

Graphs

2.3 Lines

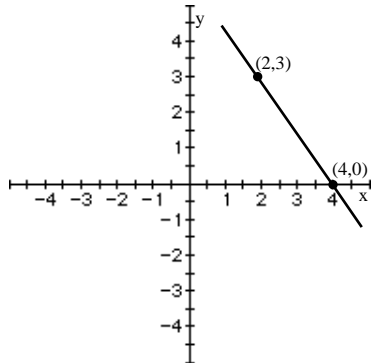
1. (a) $\text{Slope} = \frac{1-0}{2-0} = \frac{1}{2}$
 (b) If x increases by 2 units, y will increase by 1 unit.

2. (a) $\text{Slope} = \frac{1-0}{-2-0} = -\frac{1}{2}$
 (b) If x increases by 2 units, y will decrease by 1 unit.

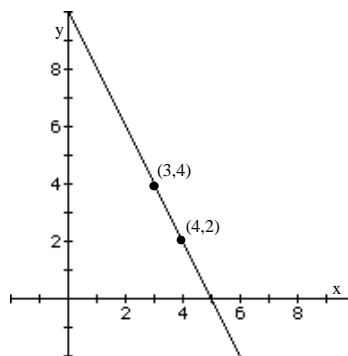
3. (a) $\text{Slope} = \frac{1-2}{1-(-2)} = \frac{-1}{3}$
 (b) If x increases by 3 units, y will decrease by 1 unit.

4. (a) $\text{Slope} = \frac{2-1}{2-(-1)} = \frac{1}{3}$
 (b) If x increases by 3 units, y will increase by 1 unit.

5. $(x_1, y_1) \quad (x_2, y_2)$
 $(2, 3) \quad (4, 0)$
 $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{4 - 2} = \frac{-3}{2}$

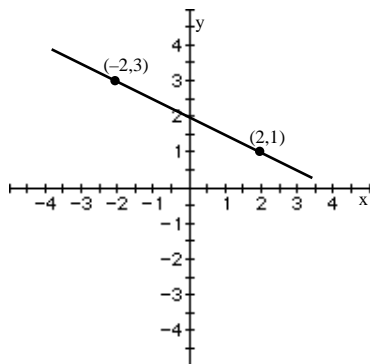


6. $(x_1, y_1) \quad (x_2, y_2)$
 $(4, 2) \quad (3, 4)$
 $\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{3 - 4} = \frac{2}{-1} = -2$



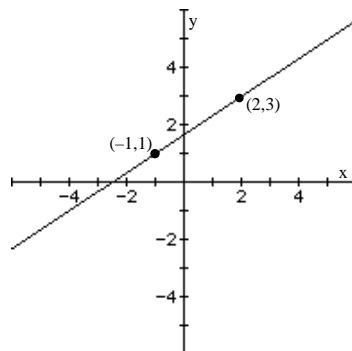
$$7. \quad \begin{matrix} (x_1, y_1) & (x_2, y_2) \\ (-2, 3) & (2, 1) \end{matrix}$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{2 - (-2)} = \frac{-2}{4} = -\frac{1}{2}$$



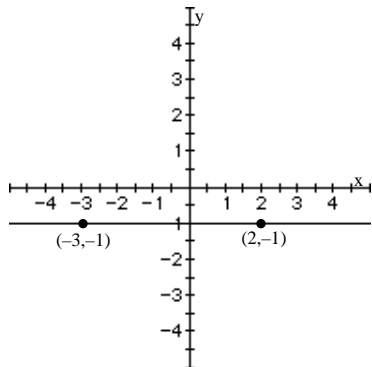
$$8. \quad \begin{matrix} (x_1, y_1) & (x_2, y_2) \\ (-1, 1) & (2, 3) \end{matrix}$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{2 - (-1)} = \frac{2}{3}$$



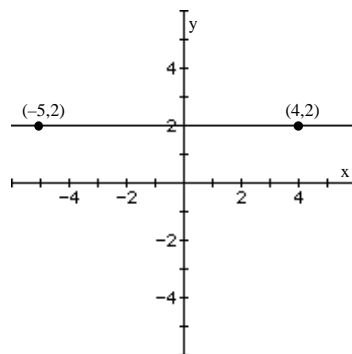
$$9. \quad \begin{matrix} (x_1, y_1) & (x_2, y_2) \\ (-3, -1) & (2, -1) \end{matrix}$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-1)}{2 - (-3)} = \frac{0}{5} = 0$$



$$10. \quad \begin{matrix} (x_1, y_1) & (x_2, y_2) \\ (4, 2) & (-5, 2) \end{matrix}$$

