Chapter 2

Graphs

2.4 Parallel and Perpendicular Lines; Circles

1. parallel line: slope = 6
   perpendicular line: slope = $-\frac{1}{6}$

2. parallel line: slope = -3
   perpendicular line: slope = $\frac{1}{3}$

3. parallel line: slope = $-\frac{1}{2}$
   perpendicular line: slope = 2

4. parallel line: slope = $\frac{2}{3}$
   perpendicular line: slope = $-\frac{3}{2}$

5. $2x - 4y + 5 = 0 \rightarrow y = \frac{1}{2}x + \frac{5}{4}$
   parallel line: slope = $\frac{1}{2}$
   perpendicular line: slope = - 2

6. $3x + y = 4 \rightarrow y = -3x + 4$
   parallel line: slope = - 3
   perpendicular line: slope = $\frac{1}{3}$

7. $3x + 5y - 10 = 0 \rightarrow y = -\frac{3}{5}x + 2$
   parallel line: slope = $-\frac{3}{5}$
   perpendicular line: slope = $\frac{5}{3}$

8. $4x - 3y + 7 = 0 \rightarrow y = \frac{4}{3}x + \frac{7}{3}$
   parallel line: slope = $\frac{4}{3}$
   perpendicular line: slope = $-\frac{3}{4}$

9. parallel line: slope is undefined
   perpendicular line: slope = 0

10. parallel line: slope = 0
    perpendicular line: slope is undefined

11. $y - y_1 = m(x - x_1), \quad m = 2$
    $y - 3 = 2(x - 3)$
    $y - 3 = 2x - 6$
    $y = 2x - 3$
    $2x - y = 3$ or $y = 2x - 3$

12. $y - y_1 = m(x - x_1), \quad m = -1$
    $y - 2 = -1(x - 1)$
    $y - 2 = -x + 1$
    $y = -x + 3$
    $x + y = 3$ or $y = -x + 3$

13. $y - y_1 = m(x - x_1),\ \ m = \frac{-1}{2}$

$$y - 2 = \frac{-1}{2}(x - 1)$$
$$y - 2 = \frac{-1}{2}x + \frac{1}{2}$$
$$y = \frac{-1}{2}x + \frac{5}{2}$$
$$x + 2y = 5 \text{ or } y = \frac{-1}{2}x + \frac{5}{2}$$

14. $y - y_1 = m(x - x_1),\ \ m = 1$

$$y - 1 = 1(x - (-1))$$
$$y - 1 = x + 1$$
$$y = x + 2$$
$$x - y = -2 \text{ or } y = x + 2$$

15. Parallel to $y = 2x$; Slope = 2
Containing (–1,2)

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 2(x - (-1))$$
$$y - 2 = 2x + 2$$
$$y = 2x + 4$$
$$2x - y = -4 \text{ or } y = 2x + 4$$

16. Parallel to $y = -3x$; Slope = –3;
Containing the point (–1,2)

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -3(x - (-1))$$
$$y - 2 = -3x - 3$$
$$y = -3x - 1$$
$$3x + y = -1 \text{ or } y = -3x - 1$$

17. Parallel to $2x - y = -2$; Slope = 2
Containing (0,0)

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x - 0)$$
$$y = 2x$$
$$2x - y = 0 \text{ or } y = 2x$$

18. Parallel to $x - 2y = -5$; Slope = $\frac{1}{2}$;
Containing the point (0,0)

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{1}{2}(x - 0)$$
$$y = \frac{1}{2}x$$
$$x - 2y = 0 \text{ or } y = \frac{1}{2}x$$

19. Parallel to $x = 5$;
Containing (4,2)
This is a vertical line.
$x = 4$
No slope intercept form.

20. Parallel to $y = 5$;
Containing the point (4,2)
This is a horizontal line. Slope = 0
$y = 2$

21. Perpendicular to $y = \frac{1}{2}x + 4$;
Slope of perpendicular = –2
Containing (1,–2)

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -2(x - 1)$$
$$y + 2 = -2x + 2$$
$$y = -2x$$
$$2x + y = 0 \text{ or } y = -2x$$

22. Perpendicular to $y = 2x - 3$;
Slope of perpendicular = $-\frac{1}{2}$
Containing the point (1,–2)

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{1}{2}(x - 1)$$
$$y + 2 = -\frac{1}{2}x + \frac{1}{2}$$
$$y = -\frac{1}{2}x - \frac{3}{2}$$
$$x + 2y = -3 \text{ or } y = -\frac{1}{2}x - \frac{3}{2}$$

23. Perpendicular to $2x + y = 2$;
Containing (–3,0)
Slope of perpendicular = $\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{1}{2}(x - (-3))$$
$$y = \frac{1}{2}x + \frac{3}{2}$$
$$x - 2y = -3 \text{ or } y = \frac{1}{2}x + \frac{3}{2}$$

24. Perpendicular to $x - 2y = -5$;
Slope of perpendicular = –2
Containing the point (0, 4)

$$y = mx + b$$
$$y = -2x + 4$$
$$2x + y = 4 \text{ or } y = -2x + 4$$

25. Perpendicular to $x = 8$;
Slope of perpendicular = 0
Containing (3,4)

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 0(x - 3)$$
$$y - 4 = 0$$
$$y = 4$$
$$y = 4 \text{ or } y = 0x + 4$$

26. Perpendicular to $y = 8$;
Slope of perpendicular is undefined.
Containing the point (3,4)
$x = 3$ No slope-intercept form.

