Chapter 1

Equations and Inequalities

1.R Chapter Review

1. $2-\frac{x}{3}=8$

$6-x=24 \rightarrow x=-18$

2. $\frac{x}{4}-2=6$

$x-8=24 \rightarrow x=32$

3. $-2(5-3x)+8=4+5x$

$-10+6x+8=4+5x$

$6x-2=4+5x$

$x=6$

4. $(6-3x)-2(1+x)=6x$

$6-3x-2-2x=6x$

$-5x+4=6x$

$-11x=-4 \rightarrow x=\frac{4}{11}$

5. $\frac{3x}{4}-\frac{x}{3}=\frac{1}{12}$

$9x-4x=1$

$5x=1 \rightarrow x=\frac{1}{5}$

6. $\frac{4-2x}{3}+\frac{1}{6}=2x$

$2(4-2x)+1=12x$

$8-4x+1=12x$

$9=16x \rightarrow x=\frac{9}{16}$

7. $\frac{x}{x-1}=\frac{6}{5}$

$5x=6x-6$

$6=x$

and since $x = 6$ does not cause a denominator to equal zero, the solution set is $\{6\}$.

8. $\frac{4x-5}{3-7x}=2$

$4x-5=6-14x$

$18x=11$

$x=\frac{11}{18}$

and since $x=\frac{11}{18}$ does not cause a denominator to equal zero, the solution set is $\left\{\frac{11}{18}\right\}$.

9. $$\begin{aligned} x(1-x) &= 6 \\ x - x^2 &= 6 \\ 0 &= x^2 - x + 6 \\ 0 &= (x-3)(x+2) \\ x = 3 &\text{ or } x = -2 \end{aligned}$$

10. $$\begin{aligned} x(1+x) &= 6 \\ x + x^2 &= 6 \\ x^2 + x - 6 &= 0 \\ (x+3)(x-2) &= 0 \\ x = -3 &\text{ or } x = 2 \end{aligned}$$

11. $$\begin{aligned} \frac{1}{2}\left(x - \frac{1}{3}\right) &= \frac{3}{4} - \frac{x}{6} \\ \frac{x}{2} - \frac{1}{6} &= \frac{3}{4} - \frac{x}{6} \\ 6x - 2 &= 9 - 2x \\ 8x &= 11 \\ x &= \frac{11}{8} \end{aligned}$$

12. $$\begin{aligned} \frac{1-3x}{4} &= \frac{x+6}{3} + \frac{1}{2} \\ 3(1-3x) &= 4(x+6) + 6 \\ 3 - 9x &= 4x + 24 + 6 \\ -13x &= 27 \\ x &= -\frac{27}{13} \end{aligned}$$

13. $$\begin{aligned} (x-1)(2x+3) &= 3 \\ 2x^2 + x - 3 &= 3 \\ 2x^2 + x - 6 &= 0 \\ (2x-3)(x+2) &= 0 \\ x = \frac{3}{2} &\text{ or } x = -2 \end{aligned}$$

14. $$\begin{aligned} x(2-x) &= 3(x-4) \\ 2x - x^2 &= 3x - 12 \\ x^2 + x - 12 &= 0 \\ (x+4)(x-3) &= 0 \\ x = -4 &\text{ or } x = 3 \end{aligned}$$

15. $$\begin{aligned} 2x + 3 &= 4x^2 \\ 0 &= 4x^2 - 2x - 3 \\ x &= \frac{2 \pm \sqrt{4+48}}{8} = \frac{2 \pm \sqrt{52}}{8} \\ &= \frac{2 \pm 2\sqrt{13}}{8} = \frac{1 \pm \sqrt{13}}{4} \end{aligned}$$

16. $$\begin{aligned} 1 + 6x &= 4x^2 \\ 0 &= 4x^2 - 6x - 1 \\ x &= \frac{6 \pm \sqrt{36+16}}{8} = \frac{6 \pm \sqrt{52}}{8} \\ &= \frac{6 \pm 2\sqrt{13}}{8} = \frac{3 \pm \sqrt{13}}{4} \end{aligned}$$

17. $$\begin{aligned} \sqrt[3]{x^2 - 1} &= 2 \\ \left(\sqrt[3]{x^2-1}\right)^3 &= (2)^3 \\ x^2 - 1 &= 8 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

18. $$\begin{aligned} \sqrt{1 + x^3} &= 3 \\ \left(\sqrt{1+x^3}\right)^2 &= (3)^2 \\ 1 + x^3 &= 9 \\ x^3 &= 8 \\ x &= 2 \end{aligned}$$

Check:

$x = -3$

$\sqrt[3]{(-3)^2 - 1} = 2$

$\sqrt[3]{9-1} = 2$

$\sqrt[3]{8} = 2$

$2 = 2$

$x = 3$

$\sqrt[3]{(3)^2 - 1} = 2$

$\sqrt[3]{9-1} = 2$

$\sqrt[3]{8} = 2$

$2 = 2$

so the solution set is {- 3, 3}.

Check:

$x = 2$

$\sqrt{1+(2)^3} = 3$

$\sqrt{9} = 3$

$3 = 3$

so the solution set is {2}.

19. $x(x+1)+2=0$

$x^2 + x + 2 = 0$

$$x = \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm \sqrt{-7}}{2}$$

no real solutions

20. $3x^2 - x + 1 = 0$

$$x = \frac{1 \pm \sqrt{1-12}}{6} = \frac{1 \pm \sqrt{-11}}{6}$$

no real solutions

21. $x^4 - 5x^2 + 4 = 0$

$(x^2-4)(x^2-1) = 0$

$x^2 - 4 = 0$ or $x^2 - 1 = 0$

$x = \pm 2$ or $x = \pm 1$

22. $3x^4 + 4x^2 + 1 = 0$

$(3x^2+1)(x^2+1) = 0$

$3x^2 + 1 = 0$

$3x^2 = -1$

but this is impossible, so there are no real solutions.