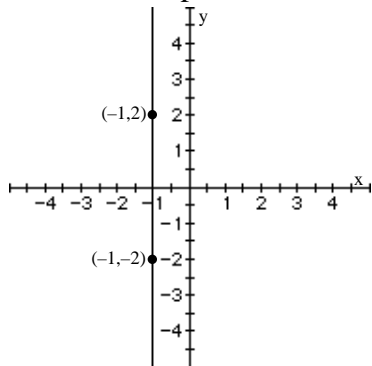
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{-5 - 4} = \frac{0}{-9} = 0$$



$$11. \quad \begin{matrix} (x_1, y_1) & (x_2, y_2) \\ (-1, 2) & (-1, -2) \end{matrix}$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{-1 - (-1)} = \frac{-4}{0}$$

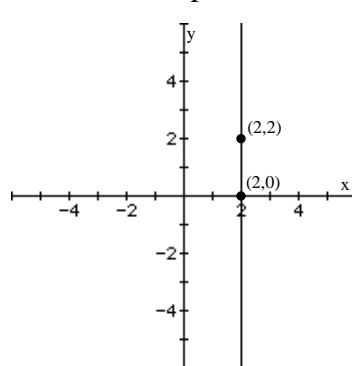
Slope is undefined.



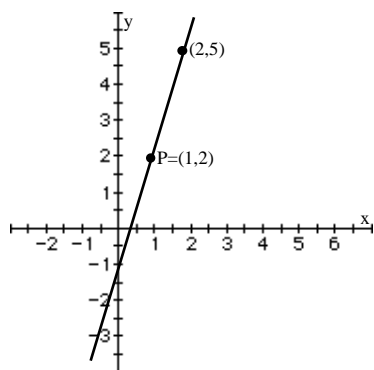
$$12. \quad \begin{matrix} (x_1, y_1) & (x_2, y_2) \\ (2, 0) & (2, 2) \end{matrix}$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{2 - 2} = \frac{2}{0}$$

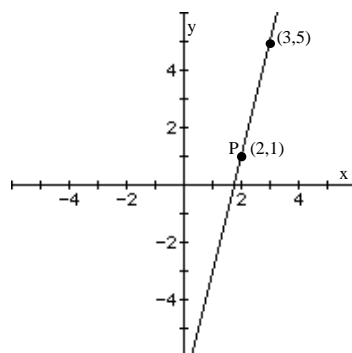
Slope is undefined.



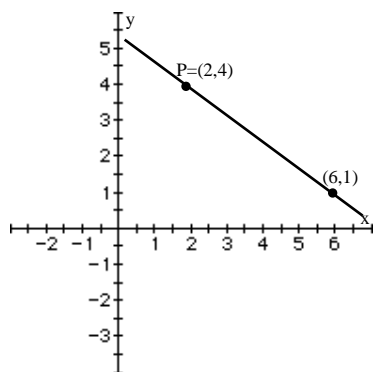
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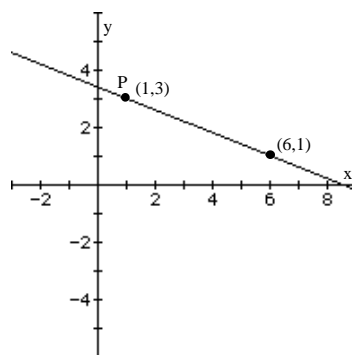
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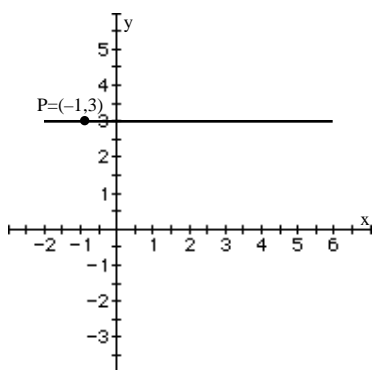
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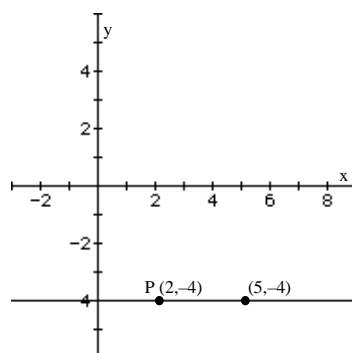
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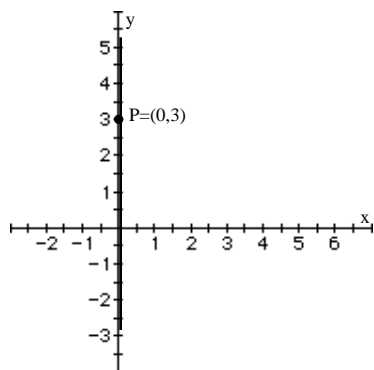
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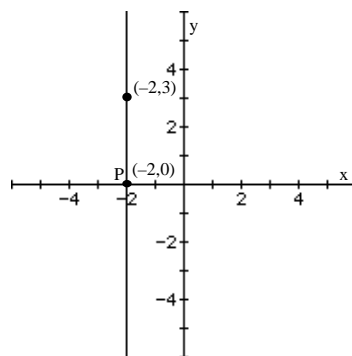
18.



