Chapter 2

Graphs

2.R Chapter Review

1. Intercepts: (0,0)
   Test for symmetry:
   x-axis: Replace y by $-y$ so $2x = 3(-y)^2$ or $2x = 3y^2$, which is equivalent to $2x = 3y^2$.
   y-axis: Replace x by $-x$ so $2(-x) = 3y^2$ or $-2x = 3y^2$, which is not equivalent to $2x = 3y^2$.
   Origin: Replace x by $-x$ and y by $-y$ so $2(-x) = 3(-y)^2$ or $-2x = 3y^2$, which is not equivalent to $2x = 3y^2$.
   Therefore, the graph is symmetric with respect to the x-axis.

2. Intercepts: (0, 0)
   Test for symmetry:
   x-axis: Replace y by $-y$ so $-y = 5x$ or $y = -5x$, which is not equivalent to $y = 5x$.
   y-axis: Replace x by $-x$ so $y = 5(-x)$ or $y = -5x$, which is not equivalent to $y = 5x$.
   Origin: Replace x by $-x$ and y by $-y$ so $-y = 5(-x)$ or $y = 5x$, which is equivalent to $y = 5x$.
   Therefore, the graph is symmetric with respect to the origin.

3. Intercepts: (0, 2), (0, -2), (4, 0), (-4, 0)
   Test for symmetry:
   x-axis: Replace y by $-y$ so $x^2 + 4(-y)^2 = 16$ or $x^2 + 4y^2 = 16$, which is equivalent to $x^2 + 4y^2 = 16$.
   y-axis: Replace x by $-x$ so $(-x)^2 + 4y^2 = 16$ or $x^2 + 4y^2 = 16$, which is equivalent to $x^2 + 4y^2 = 16$.
   Origin: Replace x by $-x$ and y by $-y$ so $(-x)^2 + 4(-y)^2 = 16$ or $x^2 + 4y^2 = 16$, which is equivalent to $x^2 + 4y^2 = 16$.
   Therefore, the graph is symmetric with respect to the x-axis, the y-axis and the origin.

4. Intercepts: (1, 0), (-1, 0)
Test for symmetry:
x-axis: Replace y by $-y$ so $9x^2 - (-y)^2 = 9$ or $9x^2 - y^2 = 9$,
which is equivalent to $9x^2 - y^2 = 9$.
y-axis: Replace x by $-x$ so $9(-x)^2 - y^2 = 9$ or $9x^2 - y^2 = 9$,
which is equivalent to $9x^2 - y^2 = 9$.
Origin: Replace x by $-x$ and y by $-y$ so $9(-x)^2 - (-y)^2 = 9$ or $9x^2 - y^2 = 9$,
which is equivalent to $9x^2 - y^2 = 9$.
Therefore, the graph is symmetric with respect to the x-axis, the y-axis and the origin.

5. Intercepts: (0, 1)
Test for symmetry:
x-axis: Replace y by $-y$ so $-y = x^4 + 2x^2 + 1$,
which is not equivalent to $y = x^4 + 2x^2 + 1$.
y-axis: Replace x by $-x$ so $y = (-x)^4 + 2(-x)^2 + 1$ or $y = x^4 + 2x^2 + 1$,
which is equivalent to $y = x^4 + 2x^2 + 1$.
Origin: Replace x by $-x$ and y by $-y$ so $-y = (-x)^4 + 2(-x)^2 + 1$ or $-y = x^4 + 2x^2 + 1$,
which is not equivalent to $y = x^4 + 2x^2 + 1$.
Therefore, the graph is symmetric with respect to the y-axis.

6. Intercepts: (0, 0), (–1, 0), and (1, 0)
Test for symmetry:
x-axis: Replace y by $-y$ so $-y = x^3 - x$, which is not
equivalent to $y = x^3 - x$.
y-axis: Replace x by $-x$ so $y = (-x)^3 - (-x)$ or $y = -x^3 + x$,
which is not equivalent to $y = x^3 - x$.
Origin: Replace x by $-x$ and y by $-y$ so $-y = (-x)^3 - (-x)$ or $-y = -x^3 + x$
or $y = x^3 - x$, which is equivalent to $y = x^3 - x$.
Therefore, the graph is symmetric with respect to the origin.