23. $\sqrt{2x-3} + x = 3$

$\sqrt{2x-3} = 3 - x$

$2x - 3 = 9 - 6x + x^2$

$x^2 - 8x + 12 = 0$

$(x-2)(x-6) = 0$

$x = 2$ or $x = 6$

Check 2: $\sqrt{2(2)-3} + 2 = \sqrt{1} + 2 = 3$

Check 6: $\sqrt{2(6)-3} + 6 = \sqrt{9} + 6$

$= 9 \neq 3$

The solution is $x = 2$.

24. $\sqrt{2x-1} = x - 2$

$2x - 1 = x^2 - 4x + 4$

$x^2 - 6x + 5 = 0$

$(x-1)(x-5) = 0$

$x = 1$ or $x = 5$

Check 1: $\sqrt{2(1)-1} = 1 - 2 \rightarrow 1 \neq -1$

Check 5: $\sqrt{2(5)-1} = 5 - 2 \rightarrow 3 = 3$

The solution is $x = 5$.

25. $\sqrt{x+1}+\sqrt{x-1}=\sqrt{2x+1}$

$$\left(\sqrt{x+1}+\sqrt{x-1}\right)^2=\left(\sqrt{2x+1}\right)^2$$
$$x+1+2\sqrt{x+1}\sqrt{x-1}+x-1=2x+1$$
$$2\sqrt{x+1}\sqrt{x-1}=1$$
$$\left(2\sqrt{x+1}\sqrt{x-1}\right)^2=1^2$$
$$4(x+1)(x-1)=1$$
$$4x^2-4=1$$
$$4x^2=5$$
$$x^2=\frac{5}{4}$$
$$x=\frac{\pm\sqrt{5}}{2}$$

Check $\frac{-\sqrt{5}}{2}$:

$$\sqrt{\frac{-\sqrt{5}}{2}+1}+\sqrt{\frac{-\sqrt{5}}{2}-1}=\sqrt{2\left(\frac{-\sqrt{5}}{2}\right)+1}$$
$$\sqrt{-0.118}+\sqrt{-2.118}=\sqrt{-1.236}$$

This is not defined.

Check $\frac{\sqrt{5}}{2}$:

$$\sqrt{\frac{\sqrt{5}}{2}+1}+\sqrt{\frac{\sqrt{5}}{2}-1}=\sqrt{2\left(\frac{\sqrt{5}}{2}\right)+1}$$
$$\sqrt{2.118}+\sqrt{0.118}=\sqrt{3.236}$$
$$1.455+0.344=1.799$$
$$1.799=1.799$$

The solution is $x=\frac{\sqrt{5}}{2}$.

26. $\sqrt{2x-1}-\sqrt{x-5}=3$

$$\sqrt{2x-1}=\sqrt{x-5}+3$$
$$\left(\sqrt{2x-1}\right)^2=\left(\sqrt{x-5}+3\right)^2$$
$$2x-1=x-5+6\sqrt{x-5}+9$$
$$x-5=6\sqrt{x-5}$$
$$(x-5)^2=\left(6\sqrt{x-5}\right)^2$$
$$x^2-10x+25=36(x-5)$$
$$x^2-10x+25=36x-180$$
$$x^2-46x+205=0$$
$$(x-41)(x-5)=0$$
$$x=41 \text{ or } x=5$$

Check 41:

$$\sqrt{2(41)-1}-\sqrt{41-5}=\sqrt{81}-\sqrt{36}=9-6=3=3$$

Check 5:

$$\sqrt{2(5)-1}-\sqrt{5-5}=\sqrt{9}-\sqrt{0}=3-0=3=3$$

The solutions are $x=5$ or $x=41$.

27.

$$\sqrt{x+1}+\sqrt{x-1}=\sqrt{2x+1}$$

$$\left(\sqrt{x+1}+\sqrt{x-1}\right)^2=\left(\sqrt{2x+1}\right)^2$$
$$x+1+2\sqrt{x+1}\sqrt{x-1}+x-1=2x+1$$
$$2x+2\sqrt{x+1}\sqrt{x-1}=2x+1$$
$$2\sqrt{x+1}\sqrt{x-1}=1$$
$$\left(2\sqrt{x+1}\sqrt{x-1}\right)^2=(1)^2$$
$$4(x+1)(x-1)=1$$
$$4x^2-4=1$$
$$4x^2=5 \rightarrow x^2=\frac{5}{4}$$
$$x=\pm\frac{\sqrt{5}}{2}$$

Check:

$$x=\frac{\sqrt{5}}{2} \rightarrow \sqrt{\frac{\sqrt{5}}{2}+1}+\sqrt{\frac{\sqrt{5}}{2}-1}=\sqrt{2\left(\frac{\sqrt{5}}{2}\right)+1}$$
$$1.79890743995=1.79890743995$$

$$x=-\frac{\sqrt{5}}{2} \rightarrow \sqrt{-\frac{\sqrt{5}}{2}+1}+\sqrt{-\frac{\sqrt{5}}{2}-1}=\sqrt{2\left(-\frac{\sqrt{5}}{2}\right)+1}$$

impossible since $-\frac{\sqrt{5}}{2}-1<0$

The solution set is $\left\{\frac{\sqrt{5}}{2}\right\}$.

