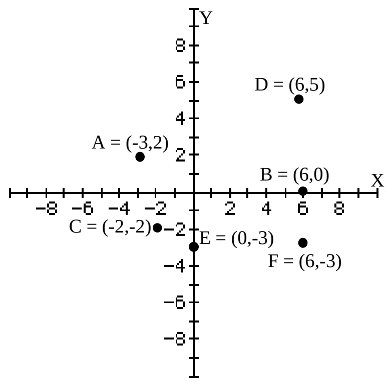
Chapter 2

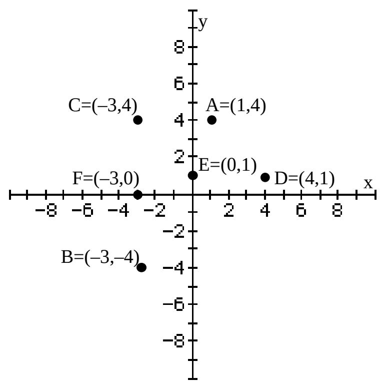
Graphs

2.1 Rectangular Coordinates

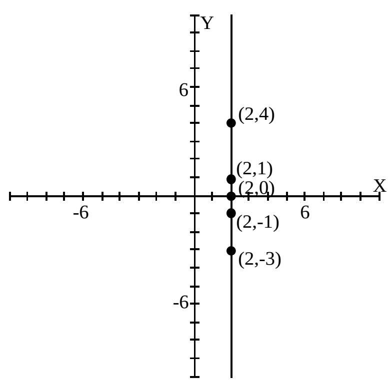
1. (a) Quadrant II
   (b) Positive x-axis
   (c) Quadrant III
   (d) Quadrant I
   (e) Negative y-axis
   (f) Quadrant IV



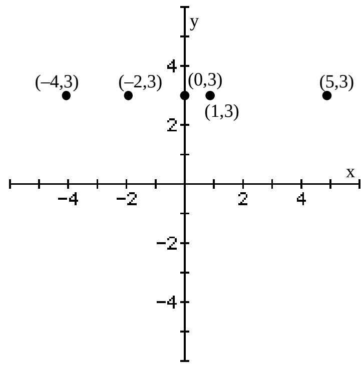
2. (a) Quadrant I
   (b) Quadrant III
   (c) Quadrant II
   (d) Quadrant I
   (e) Positive y-axis
   (f) Negative x-axis



3. The points will be on a vertical line that is two units to the right of the y-axis.



4. The points will be on a horizontal line that is three units above the x-axis.



5. $d(P_1,P_2)=\sqrt{(2-0)^2+(1-0)^2}=\sqrt{4+1}=\sqrt{5}$

6. $d(P_1,P_2)=\sqrt{(-2-0)^2+(1-0)^2}=\sqrt{4+1}=\sqrt{5}$

7. $d(P_1,P_2)=\sqrt{(-2-1)^2+(2-1)^2}=\sqrt{9+1}=\sqrt{10}$

8. $d(P_1,P_2)=\sqrt{(2-(-1))^2+(2-1)^2}=\sqrt{9+1}=\sqrt{10}$

9. $d(P_1,P_2)=\sqrt{(5-3)^2+(4-(-4))^2}=\sqrt{2^2+8^2}=\sqrt{4+64}=\sqrt{68}=2\sqrt{17}$

10. $d(P_1,P_2)=\sqrt{(2-(-1))^2+(4-0)^2}=\sqrt{3^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$

11. $d(P_1,P_2)=\sqrt{(6-(-3))^2+(0-2)^2}=\sqrt{9^2+(-2)^2}=\sqrt{81+4}=\sqrt{85}$

12. $d(P_1,P_2)=\sqrt{(4-2)^2+(2-(-3))^2}=\sqrt{2^2+5^2}=\sqrt{4+25}=\sqrt{29}$

13. $d(P_1,P_2)=\sqrt{(6-4)^2+(4-(-3))^2}=\sqrt{2^2+7^2}=\sqrt{4+49}=\sqrt{53}$

14. $d(P_1,P_2)=\sqrt{(6-(-4))^2+(2-(-3))^2}=\sqrt{10^2+5^2}=\sqrt{100+25}=\sqrt{125}=5\sqrt{5}$

