac{1}{x+4} < 1 \rightarrow a = \frac{1}{4}, b = 1$

86.
If $2 < x < 4$, then $-4 < x - 6 < -2$
$\rightarrow -\frac{1}{4} > \frac{1}{x-6} > -\frac{1}{2} \rightarrow -\frac{1}{2} < \frac{1}{x-6} < -\frac{1}{4} \rightarrow a = -\frac{1}{2}, b = -\frac{1}{4}$

87. If $6 < 3x < 12$, then $2 < x < 4 \rightarrow 4 < x^2 < 16 \rightarrow a = 4, b = 16$

88. If $0 < 2x < 6$, then $0 < x < 3 \rightarrow 0 < x^2 < 9 \rightarrow a = 0, b = 9$

89. we need $3x + 6 \geq 0 \rightarrow 3x \geq -6 \rightarrow x \geq -2$, so the domain is $\{x | x \geq -2\}$.

90. we need $8 + 2x \geq 0 \rightarrow 2x \geq -8 \rightarrow x \geq -4$, so the domain is $\{x | x \geq -4\}$

91. $21 <$ young adult's age < 30

92. $40 \leq$ middle-aged < 60

93. (a) An average 25-year-old male can expect to live at least 48.4 more years. 25 + 48.4 = 73.4. Therefore, the average age of a 25-year-old male will be ≥ 73.4.
(b) An average 25-year-old female can expect to live at least 54.7 more years. 25 + 54.7 = 79.7. Therefore, the average age of a 25-year-old female will be ≥ 79.7.
(c) By the given information, a female can expect to live 6.3 years longer.

94. $V = 20T$

$80^\circ \leq T \leq 120^\circ$
$80^\circ \leq \frac{V}{20} \leq 120^\circ$
$1600 \leq V \leq 2400$
The volume ranges from 1600 to 2400 cubic centimeters.

95. Let P represent the selling price and C represent the commission.
Calculating the commission:
$C = 45{,}000 + 0.25(P - 900{,}000) = 45{,}000 + 0.25P - 225{,}000 = 0.25P - 180{,}000$
Calculate the commission range, given the price range:

$$\begin{aligned} 900{,}000 &\le P \le 1{,}100{,}000 \\ 0.25(900{,}000) &\le 0.25P \le 0.25(1{,}100{,}000) \\ 225{,}000 &\le 0.25P \le 275{,}000 \\ 225{,}000 - 180{,}000 &\le 0.25P - 180{,}000 \le 275{,}000 - 180{,}000 \\ 45{,}000 &\le C \le 95{,}000 \end{aligned}$$

The agent's commission ranges from \$45,000 to \$95,000, inclusive.

$\frac{45{,}000}{900{,}000} = 0.05 = 5\%$ to $\frac{95{,}000}{1{,}100{,}000} = 0.086 = 8.6\%$, inclusive.

As a percent of selling price, the commission ranges from 5% to 8.6%.

96. Let C represent the commission.
Calculate the commission range:

$$\begin{aligned} 25 + 0.4(70) &\le C \le 25 + 0.4(300) \\ 53 &\le C \le 145 \end{aligned}$$

The commission varies between \$53 and \$145.

97. Let W represent the weekly wage and T represent the withholding tax.
Calculating the tax:

$$T = 69.90 + 0.28(W - 517) = 69.90 + 0.28W - 144.76 = 0.28W - 74.86$$

Calculating the withholding tax range, given the range of weekly wages:

$$\begin{aligned} 525 &\le W \le 600 \\ 0.28(525) &\le 0.28W \le 0.28(600) \\ 147 &\le 0.28W \le 168 \\ 147 - 74.86 &\le 0.28W - 74.86 \le 168 - 74.86 \\ 72.14 &\le T \le 93.14 \end{aligned}$$

The amount of withholding tax ranges from \$72.14 to \$93.14, inclusive.

98. Let W represent the weekly wage and T represent the withholding tax.
Calculating the tax:

$$T = 69.90 + 0.28(W - 517) = 69.90 + 0.28W - 144.76 = 0.28W - 74.86$$

Calculating the withholding tax range, given the range of weekly wages:

$$\begin{aligned} 600 &\le W \le 700 \\ 0.28(600) &\le 0.28W \le 0.28(700) \\ 168 &\le 0.28W \le 196 \\ 168 - 74.86 &\le 0.28W - 74.86 \le 196 - 74.86 \\ 93.14 &\le T \le 121.14 \end{aligned}$$

The amount of withholding tax ranges from \$93.14 to \$121.14, inclusive.

99. Let K represent the monthly usage in kilowatt-hours.
Let C represent the monthly customer bill.
Calculating the bill:

$$C = 0.10494K + 9.36$$

Calculating the range of kilowatt-hours, given the range of bills:

$$\begin{aligned} 80.24 &\le C \le 271.80 \\ 80.24 &\le 0.10494K + 9.36 \le 271.80 \\ 70.88 &\le 0.10494K \le 262.44 \\ 675.43 &\le K \le 2500.86 \end{aligned}$$

The range of usage in kilowatt-hours varied from 675.43 to 2500.86.

100. Let W represent the amount of water used.
Let C represent the customer charge.
Calculating the charge:
$$C = 21.60 + 1.70(W - 12) = 21.60 + 1.70W - 20.40 = 1.70W + 1.20$$
Calculating the range of water usage, given the range of charges:
$$\begin{aligned} 28.40 &\le C \le 65.75 \\ 28.40 &\le 1.70W + 1.20 \le 65.75 \\ 27.20 &\le 1.70W \le 64.55 \\ 16 &\le W \le 37.97 \end{aligned}$$
The range of water usage varied from 16,000 to 38,000 gallons.

