Functions and Their Graphs

3.1 Functions

1. Function
   Domain: {Dad, Colleen, Kaleigh, Marissa}
   Range: {Jan. 8, Mar. 15, Sept. 17}

2. Function
   Domain: {Bob, Dave, John, Chuck}
   Range: {Beth, Diane, Linda, Marcia}

3. Not a function

4. Function
   Domain: {Bob, Dave, John, Chuck}
   Range: {Diane, Linda, Marcia}

5. Function
   Domain: {2, –3, 4, 1}
   Range: {6, 9, 10}

6. Function
   Domain: {–2, –1, 3, 4}
   Range: {3, 5, 7, 12}

7. Function
   Domain: {1, 2, 3, 4}
   Range: {3}

8. Function
   Domain: {0, 1, 2, 3}
   Range: {–2, 3, 7}

9. Not a function

10. Not a function

11. Function
    Domain: {–2, –1, 0, 1}
    Range: {4, 1, 0}

12. Function
    Domain: {–2, –1, 0, 1}
    Range: {3, 4, 16}

13. $f(x) = 3x^2 + 2x - 4$

(a) $f(0) = 3(0)^2 + 2(0) - 4 = -4$

(b) $f(1) = 3(1)^2 + 2(1) - 4 = 3 + 2 - 4 = 1$

(c) $f(-1) = 3(-1)^2 + 2(-1) - 4 = 3 - 2 - 4 = -3$

(d) $f(-x) = 3(-x)^2 + 2(-x) - 4 = 3x^2 - 2x - 4$

(e) $-f(x) = -(3x^2 + 2x - 4) = -3x^2 - 2x + 4$

(f) $f(x+1) = 3(x+1)^2 + 2(x+1) - 4 = 3(x^2 + 2x + 1) + 2x + 2 - 4$
$= 3x^2 + 6x + 3 + 2x + 2 - 4 = 3x^2 + 8x + 1$

(g) $f(2x) = 3(2x)^2 + 2(2x) - 4 = 12x^2 + 4x - 4$

(h) $f(x+h) = 3(x+h)^2 + 2(x+h) - 4 = 3(x^2 + 2xh + h^2) + 2x + 2h - 4$
$= 3x^2 + 6xh + 3h^2 + 2x + 2h - 4$

14. $f(x) = -2x^2 + x - 1$

(a) $f(0) = -2(0)^2 + 0 - 1 = -1$

(b) $f(1) = -2(1)^2 + 1 - 1 = -2$

(c) $f(-1) = -2(-1)^2 + (-1) - 1 = -4$

(d) $f(-x) = -2(-x)^2 + (-x) - 1 = -2x^2 - x - 1$

(e) $-f(x) = -(-2x^2 + x - 1) = 2x^2 - x + 1$

(f) $f(x+1) = -2(x+1)^2 + (x+1) - 1 = -2(x^2 + 2x + 1) + x + 1 - 1$
$= -2x^2 - 4x - 2 + x = -2x^2 - 3x - 2$

(g) $f(2x) = -2(2x)^2 + (2x) - 1 = -8x^2 + 2x - 1$

(h) $f(x+h) = -2(x+h)^2 + (x+h) - 1 = -2(x^2 + 2xh + h^2) + x + h - 1$
$= -2x^2 - 4xh - 2h^2 + x + h - 1$

15. $f(x) = \dfrac{x}{x^2+1}$

(a) $f(0) = \dfrac{0}{0^2+1} = \dfrac{0}{1} = 0$

(b) $f(1) = \dfrac{1}{1^2+1} = \dfrac{1}{2}$

(c) $f(-1) = \dfrac{-1}{(-1)^2+1} = \dfrac{-1}{1+1} = \dfrac{-1}{2}$

(d) $f(-x) = \dfrac{-x}{(-x)^2+1} = \dfrac{-x}{x^2+1}$

(e) $-f(x) = -\dfrac{x}{x^2+1} = \dfrac{-x}{x^2+1}$

(f) $f(x+1) = \dfrac{x+1}{(x+1)^2+1} = \dfrac{x+1}{x^2+2x+1+1} = \dfrac{x+1}{x^2+2x+2}$

(g) $f(2x) = \dfrac{2x}{(2x)^2+1} = \dfrac{2x}{4x^2+1}$

(h) $f(x+h) = \dfrac{x+h}{(x+h)^2+1} = \dfrac{x+h}{x^2+2xh+h^2+1}$

16. $f(x)=\dfrac{x^2-1}{x+4}$

(a) $f(0)=\dfrac{0^2-1}{0+4}=\dfrac{-1}{4}=-\dfrac{1}{4}$

(b) $f(1)=\dfrac{1^2-1}{1+4}=\dfrac{0}{5}=0$

(c) $f(-1)=\dfrac{(-1)^2-1}{-1+4}=\dfrac{0}{3}=0$

(d) $f(-x)=\dfrac{(-x)^2-1}{-x+4}=\dfrac{x^2-1}{-x+4}$

(e) $-f(x)=-\dfrac{x^2-1}{x+4}=\dfrac{1-x^2}{x+4}$

(f) $f(x+1)=\dfrac{(x+1)^2-1}{(x+1)+4}=\dfrac{x^2+2x+1-1}{x+5}=\dfrac{x^2+2x}{x+5}$