15. $d(P_1,P_2)=\sqrt{(2.3-(-0.2))^2+(1.1-0.3)^2}=\sqrt{(2.5)^2+(0.8)^2}$
$=\sqrt{6.25+0.64}=\sqrt{6.89}\approx 2.625$

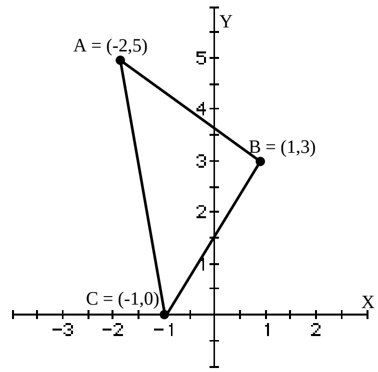
16. $d(P_1,P_2)=\sqrt{(-0.3-1.2)^2+(1.1-2.3)^2}=\sqrt{(-1.5)^2+(-1.2)^2}$
$=\sqrt{2.25+1.44}=\sqrt{3.69}$

17. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-b)^2}=\sqrt{a^2+b^2}$

18. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-a)^2}=\sqrt{a^2+a^2}=\sqrt{2a^2}=\sqrt{2}\,|a|$

19. $A = (-2, 5)$, $B = (1, 3)$, $C = (-1, 0)$

$$d(A,B) = \sqrt{(1-(-2))^2 + (3-5)^2} = \sqrt{3^2 + (-2)^2}$$
$$= \sqrt{9+4} = \sqrt{13}$$
$$d(B,C) = \sqrt{(-1-1)^2 + (0-3)^2} = \sqrt{(-2)^2 + (-3)^2}$$
$$= \sqrt{4+9} = \sqrt{13}$$
$$d(A,C) = \sqrt{(-1-(-2))^2 + (0-5)^2} = \sqrt{1^2 + (-5)^2}$$
$$= \sqrt{1+25} = \sqrt{26}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

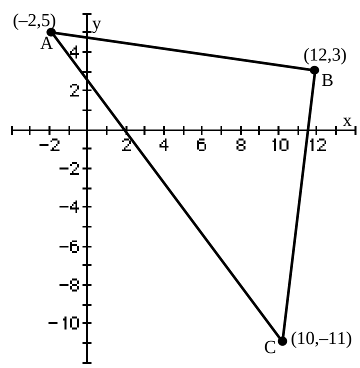
$$[d(A,B)]^2 + [d(B,C)]^2 = [d(A,C)]^2$$
$$\left(\sqrt{13}\right)^2 + \left(\sqrt{13}\right)^2 = \left(\sqrt{26}\right)^2$$
$$13 + 13 = 26$$
$$26 = 26$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \tfrac{1}{2}[d(A,B)]\,[d(B,C)] = \tfrac{1}{2}\sqrt{13}\;\sqrt{13} = \tfrac{1}{2}\;13 = \frac{13}{2} \text{ square units}$$

20. $A = (-2, 5)$, $B = (12, 3)$, $C = (10, -11)$

$$d(A,B) = \sqrt{(12-(-2))^2 + (3-5)^2} = \sqrt{14^2 + (-2)^2}$$
$$= \sqrt{196+4} = \sqrt{200} = 10\sqrt{2}$$
$$d(B,C) = \sqrt{(10-12)^2 + (-11-3)^2} = \sqrt{(-2)^2 + (-14)^2}$$
$$= \sqrt{4+196} = \sqrt{200} = 10\sqrt{2}$$
$$d(A,C) = \sqrt{(10-(-2))^2 + (-11-5)^2} = \sqrt{12^2 + (-16)^2}$$
$$= \sqrt{144+256} = \sqrt{400} = 20$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

