Chapter 3

Functions and Their Graphs

3.2 Properties of Functions

1. Yes

2. No It is increasing.

3. No It only increases on (5, 10).

4. Yes

5. f is increasing on the intervals: (–8, –2), (0, 2), (5, 10).

6. f is decreasing on the intervals: (–10, –8), (–2, 0), (2, 5).

7. Yes. The local maximum at $x = 2$ is 10.

8. No. There is a local minimum at $x = 5$.

9. f has local maxima at $x = -2$ and $x = 2$. The local maxima are 6 and 10, respectively.

10. f has local minima at $x = -8$, $x = 0$ and $x = 5$. The local minima are –4, 0, and 0, respectively.

11. (a) Intercepts: (–2,0), (2,0), and (0,3).
    (b) Domain: $\{x \mid -4 \le x \le 4\}$; Range: $\{y \mid 0 \le y \le 3\}$.
    (c) Interval notation: Increasing: (–2, 0) and (2, 4); Decreasing: (–4, –2) and (0, 2).
    Inequality notation: Increasing: $-2 < x < 0$ and $2 < x < 4$
    Decreasing: $-4 < x < -2$ and $0 < x < 2$
    (d) Since the graph is symmetric to the y-axis, the function is <u>even</u>.

12. (a) Intercepts: (–1, 0), (1, 0), and (0, 2).
    (b) Domain: $\{x \mid -3 \le x \le 3\}$; Range: $\{y \mid 0 \le y \le 3\}$.
    (c) Interval notation: Increasing: (–1, 0) and (1, 3); Decreasing: (–3, –1) and (0, 1).
    Inequality notation: Increasing: $-1 < x < 0$ and $1 < x < 3$
    Decreasing: $-3 < x < -1$ and $0 < x < 1$
    (d) Since the graph is symmetric to the y-axis, the function is <u>even</u>.

13. (a) Intercepts: (0,1).
(b) Domain: { Real Numbers }; Range: $\{y \mid y > 0\}$.
(c) Interval notation: Increasing: $(-\infty, +\infty)$; Decreasing: never.
Inequality notation: Increasing: $-\infty < x < +\infty$
Decreasing: never
(d) Since the graph is not symmetric to the y-axis or the origin, the function is neither even nor odd.

14. (a) Intercepts: (1, 0).
(b) Domain: $\{x \mid x > 0\}$; Range: { Real Numbers }.
(c) Interval notation: Increasing: $(0, +\infty)$; Decreasing: never.
Inequality notation: Increasing: $x > 0$
Decreasing: never
(d) Since the graph is not symmetric to the y-axis or the origin, the function is neither even nor odd.

15. (a) Intercepts: $(-\pi, 0), (0, 0)$, and $(\pi, 0)$.
(b) Domain: $\{x \mid -\pi \le x \le \pi\}$; Range: $\{y \mid -1 \le y \le 1\}$.
(c) Interval notation: Increasing: $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$; Decreasing: $\left(-\pi, \frac{-\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \pi\right)$.
Inequality notation: Increasing: $\frac{-\pi}{2} < x < \frac{\pi}{2}$
Decreasing: $-\pi < x < \frac{-\pi}{2}$ and $\frac{\pi}{2} < x < \pi$
(d) Since the graph is symmetric to the origin, the function is odd.

16. (a) Intercepts: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right)$ and $(0, 1)$.
(b) Domain: $\{x \mid -\pi \le x \le \pi\}$; Range: $\{y \mid -1 \le y \le 1\}$.
(c) Interval notation: Increasing: $(-\pi, 0)$; Decreasing: $(0, \pi)$.
Inequality notation: Increasing: $-\pi < x < 0$
Decreasing: $0 < x < \pi$
(d) Since the graph is symmetric to the y-axis, the function is even.

17. (a) Intercepts: $\left(0, \frac{1}{2}\right), \left(\frac{1}{3}, 0\right)$, and $\left(\frac{5}{2}, 0\right)$.
(b) Domain: $\{x \mid -3 \le x \le 3\}$; Range: $\{y \mid -1 \le y \le 2\}$.
(c) Interval notation: Increasing: $(2, 3)$; Decreasing: $(-1, 1)$;
Constant: $(-3, -1)$ and $(1, 2)$.
Inequality notation: Increasing: $2 < x < 3$; Decreasing: $-1 < x < 1$;
Constant: $-3 < x < -1$ and $1 < x < 2$.
(d) Since the graph is not symmetric to the y-axis or the origin, the function is neither even nor odd.