19.



20.



21. Slope = 4 if x increases by 1,
 y increases by 4
 original point $(1,2)$

Answers will vary. Three possible points are:

$$x = 1 + 1 = 2 \text{ and } y = 2 + 4 = 6$$

$$(2,6)$$

$$x = 2 + 1 = 3 \text{ and } y = 6 + 4 = 10$$

$$(3,10)$$

$$x = 3 + 1 = 4 \text{ and } y = 10 + 4 = 14$$

$$(4,14)$$

22. Slope = 2 if x increases by 1,
 y increases by 2
 original point $(-2,3)$

Answers will vary. Three possible points are:

$$x = -2 + 1 = -1 \text{ and } y = 3 + 2 = 5$$

$$(-1,5)$$

$$x = -1 + 1 = 0 \text{ and } y = 5 + 2 = 7$$

$$(0,7)$$

$$x = 0 + 1 = 1 \text{ and } y = 7 + 2 = 9$$

$$(1,9)$$

23. Slope = $-\frac{3}{2}$ if x increases by 2,
 y decreases by 3
 original point $(2,-4)$

Answers will vary. Three possible points are:

$$x = 2 + 2 = 4 \text{ and } y = -4 - 3 = -7$$

$$(4,-7)$$

$$x = 4 + 2 = 6 \text{ and } y = -7 - 3 = -10$$

$$(6,-10)$$

$$x = 6 + 2 = 8 \text{ and } y = -10 - 3 = -13$$

$$(8,-13)$$

24. Slope = $\frac{4}{3}$ if x increases by 3,
 y increases by 4
 original point $(-3,2)$

Answers will vary. Three possible points are:

$$x = -3 + 3 = 0 \text{ and } y = 2 + 4 = 6$$

$$(0,6)$$

$$x = 0 + 3 = 3 \text{ and } y = 6 + 4 = 10$$

$$(3,10)$$

$$x = 3 + 3 = 6 \text{ and } y = 10 + 4 = 14$$

$$(6,14)$$

25. Slope = -2 if x increases by 1,
 y decreases by 2
 original point $(-2,-3)$

Answers will vary. Three possible points are:

$$x = -2 + 1 = -1 \text{ and } y = -3 - 2 = -5$$

$$(-1,-5)$$

$$x = -1 + 1 = 0 \text{ and } y = -5 - 2 = -7$$

$$(0,-7)$$

$$x = 0 + 1 = 1 \text{ and } y = -7 - 2 = -9$$

$$(1,-9)$$

26. Slope = -1 if x increases by 1,
 y decreases by 1
 original point $(4,1)$

Answers will vary. Three possible points are:

$$x = 4 + 1 = 5 \text{ and } y = 1 - 1 = 0$$

$$(5,0)$$

$$x = 5 + 1 = 6 \text{ and } y = 0 - 1 = -1$$

$$(6,-1)$$

$$x = 6 + 1 = 7 \text{ and } y = -1 - 1 = -2$$

$$(7,-2)$$

27. (0,0) and (2,1) are points on the line.

$$\text{Slope} = \frac{1-0}{2-0} = \frac{1}{2}$$

y - intercept is 0; using $y = mx + b$:

$$y = \frac{1}{2}x + 0$$

$$2y = x$$

$$0 = x - 2y$$

$$x - 2y = 0 \text{ or } y = \frac{1}{2}x$$

28. (0,0) and (-2,1) are points on the line.

