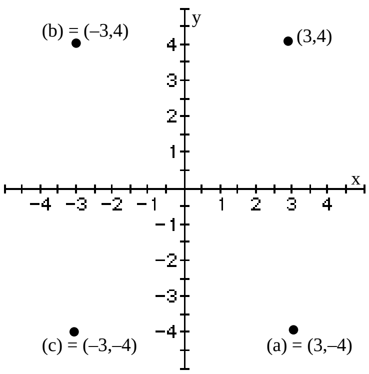
Chapter 2

Graphs

2.2 Graphs of Equations

1.



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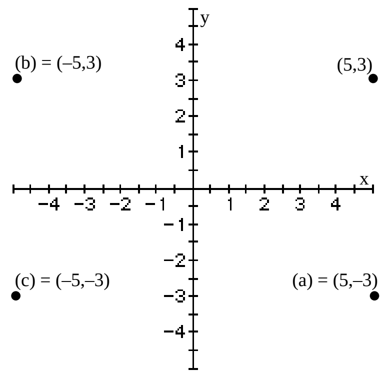

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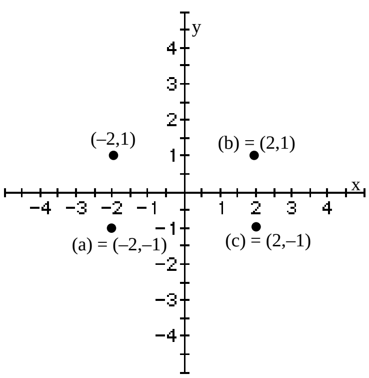

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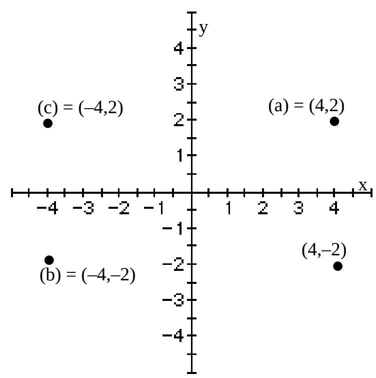

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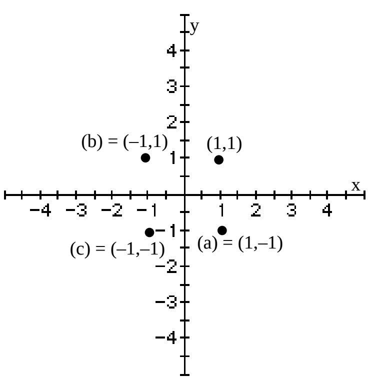

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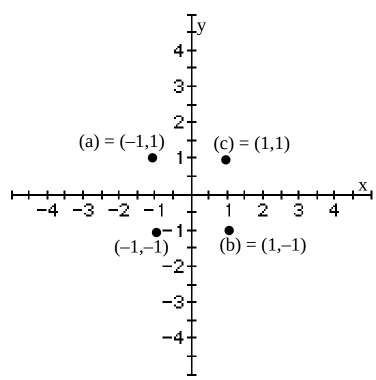

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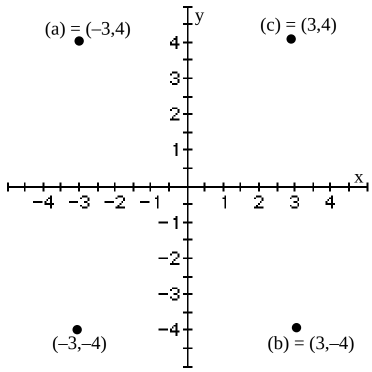

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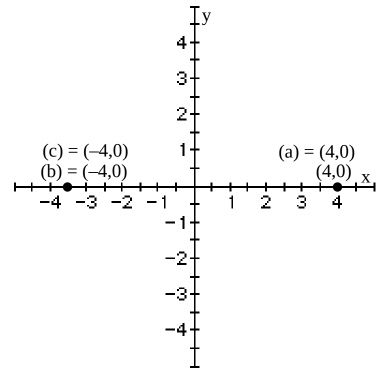