27. Center = (2, 1)
Radius = distance from (0,1) to (2,1)

$$= \sqrt{(2-0)^2 + (1-1)^2}$$
$$= \sqrt{4} = 2$$
$$(x-2)^2 + (y-1)^2 = 4$$

28. Center = (1, 2)
Radius = distance from (1,0) to (1,2)

$$= \sqrt{(1-1)^2 + (2-0)^2}$$
$$= \sqrt{4} = 2$$
$$(x-1)^2 + (y-2)^2 = 4$$

29. Center = midpoint of (1,2) and (4,2)

$$= \left(\frac{1+4}{2}, \frac{2+2}{2}\right) = \left(\frac{5}{2}, 2\right)$$

Radius = distance from $\left(\frac{5}{2}, 2\right)$ to $(4,2)$

$$= \sqrt{\left(4 - \frac{5}{2}\right)^2 + (2-2)^2}$$
$$= \sqrt{\frac{9}{4}} = \frac{3}{2}$$
$$\left(x - \frac{5}{2}\right)^2 + (y-2)^2 = \frac{9}{4}$$

30. Center = midpoint of (0,1) and (2,3)

$$= \left(\frac{0+2}{2}, \frac{1+3}{2}\right) = (1, 2)$$

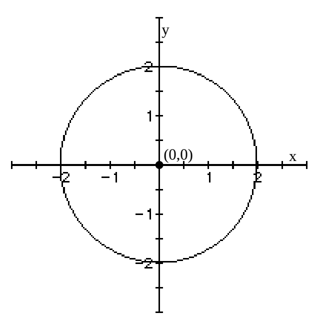
Radius = distance from $(1,2)$ to $(2,3)$

$$= \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{2}$$
$$(x-1)^2 + (y-2)^2 = 2$$

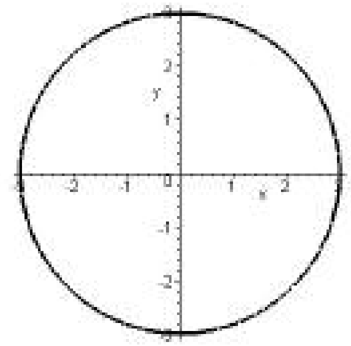
31. $(x-h)^2 + (y-k)^2 = r^2$

$$(x-0)^2 + (y-0)^2 = 2^2$$
$$x^2 + y^2 = 4$$

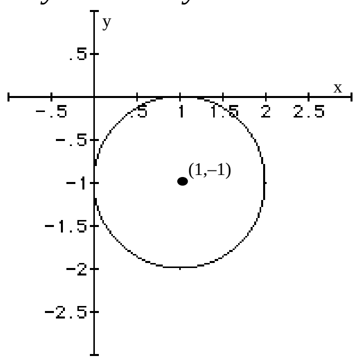
General form:
$x^2 + y^2 - 4 = 0$



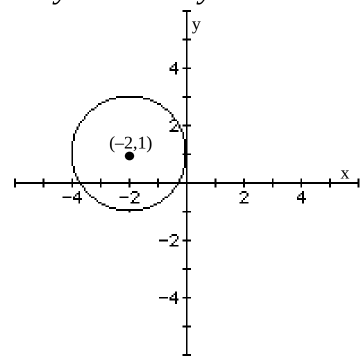
32. $(x-h)^2+(y-k)^2 = r^2$
$(x-0)^2+(y-0)^2 = 3^2$
$x^2+y^2=9$
General form:
$x^2+y^2-9=0$



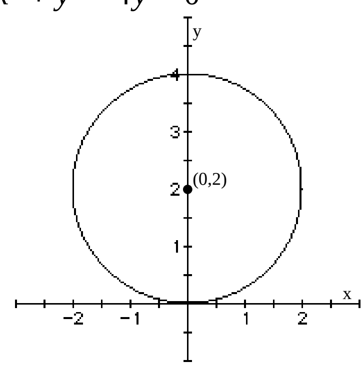
33. $(x-h)^2+(y-k)^2 = r^2$
$(x-1)^2+(y-(-1))^2 = 1^2$
$(x-1)^2+(y+1)^2 = 1$
General form:
$x^2-2x+1+y^2+2y+1=1$
$x^2+y^2-2x+2y+1=0$