(g) $f(2x)=\dfrac{(2x)^2-1}{2x+4}=\dfrac{4x^2-1}{2x+4}$

(h) $f(x+h)=\dfrac{(x+h)^2-1}{(x+h)+4}=\dfrac{x^2+2xh+h^2-1}{x+h+4}$

17. $f(x)=|x|+4$

(a) $f(0)=|0|+4=0+4=4$

(b) $f(1)=|1|+4=1+4=5$

(c) $f(-1)=|-1|+4=1+4=5$

(d) $f(-x)=|-x|+4=|x|+4$

(e) $-f(x)=-\left(|x|+4\right)=-|x|-4$

(f) $f(x+1)=|x+1|+4$

(g) $f(2x)=|2x|+4=2|x|+4$

(h) $f(x+h)=|x+h|+4$

18. $f(x)=\sqrt{x^2+x}$

(a) $f(0)=\sqrt{0^2+0}=\sqrt{0}=0$

(b) $f(1)=\sqrt{1^2+1}=\sqrt{2}$

(c) $f(-1)=\sqrt{(-1)^2+(-1)}=\sqrt{1-1}=\sqrt{0}=0$

(d) $f(-x)=\sqrt{(-x)^2+(-x)}=\sqrt{x^2-x}$

(e) $-f(x)=-\sqrt{x^2+x}$

(f) $f(x+1)=\sqrt{(x+1)^2+(x+1)}=\sqrt{x^2+2x+1+x+1}=\sqrt{x^2+3x+2}$

(g) $f(2x)=\sqrt{(2x)^2+2x}=\sqrt{4x^2+2x}$

(h) $f(x+h)=\sqrt{(x+h)^2+(x+h)}=\sqrt{x^2+2xh+h^2+x+h}$

19. $f(x)=\dfrac{2x+1}{3x-5}$

(a) $f(0)=\dfrac{2(0)+1}{3(0)-5}=\dfrac{0+1}{0-5}=\dfrac{-1}{5}$

(b) $f(1)=\dfrac{2(1)+1}{3(1)-5}=\dfrac{2+1}{3-5}=\dfrac{3}{-2}=\dfrac{-3}{2}$

(c) $f(-1)=\dfrac{2(-1)+1}{3(-1)-5}=\dfrac{-2+1}{-3-5}=\dfrac{-1}{-8}=\dfrac{1}{8}$

(d) $f(-x)=\dfrac{2(-x)+1}{3(-x)-5}=\dfrac{-2x+1}{-3x-5}=\dfrac{2x-1}{3x+5}$

(e) $-f(x)=-\dfrac{2x+1}{3x-5}=\dfrac{-2x-1}{3x-5}$

(f) $f(x+1)=\dfrac{2(x+1)+1}{3(x+1)-5}=\dfrac{2x+2+1}{3x+3-5}=\dfrac{2x+3}{3x-2}$

(g) $f(2x)=\dfrac{2(2x)+1}{3(2x)-5}=\dfrac{4x+1}{6x-5}$

(h) $f(x+h)=\dfrac{2(x+h)+1}{3(x+h)-5}=\dfrac{2x+2h+1}{3x+3h-5}$

20. $f(x)=1-\dfrac{1}{(x+2)^2}$

(a) $f(0)=1-\dfrac{1}{(0+2)^2}=1-\dfrac{1}{4}=\dfrac{3}{4}$ (b) $f(1)=1-\dfrac{1}{(1+2)^2}=1-\dfrac{1}{9}=\dfrac{8}{9}$

(c) $f(-1)=1-\dfrac{1}{(-1+2)^2}=1-\dfrac{1}{1}=0$

(d) $f(-x)=1-\dfrac{1}{(-x+2)^2}=1-\dfrac{1}{x^2-4x+4}=\dfrac{x^2-4x+4-1}{x^2-4x+4}=\dfrac{x^2-4x+3}{x^2-4x+4}$

(e) $-f(x)=-\left(1-\dfrac{1}{(x+2)^2}\right)=-1+\dfrac{1}{x^2+4x+4}=\dfrac{-x^2-4x-4+1}{x^2+4x+4}=\dfrac{-x^2-4x-3}{x^2+4x+4}$

(f) $f(x+1)=1-\dfrac{1}{(x+1+2)^2}=1-\dfrac{1}{(x+3)^2}=\dfrac{x^2+6x+9-1}{x^2+6x+9}=\dfrac{x^2+6x+8}{x^2+6x+9}$

(g) $f(2x)=1-\dfrac{1}{(2x+2)^2}=1-\dfrac{1}{4x^2+8x+4}=\dfrac{4x^2+8x+4-1}{4x^2+8x+4}=\dfrac{4x^2+8x+3}{4x^2+8x+4}$

(h) $f(x+h)=1-\dfrac{1}{(x+h+2)^2}=\dfrac{x^2+2xh+4x+h^2+4h+4-1}{x^2+2xh+4x+h^2+4h+4}$

$$=\frac{x^2+2xh+4x+h^2+4h+3}{x^2+2xh+4x+h^2+4h+4}$$

21. Graph $y=x^2$. The graph passes the vertical line test. Thus, the equation represents a function.

22. Graph $y=x^3$. The graph passes the vertical line test. Thus, the equation represents a function.

23. Graph $y=\dfrac{1}{x}$. The graph passes the vertical line test. Thus, the equation represents a function.

24. Graph $y = |x|$. The graph passes the vertical line test. Thus, the equation represents a function.

25. $y^2 = 4 - x^2$
Solve for y: $y = \pm\sqrt{4 - x^2}$
For $x = 0, y = \pm 2$. Thus, (0,2) and (0,–2) are on the graph. This is not a function, since a distinct x corresponds to two different y's.