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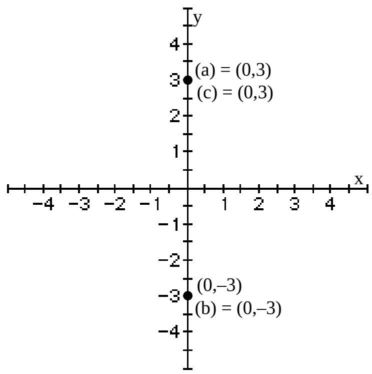

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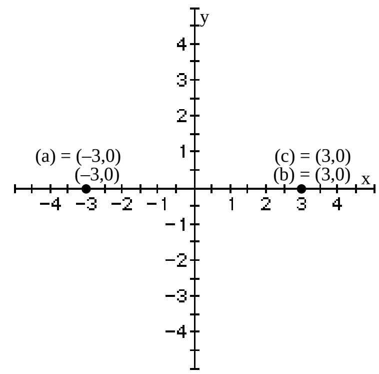


11. (a) (–1, 0), (1, 0) (b) symmetric to the x-axis, y-axis and origin

12. (a) (0, 1) (b) not symmetric to x-axis, y-axis, or origin

13. (a) $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right)$, (0, 1) (b) symmetric to the y-axis

14. (a) (–2, 0), (2, 0), (0, –3) (b) symmetric to the y-axis

15. (a) (0, 0) (b) symmetric to the x-axis

16. (a) (2, 0), (0, 2), (–2, 0), (0, –2) (b) symmetric to the x-axis, y-axis, and origin

17. (a) (1, 0) (b) not symmetric to x-axis, y-axis, or origin

18. (a) (0, 0) (b) not symmetric to x-axis, y-axis, or origin

19. (a) (–1.5, 0), (1.5, 0), (0, –2) (b) symmetric to the y-axis

20. (a) (0, 0) (b) symmetric to the origin

21. (a) none (b) symmetric to the origin

22. (a) none (b) symmetric to the x-axis

23. $y = x^4 - \sqrt{x}$

$0 = 0^4 - \sqrt{0}$ $\quad 1 = 1^4 - \sqrt{1}$ $\quad 0 = (-1)^4 - \sqrt{-1}$

$0 = 0$ $\quad 1 \neq 0$ $\quad 0 \neq 1 - \sqrt{-1}$

(0, 0) is on the graph of the equation.

24. $y = x^3 - 2\sqrt{x}$

$0 = 0^3 - 2\sqrt{0}$ $\quad 1 = 1^3 - 2\sqrt{1}$ $\quad -1 = 1^3 - 2\sqrt{1}$

$0 = 0$ $\quad 1 \neq -1$ $\quad -1 = -1$

(0, 0) and (1, –1) are on the graph of the equation.

25. $y^2 = x^2 + 9$

$3^2 = 0^2 + 9$ $\quad 0^2 = 3^2 + 9$ $\quad 0^2 = (-3)^2 + 9$

$9 = 9$ $\quad 0 \neq 18$ $\quad 0 \neq 18$

(0, 3) is on the graph of the equation.

26. $y^3 = x + 1$

$2^3 = 1 + 1$ $\quad 1^3 = 0 + 1$ $\quad 0^3 = -1 + 1$

$8 \neq 2$ $\quad 1 = 1$ $\quad 0 = 0$

(0, 1) and (–1, 0) are on the graph of the equation.

27. $x^2 + y^2 = 4$

$0^2 + 2^2 = 4$ $\quad (-2)^2 + 2^2 = 4$ $\quad \sqrt{2}^2 + \sqrt{2}^2 = 4$

$4 = 4$ $\quad 8 \neq 4$ $\quad 4 = 4$

(0, 2) and $\left(\sqrt{2}, \sqrt{2}\right)$ are on the graph of the equation.