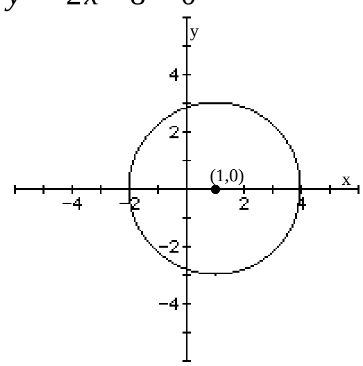
34. $(x-h)^2+(y-k)^2 = r^2$
$(x-(-2))^2+(y-1)^2 = 2^2$
$(x+2)^2+(y-1)^2 = 4$
General form:
$x^2+4x+4+y^2-2y+1=4$
$x^2+y^2+4x-2y+1=0$



35. $(x-h)^2+(y-k)^2 = r^2$
$(x-0)^2+(y-2)^2 = 2^2$
$x^2+(y-2)^2 = 4$
General form:
$x^2+y^2-4y+4=4$
$x^2+y^2-4y=0$



36. $(x-h)^2+(y-k)^2 = r^2$
$(x-1)^2+(y-0)^2 = 3^2$
$(x-1)^2+y^2 = 9$
General form:
$x^2-2x+1+y^2=9$
$x^2+y^2-2x-8=0$



37. $(x-h)^2+(y-k)^2=r^2$

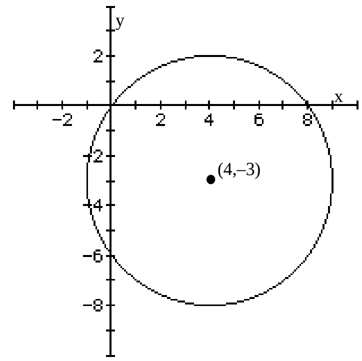
$(x-4)^2+(y-(-3))^2=5^2$

$(x-4)^2+(y+3)^2=25$

General form:

$x^2-8x+16+y^2+6y+9=25$

$x^2+y^2-8x+6y=0$



38. $(x-h)^2+(y-k)^2=r^2$

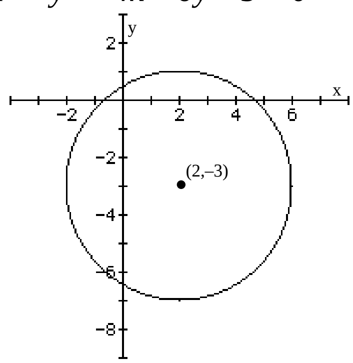
$(x-2)^2+(y-(-3))^2=4^2$

$(x-2)^2+(y+3)^2=16$

General form:

$x^2-4x+4+y^2+6y+9=16$

$x^2+y^2-4x+6y-3=0$

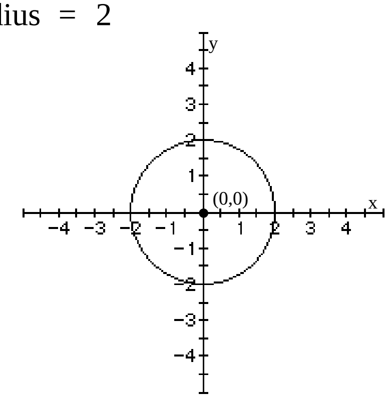


39. $x^2+y^2=4$

$x^2+y^2=2^2$

Center: $(0,0)$

Radius = 2

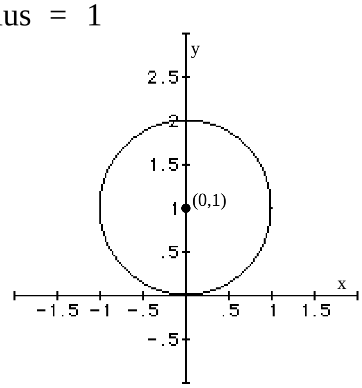


40. $x^2+(y-1)^2=1$

$x^2+(y-1)^2=1^2$

Center: $(0,1)$

Radius = 1

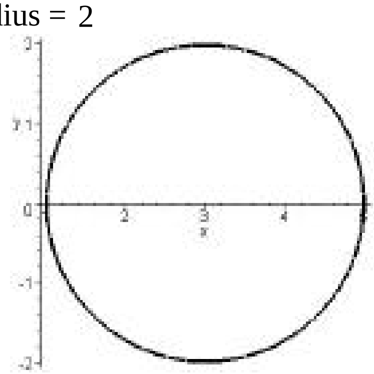


41. $2(x-3)^2+2y^2=8$

$(x-3)^2+y^2=4$

Center: (3,0)

