Chapter 3

Functions and Their Graphs

3.5 Operations on Functions; Composite Functions

1. $f(x) = 3x + 4 \qquad g(x) = 2x - 3$
   (a) $(f + g)(x) = 3x + 4 + 2x - 3 = 5x + 1$ The domain is all real numbers.
   (b) $(f - g)(x) = (3x + 4) - (2x - 3) = 3x + 4 - 2x + 3 = x + 7$
   The domain is all real numbers.
   (c) $(f \cdot g)(x) = (3x + 4)(2x - 3) = 6x^2 - 9x + 8x - 12 = 6x^2 - x - 12$
   The domain is all real numbers.
   (d) $\left(\frac{f}{g}\right)(x) = \frac{3x + 4}{2x - 3}$ The domain is all real numbers except $\frac{3}{2}$.

2. $f(x) = 2x + 1 \qquad g(x) = 3x - 2$
   (a) $(f + g)(x) = 2x + 1 + 3x - 2 = 5x - 1$ The domain is all real numbers.
   (b) $(f - g)(x) = (2x + 1) - (3x - 2) = 2x + 1 - 3x + 2 = -x + 3$
   The domain is all real numbers.
   (c) $(f \cdot g)(x) = (2x + 1)(3x - 2) = 6x^2 - 4x + 3x - 2 = 6x^2 - x - 2$
   The domain is all real numbers.
   (d) $\left(\frac{f}{g}\right)(x) = \frac{2x + 1}{3x - 2}$ The domain is all real numbers except $\frac{2}{3}$.

3. $f(x) = x - 1 \qquad g(x) = 2x^2$
   (a) $(f + g)(x) = x - 1 + 2x^2 = 2x^2 + x - 1$ The domain is all real numbers.
   (b) $(f - g)(x) = (x - 1) - (2x^2) = x - 1 - 2x^2 = -2x^2 + x - 1$
   The domain is all real numbers.
   (c) $(f \cdot g)(x) = (x - 1)(2x^2) = 2x^3 - 2x^2$
   The domain is all real numbers.
   (d) $\left(\frac{f}{g}\right)(x) = \frac{x - 1}{2x^2}$ The domain is all real numbers except 0.

4. $f(x) = 2x^2 + 3 \qquad g(x) = 4x^3 + 1$
   (a) $(f + g)(x) = 2x^2 + 3 + 4x^3 + 1 = 4x^3 + 2x^2 + 4$ The domain is all real numbers.
   (b) $(f - g)(x) = \left(2x^2 + 3\right) - \left(4x^3 + 1\right) = 2x^2 + 3 - 4x^3 - 1 = -4x^3 + 2x^2 + 2$
   The domain is all real numbers.
   (c) $(f \cdot g)(x) = \left(2x^2 + 3\right)\left(4x^3 + 1\right) = 8x^5 + 12x^3 + 2x^2 + 3$
   The domain is all real numbers.
   (d) $\left(\frac{f}{g}\right)(x) = \frac{2x^2 + 3}{4x^3 + 1}$ The domain is all real numbers except $x = \sqrt[3]{-\frac{1}{4}}$.

5. $f(x)=\sqrt{x} \qquad g(x)=3x-5$

(a) $(f+g)(x)=\sqrt{x}+3x-5$ The domain is $\{x \mid x\ge 0\}$.

(b) $(f-g)(x)=\sqrt{x}-(3x-5)=\sqrt{x}-3x+5$ The domain is $\{x \mid x\ge 0\}$.

(c) $(f\cdot g)(x)=\sqrt{x}(3x-5)=3x\sqrt{x}-5\sqrt{x}$ The domain is $\{x \mid x\ge 0\}$.

(d) $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x}}{3x-5}$ The domain is $\left\{x \,\middle|\, x\ge 0 \text{ and } x\ne \dfrac{5}{3}\right\}$.

6. $f(x)=|x| \qquad g(x)=x$

(a) $(f+g)(x)=|x|+x$ The domain is all real numbers.

(b) $(f-g)(x)=|x|-x$ The domain is all real numbers.

(c) $(f\cdot g)(x)=|x|\,x$ The domain is all real numbers.

(d) $\left(\dfrac{f}{g}\right)(x)=\dfrac{|x|}{x}$ The domain is all real numbers except 0.

7. $f(x)=1+\dfrac{1}{x} \qquad g(x)=\dfrac{1}{x}$

(a) $(f+g)(x)=1+\dfrac{1}{x}+\dfrac{1}{x}=1+\dfrac{2}{x}$ The domain is $\{x \mid x\ne 0\}$.

(b) $(f-g)(x)=1+\dfrac{1}{x}-\dfrac{1}{x}=1$ The domain is $\{x \mid x\ne 0\}$.

(c) $(f\cdot g)(x)=\left(1+\dfrac{1}{x}\right)\dfrac{1}{x}=\dfrac{1}{x}+\dfrac{1}{x^2}$ The domain is $\{x \mid x\ne 0\}$.

(d) $\left(\dfrac{f}{g}\right)(x)=\dfrac{1+\dfrac{1}{x}}{\dfrac{1}{x}}=\dfrac{\dfrac{x+1}{x}}{\dfrac{1}{x}}=\dfrac{x+1}{x}\cdot\dfrac{x}{1}=x+1$ The domain is $\{x \mid x\ne 0\}$.

8. $f(x)=\sqrt{x-2} \qquad g(x)=\sqrt{4-x}$

(a) $(f+g)(x)=\sqrt{x-2}+\sqrt{4-x}$

The domain is all $\{x \mid 2\le x\le 4\}$.

(b) $(f-g)(x)=\sqrt{x-2}-\sqrt{4-x}$

The domain is all $\{x \mid 2\le x\le 4\}$.

(c) $(f\cdot g)(x)=\left(\sqrt{x-2}\right)\left(\sqrt{4-x}\right)=\sqrt{(x-2)(4-x)}$

The domain is all $\{x \mid 2\le x\le 4\}$.