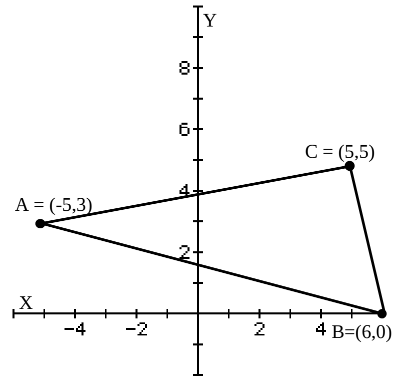
$$[d(A,B)]^2 + [d(B,C)]^2 = [d(A,C)]^2$$
$$\left(10\sqrt{2}\right)^2 + \left(10\sqrt{2}\right)^2 = (20)^2$$
$$200 + 200 = 400$$
$$400 = 400$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \tfrac{1}{2}[d(A,B)]\,[d(B,C)] = \tfrac{1}{2}\;10\sqrt{2}\;\;10\sqrt{2} = \tfrac{1}{2}\;100\;\;2 = 100 \text{ square units}$$

21. $A=(-5,3),\quad B=(6,0),\ C=(5,5)$

$$d(A,B)=\sqrt{(6-(-5))^2+(0-3)^2}=\sqrt{11^2+(-3)^2}$$
$$=\sqrt{121+9}=\sqrt{130}$$
$$d(B,C)=\sqrt{(5-6)^2+(5-0)^2}=\sqrt{(-1)^2+5^2}$$
$$=\sqrt{1+25}=\sqrt{26}$$
$$d(A,C)=\sqrt{(5-(-5))^2+(5-3)^2}=\sqrt{10^2+2^2}$$
$$=\sqrt{100+4}=\sqrt{104}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

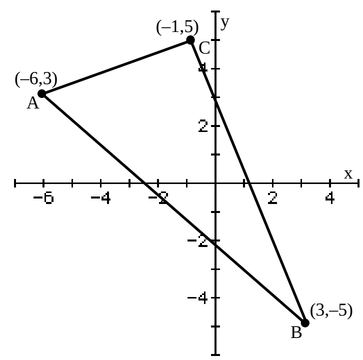
$$[d(A,C)]^2+[d(B,C)]^2=[d(A,B)]^2$$
$$\left(\sqrt{104}\right)^2+\left(\sqrt{26}\right)^2=\left(\sqrt{130}\right)^2$$
$$104+26=130$$
$$130=130$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

$$A=\tfrac{1}{2}[d(A,C)]\ [d(B,C)]=\tfrac{1}{2}\sqrt{104}\ \sqrt{26}=\tfrac{1}{2}\sqrt{2704}=\tfrac{1}{2}\ 52=26 \text{ square units}$$

22. $A=(-6,3),\quad B=(3,-5),\quad C=(-1,5)$

$$d(A,B)=\sqrt{(3-(-6))^2+(-5-3)^2}=\sqrt{9^2+(-8)^2}$$
$$=\sqrt{81+64}=\sqrt{145}$$
$$d(B,C)=\sqrt{(-1-3)^2+(5-(-5))^2}=\sqrt{(-4)^2+10^2}$$
$$=\sqrt{16+100}=\sqrt{116}$$
$$d(A,C)=\sqrt{(-1-(-6))^2+(5-3)^2}=\sqrt{5^2+2^2}$$
$$=\sqrt{25+4}=\sqrt{29}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

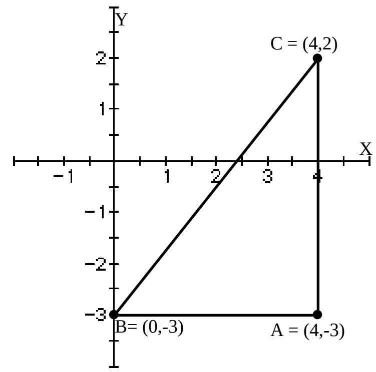
$$[d(A,C)]^2+[d(B,C)]^2=[d(A,B)]^2$$
$$\left(\sqrt{29}\right)^2+\left(\sqrt{116}\right)^2=\left(\sqrt{145}\right)^2$$
$$29+116=145$$
$$145=145$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

$$A=\tfrac{1}{2}[d(A,C)]\ [d(B,C)]=\tfrac{1}{2}\sqrt{29}\ \sqrt{116}=\tfrac{1}{2}\sqrt{3364}=\tfrac{1}{2}\ 58=29 \text{ square units}$$