18. (a) Intercepts: $(-2.3, 0)$, $(3, 0)$, and $(0, 1)$.
    (b) Domain: $\{x \mid -3 \le x \le 3\}$; Range: $\{y \mid -2 \le y \le 2\}$.
    (c) Interval notation: Increasing: $(-3, -2)$ and $(0, 2)$; Decreasing: $(2, 3)$;
    Constant: $(-2, 0)$.
    Inequality notation: Increasing: $-3 < x < -2$ and $0 < x < 2$;
    Decreasing: $2 < x < 3$; Constant: $-2 < x < 0$.
    (d) Since the graph is not symmetric to the y-axis or the origin, the function is <u>neither</u> even nor odd.

19. (a) Intercepts: $(0, 2)$, $(-2, 0)$, and $(2, 0)$.
    (b) Domain: $\{x \mid -4 \le x \le 4\}$; Range: $\{y \mid 0 \le y \le 2\}$.
    (c) Interval notation: Increasing: $(-2, 0)$ and $(2, 4)$;
    Decreasing: $(-4, -2)$ and $(0, 2)$.
    Inequality notation: Increasing: $-2 < x < 0$ and $2 < x < 4$;
    Decreasing: $-4 < x < -2$ and $0 < x < 2$.
    (d) Since the graph is symmetric to the y-axis, the function is <u>even</u>.

20. (a) Intercepts: $(0, 0)$, $(-4, 0)$, and $(4, 0)$.
    (b) Domain: $\{x \mid -4 \le x \le 4\}$; Range: $\{y \mid -2 \le y \le 2\}$.
    (c) Interval notation: Increasing: $(-2, 2)$;
    Decreasing: $(-4, -2)$ and $(2, 4)$.
    Inequality notation: Increasing: $-2 < x < 2$;
    Decreasing: $-4 < x < -2$ and $2 < x < 4$.
    (d) Since the graph is symmetric to the origin, the function is <u>odd</u>.

21. (a) f has a local maximum of 3 at $x = 0$.
    (b) f has a local minimum of 0 at both $x = -2$ and $x = 2$.

22. (a) f has a local maximum of 2 at $x = 0$.
    (b) f has a local minimum of 0 at both $x = -1$ and $x = 1$.

23. (a) f has a local maximum of 1 at $x = \frac{\pi}{2}$.
    (b) f has a local minimum of –1 at $x = \frac{-\pi}{2}$.

24. (a) f has a local maximum of 1 at $x = 0$.
    (b) f has no local minimum.

25. $f(x) = 5x$

(a) $\dfrac{f(x) - f(1)}{x-1} = \dfrac{5x-5}{x-1}$

$= \dfrac{5(x-1)}{x-1} = 5$

(b) $\dfrac{f(2)-f(1)}{2-1} = \dfrac{10-5}{2-1} = \dfrac{5}{1} = 5$

(c) Slope = 5; Containing (1, 5):

$y - 5 = 5(x-1)$

$y - 5 = 5x - 5$

$y = 5x$

26. $f(x) = -4x$

(a) $\dfrac{f(x) - f(1)}{x-1} = \dfrac{-4x+4}{x-1}$

$= \dfrac{-4(x-1)}{x-1} = -4$

(b) $\dfrac{f(2)-f(1)}{2-1} = \dfrac{-8+4}{2-1} = \dfrac{-4}{1} = -4$

(c) Slope = –4; Containing (1, –4):

$y - (-4) = -4(x-1)$

$y + 4 = -4x + 4$

$y = -4x$

27. $f(x) = 1 - 3x$

(a) $\dfrac{f(x) - f(1)}{x-1} = \dfrac{1-3x-(-2)}{x-1}$

$= \dfrac{-3x+3}{x-1} = \dfrac{-3(x-1)}{x-1} = -3$

(b) $\dfrac{f(2)-f(1)}{2-1} = \dfrac{1-3(2)-(-2)}{2-1}$

$= \dfrac{-3}{1} = -3$

(c) Slope = –3; Containing (1, –2):

$y - (-2) = -3(x-1)$

$y + 2 = -3x + 3$

$y = -3x + 1$

28. $f(x) = x^2 + 1$

(a) $\dfrac{f(x) - f(1)}{x-1} = \dfrac{x^2+1-2}{x-1}$

$= \dfrac{x^2-1}{x-1} = \dfrac{(x+1)(x-1)}{x-1} = x+1$

(b) $\dfrac{f(2)-f(1)}{2-1} = \dfrac{2^2+1-2}{2-1} = \dfrac{3}{1} = 3$

(c) Slope = 3; Containing (1, 2):