$$\text{Slope} = \frac{1-0}{-2-0} = \frac{1}{-2} = -\frac{1}{2}$$

y - intercept is 0; using $y = mx + b$:

$$y = -\frac{1}{2}x + 0$$

$$2y = -x$$

$$x + 2y = 0$$

$$x + 2y = 0 \text{ or } y = -\frac{1}{2}x$$

29. (-2,2) and (1,1) are points on the line.

$$\text{Slope} = \frac{1-2}{1-(-2)} = \frac{1}{3}$$

Using $y - y_1 = m(x - x_1)$

$$y - 1 = -\frac{1}{3}(x - 1)$$

$$y - 1 = -\frac{1}{3}x + \frac{1}{3}$$

$$y = -\frac{1}{3}x + \frac{4}{3}$$

$$x + 3y = 4 \text{ or } y = -\frac{1}{3}x + \frac{4}{3}$$

30. (-1,1) and (2,2) are points on the line.

$$\text{Slope} = \frac{2-1}{2-(-1)} = \frac{1}{3}$$

Using $y - y_1 = m(x - x_1)$

$$y - 1 = \frac{1}{3}[x - (-1)]$$

$$y - 1 = \frac{1}{3}(x + 1)$$

$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{4}{3}$$

$$x - 3y = -4 \text{ or } y = \frac{1}{3}x + \frac{4}{3}$$

31. Slope = 3; containing (-2,3)

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 3(x - (-2))$$

$$y - 3 = 3x + 6$$

$$y = 3x + 9$$

$$3x - y = -9 \text{ or } y = 3x + 9$$

32. Slope = 2; containing the point (4,-3)

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = 2(x - 4)$$

$$y + 3 = 2x - 8$$

$$y = 2x - 11$$

$$2x - y = 11 \text{ or } y = 2x - 11$$

33. Slope =
- $-\frac{2}{3}$
- ; containing (1,-1)

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = \frac{-2}{3}(x - 1)$$

$$y + 1 = \frac{-2}{3}x + \frac{2}{3}$$

$$y = \frac{-2}{3}x - \frac{1}{3}$$

$$2x + 3y = -1 \text{ or } y = \frac{-2}{3}x - \frac{1}{3}$$

34. Slope =
- $\frac{1}{2}$
- ; containing the point (3, 1)

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 3)$$

$$y - 1 = \frac{1}{2}x - \frac{3}{2}$$

$$y = \frac{1}{2}x - \frac{1}{2}$$

$$x - 2y = 1 \text{ or } y = \frac{1}{2}x - \frac{1}{2}$$

35. Containing (1,3) and (-1,2)

$$m = \frac{2-3}{-1-1} = \frac{-1}{-2} = \frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{2}(x - 1)$$

$$y - 3 = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}x + \frac{5}{2}$$

$$x - 2y = -5 \text{ or } y = \frac{1}{2}x + \frac{5}{2}$$

36. Containing the points (-3,4) and (2,5)

$$m = \frac{5-4}{2-(-3)} = \frac{1}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{1}{5}(x - 2)$$

$$y - 5 = \frac{1}{5}x - \frac{2}{5}$$

$$y = \frac{1}{5}x + \frac{23}{5}$$

$$x - 5y = -23 \text{ or } y = \frac{1}{5}x + \frac{23}{5}$$

37. Slope = -3; y-intercept = 3

$$y = mx + b$$

$$y = -3x + 3$$

$$3x + y = 3 \text{ or } y = -3x + 3$$

38. Slope = -2; y-intercept = -2

$$y = mx + b$$

$$y = -2x + (-2)$$

$$y = -2x - 2$$

$$2x + y = -2 \text{ or } y = -2x - 2$$

39. x-intercept = 2; y-intercept = -1

Points are (2,0) and (0,-1)

$$m = \frac{-1-0}{0-2} = \frac{-1}{-2} = \frac{1}{2}$$

$$y = mx + b$$

$$y = \frac{1}{2}x - 1$$

$$x - 2y = 2 \text{ or } y = \frac{1}{2}x - 1$$

40. x-intercept = -4; y-intercept = 4

Points are (-4, 0) and (0, 4)

$$m = \frac{4-0}{0-(-4)} = \frac{4}{4} = 1$$

$$y = mx + b$$

$$y = 1x + 4$$

$$y = x + 4$$

$$x - y = -4 \text{ or } y = x + 4$$

41. Slope undefined; passing through (2,4)

This is a vertical line.

$$x = 2$$

No slope intercept form.