28.

$$\sqrt{2x-1}-\sqrt{x-5}=3$$

$$\sqrt{2x-1}=3+\sqrt{x-5}$$
$$\left(\sqrt{2x-1}\right)^2=\left(3+\sqrt{x-5}\right)^2$$
$$2x-1=9+6\sqrt{x-5}+x-5$$
$$x-5=6\sqrt{x-5}$$
$$(x-5)^2=\left(6\sqrt{x-5}\right)^2$$
$$x^2-10x+25=36(x-5)$$
$$x^2-10x+25=36x-180$$
$$x^2-46x+205=0$$
$$(x-41)(x-5)=0$$
$$x=41 \text{ or } x=5$$

Check:

$$x=41 \rightarrow \sqrt{2(41)-1}-\sqrt{41-5}=3$$
$$\sqrt{81}-\sqrt{36}=3$$
$$9-6=3$$
$$3=3$$

$$x=5 \rightarrow \sqrt{2(5)-1}-\sqrt{5-5}=3-$$
$$\sqrt{9}-\sqrt{0}=3$$
$$3-0=3$$
$$3=3$$

The solution set is $\{5,41\}$

29. $2\sqrt[3]{x^2} - \sqrt[3]{x} = 1$

$$2\sqrt[3]{x^2} - \sqrt[3]{x} - 1 = 0 \rightarrow 2x^{2/3} - x^{1/3} - 1 = 0$$

$$p = x^{1/3} \rightarrow p^2 = x^{2/3}$$

$$2p^2 - p - 1 = 0 \rightarrow (2p+1)(p-1) = 0$$

$$p = -\frac{1}{2} \quad \text{or} \quad p = 1$$

$$p = -\frac{1}{2} \rightarrow x^{1/3} = -\frac{1}{2}$$

$$\rightarrow \left(x^{1/3}\right)^3 = \left(-\frac{1}{2}\right)^3 \rightarrow x = -\frac{1}{8}$$

$$p = 1 \rightarrow x^{1/3} = 1$$

$$\rightarrow \left(x^{1/3}\right)^3 = (1)^3 \rightarrow x = 1$$

Check

$$x = -\frac{1}{8}: 2\sqrt[3]{\left(-\frac{1}{8}\right)^2} - \sqrt[3]{-\frac{1}{8}} - 1 = 0$$

$$2\left(\frac{1}{4}\right) - \left(-\frac{1}{2}\right) - 1 = 0 \rightarrow \frac{1}{2} + \frac{1}{2} - 1 = 0 \rightarrow 0 = 0$$

$$x = 1: 2\sqrt[3]{1^2} - \sqrt[3]{1} - 1 = 0$$

$$2 - 1 - 1 = 0 \rightarrow 2 - 2 = 0 \rightarrow 0 = 0$$

the solution set is $\left\{-\frac{1}{8}, 1\right\}$

30. $4\sqrt[3]{x^2} = 1$

$$\left(4x^2\right)^3 = (1)^3$$

$$64x^2 = 1$$

$$64x^2 - 1 = 0$$

$$(8x+1)(8x-1) = 0$$

$$x = -\frac{1}{8} \quad \text{or} \quad x = \frac{1}{8}$$

Check

$$x = -\frac{1}{8}: 4\sqrt[3]{\left(-\frac{1}{8}\right)^2} = 1 \qquad x = \frac{1}{8}: 4\sqrt[3]{\left(\frac{1}{8}\right)^2} = 1$$

$$4\left(\frac{1}{4}\right) = 1 \qquad 4\left(\frac{1}{4}\right) = 1$$

$$1 = 1 \qquad 1 = 1$$

the solution set is $\left\{-\frac{1}{8}, \frac{1}{8}\right\}$

31. $x^{-6} - 7x^{-3} - 8 = 0$

$$p = x^{-3} \rightarrow p^2 = x^{-6}$$

$$p^2 - 7p - 8 = 0 \rightarrow (p-8)(p+1) = 0$$

$$p = 8 \quad \text{or} \quad p = -1$$

$$p = 8 \rightarrow x^{-3} = 8 \rightarrow \left(x^{-3}\right)^{-1/3} = (8)^{-1/3} \rightarrow x = \frac{1}{2}$$

$$p = -1 \rightarrow x^{-3} = -1 \rightarrow \left(x^{-3}\right)^{-1/3} = (-1)^{-1/3} \rightarrow x = -1$$

Check:

$x=\frac{1}{2}:\left(\frac{1}{2}\right)^{-6}-7(p)^{-3}-8=0\rightarrow 64-56-8=0\rightarrow 0=0$ the solution set is $\left\{\frac{1}{2},-1\right\}$

$x=-1:(-1)^{-6}-7(-1)^{-3}-8=0\rightarrow 1+7-8=0\rightarrow 0=0$

32. $6x^{-1}-5x^{-1/2}+1=0$

$p=x^{-1/2}\rightarrow p^2=x^{-1}$

$6p^2-5p+1=0\rightarrow(3p-1)(2p-1)=0$

$p=\frac{1}{3}$ or $p=\frac{1}{2}$

$p=\frac{1}{3}\rightarrow x^{-1/2}=\frac{1}{3}\rightarrow\left(x^{-1/2}\right)^{-2}=\left(\frac{1}{3}\right)^{-2}\rightarrow x=9$

$p=\frac{1}{2}\rightarrow x^{-1/2}=\frac{1}{2}\rightarrow\left(x^{-1/2}\right)^{-2}=\left(\frac{1}{2}\right)^{-2}\rightarrow x=4$

Check:

$x=9:(9)^{-1}-5(9)^{-1/2}+1=0\rightarrow\frac{1}{9}-5\left(\frac{1}{3}\right)+1=0$

$\frac{1}{9}-\frac{5}{3}+1=0\rightarrow 1-15+9=0\rightarrow 0=0$ the solution set is $\{4,9\}$

$x=4:(4)^{-1}-5(4)^{-1/2}+1=0\rightarrow\frac{1}{4}-5\left(\frac{1}{2}\right)+1=0$

