# Chapter 2

# Graphs

## 2.6 Variation

1. $y = kx$

$2 = 10k \rightarrow k = \frac{1}{5}$

$y = \frac{1}{5}x$

2. $v = kt$

$16 = 2k \rightarrow k = 8$

$v = 8t$

3. $A = kx^2$

$4\pi = 4k \rightarrow k = \pi$

$A = \pi x^2$

4. $V = kx^3$

$36\pi = 27k \rightarrow k = \frac{4}{3}\pi$

$V = \frac{4}{3}\pi x^3$

5. $F = \frac{d^2}{k}$

$10 = \frac{25}{k} \rightarrow k = 2.5$

$F = \frac{d^2}{2.5} \rightarrow F = 0.4d^2$

6. $y = \frac{\sqrt{x}}{k}$

$4 = \frac{3}{k} \rightarrow k = \frac{3}{4}$

$y = \frac{\sqrt{x}}{\left(\frac{3}{4}\right)} \rightarrow y = \frac{4}{3}\sqrt{x}$

7. $z = k\left(x^2 + y^2\right)$

$5 = k\left(3^2 + 4^2\right) \rightarrow k = \frac{1}{5}$

$z = \frac{1}{5}\left(x^2 + y^2\right)$

8. $T = k\left(\sqrt[3]{x}\right)\left(d^2\right)$

$18 = k\left(\sqrt[3]{8}\right)\left(3^2\right) \rightarrow 18 = k\,(18)$

$\rightarrow k = 1$

$T = \left(\sqrt[3]{x}\right)\left(d^2\right)$

9. $M = \frac{kd^2}{\sqrt{x}}$

$24 = \frac{k\left(4^2\right)}{\sqrt{9}} \rightarrow 24 = \frac{16k}{3} \rightarrow k = 4.5$

$M = \frac{4.5\left(d^2\right)}{\sqrt{x}}$

10. $z = k\left(x^3 + y^2\right)$

$1 = k\left(2^3 + 3^2\right) \rightarrow 1 = k\,(17)$

$\rightarrow k = \frac{1}{17}$

$z = \frac{1}{17}\left(x^3 + y^2\right)$

11. $T^2 = \dfrac{ka^3}{d^2}$

$2^2 = \dfrac{k\left(2^3\right)}{4^2} \rightarrow k = 8$

$T^2 = \dfrac{8a^3}{d^2}$

12. $z^3 = k\left(x^2 + y^2\right)$

$2^3 = k\left(9^2 + 4^2\right) \rightarrow 8 = k(97) \rightarrow k = \dfrac{8}{97}$

$z^3 = \dfrac{8}{97}\left(x^2 + y^2\right)$

13. $V = \dfrac{4}{3}\pi r^3$

14. $c^2 = a^2 + b^2$

15. $A = \dfrac{1}{2}bh$

16. $p = 2\left(l + w\right)$

17. $V = \pi r^2 h$

18. $V = \dfrac{\pi}{3}r^2 h$

19. $F = \dfrac{\left(6.67 \times 10^{-11}\right)mM}{d^2}$

20. $T = \dfrac{2\pi}{\sqrt{32}}\sqrt{l}$

21. $s = kt^2$

$16 = k(1)^2 \rightarrow k = 16$

in 3 seconds $s = (16)(9) = 144$ *feet*

$64 = 16t^2 \rightarrow t^2 = 4 \rightarrow t = \pm 2$

so it takes 2 seconds to fall 64 feet.

22. $v = kt$

$64 = k(2) \rightarrow k = 32$

in 3 seconds

$v = (32)(3) = 96$ feet per second

23. $E = kw$

$3 = k(20) \rightarrow k = \dfrac{3}{20}$

when $w = 15$, $E = \left(\dfrac{3}{20}\right)(15) = 2.25$

24. $R = \dfrac{k}{l}$

$256 = \dfrac{k}{48} \rightarrow k = 12288$

when $R = 576$,

$576 = \dfrac{12288}{l} \rightarrow l = 21.\overline{33}$ inches

25. $W = \dfrac{k}{d^2}$

$55 = \dfrac{k}{3960^2} \rightarrow k = 862488000$

when $d = 3965$,

$W = \dfrac{862488000}{3965^2} = 54.86$ pounds

26. $F = kAv^2$

$11 = k(20)(22)^2 \rightarrow k = \dfrac{11}{9680}$

when $A = 47.125$ and $v = 36.5$

$F = \left(\dfrac{11}{9680}\right)(47.125)(36.5)^2 \approx 70.37$ pounds

27. $h = ksd^3$

$36 = k(75)(2)^3 \rightarrow k = 0.06$

when $h = 45$ and $s = 125$,

$45 = (0.06)(125)(d)^3$

$\rightarrow d = \sqrt[3]{\frac{45}{7.5}} \approx 0.84$ inches

28. $W = \frac{k}{d^2}$

$200 = \frac{k}{3960^2} \rightarrow k = 3136320000$

when $d = 3961$,

$W = \frac{3136320000}{3961^2} \approx 199.89$ pounds

29. $K = kmv^2$

$400 = k(25)(100)^2 \rightarrow k = 0.0016$

when $v = 150$,

$K = (0.0016)(25)(150)^2 = 900$ foot − pounds

30. $R = \frac{kl}{d^2}$

$1.24 = \frac{k(432)}{(4)^2} \rightarrow k = \frac{(1.24)(16)}{432} \approx 0.0459$

when $R = 1.44$ and $d = 3$,

$1.44 = \frac{\left(\frac{(1.24)(16)}{432}\right)(l)}{(3)^2}$

$\rightarrow l = (9)(1.44)\left(\frac{432}{(1.24)(16)}\right) \approx 282.19$ feet