42. Slope undefined; containing the point

(3, 8)

This is a vertical line.

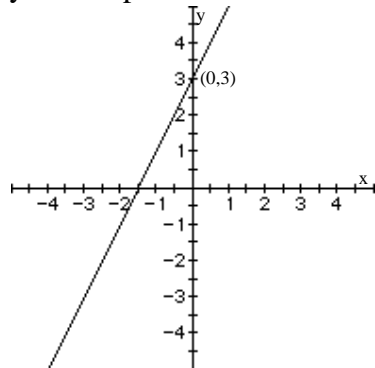
$$x = 3$$

No slope intercept form.

- 43.
- $y = 2x + 3$

Slope = 2

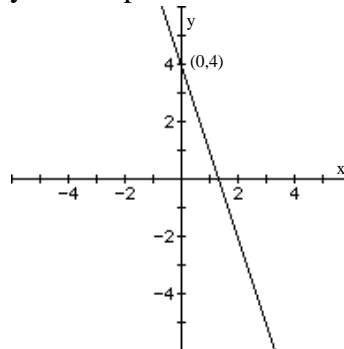
y-intercept = 3



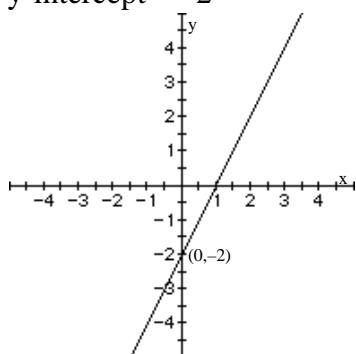
- 44.
- $y = -3x + 4$

Slope = -3

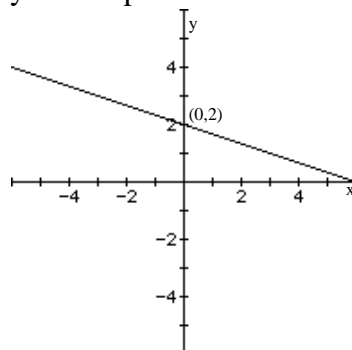
y-intercept = 4



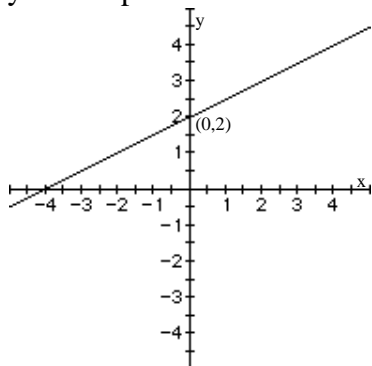
45. $\frac{1}{2}y = x - 1$
 $y = 2x - 2$
 Slope = 2
 y-intercept = -2



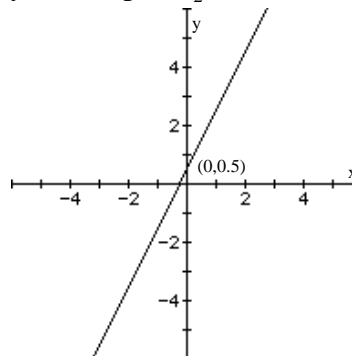
46. $\frac{1}{3}x + y = 2$
 $y = -\frac{1}{3}x + 2$
 Slope = $-\frac{1}{3}$
 y-intercept = 2



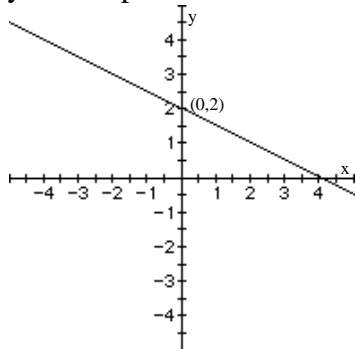
47. $y = \frac{1}{2}x + 2$
 Slope = $\frac{1}{2}$
 y-intercept = 2



