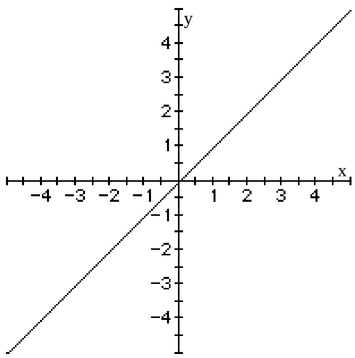
Chapter 3

Functions and Their Graphs

3.3 Library of Functions; Piecewise-Defined Functions

1.	C	2.	A	3.	E	4.	G
5.	B	6.	D	7.	F	8.	H

9.


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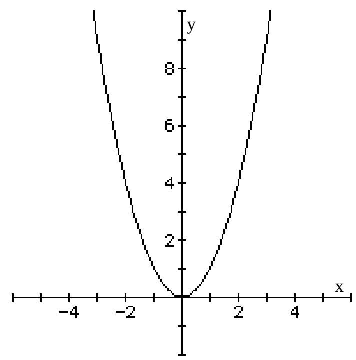

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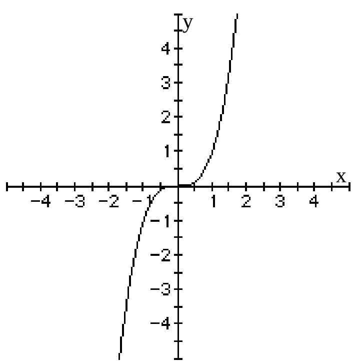

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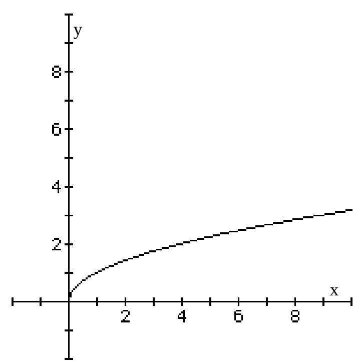

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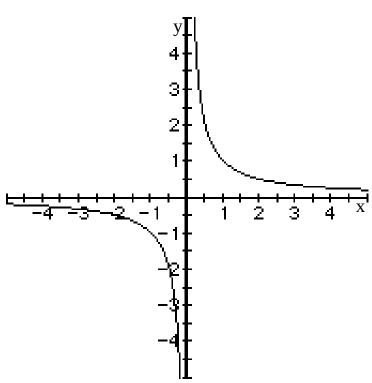

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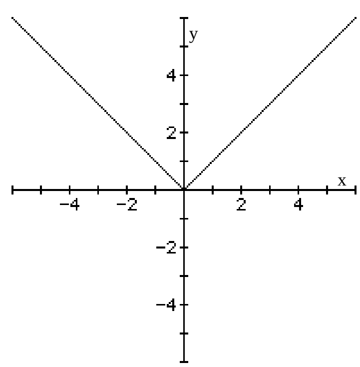
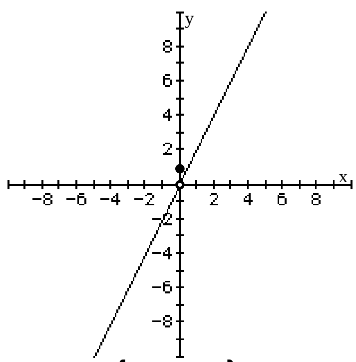


15. (a) $f(-2) = (-2)^2 = 4$
(b) $f(0) = 2$
(c) $f(2) = 2(2) + 1 = 5$

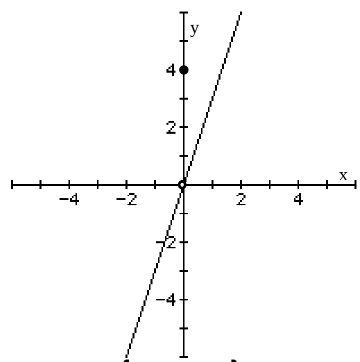
16. (a) $f(-1) = (-1)^3 = -1$
(b) $f(0) = 3(0) + 2 = 2$
(c) $f(1) = 3(1) + 2 = 5$