28. $x^2 + 4y^2 = 4$

$0^2 + 4\ 1^2 = 4$ $\quad 2^2 + 4\ 0^2 = 4$ $\quad 2^2 + 4\left(\tfrac{1}{2}\right)^2 = 4$

$4 = 4$ $\quad 4 = 4$ $\quad 5 \neq 4$

(0, 1) and (2, 0) are on the graph of the equation.

29. $x^2 = y$

y-intercept: Let $x = 0$, then $y = 0$ $\quad (0, 0)$

x-intercept: Let $y = 0$, then $x = 0$ $\quad (0, 0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $x^2 = -y$, which is not equivalent to $x^2 = y$.

y-axis: Replace x by $-x$ so $(-x)^2 = y$ or $x^2 = y$, which is equivalent to $x^2 = y$.

Origin: Replace x by $-x$ and y by $-y$ so $(-x)^2 = -y$ or $x^2 = -y$,

which is not equivalent to $x^2 = y$.

Therefore, the graph is symmetric with respect to the y-axis

30. $y^2 = x$

y-intercept: Let $x = 0$, then $y = 0$ $(0, 0)$

x-intercept: Let $y = 0$, then $x = 0$ $(0, 0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $(-y)^2 = x$ or $y^2 = x$,

which is equivalent to $y^2 = x$.

y-axis: Replace x by $-x$ so $y^2 = -x$,

which is not equivalent to $y^2 = x$.

Origin: Replace x by $-x$ and y by $-y$ so $(-y)^2 = -x$ or $y^2 = -x$,

which is not equivalent to $y^2 = x$.

Therefore, the graph is symmetric with respect to the x-axis.

31. $y = 3x$

y-intercept: Let $x = 0$, then $y = 0$ $(0, 0)$

x-intercept: Let $y = 0$, then $x = 0$ $(0, 0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $-y = 3x$, which is not equivalent to $y = 3x$.

y-axis: Replace x by $-x$ so $y = 3(-x)$ or $y = -3x$,

which is not equivalent to $y = 3x$.

Origin: Replace x by $-x$ and y by $-y$ so $-y = 3(-x)$ or $y = 3x$,

which is equivalent to $y = 3x$.

Therefore, the graph is symmetric with respect to the origin.

32. $y = -5x$

y-intercept: Let $x = 0$, then $y = 0$ $(0, 0)$

x-intercept: Let $y = 0$, then $x = 0$ $(0, 0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $-y = -5x$ or $y = 5x$,

which is not equivalent to $y = -5x$.

y-axis: Replace x by $-x$ so $y = -5(-x)$ or $y = 5x$,

which is not equivalent to $y = -5x$.

Origin: Replace x by $-x$ and y by $-y$ so $-y = -5(-x)$ or $y = -5x$,

which is equivalent to $y = -5x$.

Therefore, the graph is symmetric with respect to the origin.

33. $x^2 + y - 9 = 0$

y-intercept: Let $x = 0$, then $y = 9$ $(0,9)$

x-intercept: Let $y = 0$, then $x = \pm 3$ $(-3,0), (3,0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $x^2+(-y)-9=0$ or $x^2-y-9=0$,

which is not equivalent to $x^2+y-9=0$.

y-axis: Replace x by $-x$ so $(-x)^2+y-9=0$ or $x^2+y-9=0$,

which is equivalent to $x^2+y-9=0$.

Origin: Replace x by $-x$ and y by $-y$ so $(-x)^2+(-y)-9=0$ or $x^2-y-9=0$,

which is not equivalent to $x^2+y-9=0$.

Therefore, the graph is symmetric with respect to the y-axis.