(d) $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x-2}}{\sqrt{4-x}}=\sqrt{\dfrac{x-2}{4-x}}$ The domain is $\{x \mid 2\le x< 4\}$.

9. $f(x)=\dfrac{2x+3}{3x-2} \qquad g(x)=\dfrac{4x}{3x-2}$

(a) $(f+g)(x)=\dfrac{2x+3}{3x-2}+\dfrac{4x}{3x-2}=\dfrac{2x+3+4x}{3x-2}=\dfrac{6x+3}{3x-2}$

The domain is $\left\{x \,\middle|\, x\ne \dfrac{2}{3}\right\}$.

(b) $(f-g)(x)=\dfrac{2x+3}{3x-2}-\dfrac{4x}{3x-2}=\dfrac{2x+3-4x}{3x-2}=\dfrac{-2x+3}{3x-2}$

The domain is $\left\{x \,\middle|\, x\ne \dfrac{2}{3}\right\}$.

(c) $(f \cdot g)(x) = \left(\dfrac{2x+3}{3x-2}\right)\left(\dfrac{4x}{3x-2}\right) = \dfrac{8x^2+12x}{(3x-2)^2}$

The domain is $\left\{ x \,\middle|\, x \neq \dfrac{2}{3} \right\}$.

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{\dfrac{2x+3}{3x-2}}{\dfrac{4x}{3x-2}} = \dfrac{2x+3}{3x-2} \cdot \dfrac{3x-2}{4x} = \dfrac{2x+3}{4x}$

The domain is $\left\{ x \,\middle|\, x \neq \dfrac{2}{3} \text{ and } x \neq 0 \right\}$.

10. $f(x) = \sqrt{x+1} \qquad g(x) = \dfrac{2}{x}$

(a) $(f+g)(x) = \sqrt{x+1} + \dfrac{2}{x}$ The domain is $\{ x \mid x \geq -1,\ x \neq 0 \}$.

(b) $(f-g)(x) = \sqrt{x+1} - \dfrac{2}{x}$ The domain is $\{ x \mid x \geq -1,\ x \neq 0 \}$.

(c) $(f \cdot g)(x) = \sqrt{x+1} \cdot \dfrac{2}{x} = \dfrac{2\sqrt{x+1}}{x}$ The domain is $\{ x \mid x \geq -1,\ x \neq 0 \}$.

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x+1}}{\dfrac{2}{x}} = \dfrac{x\sqrt{x+1}}{2}$ The domain is $\{ x \mid x \geq -1,\ x \neq 0 \}$.

11. $f(x) = 3x+1 \qquad (f+g)(x) = 6 - \tfrac{1}{2}x$

$6 - \tfrac{1}{2}x = 3x + 1 + g(x)$

$5 - \tfrac{7}{2}x = g(x)$

$g(x) = 5 - \tfrac{7}{2}x$

12. $f(x) = \dfrac{1}{x} \qquad \left(\dfrac{f}{g}\right)(x) = \dfrac{x+1}{x^2-x}$

$$\frac{x+1}{x^2-x} = \frac{\frac{1}{x}}{g(x)}$$

$$g(x) = \frac{\frac{1}{x}}{\frac{x+1}{x^2-x}} = \frac{1}{x} \cdot \frac{x^2-x}{x+1} = \frac{1}{x} \cdot \frac{x(x-1)}{x+1} = \frac{x-1}{x+1}$$

13. $f(x) = 2x \qquad g(x) = 3x^2+1$

(a) $(f \circ g)(4) = f(g(4)) = f\left(3(4)^2+1\right) = f(49) = 2(49) = 98$

(b) $(g \circ f)(2) = g(f(2)) = g(2 \cdot 2) = g(4) = 3(4)^2+1 = 48+1 = 49$

(c) $(f \circ f)(1) = f(f(1)) = f(2(1)) = f(2) = 2(2) = 4$

(d) $(g \circ g)(0) = g(g(0)) = g\left(3(0)^2+1\right) = g(1) = 3(1)^2+1 = 4$

14. $f(x) = 3x+2 \qquad g(x) = 2x^2-1$

(a) $(f \circ g)(4) = f(g(4)) = f\left(2(4)^2-1\right) = f(31) = 3(31)+2 = 95$

(b) $(g \circ f)(2) = g(f(2)) = g(3(2)+2) = g(8) = 2(8)^2-1 = 128-1 = 127$

(c) $(f \circ f)(1) = f(f(1)) = f\left(3(1)+2\right) = f(5) = 3(5)+2 = 17$

(d) $(g \circ g)(0) = g(g(0)) = g\left(2(0)^2-1\right) = g(-1) = 2(-1)^2-1 = 1$

15. $f(x) = 4x^2 - 3 \qquad g(x) = 3 - \frac{1}{2}x^2$

(a) $(f \circ g)(4) = f(g(4)) = f\left(3 - \frac{1}{2}(4)^2\right) = f(-5) = 4(-5)^2 - 3 = 97$

(b) $(g \circ f)(2) = g(f(2)) = g(4(2)^2 - 3) = g(13) = 3 - \frac{1}{2}(13)^2 = 3 - \frac{169}{2} = \frac{-163}{2}$

(c) $(f \circ f)(1) = f(f(1)) = f(4(1)^2 - 3) = f(1) = 4(1)^2 - 3 = 1$

(d) $(g \circ g)(0) = g(g(0)) = g\left(3 - \frac{1}{2}(0)^2\right) = g(3) = 3 - \frac{1}{2}(3)^2 = 3 - \frac{9}{2} = \frac{-3}{2}$

16. $f(x) = 2x^2 \qquad g(x) = 1 - 3x^2$

(a) $(f \circ g)(4) = f(g(4)) = f\left(1 - 3(4)^2\right) = f(-47) = 2(-47)^2 = 4418$