23. $A=(4,-3),\quad B=(0,-3),\quad C=(4,2)$

$$d(A,B)=\sqrt{(0-4)^2+(-3-(-3))^2}=\sqrt{(-4)^2+0^2}$$
$$=\sqrt{16+0}=\sqrt{16}=4$$
$$d(B,C)=\sqrt{(4-0)^2+(2-(-3))^2}=\sqrt{4^2+5^2}$$
$$=\sqrt{16+25}=\sqrt{41}$$
$$d(A,C)=\sqrt{(4-4)^2+(2-(-3))^2}=\sqrt{0^2+5^2}$$
$$=\sqrt{0+25}=\sqrt{25}=5$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

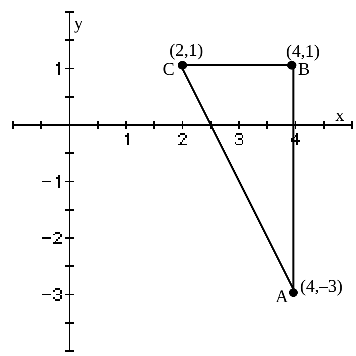
$$[d(A,B)]^2+[d(A,C)]^2=[d(B,C)]^2$$
$$4^2+5^2=\left(\sqrt{41}\right)^2 \rightarrow 16+25=41 \rightarrow 41=41$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

$$A=\tfrac{1}{2}[d(A,B)]\;[d(A,C)]=\tfrac{1}{2}\;4\;5=10 \text{ square units}$$

24. $A=(4,-3),\quad B=(4,1),\quad C=(2,1)$

$$d(A,B)=\sqrt{(4-4)^2+(1-(-3))^2}=\sqrt{0^2+4^2}$$
$$=\sqrt{0+16}=\sqrt{16}=4$$
$$d(B,C)=\sqrt{(2-4)^2+(1-1)^2}=\sqrt{(-2)^2+0^2}$$
$$=\sqrt{4+0}=\sqrt{4}=2$$
$$d(A,C)=\sqrt{(2-4)^2+(1-(-3))^2}=\sqrt{(-2)^2+4^2}$$
$$=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$[d(A,B)]^2+[d(B,C)]^2=[d(A,C)]^2$$
$$4^2+2^2=\left(2\sqrt{5}\right)^2 \rightarrow 16+4=20 \rightarrow 20=20$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

$$A=\tfrac{1}{2}[d(A,B)]\;[d(B,C)]=\tfrac{1}{2}\;4\;2=4 \text{ square units}$$

25. All points having an x-coordinate of 2 are of the form (2, y). Those which are 5 units from (–2, –1) are:

$$\sqrt{(2-(-2))^2+(y-(-1))^2}=5 \rightarrow \sqrt{4^2+(y+1)^2}=5$$

Squaring both sides: $4^2+(y+1)^2=25$

$$16+y^2+2y+1=25$$
$$y^2+2y-8=0$$
$$(y+4)(y-2)=0$$
$$y=-4 \text{ or } y=2$$

Therefore, the points are (2, –4) or (2, 2).

26. All points having a y-coordinate of –3 are of the form (x, –3). Those which are 13 units from (1, 2) are:

$$\sqrt{(x-1)^2+(-3-2)^2}=13 \rightarrow \sqrt{(x-1)^2+(-5)^2}=13$$

$$\text{Squaring both sides: } (x-1)^2+25=169$$

$$x^2-2x+1+25=169 \rightarrow x^2-2x-143=0$$

$$(x-13)(x+11)=0 \rightarrow x=13 \text{ or } x=-11$$

Therefore, the points are (–11, –3) or (13, –3).