34. $y^2-x-4=0$

y-intercept Let $x=0$, then $y=\pm 2$ $(0,2),(0,-2)$

x-intercept Let $y=0$, then $x=-4$ $(-4,0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $(-y)^2-x-4=0$ or $y^2-x-4=0$,

which is equivalent to $y^2-x-4=0$.

y-axis: Replace x by $-x$ so $y^2-(-x)-4=0$ or $y^2+x-4=0$,

which is not equivalent to $y^2-x-4=0$.

Origin: Replace x by $-x$ and y by $-y$ so $(-y)^2-(-x)-4=0$ or $y^2+x-4=0$,

which is not equivalent to $y^2-x-4=0$

Therefore, the graph is symmetric with respect to the x-axis.

35. $9x^2+4y^2=36$

y-intercept Let $x=0$, then $y=\pm 3$ $(0,-3),(0,3)$

x-intercept Let $y=0$, then $x=\pm 2$ $(-2,0),(2,0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $9x^2+4(-y)^2=36$ or $9x^2+4y^2=36$,

which is equivalent to $9x^2+4y^2=36$.

y-axis: Replace x by $-x$ so $9(-x)^2+4y^2=36$ or $9x^2+4y^2=36$,

which is equivalent to $9x^2+4y^2=36$.

Origin: Replace x by $-x$ and y by $-y$ so $9(-x)^2+4(-y)^2=36$ or $9x^2+4y^2=36$,

which is equivalent to $9x^2+4y^2=36$.

Therefore, the graph is symmetric with respect to the x-axis, the y-axis and the origin.

36. $4x^2+y^2=4$

y-intercept Let $x=0$, then $y=\pm 2$ $(0,-2),(0,2)$

x-intercept Let $y=0$, then $x=\pm 1$ $(-1,0),(1,0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $4x^2+(-y)^2=4$ or $4x^2+y^2=4$,

which is equivalent to $4x^2+y^2=4$.

y-axis: Replace x by $-x$ so $4(-x)^2+y^2=4$ or $4x^2+y^2=4$,

which is equivalent to $4x^2+y^2=4$.

Origin: Replace x by $-x$ and y by $-y$ so $4(-x)^2+(-y)^2=4$ or $4x^2+y^2=4$,

which is equivalent to $4x^2+y^2=4$.

Therefore, the graph is symmetric with respect to the *x*-axis, the *y*-axis and the origin.

37. $y=x^3-27$

y-intercept: Let $x=0$, then $y=0^3-27$

$y=-27$ $(0,-27)$

x-intercept: Let $y=0$, then $0=x^3-27$

$x^3=27$

$x=3$ $(3,0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $-y=x^3-27$, which is not

equivalent to $y=x^3-27$.

y-axis: Replace x by $-x$ so $y=(-x)^3-27$ or $y=-x^3-27$,

which is not equivalent to $y=x^3-27$.

Origin: Replace x by $-x$ and y by $-y$ so $-y=(-x)^3-27$ or

$y=x^3+27$, which is not equivalent to $y=x^3-27$.

Therefore, the graph is not symmetric to the x-axis, the y-axis, or the origin.

38. $y=x^4-1$

y-intercept: Let $x=0$, then $y=0^4-1$

$y=-1$ $(0,-1)$

x-intercept: Let $y=0$, then $0=x^4-1$

$x^4=1$

$x=\pm 1$ $(1,0),(-1,0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $-y=x^4-1$, which is not

equivalent to $y=x^4-1$.

y-axis: Replace x by $-x$ so $y=(-x)^4-1$ or $y=x^4-1$,

which is equivalent to $y=x^4-1$.

Origin: Replace x by $-x$ and y by $-y$ so $-y=(-x)^4-1$ or

$-y=x^4-1$, which is not equivalent to $y=x^4-1$.

Therefore, the graph is symmetric with respect to the y-axis.