(b) $(g \circ f)(2) = g(f(2)) = g(2(2)^2) = g(8) = 1 - 3(8)^2 = 1 - 192 = -191$

(c) $(f \circ f)(1) = f(f(1)) = f\left(2(1)^2\right) = f(2) = 2(2)^2 = 8$

(d) $(g \circ g)(0) = g(g(0)) = g\left(1 - 3(0)^2\right) = g(1) = 1 - 3(1)^2 = 1 - 3 = -2$

17. $f(x) = \sqrt{x} \qquad g(x) = 2x$

(a) $(f \circ g)(4) = f(g(4)) = f(2(4)) = f(8) = \sqrt{8} = 2\sqrt{2}$

(b) $(g \circ f)(2) = g(f(2)) = g\left(\sqrt{2}\right) = 2\sqrt{2}$

(c) $(f \circ f)(1) = f(f(1)) = f\left(\sqrt{1}\right) = f(1) = \sqrt{1} = 1$

(d) $(g \circ g)(0) = g(g(0)) = g(2(0)) = g(0) = 2(0) = 0$

18. $f(x) = \sqrt{x+1} \qquad g(x) = 3x$

(a) $(f \circ g)(4) = f(g(4)) = f(3(4)) = f(12) = \sqrt{12+1} = \sqrt{13}$

(b) $(g \circ f)(2) = g(f(2)) = g\left(\sqrt{2+1}\right) = g\left(\sqrt{3}\right) = 3\sqrt{3}$

(c) $(f \circ f)(1) = f(f(1)) = f\left(\sqrt{1+1}\right) = f\left(\sqrt{2}\right) = \sqrt{\sqrt{2}+1}$

(d) $(g \circ g)(0) = g(g(0)) = g(3(0)) = g(0) = 3(0) = 0$

19. $f(x) = |x| \qquad g(x) = \dfrac{1}{x^2+1}$

(a) $(f \circ g)(4) = f(g(4)) = f\left(\dfrac{1}{4^2+1}\right) = f\left(\dfrac{1}{17}\right) = \left|\dfrac{1}{17}\right| = \dfrac{1}{17}$

(b) $(g \circ f)(2) = g(f(2)) = g(|2|) = g(2) = \dfrac{1}{2^2+1} = \dfrac{1}{5}$

(c) $(f \circ f)(1) = f(f(1)) = f(|1|) = f(1) = |1| = 1$

(d) $(g \circ g)(0) = g(g(0)) = g\left(\dfrac{1}{0^2+1}\right) = g(1) = \dfrac{1}{1^2+1} = \dfrac{1}{2}$

20. $f(x) = |x-2| \qquad g(x) = \dfrac{3}{x^2+2}$

(a) $(f \circ g)(4) = f(g(4)) = f\left(\dfrac{3}{4^2+2}\right) = f\left(\dfrac{3}{18}\right) = f\left(\dfrac{1}{6}\right) = \left|\dfrac{1}{6} - 2\right| = \left|-\dfrac{11}{6}\right| = \dfrac{11}{6}$

(b) $(g \circ f)(2) = g(f(2)) = g(|2-2|) = g(0) = \dfrac{3}{0^2+2} = \dfrac{3}{2}$

(c) $(f \circ f)(1) = f(f(1)) = f(|1-2|) = f(1) = |1-2| = 1$

(d) $(g \circ g)(0) = g(g(0)) = g\left(\dfrac{3}{0^2+2}\right) = g\left(\dfrac{3}{2}\right) = \dfrac{3}{\left(\frac{3}{2}\right)^2+2} = \dfrac{3}{\frac{17}{4}} = \dfrac{12}{17}$

21. $f(x) = \dfrac{3}{x^2+1}$ $\quad g(x) = \sqrt{x}$

(a) $(f \circ g)(4) = f(g(4)) = f\left(\sqrt{4}\right) = f(2) = \dfrac{3}{2^2+1} = \dfrac{3}{5}$

(b) $(g \circ f)(2) = g(f(2)) = g\left(\dfrac{3}{2^2+1}\right) = g\left(\dfrac{3}{5}\right) = \sqrt{\dfrac{3}{5}} = \dfrac{\sqrt{15}}{5}$

(c) $(f \circ f)(1) = f(f(1)) = f\left(\dfrac{3}{1^2+1}\right) = f\left(\dfrac{3}{2}\right) = \dfrac{3}{\left(\frac{3}{2}\right)^2+1} = \dfrac{3}{\frac{13}{4}} = \dfrac{12}{13}$

(d) $(g \circ g)(0) = g(g(0)) = g\left(\sqrt{0}\right) = g(0) = \sqrt{0} = 0$

22. $f(x) = x^{3/2}$ $\quad g(x) = \dfrac{2}{x+1}$

(a) $(f \circ g)(4) = f(g(4)) = f\left(\dfrac{2}{4+1}\right) = f\left(\dfrac{2}{5}\right) = \left(\dfrac{2}{5}\right)^{3/2} = \left(\sqrt{\dfrac{2}{5}}\right)^3$

(b) $(g \circ f)(2) = g(f(2)) = g\left(2^{3/2}\right) = g\left(2\sqrt{2}\right) = \dfrac{2}{2\sqrt{2}+1}$

(c) $(f \circ f)(1) = f(f(1)) = f\left(1^{3/2}\right) = f(1) = 1^{3/2} = 1$

(d) $(g \circ g)(0) = g(g(0)) = g\left(\dfrac{2}{0+1}\right) = g(2) = \dfrac{2}{2+1} = \dfrac{2}{3}$

23. The domain of g is $\{x \mid x \neq 0\}$. The domain of f is $\{x \mid x \neq 1\}$.
Thus, $g(x) \neq 1$, so we solve:

$$g(x) = 1$$
$$\frac{2}{x} = 1$$
$$x = 2$$

Thus, $x \neq 2$; so the domain of $f \circ g$ is $\{x \mid x \neq 0,\ x \neq 2\}$.

