Chapter 3

Functions and Their Graphs

3.R Chapter Review

1. $f(4) = -5$ gives the ordered pair $(4, -5)$. $f(0) = 3$ gives $(0, 3)$.
   Finding the slope: $m = \dfrac{3-(-5)}{0-4} = \dfrac{8}{-4} = -2$
   Using slope-intercept form: $f(x) = -2x + 3$

2. $m = -4$, $g(-2) = 2$ gives the ordered pair $(-2, 2)$.
   Using point-slope form:
   $$\begin{aligned} y - 2 &= -4(x-(-2)) \\ y - 2 &= -4x - 8 \\ y &= -4x - 6 \\ g(x) &= -4x - 6 \end{aligned}$$

3. $f(x) = \dfrac{Ax+5}{6x-2}$ and $f(1) = 4$
   Solving:
   $$\begin{aligned} \frac{A(1)+5}{6(1)-2} &= 4 \\ \frac{A+5}{4} &= 4 \\ A + 5 &= 16 \\ A &= 11 \end{aligned}$$

4. $g(x) = \dfrac{A}{x} + \dfrac{8}{x^2}$ and $g(-1) = 0$
   Solving:
   $$\frac{A}{-1} + \frac{8}{(-1)^2} = 0 \quad \rightarrow \quad -A + 8 = 0 \quad \rightarrow \quad A = 8$$

5. (b), (c), and (d) pass the vertical line test and therefore are functions.

6. (a) Domain: $\{x \mid -5 \le x \le 4\}$ Range: $\{y \mid -3 \le y \le 1\}$
   (b) $f(-1) = 1$
   (c) Intercepts: (0, 0), (4, 0)
   (d) Increasing: (3, 4); Decreasing: (–1, 3); Constant: (–5, –1)
   (e) The function is neither even nor odd.

7. $f(x)=\dfrac{3x}{x^2-4}$

(a) $f(-x)=\dfrac{3(-x)}{(-x)^2-4}=\dfrac{-3x}{x^2-4}$

(b) $-f(x)=-\left(\dfrac{3x}{x^2-4}\right)=\dfrac{-3x}{x^2-4}$

(c) $f(x+2)=\dfrac{3(x+2)}{(x+2)^2-4}=\dfrac{3x+6}{x^2+4x+4-4}=\dfrac{3x+6}{x^2+4x}$

(d) $f(x-2)=\dfrac{3(x-2)}{(x-2)^2-4}=\dfrac{3x-6}{x^2-4x+4-4}=\dfrac{3x-6}{x^2-4x}$

(e) $f(2x)=\dfrac{3(2x)}{(2x)^2-4}=\dfrac{6x}{4x^2-4}=\dfrac{3x}{2x^2-2}$

8. $f(x)=\dfrac{x^2}{x+2}$

(a) $f(-x)=\dfrac{(-x)^2}{-x+2}=\dfrac{x^2}{-x+2}$

(b) $-f(x)=-\dfrac{x^2}{x+2}=\dfrac{-x^2}{x+2}$

(c) $f(x+2)=\dfrac{(x+2)^2}{(x+2)+2}=\dfrac{x^2+4x+4}{x+4}$

(d) $f(x-2)=\dfrac{(x-2)^2}{(x-2)+2}=\dfrac{x^2-4x+4}{x}$

(e) $f(2x)=\dfrac{(2x)^2}{(2x)+2}=\dfrac{4x^2}{2x+2}=\dfrac{2\left(2x^2\right)}{2(x+2)}=\dfrac{2x^2}{x+2}$