101. Let C represent the dealer's cost and M represent the markup over dealer's cost.
If the price is \$8800, then $8800 = C + MC = C(1 + M)$
Solving for C: $C = \dfrac{8800}{1 + M}$
Calculating the range of dealer costs, given the range of markups:
$$\begin{aligned} 0.12 &\le M \le 0.18 \\ 1.12 &\le 1 + M \le 1.18 \\ \frac{1}{1.12} &\ge \frac{1}{1 + M} \ge \frac{1}{1.18} \\ \frac{8800}{1.12} &\ge \frac{8800}{1 + M} \ge \frac{8800}{1.18} \\ 7857.14 &\ge C \ge 7457.63 \end{aligned}$$
The dealer's cost ranged from \$7457.63 to \$7857.14, inclusive.

102. Let T represent the test scores of the people in the top 2.5%.
$$T > 1.96(12) + 100 = 123.52$$
People in the top 2.5% will have test scores greater than 123.52.

103. Let T represent the score on the last test and G represent the course grade.
Calculating the course grade and solving for the last test:
$$G = \frac{68 + 82 + 87 + 89 + T}{5} = \frac{326 + T}{5}$$
$$T = 5G - 326$$
Calculating the range of scores on the last test, given the grade range:
$$\begin{aligned} 80 &\le G < 90 \\ 400 &\le 5G < 450 \\ 74 &\le 5G - 326 < 124 \\ 74 &\le T < 124 \end{aligned}$$
The fifth test must be greater than or equal to 74.

104. Let T represent the score on the last test and G represent the course grade.
Calculating the course grade and solving for the last test:
$$G = \frac{68 + 82 + 87 + 89 + 2T}{6} = \frac{326 + 2T}{6} = \frac{163 + T}{3}$$
$$T = 3G - 163$$
Calculating the range of scores on the last test, given the grade range:

$$\begin{aligned} 80 \le\ & G & < 90 \\ 240 \le\ & 3G & < 270 \\ 77 \le\ & 3G - 163 & < 107 \\ 77 \le\ & T & < 107 \end{aligned}$$

The fifth test must be greater than or equal to 77 to get a B.

105. Since $a < b$

$$\frac{a}{2} < \frac{b}{2} \qquad \frac{a}{2} < \frac{b}{2}$$
$$\frac{a}{2} + \frac{a}{2} < \frac{a}{2} + \frac{b}{2} \qquad \frac{a}{2} + \frac{b}{2} < \frac{b}{2} + \frac{b}{2} \qquad \text{Thus, } a < \frac{a+b}{2} < b$$
$$a < \frac{a+b}{2} \qquad \frac{a+b}{2} < b$$

106. $\dfrac{a+b}{2} - a = \dfrac{a+b-2a}{2} = \dfrac{b-a}{2}$

$b - \dfrac{a+b}{2} = \dfrac{2b-a-b}{2} = \dfrac{b-a}{2}$

$\dfrac{a+b}{2}$ is equidistant from a and b.

107. If $0 < a < b$, then $0 < a^2 < ab$ and $0 < ab < b^2$

$$ab - a^2 > 0 \qquad b^2 - ab > 0$$
$$ab > a^2 > 0 \qquad b^2 > ab > 0$$
$$\left(\sqrt{ab}\right)^2 > a^2 \qquad b^2 > \left(\sqrt{ab}\right)^2$$
$$\sqrt{ab} > a \qquad b > \sqrt{ab}$$

Thus, $a < \sqrt{ab} < b$

108. Show that $\sqrt{ab} < \dfrac{a+b}{2}$.

$$\frac{a+b}{2} - \sqrt{ab} = \frac{1}{2}\left(a - 2\sqrt{ab} + b\right) = \frac{1}{2}\left(\sqrt{a} - \sqrt{b}\right)^2 > 0 \quad \text{Therefore, } \sqrt{ab} < \frac{a+b}{2}.$$

109. For $0 < a < b$, $\dfrac{1}{h} = \dfrac{1}{2}\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$

$$h\ \frac{1}{h} = \frac{1}{2}\left(\frac{b+a}{ab}\right) h \quad \rightarrow \quad 1 = \frac{1}{2}\left(\frac{b+a}{ab}\right) h \quad \rightarrow \quad \frac{2ab}{a+b} = h$$

$$h - a = \frac{2ab}{a+b} - a \qquad b - h = b - \frac{2ab}{a+b}$$
$$= \frac{2ab - a(a+b)}{a+b} \qquad = \frac{b(a+b) - 2ab}{a+b}$$
$$= \frac{2ab - a^2 - ab}{a+b} \qquad = \frac{ab + b^2 - 2ab}{a+b}$$
$$= \frac{ab - a^2}{a+b} \qquad = \frac{b^2 - ab}{a+b}$$
$$= \frac{a(b-a)}{a+b} > 0 \qquad = \frac{b(b-a)}{a+b} > 0$$

Therefore, $h > a$. Therefore, $h < b$. Thus, $a < h < b$.

110. Show that $h = \dfrac{(\text{geometric mean})^2}{\text{arithmetic mean}} = \dfrac{\left(\sqrt{ab}\right)^2}{\frac{1}{2}(a+b)}$

From Problem 109 we know:

$$\frac{1}{h} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$\frac{2}{h} = \frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab}$$

$$\frac{h}{2} = \frac{ab}{a+b}$$

$$h = 2\ \frac{ab}{a+b} = \frac{\left(\sqrt{ab}\right)^2}{\frac{1}{2}(a+b)}$$

111. Answers will vary.

112. $x^2 + 1$ is a positive number. Therefore it cannot be less than a negative number.