17. (a) $f(1.2) = \text{int}(2(1.2)) = \text{int}(2.4) = 2$
(b) $f(1.6) = \text{int}(2(1.6)) = \text{int}(3.2) = 3$
(c) $f(-1.8) = \text{int}(2(-1.8)) = \text{int}(-3.6) = -4$

18. (a) $f(1.2) = \text{int}\left(\frac{1.2}{2}\right) = \text{int}(0.6) = 0$
(b) $f(1.6) = \text{int}\left(\frac{1.6}{2}\right) = \text{int}(0.8) = 0$
(c) $f(-1.8) = \text{int}\left(\frac{-1.8}{2}\right) = \text{int}(-0.9) = -1$

19. $f(x) = \begin{cases} 2x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$
(a) Domain: {Real Numbers}
(b) x-intercept: none
y-intercept: (0,1)
(c)


(d) Range: $\{y \mid y \neq 0\}$

20. $f(x) = \begin{cases} 3x & \text{if } x \neq 0 \\ 4 & \text{if } x = 0 \end{cases}$
(a) Domain: {Real Numbers}
(b) x-intercept: none
y-intercept: (0, 4)
(c)


(d) Range: $\{y \mid y \neq 0\}$

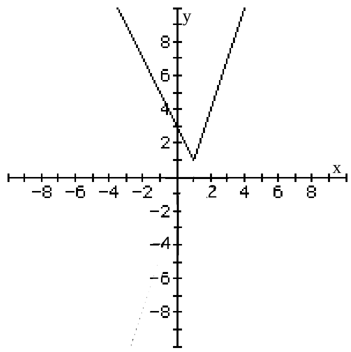
21. $f(x) = \begin{cases} -2x+3 & \text{if } x < 1 \\ 3x-2 & \text{if } x \geq 1 \end{cases}$

(a) Domain: {Real Numbers}

(b) x-intercept: none

y-intercept: (0,3)

(c)



(d) Range: $\{y | y \geq 1\}$

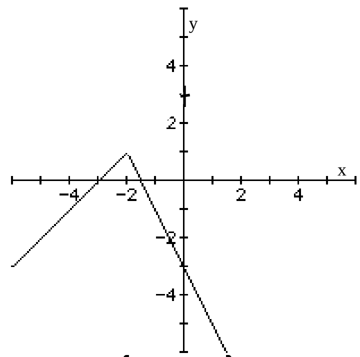
22. $f(x) = \begin{cases} x+3 & \text{if } x < -2 \\ -2x-3 & \text{if } x \geq -2 \end{cases}$

(a) Domain: {Real Numbers}

(b) x-intercept: (–3, 0), (–1.5, 0)

y-intercept: (0, –3)

(c)



(d) Range: $\{y | y \leq 1\}$

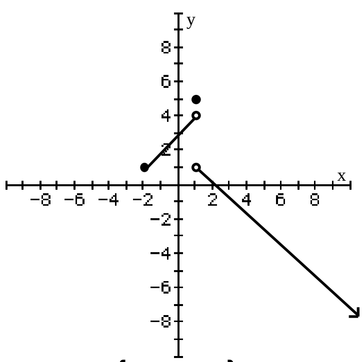
23. $f(x) = \begin{cases} x+3 & \text{if } -2 \leq x < 1 \\ 5 & \text{if } x = 1 \\ -x+2 & \text{if } x > 1 \end{cases}$

(a) Domain: $\{x | x \geq -2\}$

(b) x-intercept: (2, 0)

y-intercept: (0, 3)

(c)



(d) Range: $\{y | y < 4\} \quad \{5\}$

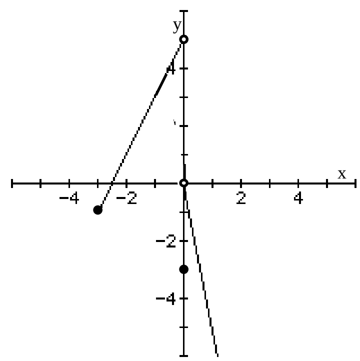
24. $f(x) = \begin{cases} 2x+5 & \text{if } -3 \leq x < 0 \\ -3 & \text{if } x = 0 \\ -5x & \text{if } x > 0 \end{cases}$

(a) Domain: $\{x | x \geq -3\}$

(b) x-intercept: (–2.5, 0)

y-intercept: (0, –3)