27. All points on the x-axis are of the form (x, 0). Those which are 5 units from (4, –3) are:

$$\sqrt{(x-4)^2+(0-(-3))^2}=5 \rightarrow \sqrt{(x-4)^2+3^2}=5$$

$$\text{Squaring both sides: } (x-4)^2+9=25$$

$$x^2-8x+16+9=25 \rightarrow x^2-8x=0$$

$$x(x-8)=0 \rightarrow x=0 \text{ or } x=8$$

Therefore, the points are (0, 0) or (8, 0).

28. All points on the y-axis are of the form (0, y). Those which are 5 units from (4, 4) are:

$$\sqrt{(0-4)^2+(y-4)^2}=5 \rightarrow \sqrt{(-4)^2+(y-4)^2}=5$$

$$\text{Squaring both sides: } (-4)^2+(y-4)^2=25$$

$$16+y^2-8y+16=25$$

$$y^2-8y+7=0 \rightarrow (y-1)(y-7)=0 \rightarrow y=1 \text{ or } y=7$$

Therefore, the points are (0, 1) or (0, 7).

29. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{5+3}{2},\frac{-4+2}{2}\right)=\left(\frac{8}{2},\frac{-2}{2}\right)=(4,-1)$$

30. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-1+2}{2},\frac{0+4}{2}\right)=\left(\frac{1}{2},\frac{4}{2}\right)=\left(\frac{1}{2},2\right)$$

31. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-3+6}{2},\frac{2+0}{2}\right)=\left(\frac{3}{2},\frac{2}{2}\right)=\left(\frac{3}{2},1\right)$$

32. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{2+4}{2},\frac{-3+2}{2}\right)=\left(\frac{6}{2},\frac{-1}{2}\right)=\left(3,-\frac{1}{2}\right)$$

33. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{4+6}{2},\frac{-3+1}{2}\right)=\left(\frac{10}{2},\frac{-2}{2}\right)=(5,-1)$$

34. The coordinates of the midpoint are:
$$(x,y)=\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)=\left(\frac{-4+2}{2},\ \frac{-3+2}{2}\right)=\left(\frac{-2}{2},\ \frac{-1}{2}\right)=\left(-1,\ -\frac{1}{2}\right)$$

35. The coordinates of the midpoint are:
$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-0.2\ +2.3}{2},\frac{0.3+1.1}{2}\right)=\left(\frac{2.1}{2},\frac{1.4}{2}\right)=(1.05,\ \ 0.7)$$

36. The coordinates of the midpoint are:
$$(x,y)=\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)=\left(\frac{1.2+(-0.3)}{2},\ \frac{2.3+1.1}{2}\right)=\left(\frac{0.9}{2},\ \frac{3.4}{2}\right)=\left(0.45,1.7\right)$$

37. The coordinates of the midpoint are:
$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{a+0}{2},\frac{b+0}{2}\right)=\left(\frac{a}{2},\frac{b}{2}\right)$$

38. The coordinates of the midpoint are:
$$(x,y)=\left(\frac{x_1+x_2}{2},\ \frac{y_1+y_2}{2}\right)=\left(\frac{a+0}{2},\ \frac{a+0}{2}\right)=\left(\frac{a}{2},\ \frac{a}{2}\right)$$

39. The midpoint of AB is: $D=\left(\frac{0+0}{2},\ \frac{0+6}{2}\right)=\left(0,3\right)$

The midpoint of AC is: $E=\left(\frac{0+4}{2},\ \frac{0+4}{2}\right)=\left(2,2\right)$

The midpoint of BC is: $F=\left(\frac{0+4}{2},\ \frac{6+4}{2}\right)=\left(2,5\right)$

$$d(C,D)=\sqrt{\left(0-4\right)^2+(3-4)^2}=\sqrt{(-4)^2+(-1)^2}=\sqrt{16+1}=\sqrt{17}$$
$$d(B,E)=\sqrt{\left(2-0\right)^2+(2-6)^2}=\sqrt{2^2+(-4)^2}=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$$
$$d(A,F)=\sqrt{(2-0)^2+(5-0)^2}=\sqrt{2^2+5^2}=\sqrt{4+25}=\sqrt{29}$$