$\frac{1}{4}-\frac{5}{2}+1=0\rightarrow 1-10+9=0\rightarrow 0=0$

33. $x^2+m^2=2mx+(nx)^2$

$x^2+m^2=2mx+n^2x^2\rightarrow x^2-n^2x^2-2mx+m^2=0$

$\left(1-n^2\right)x^2-2mx+m^2=0$

$$x=\frac{2m\pm\sqrt{4m^2-4m^2\left(1-n^2\right)}}{2\left(1-n^2\right)}=\frac{2m\pm\sqrt{4m^2\left(1-\left(1-n^2\right)\right)}}{2\left(1-n^2\right)}$$

$$=\frac{2m\pm 2m\sqrt{1-\left(1-n^2\right)}}{2\left(1-n^2\right)}=\frac{m\pm m\sqrt{n^2}}{1-n^2}=\frac{m\pm mn}{1-n^2}=\frac{m(1\pm n)}{1-n^2}$$

$$x=\frac{m(1+n)}{1-n^2}=\frac{m(1+n)}{(1+n)(1-n)}=\frac{m}{1-n}$$

or the solution set is $\left\{\frac{m}{1-n},\frac{m}{1+n}\right\}$.

$$x=\frac{m(1-n)}{1-n^2}=\frac{m(1-n)}{(1+n)(1-n)}=\frac{m}{1+n}$$

34. $b^2x^2 + 2ax = x^2 + a^2$

$b^2x^2 + 2ax - x^2 - a^2 = 0 \rightarrow b^2x^2 - x^2 + 2ax - a^2 = 0$

$\left(b^2 - 1\right)x^2 + 2ax - a^2 = 0$

$$x = \frac{-2a \pm \sqrt{4a^2 - 4\left(b^2-1\right)\left(-a^2\right)}}{2\left(b^2-1\right)} = \frac{-2a \pm \sqrt{4a^2 + 4\left(b^2-1\right)\left(a^2\right)}}{2\left(b^2-1\right)}$$

$$= \frac{-2a \pm \sqrt{4a^2\left(1 + b^2 - 1\right)}}{2\left(b^2-1\right)} = \frac{-2a \pm \sqrt{4a^2\left(b^2\right)}}{2\left(b^2-1\right)} = \frac{-2a \pm 2ab}{2\left(b^2-1\right)} = \frac{a\left(-1 \pm b\right)}{b^2 - 1}$$

$$x = \frac{a\left(-1+b\right)}{b^2-1} = \frac{a\left(b-1\right)}{\left(b+1\right)\left(b-1\right)} = \frac{a}{b+1}$$

or

$$x = \frac{a\left(-1-b\right)}{b^2-1} = \frac{-a\left(b+1\right)}{\left(b+1\right)\left(b-1\right)} = \frac{-a}{b-1}$$

the solution set is $\left\{\frac{a}{b+1}, \frac{-a}{b-1}\right\}$.

35. $10a^2x^2 - 2abx - 36b^2 = 0$

$5a^2x^2 - abx - 18b^2 = 0$

$\left(5ax + 9b\right)\left(ax - 2b\right) = 0$

$x = -\frac{9b}{5a}$ or $x = \frac{2b}{a}$

the solution set is $\left\{-\frac{9b}{5a}, \frac{2b}{a}\right\}$.

36. $\frac{1}{x-m} + \frac{1}{x-n} = \frac{2}{x}$

$x\left(x-m\right)\left(x-n\right)\left(\frac{1}{x-m} + \frac{1}{x-n}\right) = \left(\frac{2}{x}\right)x\left(x-m\right)\left(x-n\right)$

$x\left(x-n\right) + x\left(x-m\right) = 2\left(x-m\right)\left(x-n\right)$

$x^2 - xn + x^2 - xm = 2x^2 - 2xn - 2xm + 2mn$

$x^2 - xn + x^2 - xm - 2x^2 + 2xn + 2xm - 2mn = 0$

$xn + xm - 2mn = 0$

$xn + xm = 2mn$

$x\left(n + m\right) = 2mn$

$x = \frac{2mn}{n+m}$

the solution set is $\left\{\frac{2mn}{n+m}\right\}, n \neq -m, x \neq m, x \neq n, x \neq 0$

37. $\sqrt{x^2+3x+7}-\sqrt{x^2-3x+9}+2=0$

$$\sqrt{x^2+3x+7}=\sqrt{x^2-3x+9}-2$$
$$\left(\sqrt{x^2+3x+7}\right)^2=\left(\sqrt{x^2-3x+9}-2\right)^2$$
$$x^2+3x+7=x^2-3x+9-4\sqrt{x^2-3x+9}+4$$
$$6x-6=-4\sqrt{x^2-3x+9}$$

$$\left(6(x-1)\right)^2=\left(-4\sqrt{x^2-3x+9}\right)^2$$
$$36\left(x^2-2x+1\right)=16\left(x^2-3x+9\right)$$
$$36x^2-72x+36=16x^2-48x+144$$
$$20x^2-24x-108=0 \rightarrow 5x^2-6x-27=0$$
$$(5x+9)(x-3)=0$$
$$x=-\frac{9}{5} \quad \text{or} \quad x=3$$

Check $x=-\frac{9}{5}$:
$$\sqrt{\left(-\frac{9}{5}\right)^2+3\left(-\frac{9}{5}\right)+7}-\sqrt{\left(-\frac{9}{5}\right)^2-3\left(-\frac{9}{5}\right)+9}+2=0$$
$$\sqrt{\frac{81}{25}-\frac{27}{5}+7}-\sqrt{\frac{81}{25}+\frac{27}{5}+9}+2=0$$
$$\sqrt{\frac{81-135+175}{25}}-\sqrt{\frac{81+135+225}{25}}+2=0$$
$$\sqrt{\frac{121}{25}}-\sqrt{\frac{441}{25}}+2=0$$
$$\frac{11}{5}-\frac{21}{5}+2=0$$
$$0=0$$

Check:
$$x=3:\sqrt{(3)^2+3(3)+7}-\sqrt{(3)^2-3(3)+9}+2=0$$
$$\sqrt{9+9+7}-\sqrt{9-9+9}+2=0$$
$$\sqrt{25}-\sqrt{9}+2=0$$
$$2+2=0$$
$$4\neq 0$$

the solution set is $\left\{-\frac{9}{5}\right\}$.