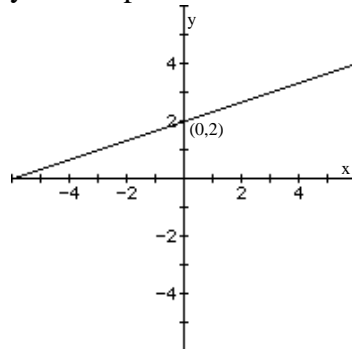
48. $y = 2x + \frac{1}{2}$
 Slope = 2
 y-intercept = $\frac{1}{2}$



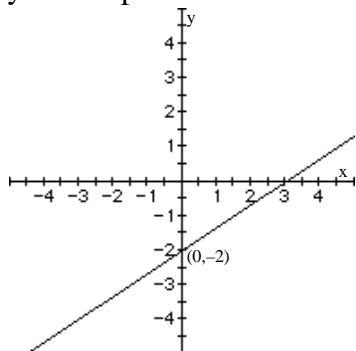
49. $x + 2y = 4$
 $2y = -x + 4$
 $y = -\frac{1}{2}x + 2$
 Slope = $-\frac{1}{2}$
 y-intercept = 2



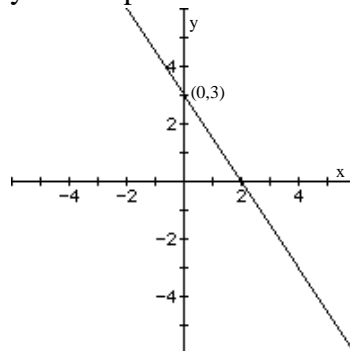
50. $-x + 3y = 6$
 $3y = x + 6$
 $y = \frac{1}{3}x + 2$
 Slope = $\frac{1}{3}$
 y-intercept = 2



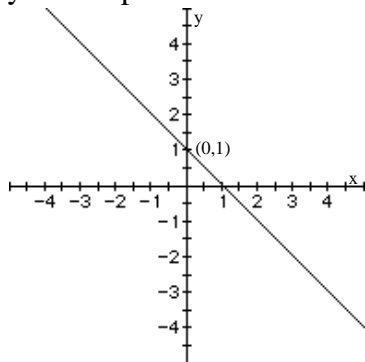
51. $2x - 3y = 6$
 $-3y = -2x + 6$
 $y = \frac{2}{3}x - 2$
Slope = $\frac{2}{3}$
y-intercept = -2



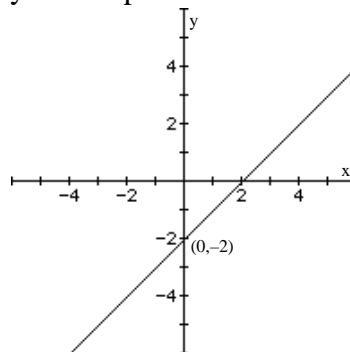
52. $3x + 2y = 6$
 $2y = -3x + 6$
 $y = -\frac{3}{2}x + 3$
Slope = $-\frac{3}{2}$
y-intercept = 3



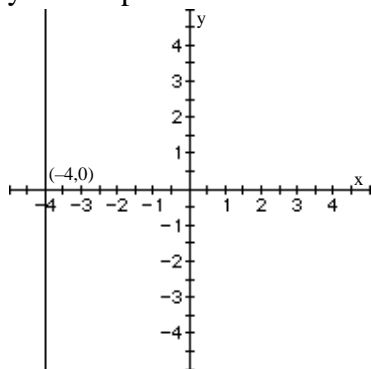
53. $x + y = 1$
 $y = -x + 1$
Slope = -1
y-intercept = 1



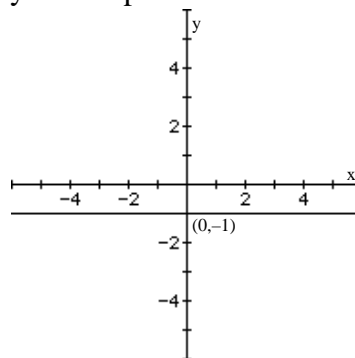
54. $x - y = 2$
 $y = x - 2$
Slope = 1
y-intercept = -2