7. Intercepts: (0,0), (0,–2), (–1,0)
Test for symmetry:
x-axis: Replace y by $-y$ so $x^2 + x + (-y)^2 + 2(-y) = 0$ or $x^2 + x + y^2 - 2y = 0$,
which is not equivalent to $x^2 + x + y^2 + 2y = 0$.

y-axis: Replace x by $-x$ so $(-x)^2+(-x)+y^2+2y=0$ or $x^2-x+y^2+2y=0$,
which is not equivalent to $x^2+x+y^2+2y=0$.

Origin: Replace x by $-x$ and y by $-y$ so $(-x)^2+(-x)+(-y)^2+2(-y)=0$ or
$x^2-x+y^2-2y=0$, which is not equivalent to
$x^2+x+y^2+2y=0$.

Therefore, the graph is not symmetric to the x-axis, the y-axis, or the origin.

8. Intercepts: (0, 0), (0, 2),and (–4, 0)
Test for symmetry:

x-axis: Replace y by $-y$ so $x^2+4x+(-y)^2-2(-y)=0$ or $x^2+4x+y^2+2y=0$,
which is not equivalent to $x^2+4x+y^2-2y=0$.

y-axis: Replace x by $-x$ so $(-x)^2+4(-x)+y^2-2y=0$ or $x^2-4x+y^2-2y=0$,
which is not equivalent to $x^2+4x+y^2-2y=0$.

Origin: Replace x by $-x$ and y by $-y$ so $(-x)^2+4(-x)+(-y)^2-2(-y)=0$ or
$x^2-4x+y^2+2y=0$, which is not equivalent to
$x^2+4x+y^2-2y=0$.

Therefore, the graph is not symmetric with respect to the x-axis, y-axis, or origin.

9. Slope = –2; containing (3,–1)

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-(-1) &= -2(x-3) \\ y+1 &= -2x+6 \\ y &= -2x+5 \end{aligned}$$

$2x+y=5$ or $y=-2x+5$

10. Slope = 0; containing the point (–5, 4)

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-4 &= 0(x-(-5)) \\ y-4 &= 0 \\ y &= 4 \end{aligned}$$

11. Slope undefined; containing (–3,4)
This is a vertical line.
$x=-3$
No slope intercept form.

12. x-intercept = 2 or the point (2, 0);
Containing the point (4, –5)

$$m=\frac{-5-0}{4-2}=\frac{-5}{2}=-\frac{5}{2}$$

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-0 &= -\frac{5}{2}(x-2) \\ y &= -\frac{5}{2}x+5 \end{aligned}$$

$5x+2y=10$ or $y=-\frac{5}{2}x+5$

13. y-intercept = –2; containing (5,–3)
Points are (5,–3) and (0,–2)

$$m=\frac{-2-(-3)}{0-5}=\frac{1}{-5}=\frac{-1}{5}$$
$$y=mx+b$$
$$y=\frac{-1}{5}x-2$$
$$x+5y=-10 \text{ or } y=\frac{-1}{5}x-2$$

14. Containing the points (3,–4) and (2,1)

$$m=\frac{1-(-4)}{2-3}=\frac{5}{-1}=-5$$
$$y-y_1=m(x-x_1)$$
$$y-(-4)=-5(x-3)$$
$$y+4=-5x+15$$
$$y=-5x+11$$
$$5x+y=11 \text{ or } y=-5x+11$$

15. Parallel to $2x-3y=-4$;
Slope = $\frac{2}{3}$; containing (–5,3)

$$y-y_1=m(x-x_1)$$
$$y-3=\frac{2}{3}(x-(-5))$$
$$y-3=\frac{2}{3}x+\frac{10}{3}$$
$$y=\frac{2}{3}x+\frac{19}{3}$$
$$2x-3y=-19 \text{ or } y=\frac{2}{3}x+\frac{19}{3}$$

16. Parallel to $x+y=2$;
Slope = –1
Containing the point (1, –3)

$$y-y_1=m(x-x_1)$$
$$y-(-3)=-1(x-1)$$
$$y+3=-x+1$$
$$y=-x-2$$
$$x+y=-2 \text{ or } y=-x-2$$