26. $y = \pm\sqrt{1 - 2x}$

For $x = 0$, $y = \pm 1$. Thus, (0,1) and (0,–1) are on the graph. This is not a function, since a distinct x corresponds to two different y's.

27. $x = y^2$
Solve for y: $y = \pm\sqrt{x}$
For $x = 1, y = \pm 1$. Thus, (1,1) and (1,–1) are on the graph. This is not a function, since a distinct x corresponds to two different y's.

28. $x + y^2 = 1$
Solve for y: $y = \pm\sqrt{1 - x}$
For $x = 0, y = \pm 1$. Thus, (0,1) and (0,–1) are on the graph. This is not a function, since a distinct x corresponds to two different y's.

29. Graph $y = 2x^2 - 3x + 4$. The graph passes the vertical line test. Thus, the equation represents a function.

30. Graph $y = \dfrac{3x - 1}{x + 2}$. The graph passes the vertical line test. Thus, the equation represents a function.

31. $2x^2 + 3y^2 = 1$
Solve for y:

$$2x^2 + 3y^2 = 1 \rightarrow 3y^2 = 1 - 2x^2 \rightarrow y^2 = \frac{1 - 2x^2}{3}$$

$$y = \pm\sqrt{\frac{1 - 2x^2}{3}}$$

For $x = 0, y = \pm\sqrt{\frac{1}{3}}$. Thus, $\left(0, \sqrt{\frac{1}{3}}\right)$ and $\left(0, -\sqrt{\frac{1}{3}}\right)$ are on the graph. This is not a function, since a distinct x corresponds to two different y's.

32. $x^2 - 4y^2 = 1$

Solve for y:

$$x^2 - 4y^2 = 1 \rightarrow 4y^2 = x^2 - 1 \rightarrow y^2 = \frac{x^2 - 1}{4}$$

$$y = \frac{\pm\sqrt{x^2 - 1}}{2}$$

For $x = \sqrt{2}, y = \pm\frac{1}{2}$. Thus, $\left(\sqrt{2}, \frac{1}{2}\right)$ and $\left(\sqrt{2}, -\frac{1}{2}\right)$ are on the graph. This is not a function, since a distinct x corresponds to two different y's.

33. $f(x) = -5x + 4$

Domain: {Real Numbers}

34. $f(x) = x^2 + 2$

Domain: {Real Numbers}

35. $f(x) = \frac{x}{x^2 + 1}$

Domain: {Real Numbers}

36. $f(x) = \frac{x^2}{x^2 + 1}$

Domain: {Real Numbers}

37. $g(x) = \frac{x}{x^2 - 16}$

$x^2 - 16 \neq 0$

$x^2 \neq 16$

$x \neq \pm 4$

Domain: $\{x \mid x \neq -4, x \neq 4\}$

38. $h(x) = \frac{2x}{x^2 - 4}$

$x^2 - 4 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$

Domain: $\{x \mid x \neq -2, x \neq 2\}$

39. $F(x) = \frac{x - 2}{x^3 + x}$

$x^3 + x \neq 0$

$x(x^2 + 1) \neq 0$

$x \neq 0, \quad x^2 \neq -1$

Domain: $\{x \mid x \neq 0\}$

40. $G(x) = \frac{x + 4}{x^3 - 4x}$

$x^3 - 4x \neq 0$

$x(x^2 - 4) \neq 0$

$x \neq 0, \quad x^2 \neq 4$

$x \neq 0, \quad x \neq \pm 2$

Domain: $\{x \mid x \neq 0, x \neq 2, x \neq -2\}$

41. $h(x) = \sqrt{3x - 12}$

$3x - 12 \geq 0$

$3x \geq 12$

$x \geq 4$

Domain: $\{x \mid x \geq 4\}$

42. $G(x) = \sqrt{1 - x}$

$1 - x \geq 0$

$-x \geq -1$

$x \leq 1$

Domain: $\{x \mid x \leq 1\}$

43. $f(x) = \frac{4}{\sqrt{x - 9}}$

$x - 9 > 0$

$x > 9$

Domain: $\{x \mid x > 9\}$

44. $f(x) = \frac{x}{\sqrt{x - 4}}$

$x - 4 > 0$

$x > 4$

Domain: $\{x \mid x > 4\}$

45. $p(x) = \sqrt{\dfrac{2}{x-1}}$

$\dfrac{2}{x-1} > 0 \quad \to \quad x-1>0 \quad \to \quad x>1$

Domain: $\{x \mid x>1\}$

46. $q(x) = \sqrt{-x-2}$

$-x-2>0 \quad \to \quad -x>2 \quad \to \quad x<-2$

Domain: $\{x \mid x<-2\}$

47. (a) $f(0)=3$ since $(0,3)$ is on the graph.
$f(-6)=-3$ since $(-6,-3)$ is on the graph.

(b) $f(6)=0$ since $(6, 0)$ is on the graph.
$f(11)=1$ since $(11, 1)$ is on the graph.

(c) $f(3)$ is positive since $f(3)\approx 3.7$.

(d) $f(-4)$ is negative since $f(-4)=-1$.

(e) $f(x)=0$ when $x=-3$, $x=6$, and $x=10$.

(f) $f(x)>0$ when $-3<x<6$, and $10<x\le 11$.