31. $S = \frac{kpd}{t}$

$100 = \frac{k(25)(5)}{(0.75)} \rightarrow k = 0.6$

when $p = 40$, $d = 8$ and $t = 0.50$

$S = \frac{(0.6)(40)(8)}{(0.50)} = 320$ pounds

32. $S = \frac{kwt^2}{l}$

$750 = \frac{k(4)(2)^2}{8} \rightarrow k = 375$

when $l = 10$, $w = 6$ and $t = 2$,

$S = \frac{(375)(6)(2)^2}{10} = 900$ pounds

33. $R = \frac{kl}{r^2}$

$10 = \frac{k(50)}{(0.006)^2} \rightarrow k = 7.2 \times 10^{-6}$

when $l = 100$ and $r = 0.007$,

$R = \frac{(7.2 \times 10^{-6})(100)}{(0.007)^2} = 14.69$ ohms

34. $V = \frac{kt}{P}$

$100 = \frac{k(300)}{15} \rightarrow k = 5$

when $V = 80$ and $t = 310$,

$80 = \frac{(5)(310)}{P} \rightarrow P = 19.375$ atmospheres

35. $v = \sqrt{g}\sqrt{r} = \sqrt{gr}$

36. $v = \sqrt{gr}$

$v = \sqrt{g(3960 + 500)} = \sqrt{(g)(4460)}$

$\approx \sqrt{(79036)(4460)} \approx 18774.998$ mph

37. 37. $v = \sqrt{gr}$

$v = \sqrt{g(3960 + 140)} = \sqrt{(g)(4100)}$

$\approx \sqrt{(79036)(4100)} \approx 18001.32$ mph

38. $v = \sqrt{gr}$

$18630 = \sqrt{g(3960 + r)}$

$(18630)^2 = g(3960 + r)$

$\frac{(18630)^2}{g} = 3960 + r$

$\frac{(18630)^2}{79036} - 3960 = r$

$r \approx 431.38$ miles

39. The satellite travels the circumference of the circular orbit once in 1.5 hours.
We also know that

$$\text{circumference of a circle} = 2\pi\,(\text{radius})$$

$$\text{distance} = (\text{rate})(\text{time})$$

$$2\pi r = vt \rightarrow v = \frac{2\pi r}{t} = \frac{2\pi r}{1.5} = \frac{4\pi}{3} r$$

so

$$v = \sqrt{gr} = \frac{4\pi}{3} r \rightarrow gr = \frac{16\pi^2}{9} r^2$$

$$0 = \frac{16\pi^2}{9} r^2 - gr \rightarrow r\left(\frac{16\pi^2}{9} r - g\right) = 0$$

$$\rightarrow r = 0 \text{ or } \frac{16\pi^2}{9} r - g = 0 \rightarrow r = (g)\left(\frac{9}{16\pi^2}\right) \approx (79036)\left(\frac{9}{16\pi^2}\right) \approx 4504.51$$

therefore the satellite is $4504.51 - 3960 = 544.51$ miles above Earth.

40. The satellite travels the circumference of the circular orbit once in 1.5 hours.
We also know that

$$\text{circumference of a circle} = 2\pi\,(\text{radius})$$

$$\text{distance} = (\text{rate})(\text{time})$$

$$2\pi r = vt \rightarrow v = \frac{2\pi r}{2} = \frac{2\pi r}{2} = \pi r$$

so

$$v = \sqrt{gr} = \pi r \rightarrow gr = \pi^2 r^2$$

$$0 = \pi^2 r^2 - gr \rightarrow r(\pi^2 r - g) = 0$$

$$\rightarrow r = 0 \text{ or } \pi^2 r - g = 0 \rightarrow r = \frac{g}{\pi^2} \approx \frac{79036}{\pi^2} \approx 8008.02$$

therefore the satellite is $8008.02 - 3960 = 4048.02$ miles above Earth.

The satellite's speed is $\sqrt{gr} \approx \sqrt{(79036)(8008.02)} \approx 25157.94$ mph .

41. $F = \frac{mv^2}{r}$

42. $F = \frac{mv^2}{r}$

$r = 0.1$ km

$F = \frac{(150)(120)^2}{0.1} = 21600000$ newtons

43. $F = \frac{mv^2}{r}$

$v = 120 + (0.10)(120) = 132$

$F = \frac{(150)(132)^2}{0.1} = 23136000$ newtons

so the force is increased by 23136000 – 21600000 = 4536000 newtons.

44. $F = \frac{mv^2}{r}$

$21600000 = \frac{(150)(v)^2}{0.05} \rightarrow v = \sqrt{7200} \approx 84.85$ kmph

45. $F = \frac{mv^2}{r}$

we compare $F = \frac{mv^2}{L}$ with $F = \frac{m(3v)^2}{L} = \frac{9mv^2}{L}$

therefore, the force needed is 9 times greater.

46. $F = \frac{mv^2}{r}$

we compare $F = \frac{mv^2}{L}$ with $F = \frac{mv^2}{2L} = \frac{1}{2}\left(\frac{mv^2}{L}\right)$

therefore, the force needed is half as great.

47 – 50. Answers will vary.