24. The domain of g is $\{x \mid x \neq 0\}$. The domain of f is $\{x \mid x \neq -3\}$.
Thus, $g(x) \neq -3$, so we solve:

$$g(x) = -3$$
$$\frac{-2}{x} = -3$$
$$x = \frac{2}{3}$$

Thus, $x \neq \dfrac{2}{3}$; so the domain of $f \circ g$ is $\left\{x \,\middle|\, x \neq 0,\ x \neq \dfrac{2}{3}\right\}$.

25. The domain of g is $\{x \mid x \neq 0\}$. The domain of f is $\{x \mid x \neq 1\}$.
Thus, $g(x) \neq 1$, so we solve:

$$g(x) = 1$$
$$\frac{-4}{x} = 1$$
$$x = -4$$

Thus, $x \neq -4$; so the domain of $f \circ g$ is $\{x \mid x \neq -4,\ x \neq 0\}$.

26. The domain of g is $\{x \mid x \neq 0\}$. The domain of f is $\{x \mid x \neq -3\}$.
Thus, $g(x) \neq -3$, so we solve:
$$g(x) = -3$$
$$\frac{2}{x} = -3$$
$$x = -\frac{2}{3}$$
Thus, $x \neq -\frac{2}{3}$; so the domain of $f \circ g$ is $\left\{x \,\middle|\, x \neq 0,\ x \neq -\frac{2}{3}\right\}$.

27. The domain of g is {Real Numbers}. The domain of f is $\{x \mid x \geq 0\}$.
Thus, $g(x) \geq 0$, so we solve:
$$g(x) \geq 0$$
$$2x + 3 \geq 0$$
$$x \geq \frac{-3}{2}$$
Thus, the domain of $f \circ g$ is $\left\{x \,\middle|\, x \geq \frac{-3}{2}\right\}$.

28. The domain of f is {Real Numbers}. The domain of g is $\{x \mid x \leq 1\}$.
Thus, the domain of $f \circ g$ is $\{x \mid x \leq 1\}$.

29. The domain of g is $\{x \mid x \geq 1\}$. The domain of f is {Real Numbers}.

Thus, the domain of $f \circ g$ is $\{x \mid x \geq 1\}$.

30. The domain of g is $\{x \mid x \geq 2\}$. The domain of f is {Real Numbers}.

Thus, the domain of $f \circ g$ is $\{x \mid x \geq 2\}$.

31. $f(x) = 2x + 3 \qquad g(x) = 3x$
The domain of f is all real numbers. The domain of g is all real numbers.
(a) $(f \circ g)(x) = f(g(x)) = f(3x) = 2(3x) + 3 = 6x + 3$ Domain: All real numbers.
(b) $(g \circ f)(x) = g(f(x)) = g(2x + 3) = 3(2x + 3) = 6x + 9$
Domain: All real numbers.
(c) $(f \circ f)(x) = f(f(x)) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 6 + 3 = 4x + 9$
Domain: All real numbers.
(d) $(g \circ g)(x) = g(g(x)) = g(3x) = 3(3x) = 9x$ Domain: All real numbers.

32. $f(x) = -x \qquad g(x) = 2x - 4$
The domain of f is all real numbers. The domain of g is all real numbers.
(a) $(f \circ g)(x) = f(g(x)) = f(2x - 4) = -(2x - 4) = -2x + 4$
Domain: All real numbers.
(b) $(g \circ f)(x) = g(f(x)) = g(-x) = 2(-x) - 4 = -2x - 4$
Domain: All real numbers.
(c) $(f \circ f)(x) = f(f(x)) = f(-x) = -(-x) = x$ Domain: All real numbers.
(d) $(g \circ g)(x) = g(g(x)) = g(2x - 4) = 2(2x - 4) - 4 = 4x - 8 - 4 = 4x - 12$
Domain: All real numbers.

33. $f(x) = 3x + 1 \qquad g(x) = x^2$

The domain of f is all real numbers. The domain of g is all real numbers.

(a) $(f \circ g)(x) = f(g(x)) = f\left(x^2\right) = 3x^2 + 1$ Domain: All real numbers.

(b) $(g \circ f)(x) = g(f(x)) = g(3x+1) = (3x+1)^2 = 9x^2 + 6x + 1$
Domain: All real numbers.

(c) $(f \circ f)(x) = f(f(x)) = f(3x+1) = 3(3x+1)+1 = 9x + 3 + 1 = 9x + 4$
Domain: All real numbers.

(d) $(g \circ g)(x) = g(g(x)) = g\left(x^2\right) = \left(x^2\right)^2 = x^4$ Domain: All real numbers.

34. $f(x) = x + 1 \qquad g(x) = x^2 + 4$

The domain of f is all real numbers. The domain of g is all real numbers.

(a) $(f \circ g)(x) = f(g(x)) = f\left(x^2 + 4\right) = x^2 + 4 + 1 = x^2 + 5$
Domain: All real numbers.

(b) $(g \circ f)(x) = g(f(x)) = g(x+1) = (x+1)^2 + 4 = x^2 + 2x + 1 + 4 = x^2 + 2x + 5$
Domain: All real numbers.

(c) $(f \circ f)(x) = f(f(x)) = f(x+1) = (x+1)+1 = x + 2$
Domain: All real numbers.

(d) $(g \circ g)(x) = g(g(x)) = g\left(x^2 + 4\right) = \left(x^2 + 4\right)^2 + 4 = x^4 + 8x^2 + 16 + 4$
$= x^4 + 8x^2 + 20$ Domain: All real numbers.

35. $f(x) = x^2 \qquad g(x) = x^2 + 4$

The domain of f is all real numbers. The domain of g is all real numbers.

(a) $(f \circ g)(x) = f(g(x)) = f\left(x^2 + 4\right) = \left(x^2 + 4\right)^2 = x^4 + 8x^2 + 16$
Domain: All real numbers.