$y - 2 = 3(x-1)$

$y - 2 = 3x - 3$

$y = 3x - 1$

29. $f(x) = x^2 - 2x$

(a) $\dfrac{f(x) - f(1)}{x-1} = \dfrac{x^2-2x-(-1)}{x-1}$

$= \dfrac{x^2-2x+1}{x-1} = \dfrac{(x-1)^2}{x-1} = x-1$

(b) $\dfrac{f(2)-f(1)}{2-1} = \dfrac{2^2-2(2)-(-1)}{2-1} = \dfrac{1}{1} = 1$

(c) Slope = 1; Containing (1, –1):

$y - (-1) = 1(x-1)$

$y + 1 = 1x - 1$

$y = x - 2$

30 $f(x) = x - 2x^2$

(a) $\dfrac{f(x) - f(1)}{x-1} = \dfrac{x-2x^2-(-1)}{x-1}$

$= \dfrac{-2x^2+x+1}{x-1} = \dfrac{(-2x-1)(x-1)}{x-1}$

$= -2x - 1$

(b) $\dfrac{f(2)-f(1)}{2-1} = \dfrac{2-2\ 2^2-(-1)}{2-1}$

$= \dfrac{-5}{1} = -5$

(c) Slope = –5; Containing (1, –1):

$y - (-1) = -5(x-1)$

$y + 1 = -5x + 5$

$y = -5x + 4$

31. $f(x) = x^3 - x$

(a) $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{x^3 - x - 0}{x - 1} = \dfrac{x^3 - x}{x - 1}$

$= \dfrac{x(x-1)(x+1)}{x-1} = x^2 + x$

(b) $\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{2^3 - 2 - 0}{2 - 1} = \dfrac{6}{1} = 6$

(c) Slope = 6; Containing (1, 0):

$y - 0 = 6(x - 1)$

$y = 6x - 6$

32. $f(x) = x^3 + x$

(a) $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{x^3 + x - 2}{x - 1}$

$= \dfrac{(x-1)(x^2 + x + 2)}{x-1} = x^2 + x + 2$

(b) $\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{2^3 + 2 - 2}{2 - 1} = \dfrac{8}{1} = 8$

(c) Slope = 8; Containing (1, 2):

$y - 2 = 8(x - 1)$

$y - 2 = 8x - 8$

$y = 8x - 6$

33. $f(x) = \dfrac{2}{x+1}$

(a) $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{\frac{2}{x+1} - 1}{x - 1} = \dfrac{\frac{2 - x - 1}{x+1}}{x - 1}$

$= \dfrac{1 - x}{(x-1)(x+1)} = \dfrac{-1}{x+1}$

(b) $\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{\frac{2}{2+1} - 1}{2 - 1}$

$= \dfrac{\frac{-1}{3}}{1} = \dfrac{-1}{3}$

(c) Slope = $\frac{-1}{3}$; Containing (1, 1):

$y - 1 = \dfrac{-1}{3}(x - 1)$

$y - 1 = \dfrac{-1}{3}x + \dfrac{1}{3}$

$y = \dfrac{-1}{3}x + \dfrac{4}{3}$

34. $f(x) = \dfrac{1}{x^2}$

(a) $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{\frac{1}{x^2} - 1}{x - 1} = \dfrac{\frac{1 - x^2}{x^2}}{x - 1}$

$= \dfrac{(1 - x)(1 + x)}{x^2(x - 1)} = \dfrac{-x - 1}{x^2}$

(b) $\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{\frac{1}{2^2} - 1}{2 - 1} = \dfrac{-\frac{3}{4}}{1} = -\dfrac{3}{4}$

(c) Slope = $-\frac{3}{4}$; Containing (1, 1):

$y - 1 = -\dfrac{3}{4}(x - 1)$

$y - 1 = -\dfrac{3}{4}x + \dfrac{3}{4}$

$y = -\dfrac{3}{4}x + \dfrac{7}{4}$

35. $f(x) = \sqrt{x}$

(a) $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{\sqrt{x} - 1}{x - 1}$

(b) $\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{\sqrt{2} - 1}{1} = \sqrt{2} - 1$