40. Let $P_1=(0,\ 0),\ \ P_2=(0,\ 4),\ \ P=(x,y)$
$$d\left(P_1,\ P_2\right)=\sqrt{(0-0)^2+(4-0)^2}=\sqrt{16}=4$$
$$d\left(P_1,\ P\right)=\sqrt{(x-0)^2+(y-0)^2}=\sqrt{x^2+y^2}=4\quad\rightarrow\quad x^2+y^2=16$$
$$d\left(P_2,\ P\right)=\sqrt{(x-0)^2+(y-4)^2}=\sqrt{x^2+(y-4)^2}=4\quad\rightarrow\quad x^2+(y-4)^2=16$$
$$x^2+y^2-8y+16=16$$
$$16-8y+16=16$$
$$-8y=-16\rightarrow y=2\rightarrow x^2+2^2=16\rightarrow x^2=12\rightarrow x=\pm2\sqrt{3}$$

Two triangles are possible. The third vertex is $\left(-2\sqrt{3},2\right)$ or $\left(2\sqrt{3},2\right)$.

41. $d(P_1,P_2) = \sqrt{(-4-2)^2+(1-1)^2} = \sqrt{(-6)^2+0^2} = \sqrt{36} = 6$
$d(P_2,P_3) = \sqrt{(-4-(-4))^2+(-3-1)^2} = \sqrt{0^2+(-4)^2} = \sqrt{16} = 4$
$d(P_1,P_3) = \sqrt{(-4-2)^2+(-3-1)^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$
Since $\left[d(P_1,P_2)\right]^2+\left[d(P_2,P_3)\right]^2=\left[d(P_1,P_3)\right]^2$, the triangle is a right triangle.

42. $d(P_1,P_2) = \sqrt{(6-(-1))^2+(2-4)^2} = \sqrt{7^2+(-2)^2} = \sqrt{49+4} = \sqrt{53}$
$d(P_2,P_3) = \sqrt{(4-6)^2+(-5-2)^2} = \sqrt{(-2)^2+(-7)^2} = \sqrt{4+49} = \sqrt{53}$
$d(P_1,P_3) = \sqrt{(4-(-1))^2+(-5-4)^2} = \sqrt{5^2+(-9)^2} = \sqrt{25+81} = \sqrt{106}$
Since $\left[d(P_1,P_2)\right]^2+\left[d(P_2,P_3)\right]^2=\left[d(P_1,P_3)\right]^2$, the triangle is a right triangle.
Since $d(P_1, P_2) = d(P_2, P_3)$, the triangle is isosceles.
Therefore, the triangle is an isosceles right triangle.

43. $d(P_1,P_2) = \sqrt{(0-(-2))^2+(7-(-1))^2} = \sqrt{2^2+8^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$
$d(P_2,P_3) = \sqrt{(3-0)^2+(2-7)^2} = \sqrt{3^2+(-5)^2} = \sqrt{9+25} = \sqrt{34}$
$d(P_1,P_3) = \sqrt{(3-(-2))^2+(2-(-1))^2} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$
Since $d(P_2,P_3) = d(P_1,P_3)$, the triangle is isosceles.
Since $\left[d(P_1,P_3)\right]^2+\left[d(P_2,P_3)\right]^2=\left[d(P_1,P_2)\right]^2$, the triangle is also a right triangle.
Therefore, the triangle is an isosceles right triangle.

44. $d(P_1,P_2) = \sqrt{(-4-7)^2+(0-2)^2} = \sqrt{(-11)^2+(-2)^2} = \sqrt{121+4} = \sqrt{125} = 5\sqrt{5}$
$d(P_2,P_3) = \sqrt{(4-(-4))^2+(6-0)^2} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100} = 10$
$d(P_1,P_3) = \sqrt{(4-7)^2+(6-2)^2} = \sqrt{(-3)^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$
Since $\left[d(P_1,P_3)\right]^2+\left[d(P_2,P_3)\right]^2=\left[d(P_1,P_2)\right]^2$, the triangle is a right triangle.