Radius = 2

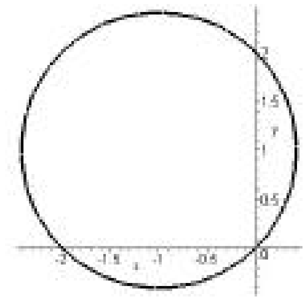


42. $3(x+1)^2 + 3(y-1)^2 = 6$

$(x+1)^2 + (y-1)^2 = 2$

Center: (–1,1)

Radius = $\sqrt{2}$



43. $x^2 + y^2 + 4x - 4y - 1 = 0$

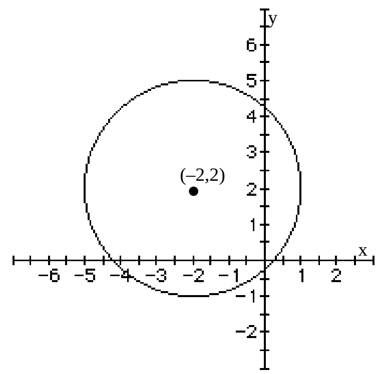
$x^2 + 4x + y^2 - 4y = 1$

$(x^2 + 4x + 4) + (y^2 - 4y + 4) = 1 + 4 + 4$

$(x+2)^2 + (y-2)^2 = 3^2$

Center: (–2,2)

Radius = 3



44. $x^2 + y^2 - 6x + 2y + 9 = 0$

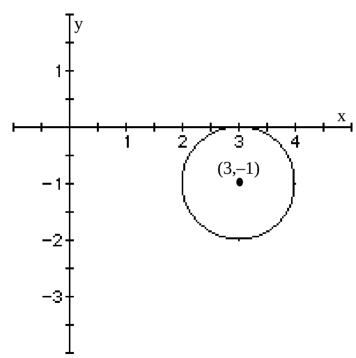
$x^2 - 6x + y^2 + 2y = -9$

$(x^2 - 6x + 9) + (y^2 + 2y + 1) = -9 + 9 + 1$

$(x-3)^2 + (y+1)^2 = 1^2$

Center: (3, –1)

Radius = 1



45. $x^2 + y^2 - x + 2y + 1 = 0$

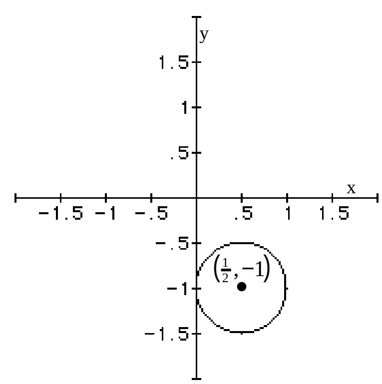
$x^2 - x + y^2 + 2y = -1$

$\left(x^2 - x + \frac{1}{4}\right) + (y^2 + 2y + 1) = -1 + \frac{1}{4} + 1$

$\left(x - \frac{1}{2}\right)^2 + (y+1)^2 = \left(\frac{1}{2}\right)^2$

Center: $\left(\frac{1}{2}, -1\right)$

Radius = $\frac{1}{2}$



46. $x^2+y^2+x+y-\frac{1}{2}=0$

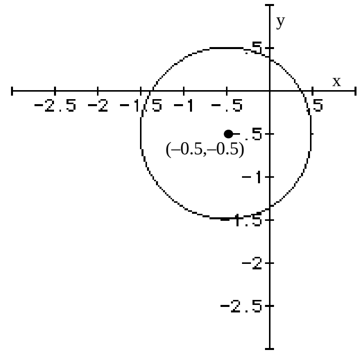
$$x^2+x+y^2+y=\tfrac{1}{2}$$

$$\left(x^2+x+\frac{1}{4}\right)+\left(y^2+y+\frac{1}{4}\right)=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$$