(g) The domain of f is $\{x \mid -6\le x\le 11\}$ or $[-6, 11]$

(h) The range of f is $\{y \mid -3\le y\le 4\}$ or $[-3, 4]$

(i) The x-intercepts are (–3, 0), (6, 0), and (11, 0).

(j) The y-intercept is (0, 3).

(k) The line $y=\frac{1}{2}$ intersects the graph 3 times.

(l) The line $x=5$ intersects the graph 1 times.

(m) $f(x)=3$ when $x=0$ and $x=4$.

(n) $f(x)=-2$ when $x=-5$ and $x=8$.

48. (a) $f(0)=0$ since $(0,0)$ is on the graph. $f(6)=0$ since $(6,0)$ is on the grapl

(b) $f(2)=-2$ since $(2,-2)$ is on the graph.
$f(-2)=1$ since $(-2,1)$ is on the graph.

(c) $f(3)$ is negative since $f(3) \approx -1$.

(d) $f(-1)$ is positive since $f(-1) \approx -0.4$.

(e) $f(x) = 0$ when $x = 0$, $x = 4$, and $x = 6$.

(f) $f(x) < 0$ when $2 < x < 4$.

(g) The domain of f is $\{x \mid -4 \le x \le 6\}$ or $[-4, 6]$

(h) The range of f is $\{y \mid -2 \le y \le 3\}$ or $[-2, 3]$

(i) The x-intercepts are (0, 0), (4, 0), and (6, 0).

(j) The y-intercept is (0, 0).

(k) The line $y = -1$ intersects the graph 2 times.

(l) The line $x = 1$ intersects the graph 1 times.

(m) $f(x) = 3$ when $x = 5$.

(n) $f(x) = -2$ when $x = 2$.

49. Not a function since vertical lines will intersect the graph in more than one point.

50. Function (a) Domain: $\{x \mid \text{Real Numbers}\}$; Range: $\{y \mid y > 0\}$
(b) (0,1)
(c) No symmetry to the x-axis, y-axis, or origin.

51. Function (a) Domain: $\{x \mid -\pi \le x \le \pi\}$; Range: $\{y \mid -1 \le y \le 1\}$
(b) $\left(\frac{-\pi}{2}, 0\right)$, $\left(\frac{\pi}{2}, 0\right)$, (0,1) (c) y-axis

52. Function (a) Domain: $\{x \mid -\pi \le x \le \pi\}$; Range: $\{y \mid -1 \le y \le 1\}$
(b) $(-\pi, 0)$, $(\pi, 0)$, (0, 0) (c) origin

53. Not a function since vertical lines will intersect the graph in more than one point.

54. Not a function since vertical lines will intersect the graph in more than one point.

55. Function (a) Domain: $\{x \mid x > 0\}$; Range: $\{y \mid y \quad \text{Real Numbers}\}$
(b) (1, 0)
(c) No symmetry to the x-axis, y-axis, or origin.

56. Function (a) Domain: $\{x \mid 0 \le x \le 4\}$; Range: $\{y \mid 0 \le y \le 3\}$
(b) (0, 0)
(c) No symmetry to the x-axis, y-axis, or origin.

57. Function (a) Domain: $\{x \mid x \quad \text{Real Numbers}\}$; Range: $\{y \mid y \le 2\}$
(b) (–3,0), (3,0), (0,2)
(c) y-axis

58. Function (a) Domain: $\{x \mid x \ge -3\}$; Range: $\{y \mid y \ge 0\}$
(b) (–3,0), (2,0), (0,2)
(c) No symmetry to the x-axis, y-axis, or origin.

59. Function (a) Domain: $\{x \mid x \quad \text{Real Numbers}\}$; Range: $\{y \mid y \ge -3\}$
(b) (1,0), (3,0), (0,9)
(c) No symmetry to the x-axis, y-axis, or origin.

60. Function (a) Domain: $\{x \mid x \quad \text{Real Numbers}\}$; Range: $\{y \mid y \le 5\}$
(b) (–1,0), (2,0), (0,4)
(c) No symmetry to the x-axis, y-axis, or origin.

61. $f(x) = 2x^2 - x - 1$

(a) $f(-1) = 2(-1)^2 - (-1) - 1 = 2$ (-1,2) is on the graph of f.

(b) $f(-2) = 2(-2)^2 - (-2) - 1 = 9$ (-2,9) is on the graph of f.

(c) Solve for x:

$$-1 = 2x^2 - x - 1$$

$$0 = 2x^2 - x$$

$$0 = x(2x - 1) \rightarrow x = 0, x = \frac{1}{2}$$

(0, -1) and $\left(\frac{1}{2}, -1\right)$ are points on the graph of f.

(d) The domain of f is: $\{x \mid x \text{ is any real number}\}$.

(e) x-intercepts:

$$f(x) = 0$$

$$2x^2 - x - 1 = 0 \rightarrow (2x + 1)(x - 1) = 0$$

$$\rightarrow x = -\frac{1}{2}, x = 1 \rightarrow \left(-\frac{1}{2}, 0\right) \text{and } (1,0)$$

(f) y-intercept: $f(0) = 2(0)^2 - 0 - 1 = 0 \rightarrow (0,0)$

62. $f(x) = -3x^2 + 5x$

(a) $f(-1) = -3(-1)^2 + 5(-1) \neq -2$ (-1,2) is not on the graph of f.

(b) $f(-2) = -3(-2)^2 + 5(-2)$ (-2,-22) is on the graph of f.