(b) $(g \circ f)(x) = g(f(x)) = g\left(x^2\right) = \left(x^2\right)^2 + 4 = x^4 + 4$ Domain: All real numbers.

(c) $(f \circ f)(x) = f(f(x)) = f\left(x^2\right) = \left(x^2\right)^2 = x^4$ Domain: All real numbers.

(d) $(g \circ g)(x) = g(g(x)) = g\left(x^2 + 4\right) = \left(x^2 + 4\right)^2 + 4 = x^4 + 8x^2 + 16 + 4$
$= x^4 + 8x^2 + 20$ Domain: All real numbers.

36. $f(x) = x^2 + 1 \qquad g(x) = 2x^2 + 3$

The domain of f is all real numbers. The domain of g is all real numbers.

(a) $(f \circ g)(x) = f(g(x)) = f\left(2x^2 + 3\right) = \left(2x^2 + 3\right)^2 + 1 = 4x^4 + 12x^2 + 9 + 1$
$= 4x^4 + 12x^2 + 10$ Domain: All real numbers.

(b) $(g \circ f)(x) = g(f(x)) = g\left(x^2 + 1\right) = 2\left(x^2 + 1\right)^2 + 3 = 2\left(x^4 + 2x^2 + 1\right) + 3$
$= 2x^4 + 4x^2 + 2 + 3 = 2x^4 + 4x^2 + 5$
Domain: All real numbers.

(c) $(f \circ f)(x) = f(f(x)) = f\left(x^2 + 1\right) = \left(x^2 + 1\right)^2 + 1 = x^4 + 2x^2 + 1 + 1 = x^4 + 2x^2 + 2$
Domain: All real numbers.

(d) $(g \circ g)(x) = g(g(x)) = g\left(2x^2 + 3\right) = 2\left(2x^2 + 3\right)^2 + 3 = 2\left(4x^4 + 12x^2 + 9\right) + 3$
$= 8x^4 + 24x^2 + 18 + 3 = 8x^4 + 24x^2 + 21$
Domain: All real numbers.

37. $f(x)=\dfrac{3}{x-1}$ $\quad g(x)=\dfrac{2}{x}$ The domain of f is $\{x \mid x \neq 1\}$.
The domain of g is $\{x \mid x \neq 0\}$.

(a) $(f\circ g)(x)=f(g(x))=f\left(\dfrac{2}{x}\right)=\dfrac{3}{\dfrac{2}{x}-1}=\dfrac{3}{\dfrac{2-x}{x}}=\dfrac{3x}{2-x}$

Domain of $f\circ g$ is $\{x \mid x\neq 0,\ x\neq 2\}$.

(b) $(g\circ f)(x)=g(f(x))=g\left(\dfrac{3}{x-1}\right)=\dfrac{2}{\dfrac{3}{x-1}}=\dfrac{2(x-1)}{3}$

Domain of $g\circ f$ is $\{x \mid x\neq 1\}$

(c) $(f\circ f)(x)=f(f(x))=f\left(\dfrac{3}{x-1}\right)=\dfrac{3}{\dfrac{3}{x-1}-1}=\dfrac{3}{\dfrac{3-(x-1)}{x-1}}=\dfrac{3(x-1)}{4-x}$

Domain of $f\circ f$ is $\{x \mid x\neq 1,\ x\neq 4\}$.

(d) $(g\circ g)(x)=g(g(x))=g\left(\dfrac{2}{x}\right)=\dfrac{2}{\dfrac{2}{x}}=\dfrac{2x}{2}=x$

Domain of $g\circ g$ is $\{x \mid x\neq 0\}$.

38. $f(x)=\dfrac{1}{x+3}$ $\quad g(x)=\dfrac{-2}{x}$

The domain of f is $\{x \mid x\neq -3\}$. The domain of g is $\{x \mid x\neq 0\}$.

(a) $(f\circ g)(x)=f(g(x))=f\left(\dfrac{-2}{x}\right)=\dfrac{1}{\dfrac{-2}{x}+3}=\dfrac{1}{\dfrac{-2+3x}{x}}=\dfrac{x}{3x-2}$

Domain of $f\circ g$ is $\left\{x \,\middle|\, x\neq 0,\ x\neq \dfrac{2}{3}\right\}$.

(b) $(g\circ f)(x)=g(f(x))=g\left(\dfrac{1}{x+3}\right)=\dfrac{-2}{\dfrac{1}{x+3}}=\dfrac{-2(x+3)}{1}=-2x-6$

Domain of $g\circ f$ is $\{x \mid x\neq -3\}$.

(c) $(f\circ f)(x)=f(f(x))=f\left(\dfrac{1}{x+3}\right)=\dfrac{1}{\dfrac{1}{x+3}+3}=\dfrac{1}{\dfrac{1+3x+9}{x+3}}=\dfrac{x+3}{3x+10}$

Domain of $f\circ f$ is $\left\{x \,\middle|\, x\neq -3,\ x\neq -\dfrac{10}{3}\right\}$.

(d) $(g\circ g)(x)=g(g(x))=g\left(\dfrac{-2}{x}\right)=\dfrac{-2}{\dfrac{-2}{x}}=\dfrac{-2x}{-2}=x$

Domain of $g\circ g$ is $\{x \mid x\neq 0\}$.

39. $f(x)=\dfrac{x}{x-1}$ $\quad g(x)=\dfrac{-4}{x}$

The domain of f is $\{x \mid x \neq 1\}$. The domain of g is $\{x \mid x \neq 0\}$.

(a) $(f \circ g)(x)=f(g(x))=f\left(\dfrac{-4}{x}\right)=\dfrac{\frac{-4}{x}}{\frac{-4}{x}-1}=\dfrac{\frac{-4}{x}}{\frac{-4-x}{x}}=\dfrac{-4}{-4-x}$

Domain of $f \circ g$ is $\{x \mid x \neq -4, x \neq 0\}$.