Slope = $\sqrt{2} - 1$; Containing (1, 1):

(c) $y - 1 = \left(\sqrt{2} - 1\right)(x - 1)$

$y - 1 = \left(\sqrt{2} - 1\right)x - \left(\sqrt{2} - 1\right)$

$y = \left(\sqrt{2} - 1\right)x - \sqrt{2} + 2$

36. $f(x) = \sqrt{x + 3}$

(a) $\dfrac{f(x) - f(1)}{x - 1} = \dfrac{\sqrt{x + 3} - 2}{x - 1}$

(b) $\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{\sqrt{5} - 2}{1} = \sqrt{5} - 2$

Slope = $\sqrt{5} - 2$; Containing (1, 2):

(c) $y - 2 = \left(\sqrt{5} - 2\right)(x - 1)$

$y - 2 = \left(\sqrt{5} - 2\right)x - \left(\sqrt{5} - 2\right)$

$y = \left(\sqrt{5} - 2\right)x - \sqrt{5} + 4$

37. $f(x) = 4x^3$

$f(-x) = 4(-x)^3 = -4x^3$

f is odd.

38. $f(x) = 2x^4 - x^2$

$f(-x) = 2(-x)^4 - (-x)^2 = 2x^4 - x^2$

f is even.

39. $g(x) = -3x^2 - 5$

$g(-x) = -3(-x)^2 - 5 = -3x^2 - 5$

g is even.

40. $h(x) = 3x^3 + 5$

$h(-x) = 3(-x)^3 + 5 = -3x^3 + 5$

h is neither even nor odd.

41. $F(x) = \sqrt[3]{x}$

$F(-x) = \sqrt[3]{-x} = -\sqrt[3]{x}$

F is odd.

42. $G(x) = \sqrt{x}$

$G(-x) = \sqrt{-x}$

G is neither even nor odd.

43. $f(x) = x + |x|$

$f(-x) = -x + |-x| = -x + |x|$

f is neither even nor odd.

44. $f(x) = \sqrt[3]{2x^2 + 1}$

$f(-x) = \sqrt[3]{2(-x)^2 + 1} = \sqrt[3]{2x^2 + 1}$

f is even.

45. $g(x) = \dfrac{1}{x^2}$

$g(-x) = \dfrac{1}{(-x)^2} = \dfrac{1}{x^2}$

g is even.

46. $h(x) = \dfrac{x}{x^2 - 1}$

$h(-x) = \dfrac{-x}{(-x)^2 - 1} = \dfrac{-x}{x^2 - 1}$

h is odd.

47. $h(x) = \dfrac{-x^3}{3x^2 - 9}$

$h(-x) = \dfrac{-(-x)^3}{3(-x)^2 - 9} = \dfrac{x^3}{3x^2 - 9}$

h is odd.

48. $F(x) = \dfrac{2x}{|x|}$

$F(-x) = \dfrac{2(-x)}{|-x|} = \dfrac{-2x}{|x|}$

F is odd.

49. $f(x) = 2x + 5$

$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{2(x+h) + 5 - 2x - 5}{h} = \frac{2h}{h} = 2$$

50. $f(x) = -3x + 2$

$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{-3(x+h) + 2 - (-3x + 2)}{h} = \frac{-3h}{h} = -3$$

51. $f(x) = x^2 + 2x$

$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h} = \frac{2xh + h^2 + 2h}{h} = 2x + h + 2$$

52. $f(x) = 2x^2 + x$

$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 + (x+h) - (2x^2 + x)}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) + x + h - 2x^2 - x}{h} = \frac{2x^2 + 4xh + 2h^2 + x + h - 2x^2 - x}{h}$$

$$= \frac{4xh + 2h^2 + h}{h} = 4x + 2h + 1$$

53. $f(x) = 2x^2 - 3x + 1$

$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$$

$$= \frac{4xh + 2h^2 - 3h}{h} = 4x + 2h - 3$$

54. $f(x) = -x^2 + 3x - 2$

$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{-(x+h)^2 + 3(x+h) - 2 - (-x^2 + 3x - 2)}{h}$$

$$= \frac{-(x^2 + 2xh + h^2) + 3x + 3h - 2 + x^2 - 3x + 2}{h}$$

$$= \frac{-x^2 - 2xh - h^2 + 3x + 3h - 2 + x^2 - 3x + 2}{h}$$

$$= \frac{-2xh - h^2 + 3h}{h} = -2x - h + 3$$

55. $f(x) = \dfrac{1}{x}$

$$m_{\text{sec}} = \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \frac{\frac{x - (x+h)}{(x+h)x}}{h} = \left(\frac{x - x - h}{(x+h)x}\right)\left(\frac{1}{h}\right) = \left(\frac{-h}{(x+h)x}\right)\left(\frac{1}{h}\right)$$