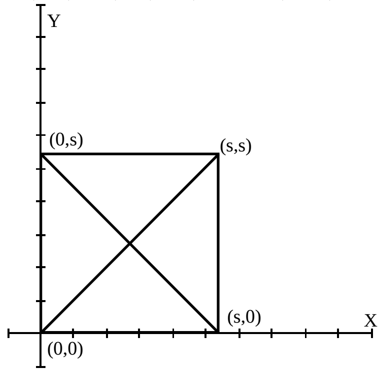
45. $P_1 = (1,3)$, $P_2 = (5,15)$
$d(P_1,P_2) = \sqrt{(5-1)^2+(15-3)^2} = \sqrt{4^2+12^2} = \sqrt{16+144} = \sqrt{160} = 4\sqrt{10}$

46. $P_1 = (-8, -4)$, $P_2 = (2, 3)$
$d(P_1,P_2) = \sqrt{(2-(-8))^2+(3-(-4))^2} = \sqrt{10^2+7^2} = \sqrt{100+49} = \sqrt{149}$

47. $P_1 = (-4, 6)$, $P_2 = (4, -8)$
$d(P_1,P_2) = \sqrt{(4-(-4))^2+(-8-6)^2} = \sqrt{8^2+(-14)^2} = \sqrt{64+196} = \sqrt{260} = 2\sqrt{65}$

48. $P_1 = (0, 6)$, $P_2 = (3, -8)$
$d(P_1,P_2) = \sqrt{(3-0)^2+(-8-6)^2} = \sqrt{3^2+(-14)^2} = \sqrt{9+196} = \sqrt{205}$

49. Plot the vertices of the square at (0, 0), (0, s), (s, s), and (s, 0).



Find the midpoints of the diagonals.

$$M_1 = \left(\frac{0+s}{2}, \frac{0+s}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$

$$M_2 = \left(\frac{0+s}{2}, \frac{s+0}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$

Since the coordinates of the midpoints are the same, the diagonals of a square intersect at their midpoints .

50. Let $A = (0, 0)$, $B = (a, 0)$, $C = \left(\frac{a}{2}, \frac{\sqrt{3}\,a}{2}\right)$

$$d(A,B) = \sqrt{(a-0)^2 + (0-0)^2} = \sqrt{a^2} = a$$

$$d(B,C) = \sqrt{\left(\frac{a}{2} - a\right)^2 + \left(\frac{\sqrt{3}\,a}{2} - 0\right)^2} = \sqrt{\left(-\frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}\,a}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{a^2} = a$$

$$d(A,C) = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{\sqrt{3}\,a}{2} - 0\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}\,a}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{a^2} = a$$

Since the sides are the same length, the triangle is equilateral.

Find the midpoints of the sides:

$$D = M_{AB} = \left(\frac{0+a}{2}, \frac{0+0}{2}\right) = \left(\frac{a}{2}, 0\right) \qquad E = M_{BC} = \left(\frac{a + \frac{a}{2}}{2}, \frac{0 + \frac{\sqrt{3}\,a}{2}}{2}\right) = \left(\frac{3a}{4}, \frac{\sqrt{3}\,a}{4}\right)$$

$$F = M_{AC} = \left(\frac{0 + \frac{a}{2}}{2}, \frac{0 + \frac{\sqrt{3}\,a}{2}}{2}\right) = \left(\frac{a}{4}, \frac{\sqrt{3}\,a}{4}\right)$$

$$d(D,E) = \sqrt{\left(\frac{3a}{4} - \frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}\,a}{4} - 0\right)^2} = \sqrt{\left(\frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}\,a}{4}\right)^2} = \sqrt{\frac{a^2}{16} + \frac{3a^2}{16}} = \frac{a}{2}$$

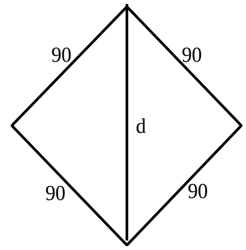
$$d(D,F) = \sqrt{\left(\frac{a}{4} - \frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}\,a}{4} - 0\right)^2} = \sqrt{\left(-\frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}\,a}{4}\right)^2} = \sqrt{\frac{a^2}{16} + \frac{3a^2}{16}} = \frac{a}{2}$$

$$d(E,F) = \sqrt{\left(\frac{3a}{4} - \frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}\,a}{4} - \frac{\sqrt{3}\,a}{4}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + 0^2} = \sqrt{\frac{a^2}{4}} = \frac{a}{2}$$

Since the sides are the same length, the triangle is equilateral.