39. $y = x^2 - 3x - 4$

y-intercept: Let $x = 0$, then $y = 0^2 - 3(0) - 4$

$y = -4 \qquad (0, -4)$

x-intercept: Let $y = 0$, then $0 = x^2 - 3x - 4$

$(x-4)(x+1) = 0$

$x = 4 \quad x = -1 \quad (4,0), (-1,0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $-y = x^2 - 3x - 4$, which is not equivalent to $y = x^2 - 3x - 4$.

y-axis: Replace x by $-x$ so $y = (-x)^2 - 3(-x) - 4$ or $y = x^2 + 3x - 4$, which is not equivalent to $y = x^2 - 3x - 4$.

Origin: Replace x by $-x$ and y by $-y$ so $-y = (-x)^2 - 3(-x) - 4$ or $y = -x^2 - 3x + 4$, which is not equivalent to $y = x^2 - 3x - 4$.

Therefore, the graph is not symmetric to the x-axis, the y-axis, or the origin.

40. $y = x^2 + 4$

y-intercept: Let $x = 0$, then $y = 0^2 + 4$

$y = 4 \qquad (0, 4)$

x-intercept: Let $y = 0$, then $0 = x^2 + 4$

$x^2 = -4$

no solution - no x-intercept

Test for symmetry:

x-axis: Replace y by $-y$ so $-y = x^2 + 4$, which is not equivalent to $y = x^2 + 4$.

y-axis: Replace x by $-x$ so $y = (-x)^2 + 4$ or $y = x^2 + 4$, which is equivalent to $y = x^2 + 4$.

Origin: Replace x by $-x$ and y by $-y$ so $-y = (-x)^2 + 4$ or $y = -x^2 - 4$, which is not equivalent to $y = x^2 + 4$.

Therefore, the graph is symmetric with respect to the y-axis.

41. $y = \dfrac{3x}{x^2 + 4}$

y-intercept: Let $x = 0$, then $y = \dfrac{0}{0 + 4}$

$y = 0 \qquad (0,0)$

x-intercept: Let $y = 0$, then $0 = \dfrac{3x}{x^2 + 4}$

$3x = 0 \rightarrow x = 0 \qquad (0,0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $-y = \dfrac{3x}{x^2+4}$, which is not equivalent to $y = \dfrac{3x}{x^2+4}$.

y-axis: Replace x by $-x$ so $y = \dfrac{3(-x)}{(-x)^2+4}$ or $y = \dfrac{-3x}{x^2+4}$, which is not equivalent to $y = \dfrac{3x}{x^2+4}$.

Origin: Replace x by $-x$ and y by $-y$ so $-y = \dfrac{-3x}{(-x)^2+4}$ or $y = \dfrac{3x}{x^2+4}$, which is equivalent to $y = \dfrac{3x}{x^2+4}$.

Therefore, the graph is symmetric with respect to the origin.

42. $y = \dfrac{x^2-4}{2x}$

y-intercept: Let $x=0$, then $y = \dfrac{-4}{0}$

undefined $\rightarrow$ no $y-$intercept

x-intercept: Let $y=0$, then $0 = \dfrac{x^2-4}{2x} \rightarrow x^2-4=0$

$x = \pm 2 \qquad (-2,0),(2,0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $-y = \dfrac{x^2-4}{2x}$, which is not equivalent to $y = \dfrac{x^2-4}{2x}$.

y-axis: Replace x by $-x$ so $y = \dfrac{(-x)^2-4}{2(-x)}$ or $y = \dfrac{x^2-4}{-2x}$, which is not equivalent to $y = \dfrac{x^2-4}{2x}$.

Origin: Replace x by $-x$ and y by $-y$ so $-y = \dfrac{(-x)^2-4}{2(-x)}$ or $-y = \dfrac{x^2-4}{-2x}$, which is equivalent to $y = \dfrac{x^2-4}{2x}$.

Therefore, the graph is symmetric with respect to the origin.