(c)



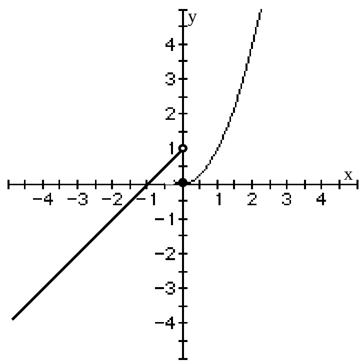
(d) Range: $\{y | y < 5\}$

25. $f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

(a) Domain: {Real Numbers}

(b) x-intercept: (–1,0), (0,0)
y-intercept: (0,0)

(c)



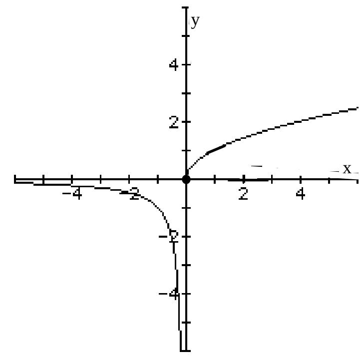
(d) Range: {Real Numbers}

26. $f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \ge 0 \end{cases}$

(a) Domain: {Real Numbers}

(b) x-intercept: (0,0)
y-intercept: (0,0)

(c)



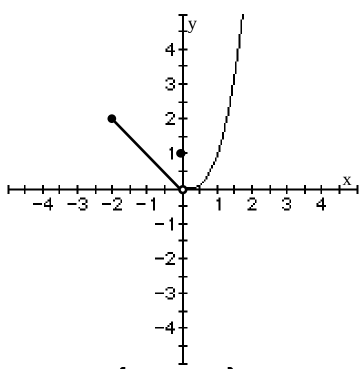
(d) Range: {Real Numbers}

27. $f(x) = \begin{cases} |x| & \text{if } -2 \le x < 0 \\ 1 & \text{if } x = 0 \\ x^3 & \text{if } x > 0 \end{cases}$

(a) Domain: $\{x \mid x \ge -2\}$

(b) x-intercept: none
y-intercept: (0, 1)

(c)



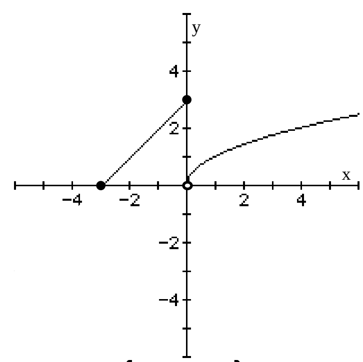
(d) Range: $\{y \mid y > 0\}$

28. $f(x) = \begin{cases} 3+x & \text{if } -3 \le x < 0 \\ 3 & \text{if } x = 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$

(a) Domain: $\{x \mid x \ge -3\}$

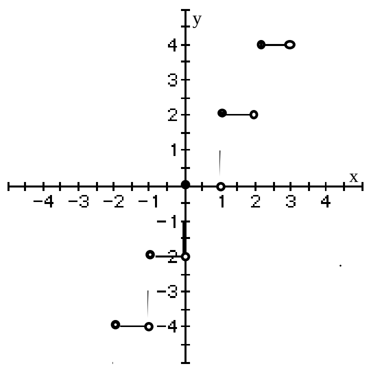
(b) x-intercept: (–3, 0)
y-intercept: (0, 3)

(c)



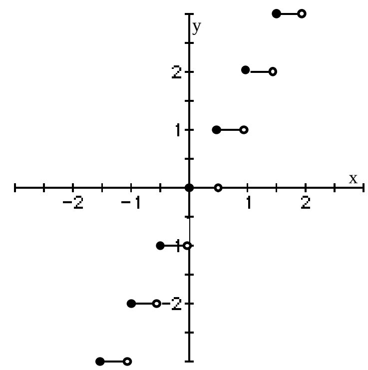
(d) Range: $\{y \mid y \ge 0\}$

29. $h(x) = 2\,\text{int}(x)$
   (a) Domain: {Real Numbers}
   (b) x-intercept: all ordered pairs $(x, 0)$ when $0 \le x < 1$.
   y-intercept: (0,0)
   (c)