(c) Solve for x:

$$-2 = -3x^2 + 5x$$

$$3x^2 - 5x - 2 = 0 \rightarrow (3x+1)(x-2) = 0$$

$$\rightarrow x = -\frac{1}{3}, x = 2$$

(2, -2) and $\left(-\frac{1}{3}, -2\right)$ are points on the graph of f.

(d) The domain of f is: $\{x \mid x \text{ is any real number}\}$.

(e) x-intercepts:

$$f(x) = 0$$

$$-3x^2 + 5x = 0 \rightarrow x(-3x+5) = 0$$

$$\rightarrow x = 0, x = \frac{5}{3} \rightarrow (0,0) \text{ and } \left(\frac{5}{3}, 0\right)$$

(f) y-intercept: $f(0) = -3(0)^2 + 5(0) = 0 \rightarrow (0,0)$

63. $f(x) = \dfrac{x+2}{x-6}$

(a) $f(3) = \dfrac{3+2}{3-6} = \dfrac{5}{-3} \neq 14$ (3,14) is not on the graph of f.

(b) $f(4) = \dfrac{4+2}{4-6} = \dfrac{6}{-2} = -3$ (4, –3) is the point on the graph of f.

(c) Solve for x:

$$2 = \frac{x+2}{x-6}$$

$$2x - 12 = x + 2$$

$$x = 14$$

(14, 2) is a point on the graph of f.

(d) The domain of f is: $\{x \mid x \neq 6\}$.

(e) x-intercepts:

$$f(x) = 0$$

$$\frac{x+2}{x-6} = 0 \rightarrow x + 2 = 0$$

$$\rightarrow x = -2 \rightarrow (-2,0)$$

(f) y-intercept: $f(0) = \dfrac{0+2}{0-6} = -\dfrac{1}{3} \rightarrow \left(0, -\dfrac{1}{3}\right)$

64. $f(x)=\dfrac{x^2+2}{x+4}$

(a) $f(1)=\dfrac{1^2+2}{1+4}=\dfrac{3}{5}=\dfrac{3}{5}$ $\left(1,\dfrac{3}{5}\right)$ is on the graph of f.

(b) $f(0)=\dfrac{0^2+2}{0+4}=\dfrac{2}{4}=\dfrac{1}{2}$ $\left(0,\dfrac{1}{2}\right)$ is the point on the graph of f.

(c) Solve for x:

$$\frac{1}{2}=\frac{x^2+2}{x+4}$$
$$x+4=2x^2+4$$
$$0=2x^2-x$$
$$x(2x-1)=0$$
$$x=0 \text{ or } x=\frac{1}{2}$$

$\left(0,\dfrac{1}{2}\right)$ and $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ are points on the graph of f.

(d) The domain of f is: $\{x \mid x\neq -4\}$.

(e) x-intercepts:

$$f(x)=0$$
$$\frac{x^2+2}{x+4}=0 \rightarrow x^2+2=0 \text{ which is impossible}$$
$$\rightarrow \text{no } x-\text{intercepts}$$

(f) y-intercept: $f(0)=\dfrac{x^2+2}{x+4}=\dfrac{2}{4}=\dfrac{1}{2}\rightarrow\left(0,\dfrac{1}{2}\right)$

65. $f(x)=\dfrac{2x^2}{x^4+1}$

(a) $f(-1)=\dfrac{2(-1)^2}{(-1)^4+1}=\dfrac{2}{2}=1$ (–1,1) is a point on the graph of f.

(b) $f(2)=\dfrac{2(2)^2}{(2)^4+1}=\dfrac{8}{17}$ $\left(2,\dfrac{8}{17}\right)$ is a point on the graph of f.

(c) Solve for x:

$$1=\frac{2x^2}{x^4+1}$$
$$x^4+1=2x^2$$
$$x^4-2x^2+1=0$$
$$(x^2-1)^2=0$$
$$x^2-1=0 \rightarrow x=\pm 1$$

(1,1) and (–1,1) are points on the graph of f.

(d) The domain of f is: $\{\text{Real Numbers}\}$.

(e) x-intercepts:

$$f(x)=0$$
$$\frac{2x^2}{x^4+1}=0 \rightarrow 2x^2=0 \rightarrow x=0 \rightarrow (0,0)$$

(f) y-intercept: $f(0)=\dfrac{2x^2}{x^4+1}=\dfrac{0}{0+1}=0\rightarrow(0,0)$

66. $f(x)=\dfrac{2x}{x-2}$

(a) $f\left(\frac{1}{2}\right)=\dfrac{2\left(\frac{1}{2}\right)}{\frac{1}{2}-2}=\dfrac{1}{-\frac{3}{2}}=-\dfrac{2}{3}$ $\left(\frac{1}{2},-\frac{2}{3}\right)$ is a point on the graph of f.

(b) $f(4)=\dfrac{2(4)}{4-2}=\dfrac{8}{2}=4$ $(4,4)$ is a point on the graph of f.

(c) Solve for x:

$$1=\frac{2x}{x-2}$$
$$x-2=2x$$
$$-2=x$$

$(-2,1)$ is a point on the graph of f.

(d) The domain of f is: $\{x \mid x\neq 2\}$.