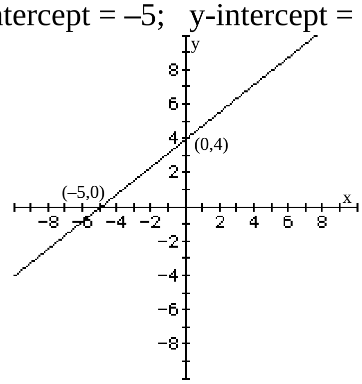
17. Perpendicular to $x+y=2$;
Containing (4,–3)
Slope of perpendicular = 1

$$y-y_1=m(x-x_1)$$
$$y-(-3)=1(x-4)$$
$$y+3=x-4$$
$$y=x-7$$
$$x-y=7 \text{ or } y=x-7$$

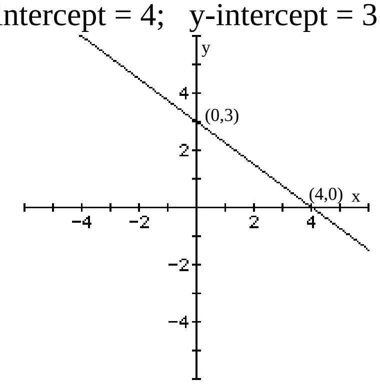
18. Perpendicular to $3x-y=-4$;
Slope of perpendicular = $-\frac{1}{3}$
Containing the point (–2, 4)

$$y-y_1=m(x-x_1)$$
$$y-4=-\frac{1}{3}(x-(-2))$$
$$y-4=-\frac{1}{3}x-\frac{2}{3}$$
$$y=-\frac{1}{3}x+\frac{10}{3}$$
$$x+3y=10 \text{ or } y=-\frac{1}{3}x+\frac{10}{3}$$

19. $4x-5y=-20$
x-intercept = –5; y-intercept = 4

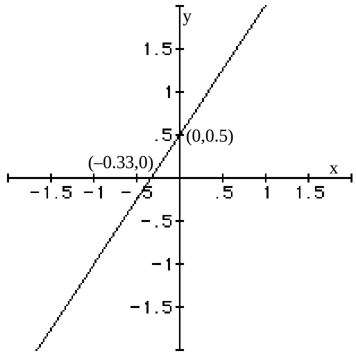


20. $3x+4y=12$
x-intercept = 4; y-intercept = 3



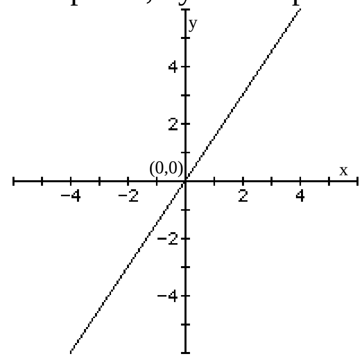
21. $\frac{1}{2}x - \frac{1}{3}y = -\frac{1}{6}$