   (d) Range: {Even Integers}

30. $f(x) = \text{int}(2x)$
   (a) Domain: {Real Numbers}
   (b) x-intercept: all ordered pairs $(x, 0)$ when $0 \le x < \frac{1}{2}$.
   y-intercept: (0,0)
   (c)



   (d) Range: {Integers}

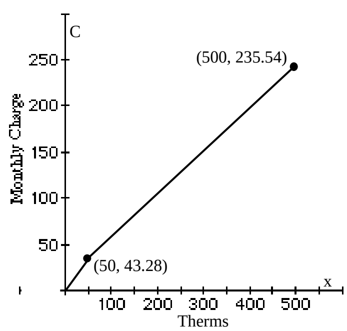
31. $f(x) = \begin{cases} -x & \text{if } -1 \le x \le 0 \\ \frac{1}{2}x & \text{if } 0 < x \le 2 \end{cases}$

32. $f(x) = \begin{cases} x & \text{if } -1 \le x \le 0 \\ 1 & \text{if } 0 < x \le 2 \end{cases}$

33. $f(x) = \begin{cases} -x & \text{if } x \le 0 \\ -x + 2 & \text{if } 0 < x \le 2 \end{cases}$

34. $f(x) = \begin{cases} 2x + 2 & \text{if } -1 \le x \le 0 \\ x & \text{if } x > 0 \end{cases}$

35. (a) Charge for 50 therms: $C = 9.45 + 0.36375(50) + 0.3128(50) = \$43.28$
   (b) Charge for 500 therms:
   $$C = 9.45 + 0.36375(50) + 0.11445(450) + 0.3128(500) = \$235.54$$
   (c) The monthly charge function:
   $$C = \begin{cases} 9.45 + 0.36375x + 0.3128x & \text{for } 0 \le x \le 50 \\ 9.45 + 0.36375(50) + 0.11445(x - 50) + 0.3128x & \text{for } x > 50 \end{cases}$$
   $$= \begin{cases} 9.45 + 0.67655x & \text{for } 0 \le x \le 50 \\ 9.45 + 18.1875 + 0.11445x - 5.7225 + 0.3128x & \text{for } x > 50 \end{cases}$$
   $$= \begin{cases} 9.45 + 0.67655x & \text{for } 0 \le x \le 50 \\ 21.915 + 0.42725x & \text{for } x > 50 \end{cases}$$
   (d)



36. (a) Charge for 40 therms:

$$C = 6.45 + 0.2012(20) + 0.1117(20) + 0.3209(40) = \$25.54$$

(b) Charge for 202 therms:

$$C = 6.45 + 0.2012(20) + 0.1117(30) + 0.0374(152) + 0.3209(202) = \$84.33$$

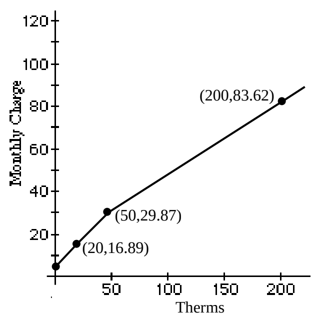
(c) The monthly charge function:

$$C = \begin{cases} 6.45 + 0.2012x + 0.3209x & \text{for } 0 \le x \le 20 \\ 6.45 + 0.2012(20) + 0.1117(x-20) + 0.3209x & \text{for } 20 < x \le 50 \\ 6.45 + 0.2012(20) + 0.1117(30) + 0.0374(x-50) + 0.3209x & \text{for } x > 50 \end{cases}$$

$$= \begin{cases} 6.45 + 0.5221x & \text{for } 0 \le x \le 20 \\ 6.45 + 4.024 + 0.1117x - 2.234 + 0.3209x & \text{for } 20 < x \le 50 \\ 6.45 + 4.024 + 3.351 + 0.0374x - 1.87 + 0.3209x & \text{for } x > 50 \end{cases}$$

$$= \begin{cases} 6.45 + 0.5221x & \text{for } 0 \le x \le 20 \\ 8.24 + 0.4326x & \text{for } 20 < x \le 50 \\ 11.955 + 0.3583x & \text{for } x > 50 \end{cases}$$

(d)



37. (a) $W = 10°C$

(b) $W = 33 - \dfrac{(10.45 + 10\sqrt{5} - 5)(33 - 10)}{22.04} = 3.98°C$

(c) $W = 33 - \dfrac{(10.45 + 10\sqrt{15} - 15)(33 - 10)}{22.04} = -2.67°C$

(d) $W = 33 - 1.5958(33 - 10) = -3.7°C$

(e) When $0 \le v < 1.79$, the wind speed is so small that there is no effect on the temperature.