$$\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$$

Center: $\left(-\frac{1}{2},-\frac{1}{2}\right)$

Radius = 1



47. $2x^2+2y^2-12x+8y-24=0$

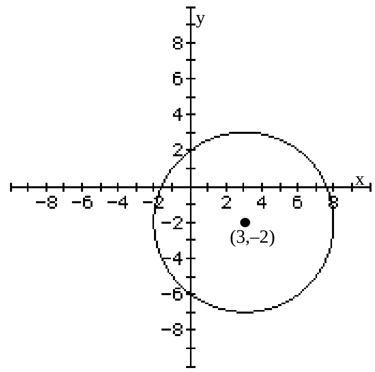
$$x^2+y^2-6x+4y=12$$

$$x^2-6x+y^2+4y=12$$

$$(x^2-6x+9)+(y^2+4y+4)=12+9+4$$

$$(x-3)^2+(y+2)^2=5^2$$

Center: (3,–2)
Radius = 5



48. $2x^2+2y^2+8x+7=0$

$$x^2+y^2+4x=-\tfrac{7}{2}$$

$$x^2+4x+y^2=-\tfrac{7}{2}$$

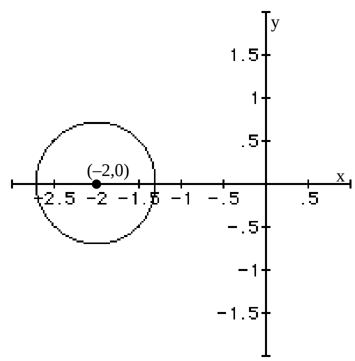
$$(x^2+4x+4)+y^2=-\tfrac{7}{2}+4$$

$$(x+2)^2+y^2=\tfrac{1}{2}$$

$$(x+2)^2+y^2=\left(\frac{\sqrt{2}}{2}\right)^2$$

Center: (–2, 0)

Radius = $\frac{\sqrt{2}}{2}$



49. Center at (0,0); containing point (- 3, 2).

$$r=\sqrt{(-3-0)^2+(2-0)^2}=\sqrt{9+4}=\sqrt{13}$$

Equation:

$$(x+3)^2+(y-2)^2=\left(\sqrt{13}\right)^2$$

$$x^2+6x+9+y^2-4y+4=13$$

$$x^2+y^2+6x-4y=0$$

50. Center at (1,0); containing point (- 2, 3).

$$r = \sqrt{(-2-1)^2 + (3-0)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

Equation:

$$(x-1)^2 + (y-0)^2 = \left(\sqrt{18}\right)^2$$
$$x^2 - 2x + 1 + y^2 = 18$$
$$x^2 + y^2 - 2x - 17 = 0$$

51. Center at (2,3); tangent to the x-axis.

$$r = 3$$

Equation:

$$(x-2)^2 + (y-3)^2 = 3^2$$
$$x^2 - 4x + 4 + y^2 - 6y + 9 = 9$$
$$x^2 + y^2 - 4x - 6y + 4 = 0$$

52. Center at (–3, 1); tangent to the y-axis.

$$r = 3$$

Equation:

$$(x+3)^2 + (y-1)^2 = 3^2$$
$$x^2 + 6x + 9 + y^2 - 2y + 1 = 9$$
$$x^2 + y^2 + 6x - 2y + 1 = 0$$

53. Endpoints of a diameter are (1,4) and (–3,2).
The center is at the midpoint of the diameter:

Center: $\left(\frac{1+(-3)}{2}, \frac{4+2}{2}\right) = (-1,3)$

Radius: $r = \sqrt{(1-(-1))^2 + (4-3)^2} = \sqrt{4+1} = \sqrt{5}$

Equation:

$$(x-(-1))^2 + (y-3)^2 = \left(\sqrt{5}\right)^2$$
$$x^2 + 2x + 1 + y^2 - 6y + 9 = 5$$
$$x^2 + y^2 + 2x - 6y + 5 = 0$$

54. Endpoints of a diameter are (4, 3) and (0, 1).
The center is at the midpoint of the diameter:

Center: $\left(\frac{4+0}{2}, \frac{3+1}{2}\right) = (2, 2)$

Radius: $r = \sqrt{(4-2)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$

Equation:

$$(x-2)^2 + (y-2)^2 = \left(\sqrt{5}\right)^2$$
$$x^2 - 4x + 4 + y^2 - 4y + 4 = 5$$
$$x^2 + y^2 - 4x - 4y + 3 = 0$$

55. Consider the points $A(-2,5)$, $B(1,3)$ and $C(-1,0)$

$$\text{slope of } \overline{AB} = \frac{3-5}{1-(-2)} = -\frac{2}{3};\ \text{slope of } \overline{AC} = \frac{0-5}{-1-(-2)} = -\frac{5}{3};\ \text{slope of } \overline{BC} = \frac{0-3}{-1-1} = \frac{3}{2}$$

Therefore, ΔABC has a right angle at vertex B since

$$\text{slope } \overline{AB} = -\frac{2}{3} \text{ and slope } \overline{BC} = \frac{3}{2} \rightarrow \overline{AB} \quad \overline{BC}$$

56. Consider the points $A(1,-1)$, $B(4,1)$, $C(2,2)$ and $D(5,4)$

$$\text{slope of } \overline{AB} = \frac{1-(-1)}{4-1} = \frac{2}{3};\ \text{slope of } \overline{CD} = \frac{4-2}{5-2} = \frac{2}{3}$$
$$\text{slope of } \overline{AC} = \frac{2-(-1)}{2-1} = 3;\ \text{slope of } \overline{BD} = \frac{4-1}{5-4} = 3$$