x-intercept = $\frac{-1}{3}$; y-intercept = $\frac{1}{2}$



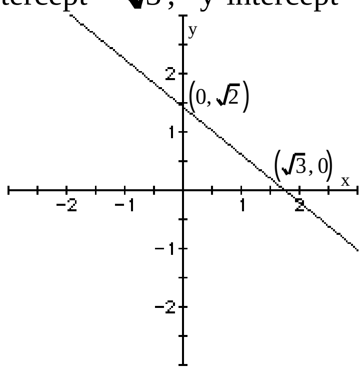
22. $-\frac{3}{4}x + \frac{1}{2}y = 0$

x-intercept = 0; y-intercept = 0



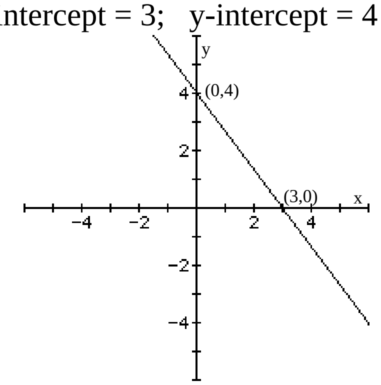
23. $\sqrt{2}x + \sqrt{3}y = \sqrt{6}$

x-intercept = $\sqrt{3}$; y-intercept = $\sqrt{2}$



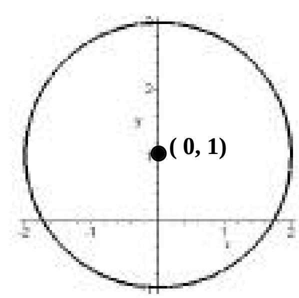
24. $\frac{x}{3} + \frac{y}{4} = 1$

x-intercept = 3; y-intercept = 4



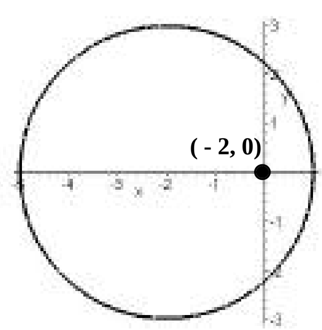
25. $x^2 + (y-1)^2 = 4$

Center: (0,1)
Radius = 2



26. $(x+2)^2 + y^2 = 9$

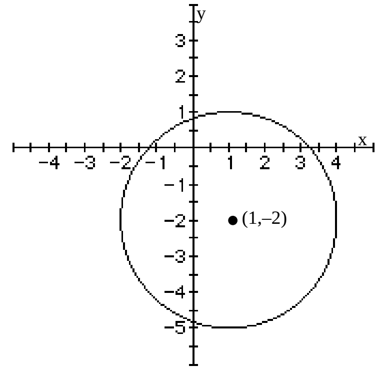
Center: (-2,0)
Radius = 3



27. $x^2 + y^2 - 2x + 4y - 4 = 0$

$$x^2 - 2x + y^2 + 4y = 4$$
$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 4 + 1 + 4$$
$$(x-1)^2 + (y+2)^2 = 3^2$$

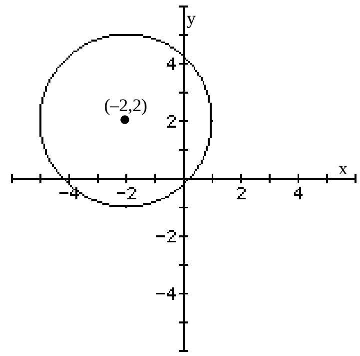
Center: (1,–2) Radius = 3



28. $x^2 + y^2 + 4x - 4y - 1 = 0$

$$x^2 + 4x + y^2 - 4y = 1$$
$$(x^2 + 4x + 4) + (y^2 - 4y + 4) = 1 + 4 + 4$$
$$(x+2)^2 + (y-2)^2 = 3^2$$

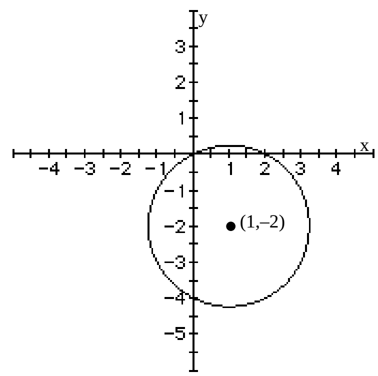
Center: (–2, 2) Radius = 3



29. $3x^2 + 3y^2 - 6x + 12y = 0$

$$x^2 + y^2 - 2x + 4y = 0$$
$$x^2 - 2x + y^2 + 4y = 0$$
$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 1 + 4$$
$$(x-1)^2 + (y+2)^2 = \left(\sqrt{5}\right)^2$$

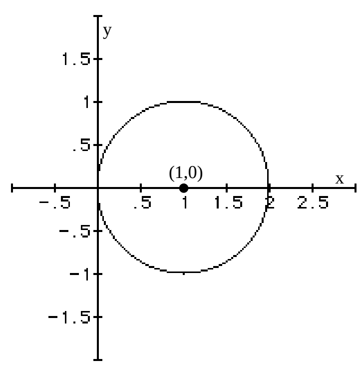
Center: (1,–2) Radius = $\sqrt{5}$



30. $2x^2 + 2y^2 - 4x = 0$

$$x^2 + y^2 - 2x = 0$$
$$x^2 - 2x + y^2 = 0$$
$$(x^2 - 2x + 1) + y^2 = 0 + 1$$
$$(x-1)^2 + y^2 = 1^2$$

Center: (1, 0) Radius = 1



31. Given the points (7,4) and (–3,2).

Slope: $m = \dfrac{2-4}{-3-7} = \dfrac{-2}{-10} = \dfrac{1}{5}$

Distance: $d = \sqrt{(-3-7)^2 + (2-4)^2} = \sqrt{100+4} = \sqrt{104} = 2\sqrt{26}$

Midpoint: $\left(\dfrac{7+(-3)}{2}, \dfrac{4+2}{2}\right) = (2, 3)$

32. Find the distance between each pair of points.

$d_{A,B} = \sqrt{(1-3)^2 + (1-4)^2} = \sqrt{4+9} = \sqrt{13}$

$d_{B,C} = \sqrt{(-2-1)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$

$d_{A,C} = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{25+1} = \sqrt{26}$

Since $AB = BC$, triangle ABC is isosceles.