(b) $(g \circ f)(x)=g(f(x))=g\left(\dfrac{x}{x-1}\right)=\dfrac{-4}{\frac{x}{x-1}}=\dfrac{-4(x-1)}{x}$

Domain of $g \circ f$ is $\{x \mid x \neq 0, x \neq 1\}$.

(c) $(f \circ f)(x)=f(f(x))=f\left(\dfrac{x}{x-1}\right)=\dfrac{\frac{x}{x-1}}{\frac{x}{x-1}-1}=\dfrac{\frac{x}{x-1}}{\frac{x-(x-1)}{x-1}}=\dfrac{x}{1}=x$

Domain of $f \circ f$ is $\{x \mid x \neq 1\}$.

(d) $(g \circ g)(x)=g(g(x))=g\left(\dfrac{-4}{x}\right)=\dfrac{-4}{\frac{-4}{x}}=\dfrac{-4x}{-4}=x$

Domain of $g \circ g$ is $\{x \mid x \neq 0\}$.

40. $f(x)=\dfrac{x}{x+3}$ $\quad g(x)=\dfrac{2}{x}$

The domain of f is $\{x \mid x \neq -3\}$. The domain of g is $\{x \mid x \neq 0\}$.

(a) $(f \circ g)(x)=f(g(x))=f\left(\dfrac{2}{x}\right)=\dfrac{\frac{2}{x}}{\frac{2}{x}+3}=\dfrac{\frac{2}{x}}{\frac{2+3x}{x}}=\dfrac{2}{2+3x}$

Domain of $f \circ g$ is $\left\{x \,\middle|\, x \neq 0, x \neq -\dfrac{2}{3}\right\}$.

(b) $(g \circ f)(x)=g(f(x))=g\left(\dfrac{x}{x+3}\right)=\dfrac{2}{\frac{x}{x+3}}=\dfrac{2(x+3)}{x}=\dfrac{2x+6}{x}$

Domain of $g \circ f$ is $\{x \mid x \neq 0, x \neq -3\}$.

(c) $(f \circ f)(x)=f(f(x))=f\left(\dfrac{x}{x+3}\right)=\dfrac{\frac{x}{x+3}}{\frac{x}{x+3}+3}=\dfrac{\frac{x}{x+3}}{\frac{x+3x+9}{x+3}}=\dfrac{x}{4x+9}$

Domain of $f \circ f$ is $\left\{x \,\middle|\, x \neq -3, x \neq \dfrac{-9}{4}\right\}$.

(d) $(g \circ g)(x)=g(g(x))=g\left(\dfrac{2}{x}\right)=\dfrac{2}{\frac{2}{x}}=\dfrac{2x}{2}=x$ Domain of $g \circ g$ is $\{x \mid x \neq 0\}$.

41. $f(x)=\sqrt{x}$ $\quad g(x)=2x+3$

The domain of f is $\{x \mid x \geq 0\}$. The domain of g is $\{\text{Real Numbers}\}$.

(a) $(f \circ g)(x)=f(g(x))=f(2x+3)=\sqrt{2x+3}$ Domain of $f \circ g$ is $\left\{x \,\middle|\, x \geq \dfrac{-3}{2}\right\}$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = 2\sqrt{x} + 3$ Domain of $g \circ f$ is $\{x \mid x \ge 0\}$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(\sqrt{x}\right) = \sqrt{\sqrt{x}} = x^{\frac{1}{4}} = \sqrt[4]{x}$

Domain of $f \circ f$ is $\{x \mid x \ge 0\}$.

(d) $(g \circ g)(x) = g(g(x)) = g(2x+3) = 2(2x+3)+3 = 4x+6+3 = 4x+9$

Domain of $g \circ g$ is $\{\text{Real Numbers}\}$.

42. $f(x) = \sqrt{x-2}$ $g(x) = 1-2x$

The domain of f is $\{x \mid x \ge 2\}$. The domain of g is $\{\text{Real Numbers}\}$.

(a) $(f \circ g)(x) = f(g(x)) = f(1-2x) = \sqrt{1-2x-2} = \sqrt{-2x-1}$

Domain of $f \circ g$ is $\left\{x \,\middle|\, x \le -\frac{1}{2}\right\}$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x-2}\right) = 1 - 2\sqrt{x-2}$ Domain of $g \circ f$ is $\{x \mid x \ge 2\}$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2}-2}$ Domain of $f \circ f$ is $\{x \mid x \ge 6\}$.

(d) $(g \circ g)(x) = g(g(x)) = g(1-2x) = 1-2(1-2x) = 1-2+4x = 4x-1$

Domain of $g \circ g$ is $\{\text{Real Numbers}\}$.

43. $f(x) = x^2+1$ $g(x) = \sqrt{x-1}$

The domain of f is $\{\text{Real Numbers}\}$. The domain of g is $\{x \mid x \ge 1\}$.

(a) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x-1}\right) = \left(\sqrt{x-1}\right)^2 + 1 = x-1+1 = x$

Domain of $f \circ g$ is $\{x \mid x \ge 1\}$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(x^2+1\right) = \sqrt{x^2+1-1} = \sqrt{x^2} = |x|$

Domain of $g \circ f$ $\{\text{Real Numbers}\}$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(x^2+1\right) = \left(x^2+1\right)^2 + 1 = x^4+2x^2+1+1 = x^4+2x^2+2$

Domain of $f \circ f$ is $\{\text{Real Numbers}\}$.

(d) $(g \circ g)(x) = g(g(x)) = g\left(\sqrt{x-1}\right) = \sqrt{\sqrt{x-1}-1}$ Domain of $g \circ g$ is $\{x \mid x \ge 2\}$.