Therefore, the quadrilateral $ACDB$ is a parallelogram since

$$\text{slope } \overline{AB} = \frac{2}{3} \text{ and slope } \overline{CD} = \frac{2}{3} \rightarrow \overline{AB} \text{ is parallel to } \overline{CD}$$
$$\text{slope } \overline{AC} = 3 \text{ and slope } \overline{BD} = 3 \rightarrow \overline{AC} \text{ is parallel to } \overline{BD}$$

57. Consider the points $A(-1,0)$, $B(2,3)$, $C(1,\text{-}2)$ and $D(4,1)$

$$\text{slope of } \overline{AB} = \frac{3-0}{2-(-1)} = 1;\ \text{slope of } \overline{CD} = \frac{1-(-2)}{4-1} = 1$$
$$\text{slope of } \overline{AC} = \frac{-2-0}{1-(-1)} = -1;\ \text{slope of } \overline{BD} = \frac{1-3}{4-2} = -1$$

Therefore, the quadrilateral $ACDB$ is a parallelogram since

$$\text{slope } \overline{AB} = 1 \text{ and slope } \overline{CD} = 1 \rightarrow \overline{AB} \text{ is parallel to } \overline{CD}$$
$$\text{slope } \overline{AC} = -1 \text{ and slope } \overline{BD} = -1 \rightarrow \overline{AC} \text{ is parallel to } \overline{BD}$$

Furthermore,

$$\text{slope } \overline{AB} = 1 \text{ and slope } \overline{BD} = -1 \rightarrow \overline{AB} \quad \overline{BD}$$
$$\text{slope } \overline{AC} = -1 \text{ and slope } \overline{CD} = 1 \rightarrow \overline{AC} \quad \overline{CD}$$

So the quadrilateral $ACDB$ is a rectangle.

58. Consider the points $A(0,0)$, $B(1,3)$, $C(4,2)$ and $D(3,-1)$

$$\text{slope of } \overline{AB} = \frac{3-0}{1-0} = 3; \text{ slope of } \overline{CD} = \frac{-1-2}{3-4} = 3$$
$$\text{slope of } \overline{AD} = \frac{-1-0}{3-0} = -\frac{1}{3}; \text{ slope of } \overline{BC} = \frac{2-3}{4-1} = -\frac{1}{3}$$

Therefore, the quadrilateral $ABCD$ is a parallelogram since

slope $\overline{AB} = 3$ and slope $\overline{CD} = 3 \rightarrow \overline{AB}$ is parallel to $\overline{CD}$
slope $\overline{AD} = -\frac{1}{3}$ and slope $\overline{BC} = -\frac{1}{3} \rightarrow \overline{BC}$ is parallel to $\overline{AD}$

Furthermore,

slope $\overline{AB} = 3$ and slope $\overline{BC} = -\frac{1}{3} \rightarrow \overline{AB}$ $\overline{BC}$
slope $\overline{AD} = -\frac{1}{3}$ and slope $\overline{CD} = 3 \rightarrow \overline{AD}$ $\overline{CD}$

So the quadrilateral $ABCD$ is a rectangle.

Finally, the quadrilateral is a square since

$$d(A,B) = \sqrt{(1-0)^2 + (3-0)^2} = \sqrt{10}$$
$$d(A,D) = \sqrt{(-1-0)^2 + (3-0)^2} = \sqrt{10}$$
$$d(B,C) = \sqrt{(4-1)^2 + (2-3)^2} = \sqrt{10}$$
$$d(C,D) = \sqrt{(3-4)^2 + (-1-2)^2} = \sqrt{10}$$

59. (c) 60. (d) 61. (b) 62. (a)

63. $(x+3)^2 + (y-1)^2 = 16$

64. $(x-4)^2 + (y+2)^2 = 9$

65. $(x-2)^2 + (y-2)^2 = 9$

66. $(x-1)^2 + (y-3)^2 = 4$

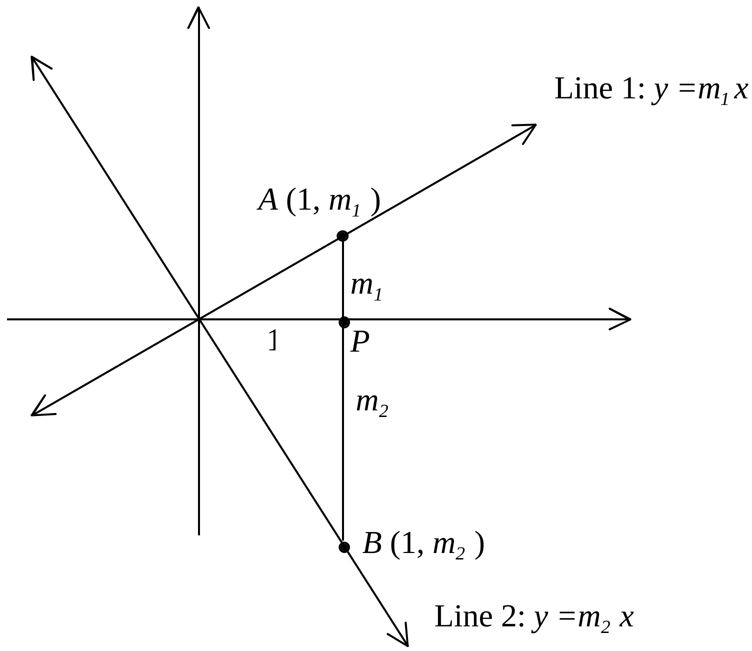
67. (b), (c), (e) and (g)