33. Given the points $A = (-2, 0)$, $B = (-4, 4)$, $C = (8, 5)$.

(a) Find the distance between each pair of points.

$$d_{A,B} = \sqrt{(-4-(-2))^2 + (4-0)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$
$$d_{B,C} = \sqrt{(8-(-4))^2 + (5-4)^2} = \sqrt{144+1} = \sqrt{145}$$
$$d_{A,C} = \sqrt{(8-(-2))^2 + (5-0)^2} = \sqrt{100+25} = \sqrt{125} = 5\sqrt{5}$$
$$\left(\sqrt{20}\right)^2 + \left(\sqrt{125}\right)^2 = \left(\sqrt{145}\right)^2 \quad \rightarrow \quad 20 + 125 = 145 \quad \rightarrow \quad 145 = 145$$

The Pythagorean theorem is satisfied, so this is a right triangle.

(b) Find the slopes:

$$m_{AB} = \frac{4-0}{-4-(-2)} = \frac{4}{-2} = -2$$
$$m_{BC} = \frac{5-4}{8-(-4)} = \frac{1}{12}$$
$$m_{AC} = \frac{5-0}{8-(-2)} = \frac{5}{10} = \frac{1}{2}$$
$$m_{AB}\ \ m_{AC} = -2\ \ \frac{1}{2} = -1$$

Since the product of the slopes is –1, the sides of the triangle are perpendicular and the triangle is a right triangle.

34. Endpoints of the diameter are (–3, 2) and (5,–6).
The center is at the midpoint of the diameter:

Center: $\left(\dfrac{-3+5}{2}, \dfrac{2+(-6)}{2}\right) = (1, -2)$

Radius: $r = \sqrt{(1-(-3))^2 + (-2-2)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

Equation:

$$(x-1)^2 + (y+2)^2 = \left(4\sqrt{2}\right)^2$$
$$x^2 - 2x + 1 + y^2 + 4y + 4 = 32$$
$$x^2 + y^2 - 2x + 4y - 27 = 0$$

35. slope of $\overline{AB} = \frac{1-5}{6-2} = -1$; slope of $\overline{AC} = \frac{-1-5}{8-2} = -1$; slope of $\overline{BC} = \frac{-1-1}{8-6} = -1$
therefore the points are collinear.

36. A circle with center (-1,2) has equation $(x+1)^2 + (y-2)^2 = r^2$
point $A(1,5) \rightarrow (1+1)^2 + (5-2)^2 = r^2$
$4+9 = r^2 \rightarrow r = \sqrt{13}$
point $B(2,4) \rightarrow (2+1)^2 + (4-2)^2 = r^2$
$9+4 = r^2 \rightarrow r = \sqrt{13}$
point $C(-3,5) \rightarrow (-3+1)^2 + (5-2)^2 = r^2$
$4+9 = r^2 \rightarrow r = \sqrt{13}$

Therefore the points *A*, *B* and *C* lie on a circle with center point (-1, 2) and radius = $\sqrt{13}$.

37. $Area = A = kx^2, x =$ length of a side of the triangle

$$A = \frac{\sqrt{3}}{4}, x = 1 \rightarrow \frac{\sqrt{3}}{4} = (k)(1) \rightarrow k = \frac{\sqrt{3}}{4}$$

$$A = 16 \rightarrow A = \left(\frac{\sqrt{3}}{4}\right)(x) = 16 \rightarrow x = \frac{48}{\sqrt{3}} = 16\sqrt{3} \text{ cm.}$$

38. $Pitch = P = k\sqrt{t}, t =$ tension of the string

$$P = 300,\ t = 9 \rightarrow 300 = (k)\sqrt{9} \rightarrow k = 100$$

$$P = 400 \rightarrow P = 100\sqrt{t} = 400 \rightarrow \sqrt{t} = 4 \rightarrow t = 16 \text{ pounds.}$$

39. period (in days) $= T$, mean distance (in millon miles) from sun $= a$
$T^2 = ka^3$

$$T = 365, a = 93 \rightarrow 365^2 = (k)(93)^3 \rightarrow k = \frac{365^2}{93^3} \approx 0.1656292013$$