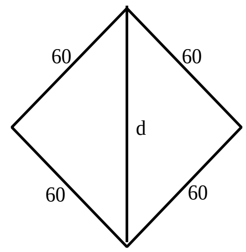
51. Using the Pythagorean Theorem:

$$\begin{aligned} 90^2 + 90^2 &= d^2 \\ 8100 + 8100 &= d^2 \\ 16200 &= d^2 \\ d &= \sqrt{16200} \\ d &= 90\sqrt{2} \approx 127.28 \text{ feet} \end{aligned}$$

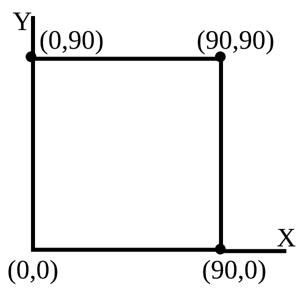


52. Using the Pythagorean Theorem:

$$\begin{aligned} 60^2 + 60^2 &= d^2 \\ 3600 + 3600 &= d^2 \\ 7200 &= d^2 \\ d &= \sqrt{7200} \\ d &= 60\sqrt{2} \approx 84.85 \text{ feet} \end{aligned}$$



53. (a) First: (90, 0), Second: (90, 90)
Third: (0, 90)



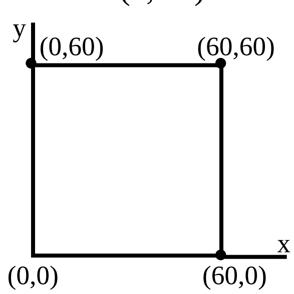
(b) Using the distance formula:

$$\begin{aligned} d &= \sqrt{(310-90)^2 + (15-90)^2} \\ &= \sqrt{220^2 + (-75)^2} \\ &= \sqrt{54025} \approx 232.4 \text{ feet} \end{aligned}$$

(c) Using the distance formula:

$$\begin{aligned} d &= \sqrt{(300-0)^2 + (300-90)^2} \\ &= \sqrt{300^2 + 210^2} \\ &= \sqrt{134100} \approx 366.2 \text{ feet} \end{aligned}$$

54. (a) First: (60, 0), Second: (60, 60)
Third: (0, 60)



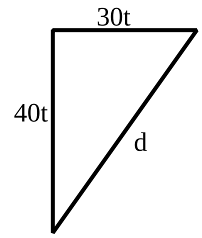
(b) Using the distance formula:

$$\begin{aligned} d &= \sqrt{(180-60)^2 + (20-60)^2} \\ &= \sqrt{120^2 + (-40)^2} \\ &= \sqrt{16000} \approx 126.5 \text{ feet} \end{aligned}$$

(c) Using the distance formula:

$$\begin{aligned} d &= \sqrt{(220-0)^2 + (220-60)^2} \\ &= \sqrt{220^2 + 160^2} \\ &= \sqrt{74000} \approx 272.0 \text{ feet} \end{aligned}$$

55. The Intrepid heading east moves a distance $30t$ after t hours. The truck heading south moves a distance $40t$ after t hours. Their distance apart after t hours is:



$$\begin{aligned} d &= \sqrt{(30t)^2 + (40t)^2} \\ &= \sqrt{900t^2 + 1600t^2} \\ &= \sqrt{2500t^2} \\ &= 50t \end{aligned}$$

56. $d = \sqrt{100^2 + \left(15 \cdot \frac{5280}{1} \cdot \frac{1}{3600}\, t\right)^2} = \sqrt{100^2 + (22t)^2} = \sqrt{10000 + 484t^2}$