38. $\sqrt{x^2+3x+7}-\sqrt{x^2+3x+9}=2$

$$\sqrt{x^2+3x+7}=\sqrt{x^2+3x+9}+2$$
$$\left(\sqrt{x^2+3x+7}\right)^2=\left(\sqrt{x^2+3x+9}+2\right)^2$$
$$x^2+3x+7=x^2+3x+9+4\sqrt{x^2+3x+9}+4$$
$$-6=4\sqrt{x^2+3x+9}$$

but this is impossible since the principal square root always yields a non-negative number. Therefore, there is no real solution.

39. $|2x+3|=7$

$2x+3=7$ or $2x+3=-7$

$2x=4$ or $2x=-10$

$x=2$ or $x=-5$

The solution set is $\{-5, 2\}$.

40. $|3x-1|=5$

$3x-1=5$ or $3x-1=-5$

$3x=6$ or $3x=-4$

$x=2$ or $x=-\frac{4}{3}$

The solution set is $\left\{-\frac{4}{3}, 2\right\}$.

41. $|2-3x|+2=9 \rightarrow |2-3x|=7$

$2-3x=7$ or $2-3x=-7$

$3x=-5$ or $3x=9$

$x=-\frac{5}{2}$ or $x=3$

The solution set is $\left\{-\frac{5}{2}, 3\right\}$

42. $|1-2x|+1=4 \rightarrow |1-2x|=3$

$1-2x=3$ or $1-2x=-3$

$2x=-2$ or $2x=4$

$x=-1$ or $x=2$

The solution set is $\{-1, 2\}$.

43. $\frac{2x-3}{5}+2\le\frac{x}{2}$

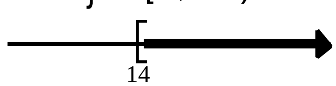
$2(2x-3)+10(2)\le 5x$

$4x-6+20\le 5x$

$14\le x$

$x\ge 14$

$\{x \mid x\ge 14\}$ or $[14, +\infty)$



44. $\frac{5-x}{3}\le 6x-4$

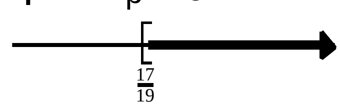
$5-x\le 3(6x-4)$

$5-x\le 18x-12$

$-19x\le -17$

$x\ge\frac{17}{19}$

$\left\{x \mid x\ge\frac{17}{19}\right\}$ or $\left[\frac{17}{19}, +\infty\right)$



45. $-9 \le \dfrac{2x+3}{-4} \le 7$

$$36 \ge 2x+3 \ge -28$$
$$33 \ge 2x \ge -31$$
$$\frac{33}{2} \ge x \ge \frac{-31}{2}$$
$$\frac{-31}{2} \le x \le \frac{33}{2}$$
$$\left\{x \,\middle|\, \frac{-31}{2} \le x \le \frac{33}{2}\right\} \text{ or } \left[\frac{-31}{2}, \frac{33}{2}\right]$$

46. $-4 < \dfrac{2x-2}{3} < 6$

$$-12 < 2x-2 < 18$$
$$-10 < 2x < 20$$
$$-5 < x < 10$$
$$\{x \mid -5 < x < 10\} \text{ or } (-5,\ 10)$$

47. $6 > \dfrac{3-3x}{12} > 2$

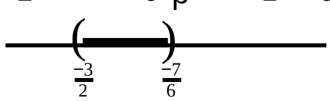
$$72 > 3-3x > 24$$
$$69 > -3x > 21$$
$$-23 < x < -7$$
$$\{x \mid -23 < x < -7\} \text{ or } (-23,\ -7)$$

48. $6 > \dfrac{5-3x}{2} \ge -3$

$$12 > 5-3x \ge -6$$
$$7 > -3x \ge -11$$
$$-\frac{7}{3} < x \le \frac{11}{3}$$
$$\left\{x \,\middle|\, -\frac{7}{3} < x \le \frac{11}{3}\right\} \text{ or } \left(-\frac{7}{3}, \frac{11}{3}\right]$$

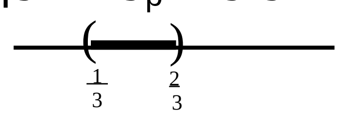
49. $|3x+4| < \frac{1}{2}$

$$\frac{-1}{2} < 3x+4 < \frac{1}{2}$$
$$\frac{-9}{2} < 3x < \frac{-7}{2}$$
$$\frac{-3}{2} < x < \frac{-7}{6}$$
$$\left\{x \,\middle|\, \frac{-3}{2} < x < \frac{-7}{6}\right\} \text{ or } \left(\frac{-3}{2}, \frac{-7}{6}\right)$$



50. $|1-2x| < \frac{1}{3}$

$$-\frac{1}{3} < 1-2x < \frac{1}{3}$$
$$-\frac{4}{3} < -2x < -\frac{2}{3}$$
$$\frac{2}{3} > x > \frac{1}{3}$$
$$\left\{x \,\middle|\, \frac{1}{3} < x < \frac{2}{3}\right\} \text{ or } \left(\frac{1}{3}, \frac{2}{3}\right)$$