43. $y = \dfrac{-x^3}{x^2 - 9}$

y-intercept: Let $x = 0$, then $y = \dfrac{0}{-9} = 0$

$(0,0)$

x-intercept: Let $y = 0$, then $0 = \dfrac{-x^3}{x^2 - 9} \rightarrow -x^3 = 0$

$x = 0 \quad (0,0)$

Test for symmetry:

x-axis: Replace y by $-y$ so $-y = \dfrac{-x^3}{x^2 - 9}$, which is not equivalent to $y = \dfrac{-x^3}{x^2 - 9}$.

y-axis: Replace x by $-x$ so $y = \dfrac{-(-x)^3}{(-x)^2 - 9}$ or $y = \dfrac{x^3}{x^2 - 9}$, which is not equivalent to $y = \dfrac{-x^3}{x^2 - 9}$.

Origin: Replace x by $-x$ and y by $-y$ so $-y = \dfrac{-(-x)^3}{(-x)^2 - 9}$ or

$-y = \dfrac{x^3}{x^2 - 9}$, which is equivalent to $y = \dfrac{-x^3}{x^2 - 9}$.

Therefore, the graph is symmetric with respect to the origin.

44. $y = \dfrac{x^4 + 1}{2x^5}$

y-intercept: Let $x = 0$, then $y = \dfrac{1}{0}$

undefined $\rightarrow$ no $y-$intercept

x-intercept: Let $y = 0$, then $0 = \dfrac{x^4 + 1}{2x^5} \rightarrow x^4 + 1 = 0$

no solution $\rightarrow$ no $x-$ intercept

Test for symmetry:

x-axis: Replace y by $-y$ so $-y = \dfrac{x^4 + 1}{2x^5}$, which is not equivalent to $y = \dfrac{x^4 + 1}{2x^5}$.

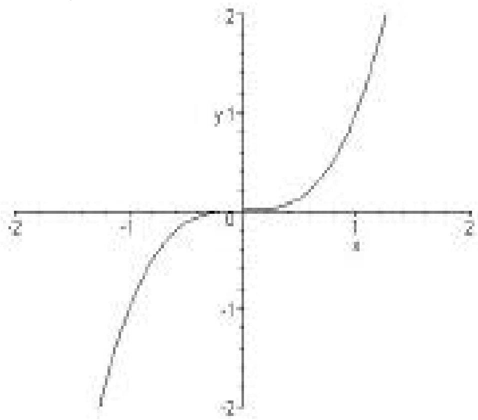
y-axis: Replace x by $-x$ so $y = \dfrac{(-x)^4 + 1}{2(-x)^5}$ or $y = \dfrac{x^4 + 1}{-2x^5}$, which is not equivalent to $y = \dfrac{x^4 + 1}{2x^5}$.

Origin: Replace x by $-x$ and y by $-y$ so $-y = \dfrac{(-x)^4+1}{2(-x)^5}$ or $-y = \dfrac{x^4+1}{-2x^5}$,

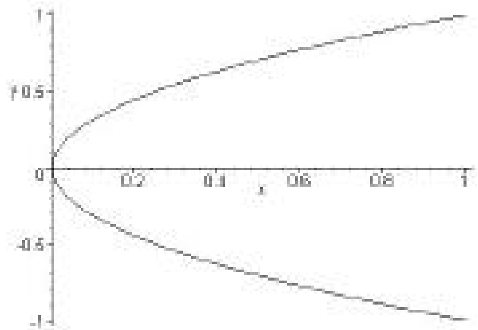
which is equivalent to $y = \dfrac{x^4+1}{2x^5}$.

Therefore, the graph is symmetric with respect to the origin.