55. $x = -4$
Slope is undefined
y-intercept - none



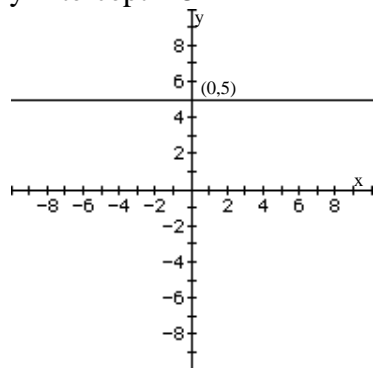
56. $y = -1$
Slope = 0
y-intercept = -1



57. $y = 5$

Slope = 0

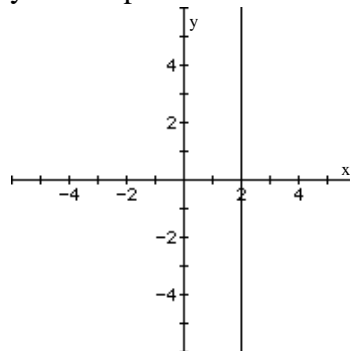
y-intercept = 5



58. $x = 2$

Slope is undefined

y-intercept - none

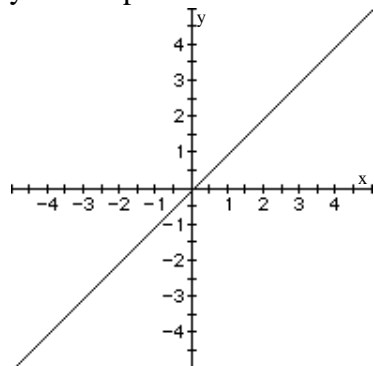


59. $y - x = 0$

 $y = x$

Slope = 1

y-intercept = 0

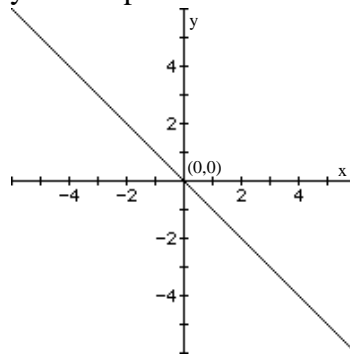


60. $x + y = 0$

 $y = -x$

Slope = -1

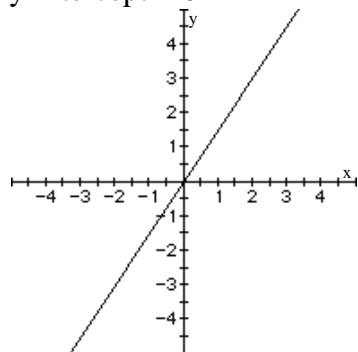
y-intercept = 0



61. $2y - 3x = 0$

 $2y = 3x$ $y = \frac{3}{2}x$ Slope = $\frac{3}{2}$

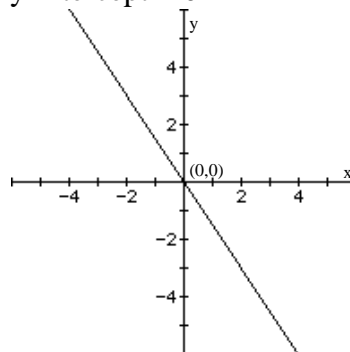
y-intercept = 0



62. $3x + 2y = 0$

 $2y = -3x$ $y = -\frac{3}{2}x$ Slope = $-\frac{3}{2}$

y-intercept = 0

63. The equation of the x-axis is $y = 0$. (The slope is 0 and the y-intercept is 0.)64. The equation of the y-axis is $x = 0$. (The slope is undefined.)

65. $(^{\circ}C, ^{\circ}F) = (0, 32); (^{\circ}C, ^{\circ}F) = (100, 212)$

$$\text{slope} = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$$

$$^{\circ}F - 32 = \frac{9}{5}(^{\circ}C - 0)$$

$$^{\circ}F - 32 = \frac{9}{5}(^{\circ}C)$$

$$^{\circ}C = \frac{5}{9}(^{\circ}F - 32)$$

 If $^{\circ}F = 70$, then

$$^{\circ}C = \frac{5}{9}(70 - 32) = \frac{5}{9}(38)$$

$$^{\circ}C = 21^{\circ}$$

66. (a) $K = ^{\circ}C + 273$

(b) $^{\circ}C = \frac{5}{9}(^{\circ}F - 32)$

$$K = \frac{5}{9}(^{\circ}F - 32) + 273$$

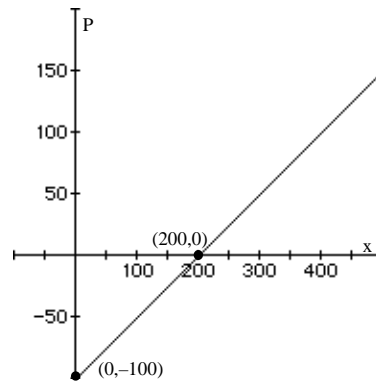
$$K = \frac{5}{9}^{\circ}F - \frac{160}{9} + 273$$