(e) x-intercepts:

$$f(x)=0$$
$$\frac{2x}{x-2}=0\rightarrow 2x=0$$
$$\rightarrow x=0\rightarrow (0,0)$$

(f) y-intercept: $f(0)=\dfrac{0}{0-2}=0\rightarrow (0,0)$

67. Solving for A:

$f(x)=2x^3+Ax^2+4x-5$ and $f(2)=5$

$$f(2)=2(2)^3+A(2)^2+4(2)-5$$
$$5=16+4A+8-5$$
$$5=4A+19$$
$$-14=4A$$
$$A=\frac{-7}{2}$$

68. Solve for B:

$f(x)=3x^2-Bx+4$ and $f(-1)=12$

$$f(-1)=3(-1)^2-B(-1)+4$$
$$12=3+B+4$$
$$B=5$$

69. Solving for A:

$f(x)=\dfrac{3x+8}{2x-A}$ and $f(0)=2$

$$f(0)=\frac{3(0)+8}{2(0)-A}$$
$$2=\frac{8}{-A}$$
$$-2A=8$$
$$A=-4$$

70. Solve for B:

$f(x)=\dfrac{2x-B}{3x+4}$ and $f(2)=\dfrac{1}{2}$

$$f(2)=\frac{2(2)-B}{3(2)+4}$$
$$\frac{1}{2}=\frac{4-B}{10}$$
$$5=4-B$$
$$B=-1$$

71. Solving for A:

$f(x) = \dfrac{2x - A}{x - 3}$ and $f(4) = 0$

$f(4) = \dfrac{2(4) - A}{4 - 3}$

$0 = \dfrac{8 - A}{1}$

$0 = 8 - A$

$A = 8$

f is undefined when $x = 3$.

72. Solve for A and B:

$f(x) = \dfrac{x - B}{x - A}$ and $f(2) = 0$

$f(1)$ is undefined

$1 - A = 0 \quad \rightarrow \quad A = 1$

$f(2) = \dfrac{2 - B}{2 - 1}$

$0 = \dfrac{2 - B}{1}$

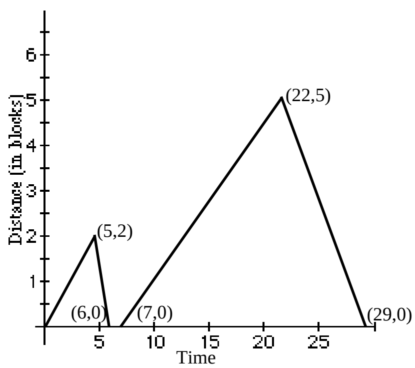
$0 = 2 - B$

$B = 2$

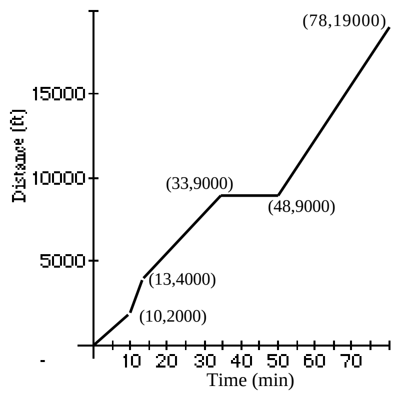
73. (a) III (b) IV (c) I (d) V (e) II

74. (a) II (b) V (c) IV (d) III (e) I

75.



76.



77. (a) Two times
(b) Kevin's distance from home increased steadily at a rate of 1.5 miles per hour.
(c) Kevin's distance from home did not change.
(d) Kevin's distance from home decreased steadily at a rate of 10 miles per hour.
(e) Kevin stayed at home for 0.2 hours.
(f) Kevin's distance from home increased rapidly at the beginning and then tapered off to a very slow change in distance from home.
(g) Kevin's distance from home did not change.
(h) Kevin's distance from home decreased rapidly at the beginning of the interval and then tapered off as he got closer to home.
(i) The furthest distance Kevin is from home is 3 miles.

78. (a) Michael travels fastest between 7 and 7.4 minutes.
(b) Michael's speed is zero between 4.2 and 6 minutes.
(c) Between 0 and 2 minutes, Michael's speed increased from 0 to 30 miles/hour.
(d) Between 4.2 and 6 minutes, Michael was stopped.
(e) Between 7 and 7.4 minutes, Michael was traveling at a steady rate of 50 miles/hr.
(f) Michael's speed is constant between 2 and 4 minutes, between 4.2 and 6 minutes, between 7 and 7.4 minutes, and between 7.6 and 8 minutes.

79. (a) $H(1) = 20 - 4.9(1)^2 = 20 - 4.9 = 15.1$ meters

$H(1.1) = 20 - 4.9(1.1)^2 = 20 - 4.9(1.21) = 20 - 5.929 = 14.071$ meters

$H(1.2) = 20 - 4.9(1.2)^2 = 20 - 4.9(1.44) = 20 - 7.056 = 12.944$ meters

$H(1.3) = 20 - 4.9(1.3)^2 = 20 - 4.9(1.69) = 20 - 8.281 = 11.719$ meters

(b)

$$\begin{aligned} H(x) &= 15 \\ 15 &= 20 - 4.9x^2 \\ -5 &= -4.9x^2 \\ x^2 &= 1.0204 \\ x &= 1.01 \text{ seconds} \end{aligned} \qquad \begin{aligned} H(x) &= 10 \\ 10 &= 20 - 4.9x^2 \\ -10 &= -4.9x^2 \\ x^2 &= 2.0408 \\ x &= 1.43 \text{ seconds} \end{aligned} \qquad \begin{aligned} H(x) &= 5 \\ 5 &= 20 - 4.9x^2 \\ -15 &= -4.9x^2 \\ x^2 &= 3.0612 \\ x &= 1.75 \text{ seconds} \end{aligned}$$