68. (b), (e) and (g)

69. (c)

70. (d)

71. Consider the diagram



$$\text{length of } \overline{OA} = \sqrt{1 + m_1^2}$$
$$\text{length of } \overline{OB} = \sqrt{1 + m_2^2}$$
$$\text{length of } \overline{AB} = m_1 - m_2$$

Now consider the equation

$$\left(\sqrt{1+m_1^2}\right)^2 + \left(\sqrt{1+m_2^2}\right)^2 = \left(m_1 - m_2\right)^2$$

If this equation is valid, then ΔAOB is a right triangle with right angle at vertex *O*.

$$\left(\sqrt{1+m_1^2}\right)^2 + \left(\sqrt{1+m_2^2}\right)^2 = \left(m_1 - m_2\right)^2$$
$$1 + m_1^2 + 1 + m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$$
$$2 + m_1^2 + m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$$

but we are assuming that $m_1m_2 = -1$, so we have

$$2 + m_1^2 + m_2^2 = m_1^2 - 2(-1) + m_2^2$$
$$2 + m_1^2 + m_2^2 = m_1^2 + 2 + m_2^2$$
$$0 = 0$$

Therefore, by the converse of the Pythagorean Theorem, ΔAOB is a right triangle with right angle at vertex *O*. Thus $\text{Line1} \quad \text{Line2}$.

72. $x^2 + y^2 + 2x + 4y - 4091 = 0$

$x^2 + 2x + y^2 + 4y - 4091 = 0$

$x^2 + 2x + 1 + y^2 + 4y + 4 = 4091 + 5$

$(x+2)^2 + (y+2)^2 = 4096$

The circle representing Earth has center (- 2, - 2) and radius = $\sqrt{4096} = 64$

So the radius of the satellite's orbit is $64 + 0.6 = 64.06$ units.

The equation of the orbit is $(x+2)^2 + (y+2)^2 = (64.06)^2$.

73. (a) $x^2 + (mx+b)^2 = r^2$

$x^2 + m^2x^2 + 2bmx + b^2 = r^2$

$(1+m^2)x^2 + 2bmx + b^2 - r^2 = 0$

There is one solution if and only if the discriminant is zero.

$$(2bm)^2 - 4(1+m^2)(b^2 - r^2) = 0$$
$$4b^2m^2 - 4b^2 + 4r^2 - 4b^2m^2 + 4m^2r^2 = 0$$
$$-4b^2 + 4r^2 + 4m^2r^2 = 0$$
$$-b^2 + r^2 + m^2r^2 = 0$$
$$r^2(1+m^2) = b^2$$

(b) Using the quadratic formula, knowing that the discriminant is zero:

$$x = \frac{-2bm}{2(1+m^2)} = \frac{-bm}{\frac{b^2}{r^2}} = \frac{-bmr^2}{b^2} = \frac{-mr^2}{b}$$

$$y = m\left(\frac{-mr^2}{b}\right) + b = \frac{-m^2r^2}{b} + b = \frac{-m^2r^2 + b^2}{b} = \frac{r^2}{b}$$

(c) The slope of the tangent line is m.

The slope of the line joining the point of tangency and the center is:

$$\frac{\frac{r^2}{b} - 0}{\frac{-mr^2}{b} - 0} = \frac{r^2}{b} \cdot \frac{b}{-mr^2} = -\frac{1}{m}$$

74. $x^2 + y^2 = 9$

Center: (0, 0)

Slope from center to $\left(1, 2\sqrt{2}\right)$ is $\frac{2\sqrt{2} - 0}{1 - 0} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$.

Slope of the tangent line is $\frac{-1}{2\sqrt{2}} = \frac{-\sqrt{2}}{4}$.