$$K = \frac{5}{9}^{\circ}F + \frac{2297}{9}$$

67. (a) Since there is only a profit of \$0.50 per copy and the expense of \$100 must be deducted, the profit is:

$$\begin{aligned} P &= 0.50x - 100 \\ \text{(b)} \quad P &= 0.50(1000) - 100 \\ &= 500 - 100 \\ &= \$400 \end{aligned}$$

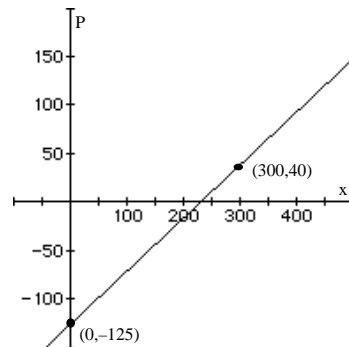
$$\begin{aligned} \text{(c)} \quad P &= 0.50(5000) - 100 \\ &= 2500 - 100 \\ &= \$2400 \end{aligned}$$



68. (a) Since there is only a profit of \$0.55 per copy and the expense of \$125 must be deducted, the profit is:

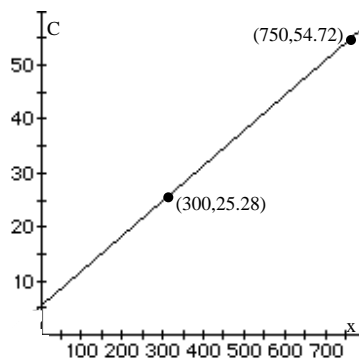
$$\begin{aligned} P &= 0.55x - 125 \\ \text{(b)} \quad P &= 0.55(1000) - 125 \\ &= 550 - 125 \\ &= \$425 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad P &= 0.55(5000) - 125 \\ &= 2750 - 125 \\ &= \$2625 \end{aligned}$$



Chapter 2 Graphs

69. $C = 0.06543x + 5.65$
 For 300 kWh,
 $C = 0.06543(300) + 5.65$
 $= \$25.28$
 For 750 kWh,
 $C = 0.06543(750) + 5.65$
 $= \$54.72$



70. given x -intercept $(a,0)$ and y -intercept $(0,b)$,

$$\text{slope} = \frac{b-0}{0-a} = -\frac{b}{a}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{b}{a}(x - a)$$

$$y = -\frac{b}{a}x + b$$

$$ay = -x + ba \quad bx + ay = ba$$

now divide each term by ab to get $\frac{bx}{ab} + \frac{ay}{ab} = \frac{ba}{ab} \quad \frac{x}{a} + \frac{y}{b} = 1$

71. (b) 72. (c) 73. (d) 74. (a)
75. Slope = 1; y -intercept = 2
 $y = x + 2$ or $x - y = -2$
76. Slope = -1; y -intercept = 1
 $y = -x + 1$ or $x + y = 1$
77. Slope = $-\frac{1}{3}$; y -intercept = 1
 $y = -\frac{1}{3}x + 1$ or $x + 3y = 3$
78. Slope = $-\frac{1}{2}$; y -intercept = -1
 $y = -\frac{1}{2}x - 1$ or $x + 2y = -2$
79. (b), (c), (e) and (g) 80. (a), (c), (f) and (g)
81. Answers will vary.
82. A vertical line cannot be written in slope-intercept form since the slope is undefined.
83. Not every line has two distinct intercepts since a horizontal line might not touch the x -axis and a vertical line might not touch the y -axis.
 A line must have at least one intercept since a vertical line always crosses the x -axis, a horizontal line always crosses the y -axis and a non-vertical, non-horizontal line always crosses both axes.
84. Two lines that have equal slopes and equal y -intercepts must be represented by equivalent equations.

- 85. Two lines with the same non-zero x -intercept and the same y -intercept must have the same slope and therefore must be represented by equivalent equations.
- 86. Two lines that have the same slope but different x -intercepts cannot have the same y -intercept.
- 87. Two lines that have the same y -intercept but different slopes can only have the same x -intercept if the y -intercept is zero.
- 88 – 90. Answers will vary.