51. $|2x-5| \ge 9$

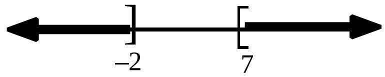
$2x-5 \le -9$ or $2x-5 \ge 9$

$2x \le -4$ or $2x \ge 14$

$x \le -2$ or $x \ge 7$

$\{x \mid x \le -2 \text{ or } x \ge 7\}$

or $(-\infty,-2] \quad [7,+\infty)$



52. $|3x+1| \ge 10$

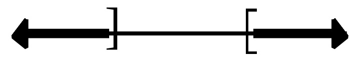
$3x+1 \le -10$ or $3x+1 \ge 10$

$3x \le -11$ or $3x \ge 9$

$x \le -\frac{11}{3}$ or $x \ge 3$

$\left\{x \,\middle|\, x \le -\frac{11}{3} \text{ or } x \ge 3\right\}$

or $\left(-\infty, -\frac{11}{3}\right] \quad [3, +\infty)$



53. $2+|2-3x| \le 4$

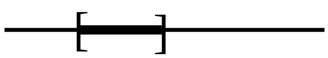
$|2-3x| \le 2$

$-2 \le 2-3x \le 2$

$-4 \le -3x \le 0 \rightarrow \frac{4}{3} \ge x \ge 0$

$0 \le x \le \frac{4}{3}$

$\left\{x \,\middle|\, 0 \le x \le \frac{4}{3}\right\}$ or $\left[0, \frac{4}{3}\right]$



54. $\frac{1}{2} + \left|\frac{2x-1}{3}\right| \le 1$

$\left|\frac{2x-1}{3}\right| \le \frac{1}{2}$

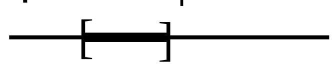
$-\frac{1}{2} \le \frac{2x-1}{3} \le \frac{1}{2}$

$-\frac{3}{2} \le 2x-1 \le \frac{3}{2}$

$-\frac{1}{2} \le 2x \le \frac{5}{2}$

$-\frac{1}{4} \le x \le \frac{5}{4}$

$\left\{x \,\middle|\, -\frac{1}{4} \le x \le \frac{5}{4}\right\}$ or $\left[-\frac{1}{4}, \frac{5}{4}\right]$



55. $1-|2-3x|<-4$

$-|2-3x|<-5 \rightarrow |2-3x|>5$

$2-3x<-5$ or $2-3x>5$

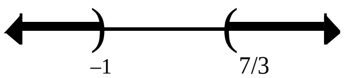
$7<3x$ or $-3>3x$

$\frac{7}{3}<x$ or $-1>x$

$\rightarrow x<-1$ or $x>\frac{7}{3}$

$\left\{x \,\middle|\, x<-1 \text{ or } x>\frac{7}{3}\right\}$

or $(-\infty,-1)\cup\left(\frac{7}{3},+\infty\right)$



56. $1-\left|\frac{2x-1}{3}\right|<-2$

$-\left|\frac{2x-1}{3}\right|<-3 \rightarrow \left|\frac{2x-1}{3}\right|>3$

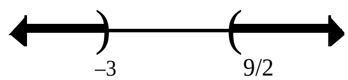
$\frac{2x-1}{3}<-3$ or $\frac{2x-1}{3}>3$

$2x<-6$ or $2x>9$

$x<-3$ or $x>\frac{9}{2}$

$\left\{x \,\middle|\, x<-3 \text{ or } x>\frac{9}{2}\right\}$

or $(-\infty,-3)\cup\left(\frac{9}{2},+\infty\right)$



57. $\left(\frac{6}{2}\right)^2=9$

58. $\left(\frac{-10}{2}\right)^2=25$

59. $\left(\frac{\left(-\frac{4}{3}\right)}{2}\right)^2=\frac{4}{9}$

60. $\left(\frac{\left(\frac{4}{5}\right)}{2}\right)^2=\frac{4}{25}$

61. Using $s=vt$, we have $t=3$ and $v=1100$. Finding the distance s in feet:

$s=1100(3)$

$s=3300$

The storm is 3300 feet away.

62. $1600 \le I \le 3600$

$1600 \le \frac{900}{x^2} \le 3600$

$\frac{1}{1600} \ge \frac{x^2}{900} \ge \frac{1}{3600}$

$\frac{9}{16} \ge x^2 \ge \frac{1}{4}$

$\frac{3}{4} \ge x \ge \frac{1}{2}$

The range of distances is from 0.5 meters to 0.75 meters, inclusive.

63. Let s represent the distance the plane can travel.

	Rate	Time	Distance
With wind	250+30=280	$\dfrac{\left(\frac{s}{2}\right)}{280}$	$\frac{s}{2}$
Against wind	250–30=220	$\dfrac{\left(\frac{s}{2}\right)}{220}$	$\frac{s}{2}$

Since the total time is at most 5 hours, we have:

$$\frac{\left(\frac{s}{2}\right)}{280}+\frac{\left(\frac{s}{2}\right)}{220}\le 5$$

$$\frac{s}{560}+\frac{s}{440}\le 5$$

$$11s+14s\le 5(6160)$$

$$25s\le 30800$$

$$s\le 1232$$

The plane can travel at most 1232 miles or 616 miles one way and return 616 miles.

64. Let s represent the distance the plane can travel.

	Rate	Time	Distance
With wind	250+30=280	$\dfrac{\left(\frac{s}{2}\right)}{280}$	$\frac{s}{2}$
Against wind	250–30=220	$\dfrac{\left(\frac{s}{2}\right)}{220}$	$\frac{s}{2}$

Since the total time is at most 7 hours, we have:

$$\frac{\left(\frac{s}{2}\right)}{280}+\frac{\left(\frac{s}{2}\right)}{220}\le 7$$

$$\frac{s}{560}+\frac{s}{440}\le 7$$

$$11s+14s\le 7(6160)$$

$$25s\le 43120$$

$$s\le 1724.8$$

The plane can travel at most 1724.8 miles or 862.4 miles one way. This is 246.4 miles further than in Problem 63.