44. $f(x) = x^2+4$ $g(x) = \sqrt{x-2}$

The domain of f is $\{\text{Real Numbers}\}$. The domain of g is $\{x \mid x \ge 2\}$.

(a) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x-2}\right) = \left(\sqrt{x-2}\right)^2 + 4 = x-2+4 = x+2$

Domain of $f \circ g$ is $\{x \mid x \ge 2\}$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(x^2+4\right) = \sqrt{x^2+4-2} = \sqrt{x^2+2}$

Domain of $g \circ f$ $\{\text{Real Numbers}\}$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(x^2+4\right) = \left(x^2+1\right)^2 + 4 = x^4+2x^2+1+4$

$= x^4+2x^2+5$ Domain of $f \circ f$ is $\{\text{Real Numbers}\}$.

(d) $(g \circ g)(x) = g(g(x)) = g\left(\sqrt{x-1}\right) = \sqrt{\sqrt{x-1}-1}$ Domain of $g \circ g$ is $\{x \mid x \ge 1\}$.

45. $f(x) = ax + b \qquad g(x) = cx + d$ The domain of f is $\{\text{Real Numbers}\}$.
The domain of g is $\{\text{Real Numbers}\}$.
(a) $(f \circ g)(x) = f(g(x)) = f(cx+d) = a(cx+d) + b = acx + ad + b$
Domain of $f \circ g$ is $\{\text{Real Numbers}\}$.
(b) $(g \circ f)(x) = g(f(x)) = g(ax+b) = c(ax+b) + d = acx + bc + d$
Domain of $g \circ f$ is $\{\text{Real Numbers}\}$.
(c) $(f \circ f)(x) = f(f(x)) = f(ax+b) = a(ax+b) + b = a^2x + ab + b$
Domain of $f \circ f$ is $\{\text{Real Numbers}\}$.
(d) $(g \circ g)(x) = g(g(x)) = g(cx+d) = c(cx+d) + d = c^2x + cd + d$
Domain of $g \circ g$ is $\{\text{Real Numbers}\}$.

46. $f(x) = \dfrac{ax+b}{cx+d} \qquad g(x) = mx$ The domain of f is $\left\{ x \,\middle|\, x \neq -\dfrac{d}{c} \right\}$.
The domain of g is $\{\text{Real Numbers}\}$.
(a) $(f \circ g)(x) = f(g(x)) = f(mx) = \dfrac{a(mx)+b}{c(mx)+d} = \dfrac{amx+b}{cmx+d}$
Domain of $f \circ g$ is $\left\{ x \,\middle|\, x \neq -\dfrac{d}{cm} \right\}$.
(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{ax+b}{cx+d}\right) = m\left(\dfrac{ax+b}{cx+d}\right) = \dfrac{amx+bm}{cx+d}$
Domain of $g \circ f$ is $\left\{ x \,\middle|\, x \neq -\dfrac{d}{c} \right\}$.
(c) $(f \circ f)(x) = f(f(x)) = f\left(\dfrac{ax+b}{cx+d}\right) = \dfrac{a\left(\dfrac{ax+b}{cx+d}\right)+b}{c\left(\dfrac{ax+b}{cx+d}\right)+d} = \dfrac{\dfrac{a^2x+ab+bcx+bd}{cx+d}}{\dfrac{acx+bc+cdx+d^2}{cx+d}}$

$$= \frac{a^2x+ab+bcx+bd}{acx+bc+cdx+d^2}$$

$$acx + bc + cdx + d^2 \neq 0$$
$$(ac+cd)x \neq -bc - d^2$$
$$x \neq \frac{-bc-d^2}{ac+cd}$$

Domain of $f \circ f$ is $\left\{ x \,\middle|\, x \neq \dfrac{-bc-d^2}{ac+cd},\ x \neq -\dfrac{d}{c} \right\}$.

(d) $(g \circ g)(x) = g(g(x)) = g(mx) = m(mx) = m^2x$
Domain of $g \circ g$ is $\{\text{Real Numbers}\}$.

47. $(f \circ g)(x) = f(g(x)) = f\left(\tfrac{1}{2}x\right) = 2\left(\tfrac{1}{2}x\right) = x$
$(g \circ f)(x) = g(f(x)) = g(2x) = \tfrac{1}{2}(2x) = x$

48. $(f \circ g)(x) = f(g(x)) = f\left(\tfrac{1}{4}x\right) = 4\left(\tfrac{1}{4}x\right) = x$
$(g \circ f)(x) = g(f(x)) = g(4x) = \tfrac{1}{4}(4x) = x$

49. $(f \circ g)(x) = f(g(x)) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$
$(g \circ f)(x) = g(f(x)) = g\left(x^3\right) = \sqrt[3]{x^3} = x$

50. $(f \circ g)(x) = f(g(x)) = f\left(x - 5\right) = x - 5 + 5 = x$
$(g \circ f)(x) = g(f(x)) = g\left(x + 5\right) = x + 5 - 5 = x$

51. $(f \circ g)(x) = f(g(x)) = f\left(\tfrac{1}{2}(x + 6)\right) = 2\left(\tfrac{1}{2}(x + 6)\right) - 6 = x + 6 - 6 = x$
$(g \circ f)(x) = g(f(x)) = g\left(2x - 6\right) = \tfrac{1}{2}\left((2x - 6) + 6\right) = \tfrac{1}{2}(2x) = x$

52. $(f \circ g)(x) = f(g(x)) = f\left(\tfrac{1}{3}(4 - x)\right) = 4 - 3\left(\tfrac{1}{3}(4 - x)\right) = 4 - 4 + x = x$
$(g \circ f)(x) = g(f(x)) = g\left(4 - 3x\right) = \tfrac{1}{3}\left(4 - (4 - 3x)\right) = \tfrac{1}{3}(3x) = x$