9. $f(x)=\sqrt{x^2-4}$

(a) $f(-x)=\sqrt{(-x)^2-4}=\sqrt{x^2-4}$

(b) $-f(x)=-\sqrt{x^2-4}$

(c) $f(x+2)=\sqrt{(x+2)^2-4}=\sqrt{x^2+4x+4-4}=\sqrt{x^2+4x}$

(d) $f(x-2)=\sqrt{(x-2)^2-4}=\sqrt{x^2-4x+4-4}=\sqrt{x^2-4x}$

(e) $f(2x)=\sqrt{(2x)^2-4}=\sqrt{4x^2-4}=2\sqrt{x^2-1}$

10. $f(x)=\left|x^2-4\right|$

(a) $f(-x)=\left|(-x)^2-4\right|=\left|x^2-4\right|$

(b) $-f(x)=-\left|x^2-4\right|$

(c) $f(x+2)=\left|(x+2)^2-4\right|=\left|x^2+4x+4-4\right|=\left|x^2+4x\right|$

(d) $f(x-2)=\left|(x-2)^2-4\right|=\left|x^2-4x+4-4\right|=\left|x^2-4x\right|$

(e) $f(2x)=\left|(2x)^2-4\right|=\left|4x^2-4\right|=4\left|x^2-1\right|$

11. $f(x) = \dfrac{x^2 - 4}{x^2}$

(a) $f(-x) = \dfrac{(-x)^2 - 4}{(-x)^2} = \dfrac{x^2 - 4}{x^2}$

(b) $-f(x) = -\left(\dfrac{x^2 - 4}{x^2}\right) = \dfrac{4 - x^2}{x^2}$

(c) $f(x+2) = \dfrac{(x+2)^2 - 4}{(x+2)^2} = \dfrac{x^2 + 4x + 4 - 4}{x^2 + 4x + 4} = \dfrac{x^2 + 4x}{x^2 + 4x + 4}$

(d) $f(x-2) = \dfrac{(x-2)^2 - 4}{(x-2)^2} = \dfrac{x^2 - 4x + 4 - 4}{x^2 - 4x + 4} = \dfrac{x^2 - 4x}{x^2 - 4x + 4}$

(e) $f(2x) = \dfrac{(2x)^2 - 4}{(2x)^2} = \dfrac{4x^2 - 4}{4x^2} = \dfrac{x^2 - 1}{x^2}$

12. $f(x) = \dfrac{x^3}{x^2 - 4}$

(a) $f(-x) = \dfrac{(-x)^3}{(-x)^2 - 4} = \dfrac{-x^3}{x^2 - 4}$

(b) $-f(x) = -\dfrac{x^3}{x^2 - 4} = \dfrac{-x^3}{x^2 - 4}$

(c) $f(x+2) = \dfrac{(x+2)^3}{(x+2)^2 - 4} = \dfrac{x^3 + 6x^2 + 12x + 8}{x^2 + 4x + 4 - 4} = \dfrac{x^3 + 6x^2 + 12x + 8}{x^2 + 4x}$

(d) $f(x-2) = \dfrac{(x-2)^3}{(x-2)^2 - 4} = \dfrac{x^3 - 6x^2 + 12x - 8}{x^2 - 4x + 4 - 4} = \dfrac{x^3 - 6x^2 + 12x - 8}{x^2 - 4x}$

(e) $f(2x) = \dfrac{(2x)^3}{(2x)^2 - 4} = \dfrac{8x^3}{4x^2 - 4} = \dfrac{4\left(2x^3\right)}{4\left(x^2 - 1\right)} = \dfrac{2x^3}{x^2 - 1}$

13. $f(x) = \dfrac{x}{x^2 - 9}$

The denominator cannot be zero:

$$x^2 - 9 \neq 0$$
$$(x+3)(x-3) \neq 0$$
$$x \neq -3 \text{ or } 3$$

Domain: $\{x \mid x \neq -3,\ x \neq 3\}$

14. $f(x) = \dfrac{3x^2}{x - 2}$

The denominator cannot be zero:

$$x - 2 \neq 0$$
$$x \neq 2$$

Domain: $\{x \mid x \neq 2\}$

15. $f(x) = \sqrt{2 - x}$

The radicand must be positive:

$$2 - x \geq 0$$
$$x \leq 2$$

Domain: $\{x \mid x \leq 2\}$ or $(-\infty, 2]$

16. $f(x) = \sqrt{x + 2}$

The radicand must be positive:

$$x + 2 \geq 0$$
$$x \geq 2$$

Domain: $\{x \mid x \geq 2\}$ or $[2, +\infty)$

17. $f(x) = \dfrac{\sqrt{x}}{|x|}$

The radicand must be positive and the denominator cannot be zero: $x > 0$

Domain: $\{x \mid x > 0\}$ or $(0, +\infty)$

18. $g(x) = \dfrac{|x|}{x}$

The denominator cannot be zero:

$$x \neq 0$$

Domain: $\{x \mid x \neq 0\}$

19. $f(x) = \dfrac{x}{x^2 + 2x - 3}$

The denominator cannot be zero:

$$x^2 + 2x - 3 \neq 0$$
$$(x+3)(x-1) \neq 0$$
$$x \neq -3 \text{ or } 1$$

Domain: $\{x \mid x \neq -3,\ x \neq 1\}$

20. $F(x) = \dfrac{1}{x^2 - 3x - 4}$

The denominator cannot be zero:

$$x^2 - 3x - 4 \neq 0$$
$$(x+1)(x-4) \neq 0$$
$$x \neq -1 \text{ or } 4$$

Domain: $\{x \mid x \neq -1,\ x \neq 4\}$

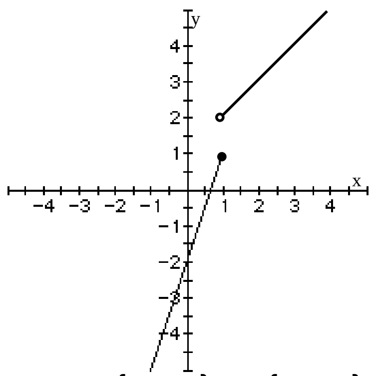
21. $f(x) = \begin{cases} 3x - 2 & \text{if } x \leq 1 \\ x + 1 & \text{if } x > 1 \end{cases}$

(a) Domain: {Real Numbers}

(b) x-intercept: $\left(\frac{2}{3}, 0\right)$

y-intercept: (0,–2)

(c)



(d) Range: $\{ y > 2\}$ $\{ y \leq 1\}$

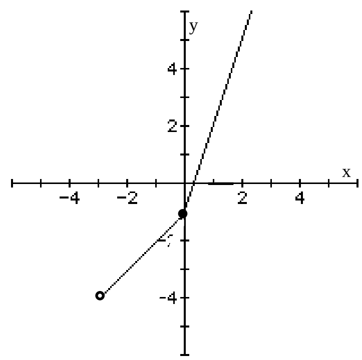
22. $f(x) = \begin{cases} x - 1 & \text{if } -3 < x < 0 \\ 3x - 1 & \text{if } x \geq 0 \end{cases}$

(a) Domain: $\{ x \mid x > -3\}$

(b) x-intercept: $\left(\frac{1}{3}, 0\right)$

y-intercept: (0, –1)

(c)



(d) Range: $\{ y > -4\}$

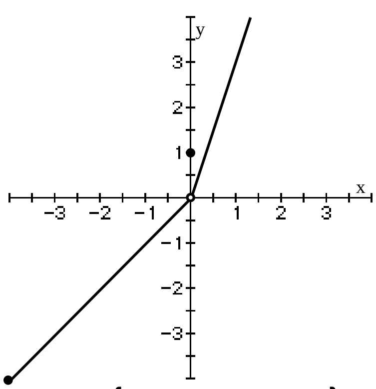
23. $f(x) = \begin{cases} x & \text{if } -4 \leq x < 0 \\ 1 & \text{if } x = 0 \\ 3x & \text{if } x > 0 \end{cases}$

(a) Domain: $\{x \mid x \geq -4\}$

(b) x-intercept: none

y-intercept: (0, 1)

(c)



(d) Range: $\{y \mid y \geq -4,\ y \neq 0\}$

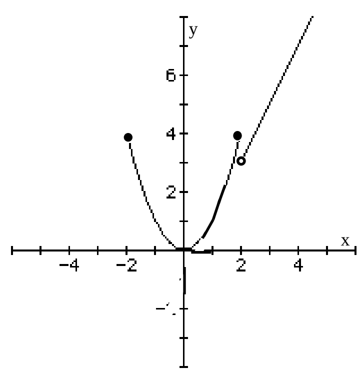
24. $f(x) = \begin{cases} x^2 & \text{if } -2 \leq x \leq 2 \\ 2x - 1 & \text{if } x > 2 \end{cases}$