65. Let t represent the time it takes the helicopter to reach the raft.

	Rate	Time	Distance
Raft	5	t	$5t$
Helicopter	90	t	$90t$

Since the total distance is 150 miles, we have:

$$5t+90t=150 \rightarrow 95t=150 \rightarrow t=1.58 \text{ hours } = 1 \text{ hour and } 35 \text{ minute}$$

The helicopter will reach the raft in 1 hour and 35 minutes.

66. Let d represent the distance flown by the bee traveling at 3 meters per second.

$$\frac{d}{3}=\frac{150-d}{5} \quad \text{(Times needed to meet are equal.)}$$

$$5d=450-3d \rightarrow 8d=450 \rightarrow d=56.25\text{meters}$$

$$t=\frac{56.25}{3}=18.75\text{seconds}$$

The bees meet for the first time after 18.75 seconds.
The bees will meet a second time on the second lap. The first bee will have traveled 150 + x meters and the second bee will have traveled 150 + (150 – x) meters.
Solving for time, we have:

$$\frac{150+x}{3}=\frac{150+(150-x)}{5} \rightarrow \frac{150+x}{3}=\frac{300-x}{5}$$

$$750+5x=900-3x \rightarrow 8x=150$$

$$x=18.75 \text{ meters into the second lap}$$

$$t=\frac{168.75}{3}=56.25 \text{ seconds}$$

The bees meet the second time after 56.25 seconds.

67. Let t represent the time it takes Clarissa to complete the job by herself.

	Time to do job alone	Part of job done in one day	Time on Job	Part of total job done by each person
Clarissa	t	$\frac{1}{t}$	6	$\frac{6}{t}$
Shawna	$t+5$	$\frac{1}{t+5}$	6	$\frac{6}{t+5}$

Since the two people paint one house, we have:

$$\frac{6}{t}+\frac{6}{t+5}=1 \rightarrow 6(t+5)+6t=t(t+5) \rightarrow 6t+30+6t=t^2+5t$$

$$t^2-7t-30=0 \rightarrow (t-10)(t+3)=0$$

$$t=10 \text{ or } t=-3$$

It takes Clarissa 10 days to paint the house when working by herself.

68. Let t represent the time it takes the smaller pump to empty the tank.

	Time to do job alone	Part of job done in one hour	Time on Job	Part of total job done by each pump
Small Pump	t	$\frac{1}{t}$	5	$\frac{5}{t}$
Large Pump	$t-4$	$\frac{1}{t-4}$	5	$\frac{5}{t-4}$

Since the two pumps empty one tank, we have:

$$\frac{5}{t}+\frac{5}{t-4}=1$$

$$5(t-4)+5t=t(t-4) \rightarrow 5t-20+5t=t^2-4t \rightarrow t^2-14t+20=0$$

$$t=\frac{-(-14)\pm\sqrt{(-14)^2-4(1)(20)}}{2(1)}=\frac{14\pm\sqrt{196-80}}{2}=\frac{14\pm\sqrt{116}}{2}$$

$$=\frac{14\pm2\sqrt{29}}{2}=7\pm\sqrt{29}\approx 7\pm5.385$$

$t=12.385$ or $t=1.615$(extraneous)

It takes the small pump approximately 12.385 hours to empty the tank.

69.

% acid	amount	amount of acid
40%	60	$(0.40)(60)$
15%	x	$(0.15)(x)$
25%	$60+x$	$(0.25)(60+x)$

$$(0.40)(60)+(0.15)(x)=(0.25)(60+x)$$

$$24+.15x=15+.25x \rightarrow 9=0.1x \rightarrow x=90$$

90 cubic centimeters of the 15% solution must be added, producing 150 cubic centimeters of the 25% solution.

70.

Amount of coffee (lbs)	Price (dollars)	Total money
20	4	$(20)(4)$
x	8	$(8)(x)$
$20+x$	5	$(5)(20+x)$

$$80+8x=(5)(20+x)$$

$$80+8x=100+5x$$

$$3x=20$$

Add $6\frac{2}{3}$ pounds of \$8/lb coffee to get $26\frac{2}{3}$ pounds of \$5/lb coffee.

$$x=\frac{20}{3}=6\tfrac{2}{3}$$

71.

% salt	amount	amount of salt
10%	64	$(0.10)(64)$
0%	x	$(0.00)(x)$
2%	$64+x$	$(0.02)(64+x)$

$$(0.10)(64)+(0.00)(x)=(0.02)(64+x)$$
$$6.4=1.28+.02x$$
$$5.12=0.2x$$
$$x=256$$

256 ounces of water must be added.

72.

% salt	amount	amount of salt
2%	64	$(0.02)(64)$
0%	x	$(0.00)(x)$
10%	$64-x$	$(0.10)(64-x)$

$$(0.02)(64)-(0.00)(x)=(0.10)(64-x)$$
$$1.28=6.4-.10x$$
$$.10x=5.12$$
$$x=51.2$$

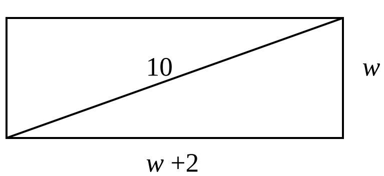
51.2 ounces of water must be evaporated

73. length of $\text{leg}_1=x$, length of $\text{leg}_2=17-x$, by the Pythagorean Theorem we have

$$x^2+(17-x)^2=(13)^2$$
$$x^2+x^2-34x+289=169$$
$$2x^2-34x+120=0 \rightarrow x^2-17x+60=0 \rightarrow (x-12)(x-5)=0$$
$$x=12 \quad \text{or} \quad x=5$$

the legs are 5 inches and 12 inches long.