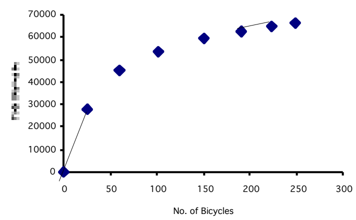
$$= \frac{-1}{(x+h)x}$$

56. $f(x)=\dfrac{1}{x^2}$

$$m_{\text{sec}}=\frac{f(x+h)-f(x)}{h}=\frac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}$$

$$=\frac{\dfrac{x^2-(x+h)^2}{(x+h)^2x^2}}{h}=\left(\frac{x^2-(x^2+2xh+h^2)}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)=\left(\frac{-2xh-h^2}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)=\frac{-2x-h}{(x+h)^2x^2}$$

57. (a), (b), (e)



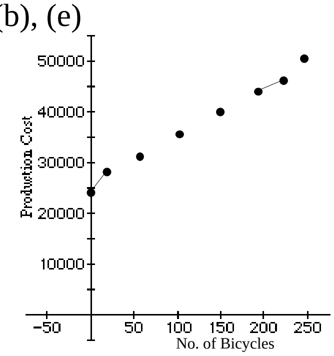
(c) Average rate of change $=\dfrac{28000-0}{25-0}=\dfrac{28000}{25}=1120$

(d) For each additional bicycle sold between 0 and 25, the total revenue increases by \$1120.

(f) Average rate of change $=\dfrac{64835-62360}{223-190}=\dfrac{2475}{33}=75$

(g) For each additional bicycle sold between 190 and 223, the total revenue increases by \$75.

58. (a), (b), (e)



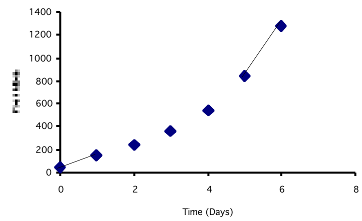
(c) Average rate of change $=\dfrac{27750-24000}{25-0}=\dfrac{3750}{25}=150$

(d) For each additional bicycle made between 0 and 25, the total production cost increases by \$150.

(f) Average rate of change $=\dfrac{46500-42750}{223-190}=\dfrac{3750}{33}=113.64$

(g) For each additional bicycle made between 190 and 223, the total production cost increases by \$113.64.

59. (a), (b), (e)



(c) Average rate of change = $\frac{153-50}{1-0} = \frac{103}{1} = 103$

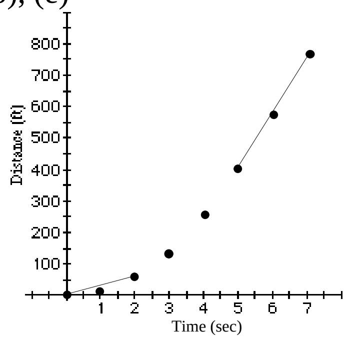
(d) The population is increasing at a rate of 103 per day between day 0 and day 1.

(f) Average rate of change = $\frac{1280-839}{6-5} = \frac{441}{1} = 441$

(g) The population is increasing at a rate of 441 per day between day 5 and day 6.

(h) As time passes, the average rate of change of the population is increasing.

60. (a), (b), (e)



(c) Average rate of change = $\frac{64-0}{2-0} = \frac{64}{2} = 32$ ft per sec

(d) The distance is increasing at a rate of 32 feet per second between 1 and 2 seconds.

(f) Average rate of change = $\frac{784-400}{7-5} = \frac{384}{2} = 192$ ft per sec

(g) The distance is increasing at a rate of 192 feet per second between 5 and 7 seconds.

(h) As time passes, the average rate of change of the distance is increasing.

61. One at most because if f is increasing it could only cross the x-axis at most one time. It could not "turn" and cross it again or it would start to decrease.

62. A function cannot be both even and odd, since its graph would have both y-axis and origin symmetry, thus, by Problem 55 in Section 2.3, the graph would also have x-axis symmetry. But a graph with x-axis symmetry will fail the vertical line test.