(f) For each drop of 1˚ in temperature, the wind chill factor drops approximately 1.6˚C. When the wind speed exceeds 20, there is a constant drop in temperature.

38. (a) $W = -10°C$

(b) $W = 33 - \dfrac{(10.45 + 10\sqrt{5} - 5)(33 - (-10))}{22.04} = -21.26°C$

(c) $W = 33 - \dfrac{(10.45 + 10\sqrt{15} - 15)(33 - (-10))}{22.04} = -33.68°C$

(d) $W = 33 - 1.5958(33 - (-10)) = -35.62°C$

39. Each graph is that of $y = x^2$, but shifted vertically. If $y = x^2 + k,\ k > 0$, the shift is up k units; if $y = x^2 + k,\ k < 0$, the shift is down $|k|$ units. The graph of $y = x^2 - 4$ is the same as the graph of $y = x^2$, but shifted down 4 units. The graph of $y = x^2 + 5$ is the graph of $y = x^2$, but shifted up 5 units.

40. Each graph is that of $y = x^2$, but shifted horizontally. If $y = (x-k)^2,\ k > 0$, the shift is to the right k units; if $y = (x-k)^2,\ k < 0$, the shift is to the left $|k|$ units. The graph of $y = (x+4)^2$ is the same as the graph of $y = x^2$, but shifted to the left 4 units. The graph of $y = (x-5)^2$ is the graph of $y = x^2$, but shifted to the right 5 units.

41. Each graph is that of $y = |x|$, but either compressed or stretched. If $y = k|x|$ and $k > 1$, the graph is stretched; if $y = k|x|$ and $0 < k < 1$, the graph is compressed. The graph of $y = \frac{1}{4}|x|$ is the same as the graph of $y = |x|$, but compressed. The graph of $y = 5|x|$ is the same as the graph of $y = |x|$, but stretched.

42. The graph of $y = -x^2$ is the reflection of the graph of $y = x^2$ on the x-axis. The graph of $y = -|x|$ is the reflection of the graph of $y = |x|$ on the x-axis. Multiplying a function by –1 causes the graph to be a reflection on the x-axis of the original function's graph.

43. The graph of $y = \sqrt{-x}$ is the reflection about the y-axis of the graph of $y = \sqrt{x}$. The same type of reflection occurs when graphing $y = 2x + 1$ and $y = 2(-x) + 1$. The conclusion is that the graph of $y = f(-x)$ is the reflection about the y-axis of the graph of $y = f(x)$.

44. The graph of $y = (x-1)^3 + 2$ is a shifting of the graph of $y = x^3$ one unit to the right and two units up.

45. For the graph of $y = x^n$, n a positive even integer, as n increases, the graph of the function is narrower for $|x| > 1$ and flatter for $|x| < 1$.

46. For the graph of $y = x^n$, n a positive odd integer, as n increases, the graph of the function increases at a greater rate for $|x| > 1$ and is closer to zero for $|x| < 1$. They have the same basic shape.

47. $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$ Domain = { all real numbers} Range = {0,1}

y-intercept: $x = 0 \rightarrow x$ is rational $\rightarrow y = 1$, so the y-intercept is (0, 1).

x-intercept: $y = 0 \rightarrow x$ is irrational, so the graph has infinitely many x-intercepts, namely, there is an x-intercept at each irrational value for x.

$f(-x) = 1 = f(x)$ when x is rational; $f(-x) = 0 = f(x)$ when x is irrational

f is even.

The graph of f consists of 2 infinite clusters of distinct points, extending horizontally in both directions.

One cluster is located 1 unit above the x-axis, and the other is located along the x-axis.

48. For $0 < x < 1$, the graph of $y = x^n$ flattens down toward the x-axis as n gets bigger.

For $1 < x$, the graph of $y = x^n$ grows more steeply as n gets bigger.