Equation of the tangent line is:

$$y - 2\sqrt{2} = \frac{-\sqrt{2}}{4}(x-1) \rightarrow y - 2\sqrt{2} = \frac{-\sqrt{2}}{4}x + \frac{\sqrt{2}}{4}$$

$$4y - 8\sqrt{2} = -\sqrt{2}\,x + \sqrt{2} \rightarrow \sqrt{2}\,x + 4y = 9\sqrt{2}$$

75. $x^2 + y^2 - 4x + 6y + 4 = 0$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -4 + 4 + 9$$
$$(x-2)^2 + (y+3)^2 = 9$$

Center: (2, –3)

Slope from center to $\left(3, 2\sqrt{2} - 3\right)$ is $\dfrac{2\sqrt{2} - 3 - (-3)}{3-2} = \dfrac{2\sqrt{2}}{1} = 2\sqrt{2}$

Slope of the tangent line is: $\dfrac{-1}{2\sqrt{2}} = \dfrac{-\sqrt{2}}{4}$

Equation of the tangent line:

$$y - \left(2\sqrt{2} - 3\right) = \frac{-\sqrt{2}}{4}(x-3) \rightarrow y - 2\sqrt{2} + 3 = \frac{-\sqrt{2}}{4}x + \frac{3\sqrt{2}}{4}$$
$$4y - 8\sqrt{2} + 12 = -\sqrt{2}x + 3\sqrt{2} \rightarrow \sqrt{2}x + 4y = 11\sqrt{2} - 12$$

76. Let (h, k) be the center of the circle.

$$x - 2y + 4 = 0$$
$$2y = x + 4$$
$$y = \tfrac{1}{2}x + 2$$

The slope of the tangent line is $\tfrac{1}{2}$. The slope from (h, k) to (0, 2) is –2.

$$\frac{2-k}{0-h} = -2 \quad \rightarrow \quad 2 - k = 2h$$

The other tangent line is $y = 2x - 7$. Its slope is 2.

The slope from (h, k) to (3, –1) is $-\tfrac{1}{2}$.

$$\frac{-1-k}{3-h} = -\frac{1}{2} \quad \rightarrow \quad 2 + 2k = 3 - h \quad \rightarrow \quad 2k = 1 - h \quad \rightarrow \quad h = 1 - 2k$$

Solve the two equations in h and k:

$$2 - k = 2(1 - 2k) \quad \rightarrow \quad 2 - k = 2 - 4k \quad \rightarrow \quad 3k = 0 \quad \rightarrow \quad k = 0$$
$$h = 1 - 2(0) = 1$$

The center of the circle is (1, 0).

77. Find the centers of the two circles:

$$x^2 + y^2 - 4x + 6y + 4 = 0$$
$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -4 + 4 + 9$$
$$(x-2)^2 + (y+3)^2 = 9 \qquad \text{Center: } (2, -3)$$
$$x^2 + y^2 + 6x + 4y + 9 = 0$$
$$(x^2 + 6x + 9) + (y^2 + 4y + 4) = -9 + 9 + 4$$
$$(x+3)^2 + (y+2)^2 = 4 \qquad \text{Center: } (-3, -2)$$

Find the slope of the line containing the centers:

$$m = \frac{-2 - (-3)}{-3 - 2} = \frac{1}{-5}$$

Find the equation of the line containing the centers:

$$y + 3 = \frac{-1}{5}(x - 2)$$
$$5y + 15 = -x + 2$$
$$x + 5y = -13$$

78. Find the slope of the line containing (a,b) and (b,a): $\frac{a-b}{b-a} = -1$

The slope of the line $y = x$ is 1.

Since $-1 \cdot 1 = -1$, the line containing the points (a,b) and (b,a) is perpendicular to the line $y = x$.

The midpoint of (a,b) and (b,a) $= \left(\frac{a+b}{2}, \frac{b+a}{2}\right)$.

Since the coordinates are the same, the midpoint lies on the line $y = x$.

79. $2x - y = C$

Graph the lines:

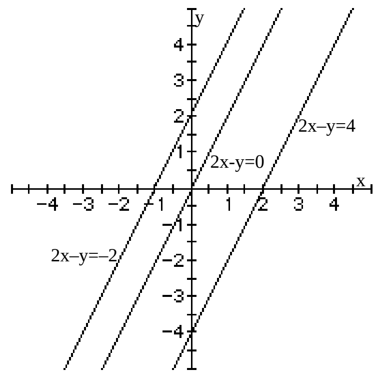
$2x - y = -2$

$2x - y = 0$

$2x - y = 4$

All the lines have the same slope, 2.

The lines are parallel.



80. $Cx + y = -4$

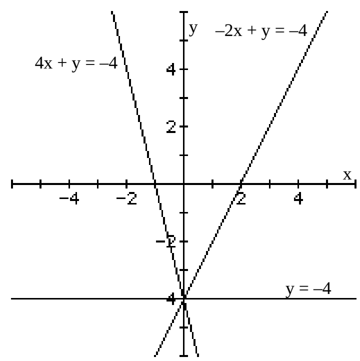
Graph the lines:

$-2x + y = -4$

$0x + y = -4$

$4x + y = -4$

All the lines have the same y-intercept, –4.



81. $y = 2$