53. $(f \circ g)(x) = f(g(x)) = f\left(\tfrac{1}{a}(x - b)\right) = a\left(\tfrac{1}{a}(x - b)\right) + b = x - b + b = x$
$(g \circ f)(x) = g(f(x)) = g\left(ax + b\right) = \tfrac{1}{a}\left((ax + b) - b\right) = \tfrac{1}{a}(ax) = x$

54. $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = x$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = x$

55. $H(x) = (2x + 3)^4$
$f(x) = x^4, \quad g(x) = 2x + 3$

56. $H(x) = \left(1 + x^2\right)^3 \qquad f(x) = x^3, \quad g(x) = 1 + x^2$

57. $H(x) = \sqrt{x^2 + 1}$
$f(x) = \sqrt{x}, \quad g(x) = x^2 + 1$

58. $H(x) = \sqrt{1 - x^2} \qquad f(x) = \sqrt{x}, \quad g(x) = 1 - x^2$

59. $H(x) = |2x + 1|$
$f(x) = |x|, \quad g(x) = 2x + 1$

60. $H(x) = |2x^2 + 3| \qquad f(x) = |x|, \quad g(x) = 2x^2 + 3$

61. $f(x) = 2x^3 - 3x^2 + 4x - 1 \qquad g(x) = 2$
$(f \circ g)(x) = f(g(x)) = f(2) = 2(2)^3 - 3(2)^2 + 4(2) - 1 = 16 - 12 + 8 - 1 = 11$
$(g \circ f)(x) = g(f(x)) = g\left(2x^3 - 3x^2 + 4x - 1\right) = 2$

62. $f(x) = \dfrac{x}{x-1}$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{x}{x-1}\right) = \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = \frac{\frac{x}{x-1}}{\frac{x-x+1}{x-1}} = \frac{x}{1} = x$$

63. $f(x) = 2x^2 + 5 \qquad g(x) = 3x + a$

$(f \circ g)(x) = f(g(x)) = f(3x + a) = 2(3x + a)^2 + 5$

When $x = 0$, $(f \circ g)(0) = 23$

Solving:

$$2(3\ 0 + a)^2 + 5 = 23$$
$$2a^2 + 5 = 23$$
$$2a^2 = 18 \rightarrow a^2 = 9 \rightarrow a = -3 \text{ or } 3$$

64. $f(x) = 3x^2 - 7 \qquad g(x) = 2x + a$

$(f \circ g)(x) = f(g(x)) = f(2x + a) = 3(2x + a)^2 - 7$

When $x = 0$, $(f \circ g)(0) = 68$

Solving:

$$3(2\ 0 + a)^2 - 7 = 68$$
$$3a^2 - 7 = 68$$
$$3a^2 = 75 \rightarrow a^2 = 25 \rightarrow a = -5 \text{ or } 5$$

65. $S(r) = 4\pi r^2 \qquad r(t) = \frac{2}{3}t^3,\ t \ge 0$

$S(r(t)) = S\left(\frac{2}{3}t^3\right) = 4\pi\left(\frac{2}{3}t^3\right)^2 = 4\pi\left(\frac{4}{9}t^6\right) = \frac{16}{9}\pi t^6$

66. $V(r) = \dfrac{4}{3}\pi r^3 \qquad r(t) = \frac{2}{3}t^3,\ t \ge 0$

$$V(r(t)) = V\left(\frac{2}{3}t^3\right) = \frac{4}{3}\pi\left(\frac{2}{3}t^3\right)^3 = \frac{4}{3}\pi\left(\frac{8}{27}t^9\right) = \frac{32}{81}\pi t^9$$

67. $N(t) = 100t - 5t^2,\ 0 \le t \le 10 \qquad C(N) = 15000 + 8000N$

$C(N(t)) = C\left(100t - 5t^2\right) = 15000 + 8000\left(100t - 5t^2\right)$

$\qquad = 15{,}000 + 800{,}000t - 40{,}000t^2$

68. $A(r) = \pi r^2 \qquad r(t) = 200\sqrt{t}$

$A(r(t)) = A\left(200\sqrt{t}\right) = \pi\left(200\sqrt{t}\right)^2 = 40000\pi t$

69. $p = \frac{-1}{4}x + 100 \qquad 0 \le x \le 400$

$\frac{1}{4}x = 100 - p$

$x = 4(100 - p)$

$$C = \frac{\sqrt{x}}{25} + 600 = \frac{\sqrt{4(100-p)}}{25} + 600 = \frac{2\sqrt{100-p}}{25} + 600$$

70. $p = \frac{-1}{5}x + 200 \qquad 0 \le x \le 1000$

$\frac{1}{5}x = 200 - p$

$x = 5(200 - p)$

$C = \frac{\sqrt{x}}{10} + 400 = \frac{\sqrt{5(200-p)}}{10} + 400 = \frac{\sqrt{1000-5p}}{10} + 400$

71. $V = \pi r^2 h \qquad h = 2r$

$V(r) = \pi r^2(2r) = 2\pi r^3$

72. $V = \frac{1}{3}\pi r^2 h \qquad h = 2r$

$V(r) = \frac{1}{3}\pi r^2(2r) = \frac{2}{3}\pi r^3$

73. Given that f and g are odd functions, we know that

$f(-x) = -f(x)$ and $g(-x) = -g(x)$ for all x in the domain of f and g respectively.

The composite function $f \circ g = f(g(x))$ has the following property:

$f(g(-x)) = f(-g(x))$ since g is odd

$= -f(g(x))$ since f is odd, $f \circ g$ is odd

74. Given that f is odd and g is even, we know that

$f(-x) = -f(x)$ and $g(-x) = g(x)$ for all x in the domain of f and g respectively.

The composite function $f \circ g = f(g(x))$ has the following property:

$f(g(-x)) = f(g(x))$ since g is even, $f \circ g$ is even

The composite function $g \circ f = g(f(x))$ has the following property:

$g(f(-x)) = g(-f(x))$ since f is odd

$= g(f(x))$ since g is even, $g \circ f$ is even