(c)

$$\begin{aligned} H(x) &= 0 \\ 0 &= 20 - 4.9x^2 \\ -20 &= -4.9x^2 \\ x^2 &= 4.0816 \\ x &= 2.02 \text{ second} \end{aligned}$$

80. (a) $H(1) = 20 - 13(1)^2 = 20 - 13 = 7$ meters

$H(1.1) = 20 - 13(1.1)^2 = 20 - 13(1.21) = 20 - 15.73 = 4.27$ meters

$H(1.2) = 20 - 13(1.2)^2 = 20 - 13(1.44) = 20 - 18.72 = 1.28$ meters

(b)

$$\begin{aligned} H(x) &= 15 \\ 15 &= 20 - 13x^2 \\ -5 &= -13x^2 \\ x^2 &= 0.3846 \\ x &= 0.62 \text{ second} \end{aligned} \qquad \begin{aligned} H(x) &= 10 \\ 10 &= 20 - 13x^2 \\ -10 &= -13x^2 \\ x^2 &= 0.7692 \\ x &= 0.88 \text{ seconds} \end{aligned} \qquad \begin{aligned} H(x) &= 5 \\ 5 &= 20 - 13x^2 \\ -15 &= -13x^2 \\ x^2 &= 1.1538 \\ x &= 1.07 \text{ seconds} \end{aligned}$$

(c)

$$\begin{aligned} H(x) &= 0 \\ 0 &= 20 - 13x^2 \\ -20 &= -13x^2 \\ x^2 &= 1.5385 \\ x &= 1.24 \text{ seconds} \end{aligned}$$

81. $h(x) = \dfrac{-32x^2}{130^2} + x$

(a) $h(100) = \dfrac{-32(100)^2}{130^2} + 100 = \dfrac{-320000}{16900} + 100 = -18.93 + 100 = 81.07$ feet

(b) $h(300) = \dfrac{-32(300)^2}{130^2} + 300 = \dfrac{-2880000}{16900} + 300 = -170.41 + 300 = 129.59$ feet

(c) $h(500) = \dfrac{-32(500)^2}{130^2} + 500 = \dfrac{-8000000}{16900} + 500 = -473.37 + 500 = 26.63$ feet

(d) Solve $h(x) = \dfrac{-32x^2}{130^2} + x = 0$

$$x\left(\frac{-32x}{130^2} + 1\right) = 0$$

$x = 0$ or $\dfrac{-32x}{130^2} + 1 = 0 \rightarrow 1 = \dfrac{32x}{130^2} \rightarrow x = \dfrac{130^2}{32} = 528.125$ feet

82. $A(x) = 4x\sqrt{1-x^2}$

(a) $A\left(\frac{1}{3}\right) = 4\left(\frac{1}{3}\right)\sqrt{1-\left(\frac{1}{3}\right)^2} = \frac{4}{3}\sqrt{\frac{8}{9}} = \frac{4}{3}\left(\frac{2\sqrt{2}}{3}\right) = \frac{8\sqrt{2}}{9} \approx 1.26\text{ ft}^2$

(b) $A\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)\sqrt{1-\left(\frac{1}{2}\right)^2} = 2\sqrt{\frac{3}{4}} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3} \approx 1.73\text{ ft}^2$

(c) $A\left(\frac{2}{3}\right) = 4\left(\frac{2}{3}\right)\sqrt{1-\left(\frac{2}{3}\right)^2} = \frac{8}{3}\sqrt{\frac{5}{9}} = \frac{8}{3}\left(\frac{\sqrt{5}}{3}\right) = \frac{8\sqrt{5}}{9} \approx 1.99\text{ ft}^2$

83. $C(x) = 100 + \frac{x}{10} + \frac{36000}{x}$

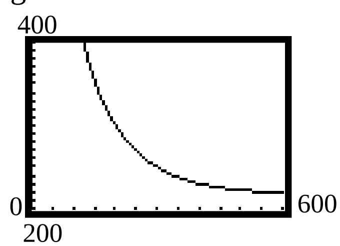
(a) $C(500) = 100 + \frac{500}{10} + \frac{36000}{500} = 100 + 50 + 72 = \$222$

(b) $C(450) = 100 + \frac{450}{10} + \frac{36000}{450} = 100 + 45 + 80 = \$225$

(c) $C(600) = 100 + \frac{600}{10} + \frac{36000}{600} = 100 + 60 + 60 = \$220$

(d) $C(400) = 100 + \frac{400}{10} + \frac{36000}{400} = 100 + 40 + 90 = \$230$

(e) Graphing:



(f) As x varies from 400 to 600 mph, the cost decreases from \$230 to \$220.

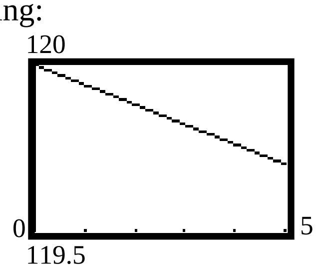
84. $W(h) = m\left(\frac{4000}{4000+h}\right)^2$

(a) $h = 14110$ feet ≈ 2.67 miles

$$W(2.67) = 120\left(\frac{4000}{4000+2.67}\right)^2 \approx 119.84$$

On Pike's Peak, Amy will weigh about 119.84 pounds.