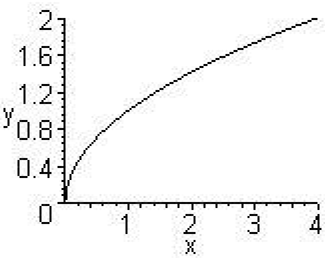
45. $y = x^3$



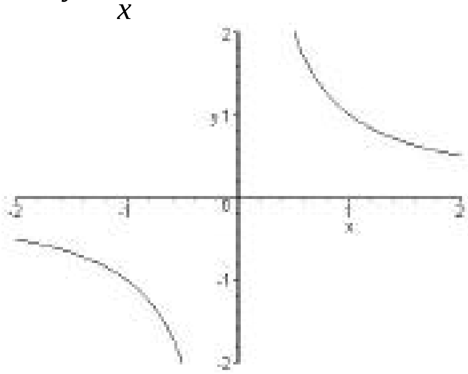
46. $x = y^2$



47. $y = \sqrt{x}$



48. $y = \dfrac{1}{x}$



49. $y = 3x + 5$

$$2 = 3a + 5$$
$$3a = -3$$
$$a = -1$$

50. $y = x^2 + 4x$

$$b = 2^2 + 4(2) = 4 + 8 = 12$$

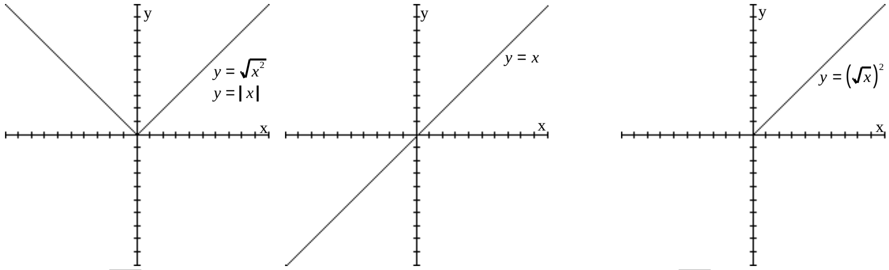
51. $2x + 3y = 6$

$$2a + 3b = 6$$

52. $y = mx + b$

$$0 = m(2) + b$$
$$5 = m(0) + b \rightarrow 5 = b$$
$$0 = 2m + 5$$
$$-2m = 5 \rightarrow m = -\frac{5}{2}$$

53. (a)



(b) Since $\sqrt{x^2} = |x|$, then for all x, the graphs of $y = \sqrt{x^2}$ and $y = |x|$ are the same.

(c) For $y = \left(\sqrt{x}\right)^2$, the domain of the variable x is $x \geq 0$; for $y = x$, the domain of the variable x is all real numbers. Thus, $\left(\sqrt{x}\right)^2 = x$ only for $x \geq 0$.

(d) For $y = \sqrt{x^2}$, the range of the variable y is $y \geq 0$; for $y = x$, the range of the variable y is all real numbers. Also, $\sqrt{x^2} = x$ only if $x \geq 0$.

54. Answers will vary

55. If the equation has *x*-axis and *y*-axis symmetry, then we have the following:

$x-$axis symmetry means $(x, y) \leftrightarrow (x, -y)$

$y-$axis symmetry means $(x.y) \leftrightarrow (-x, y)$

$(x, -y) \leftrightarrow (-x, y)$

but the third statement is equivalent to origin symmetry.

If the equation has *x*-axis and origin symmetry, then we have the following:

$x-$axis symmetry means $(x, y) \leftrightarrow (x, -y)$

origin symmetry means $(x.y) \leftrightarrow (-x, -y)$

$(x, -y) \leftrightarrow (-x, -y)$

but the third statement is equivalent to *y*-axis symmetry.

If the equation has *y*-axis and origin symmetry, then we have the following:

$y-$axis symmetry means $(x, y) \leftrightarrow (-x, y)$

origin symmetry means $(x.y) \leftrightarrow (-x, -y)$

$(-x, y) \leftrightarrow (-x, -y)$

but the third statement is equivalent to *x*-axis symmetry.

56. Answers will vary.