74.



by the Pythagorean Theorem we have

$$w^2+(w+2)^2=(10)^2$$
$$w^2+w^2+4w+4=100$$
$$2w^2+4w-96=0 \rightarrow w^2+2w-48=0$$
$$(w+8)(w-6)=0$$
$$w=-8 \quad \text{or} \quad w=6$$

the width is 6 inches and the length is 8 inches.

75. The effective speed of the train (i.e., relative to the man) is 30 – 4 = 26 miles per hour. The time is 5 seconds $= \frac{5}{60}$ minutes $= \frac{5}{3600}$ hours $= \frac{1}{720}$ hours.

$$s = vt = 26\left(\frac{1}{720}\right) = \frac{26}{720}\text{ miles } = \frac{26}{720}\ 5280 = 190.67\text{ feet}$$

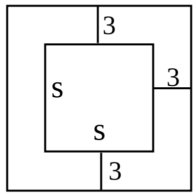
The freight train is 190.67 feet long.

76. (a) $4(s+6)=50$

$4s+24=50$

$4s=26$

$s=6.5$



The painting is 6.5 inches by 6.5 inches.

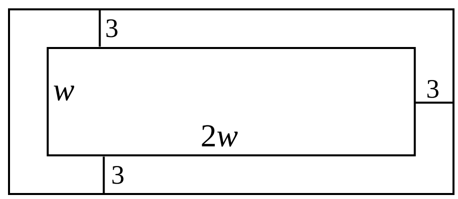
$s+6=12.5$ So the frame is 12.5 inches by 12.5 inches.

(b) $2(2w+6)+2(w+6)=50$

$4w+12+2w+12=50$

$6w=26$

$w=\frac{26}{6}=4\frac{1}{3} \to l=2w=8\frac{2}{3}$



The painting is $8\frac{2}{3}$ inches by $4\frac{1}{3}$ inches.

The frame is $14\frac{2}{3}$ inches by $10\frac{1}{3}$ inches.

77. Let t represent the time it takes the smaller pump to fill the tank.

	Time to do job alone	Part of job done in one hour	Time on Job	Part of total job done by each pump
3hp Pump	12	$\frac{1}{12}$	$t+4$	$\frac{t+4}{12}$
8hp Pump	8	$\frac{1}{8}$	4	$\frac{4}{8}$

Since the two pumps fill one tank, we have:

$$\frac{t+4}{12}+\frac{4}{8}=1$$

$$\frac{t+4}{12}=\frac{1}{2}$$

$$2t+8=12$$

$$2t=4$$

$$t=2$$

It takes the small pump a total of 2 more hours to fill the tank.

78. Let $w = 4$. Solve for the length:

$$l^2 = 4(l+4)$$
$$l^2 = 4l + 16$$
$$l^2 - 4l - 16 = 0$$
$$l = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-16)}}{2(1)} = \frac{4 \pm \sqrt{80}}{2} = 2 \pm 2\sqrt{5} \approx 6.47$$

The length of the plasterboard should be 6.47 feet.

79. Let x represent the number of passengers over 20.
Then $20 + x$ represents the total number of passengers.
$15 - 0.1x$ represents the fare for each passenger.
Solving the equation for total cost ($482.40), we have:

$$(20 + x)(15 - 0.1x) = 482.40$$
$$300 + 13x - 0.1x^2 = 482.40$$
$$-0.1x^2 + 13x - 182.40 = 0$$
$$x^2 - 130x + 1824 = 0$$
$$(x - 114)(x - 16) = 0 \rightarrow x = 114 \text{ or } x = 16$$

Since the capacity of the bus is 44, we discard the 114. The total number of passengers is 20 + 16 = 36, and the ticket price per passenger is 15 – 0.1(16) = $13.40.
So 36 people went on the trip; each person paid $13.40.

80. Let t represent the time it takes the older machine to complete the job by itself.

	Time to do job alone	Part of job done in one hour	Time on Job	Part of total job done by each machine
Old Copier	t	$\frac{1}{t}$	1.2	$\frac{1.2}{t}$
New Copier	$t-1$	$\frac{1}{t-1}$	1.2	$\frac{1.2}{t-1}$

Since the two copiers complete one job, we have:

$$\frac{1.2}{t} + \frac{1.2}{t-1} = 1$$
$$1.2(t-1) + 1.2t = t(t-1)$$
$$1.2t - 1.2 + 1.2t = t^2 - t$$
$$t^2 - 3.4t + 1.2 = 0$$
$$5t^2 - 17t + 6 = 0 \rightarrow (5t-2)(t-3) = 0 \rightarrow t = 0.4 \text{ or } t = 3$$

It takes the old copier 3 hours to do the job by itself. (0.4 hours is impossible since together it takes 1.2 hours.)

81. Let r_S represent Scott's rate and let r_T represent Todd's rate.
The time for Scott to run 95 meters is the same as for Todd to run 100 meters.

$$\frac{95}{r_S} = \frac{100}{r_T}$$
$$r_S = 0.95r_T$$
$$d_S = 0.95d_T$$

If Todd starts from 5 meters behind the start:
$$d_T = 105$$
$$d_S = 0.95d_T = 0.95(105) = 99.75$$

(a) The race does not end in a tie.
(b) Todd wins the race.
(c) Todd wins by 0.25 meters.
(d) To end in a tie:
$$100 = 0.95(100 + x)$$
$$100 = 95 + 0.95x$$
$$5 = 0.95x$$
$$x = 5.263 \text{ meters}$$
(e) $95 = 0.95(100)$ Therefore, the race ends in a tie.