(b) Graphing:



(c) Create a TABLE:

X	Y1
0	120
.5	119.97
1	119.94
1.5	119.91
2	119.88
2.5	119.85
3	119.82

X=0

X	Y1
2	119.88
2.5	119.85
3	119.82
3.5	119.79
4	119.76
4.5	119.73
5	119.7

X=5

(d) By refining the table, Amy will weigh 119.95 lbs at a height of about 0.8 miles.

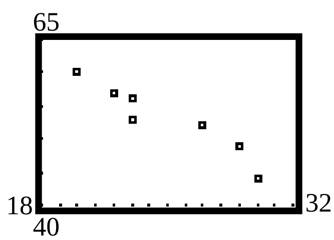
85. Let x represent the length of the rectangle.
Then $\frac{x}{2}$ represents the width of the rectangle, since the length is twice the width.
The function for the area is: $A(x) = x \;\; \frac{x}{2} = \frac{x^2}{2} = \frac{1}{2}x^2$

86. Let x represent the length of one of the two equal sides.
The function for the area is: $A(x) = \frac{1}{2} \;\; x \;\; x = \frac{1}{2}x^2$

87. Let x represent the number of hours worked.
The function for the gross salary is: $G(x) = 10x$

88. Let x represent the number of items sold.
The function for the gross salary is: $G(x) = 10x + 100$

89. (a) The relation is not a function because 23 is paired with both 56 and 53.

(b)


(c) Using the points (20,60) and (30, 44) we get

$D = -1.6p + 92$

(d) As the price of the jeans increases by \$1, the demand for the jeans decreases by 1.6.

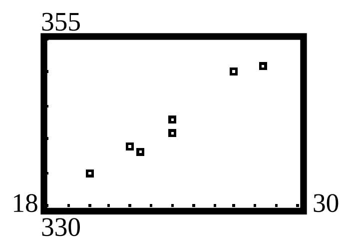
(e) $D(p) = -1.6p + 92$

(f) Domain: $\{p \mid p > 0\}$

(g) $D(28) = -1.6(28) + 92$
$= 47.2 \approx 47$
Demand is about 47 pairs.

(h) $D = -1.3355p + 86.1974$

90. (a) The relation is not a function because 24 is paired with both 343 and 341.

(b)


(c) Using the points (20,335) and (28.3, 351) we get

$S = 1.9277A + 296.4458$

(d) As the advertising expenditure increases by \$1, the sales increase by \$1.9277.

(e) $S(A) = 1.9277A + 296.4458$

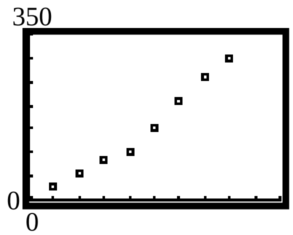
(f) Domain: $\{A \mid A \geq 0\}$

(g) $S(25) = 1.9277(25) + 296.4458$
≈ 344.638
Sales are about \$344,638.

(h) $S = 2.0667A + 292.8869$

91. (a) The relation is a function. Each time value is paired with exactly one distance value.

(b)



(c) Using the points (0,0) and (8, 300) we get

$s = 37.5t$

(d) As the time increases by 1 hour, the distances increases by 37.5 miles.

(e) $s(t) = 37.5t$

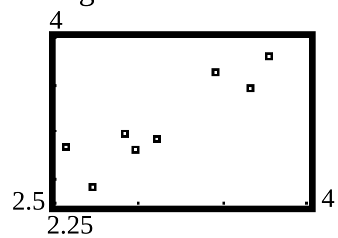
(f) Domain: $\{t \mid t \geq 0\}$

(g) $s(11) = 37.5(11)$
$= 412.5$ miles

(h) $s = 37.7833t - 19.1333$

92. (a) The relation is a function. Each HS GPA value is paired with exactly one College GPA value.

(b)



(c) Using the points (2.73,2.43) and (3.10, 2.93) we get

$G = 1.3514x - 1.259$

(d) As the high school GPA increases by 1 , the college GPA increases by 1.3514.

(e) $G(x) = 1.3514x - 1.259$

(f) Domain: $\{x \mid x \geq 0\}$

(g) $G(3.23) = 1.3514(3.23) - 1.259$
$= 3.1058 \approx 3.12$ The college GPA is about 3.12.

(h) $G = 0.9639x + 0.0724$

93. (a) $h(x) = 2x$
$h(a+b) = 2(a+b) = 2a + 2b = h(a) + h(b)$
$h(x) = 2x$ has the property.

(b) $g(x) = x^2$
$g(a+b) = (a+b)^2 = a^2 + 2ab + b^2 \neq a^2 + b^2 = h(a) + h(b)$
$g(x) = x^2$ does not have the property.

(c) $F(x) = 5x - 2$
$F(a+b) = 5(a+b) - 2 = 5a + 5b - 2 \neq 5a - 2 + 5b - 2 = h(a) + h(b)$
$F(x) = 5x - 2$ does not have the property.

(d) $G(x) = \dfrac{1}{x}$
$G(a+b) = \dfrac{1}{a+b} \neq \dfrac{1}{a} + \dfrac{1}{b} = h(a) + h(b)$
$G(x) = \dfrac{1}{x}$ does not have the property.

94. All points of the form (5, y) and of the form (x, 0) cannot be on the graph of the function.
95. No, $x = -1$ is not in the domain of g, but it is in the domain of f.
96. Answers will vary
97. A function may have any number of x-intercepts; it can have only one y-intercept.
98. Yes. One example is $G(x) = 2$,where the domain of G is {1}.
99. A function cannot be symmetric to the x-axis, because it would then fail the vertical line test.