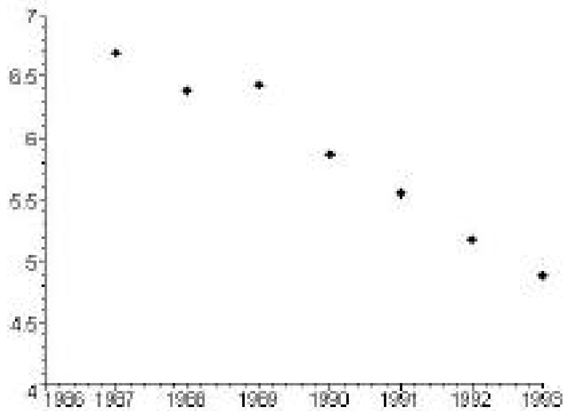
$$T = 88 \rightarrow 88^2 = \left(\frac{365^2}{93^3}\right)(a)^3 \rightarrow a^3 = (88^2)\left(\frac{93^3}{365^2}\right)$$

$$\rightarrow a = \sqrt[3]{(88^2)\left(\frac{93^3}{365^2}\right)} \approx 36.025 \text{ million miles}$$

40. $T = \left(5\sqrt{5}\right)(365) \rightarrow \left(\left(5\sqrt{5}\right)(365)\right)^2 = \left(\dfrac{365^2}{93^3}\right)(a)^3 \rightarrow a^3 = \left(\left(5\sqrt{5}\right)(365)\right)^2\left(\dfrac{93^3}{365^2}\right)$

$a = \sqrt[3]{125\left(93^3\right)} = 465$ million miles

41. (a)



(b) slope = $\dfrac{5.87-6.69}{1990-1987} = -0.27\bar{3}$

(c) for each 1 year increase, the concentration decreases by $0.27\bar{3}$ ppm.

(d) slope = $\dfrac{4.88-5.87}{1993-1990} = -0.33$

(e) for each 1 year increase, the concentration decreases by 0.33 ppm.

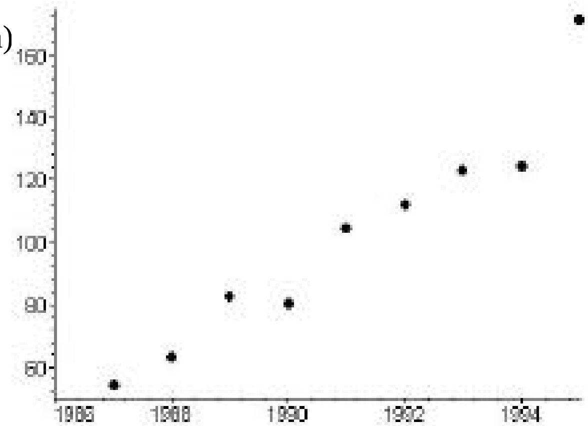
(f) $y = 618.477 - 0.308x$

(g) for each 1 year increase, the concentration decreases by 0.308 ppm.

(h) Answers will vary.

(i) As time passes, the average level of carbon monoxide is decreasing more rapidly.

42. (a)



(b) slope = $\dfrac{104.28-54.26}{1991-1987}=12.505$

(c) for each 1 year increase, the index value increases by 12.505 dollars

(d) slope = $\dfrac{171.20-104.28}{1995-1991}=16.73$

(e) for each 1 year increase, the index value increases by 16.73 dollars.

(f) $y=-25275.241+12.746x$

(g) for each 1 year increase, the index value increases by 12.746 dollars.

(h) the slope from part (d), since it predicts the greatest yearly index increase of the three slopes

(i) As time passes, the portfolio value is increasing at a higher rate.

43. Answers will vary.

44. (a) $x=0$ is a vertical line passing through the origin, that is $x=0$ is the equation of the *y*-axis

(b) $y=0$ is a horizontal line passing through the origin, that is $y=0$ is the equation of the *x*-axis

(c) $x+y=0\rightarrow y=-x$ is line passing through the origin with slope = - 1.

(d) $xy=0\rightarrow y=0$ or $x=0$ is a graph consisting of the coordinate axes.

(e) $x^2+y^2=0\rightarrow y=0$ and $x=0$ is a graph consisting of the origin.

45. Answers will vary.