(a) Domain: $\{x \mid x \geq -2\}$

(b) x-intercept: (0, 0)

y-intercept: (0, 0)

(c)



(d) Range: $\{y \mid y \geq 0\}$

25. $f(x) = 2 - 5x$

$$\frac{f(x) - f(2)}{x - 2} = \frac{2 - 5x - (-8)}{x - 2} = \frac{-5x + 10}{x - 2} = \frac{-5(x - 2)}{x - 2} = -5$$

26. $f(x) = 2x^2 + 7$

$$\frac{f(x) - f(2)}{x - 2} = \frac{2x^2 + 7 - 15}{x - 2} = \frac{2x^2 - 8}{x - 2} = \frac{2(x - 2)(x + 2)}{x - 2} = 2x + 4$$

27. $f(x) = 3x - 4x^2$

$$\frac{f(x) - f(2)}{x - 2} = \frac{3x - 4x^2 - (-10)}{x - 2} = \frac{-4x^2 + 3x + 10}{x - 2}$$

$$= \frac{-(4x^2 - 3x - 10)}{x - 2} = \frac{-(4x + 5)(x - 2)}{x - 2} = -4x - 5$$

28. $f(x) = x^2 - 3x + 2$

$$\frac{f(x) - f(2)}{x - 2} = \frac{x^2 - 3x + 2 - 0}{x - 2} = \frac{x^2 - 3x + 2}{x - 2} = \frac{(x - 2)(x - 1)}{x - 2} = x - 1$$

29. $f(x) = x^3 - 4x$

$$f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x = -\left(x^3 - 4x\right) = -f(x)$$

f is odd.

30. $g(x) = \dfrac{4 + x^2}{1 + x^4}$

$$g(-x) = \frac{4 + (-x)^2}{1 + (-x)^4} = \frac{4 + x^2}{1 + x^4} = g(x)$$ g is even.

31. $h(x) = \dfrac{1}{x^4} + \dfrac{1}{x^2} + 1$

$$h(-x) = \frac{1}{(-x)^4} + \frac{1}{(-x)^2} + 1 = \frac{1}{x^4} + \frac{1}{x^2} + 1 = h(x)$$ h is even.

32. $F(x) = \sqrt{1 - x^3}$

$$F(-x) = \sqrt{1 - (-x)^3} = \sqrt{1 + x^3} \neq F(x) \text{ or } -F(x)$$ F is neither even nor odd.

33. $G(x) = 1 - x + x^3$

$$G(-x) = 1 - (-x) + (-x)^3 = 1 + x - x^3 \neq -G(x) \neq G(x)$$

G is neither even nor odd.

34. $H(x) = 1 + x + x^2$

$$H(-x) = 1 + (-x) + (-x)^2 = 1 - x + x^2 \neq -H(x) \text{ or } H(x)$$

H is neither even nor odd.

35. $f(x)=\dfrac{x}{1+x^2}$

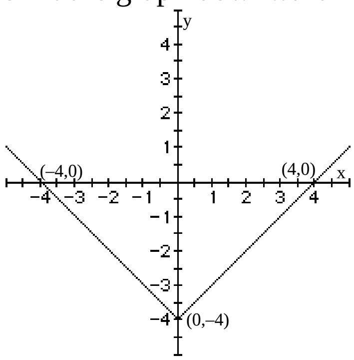
$$f(-x)=\frac{-x}{1+(-x)^2}=\frac{-x}{1+x^2}=-f(x) \qquad f \text{ is odd.}$$

36. $g(x)=\dfrac{1+x^2}{x^3}$

$$g(-x)=\frac{1+(-x)^2}{(-x)^3}=\frac{1+x^2}{-x^3}=-\frac{1+x^2}{x^3}=-g(x) \qquad g \text{ is odd.}$$

37. $F(x)=|x|-4$

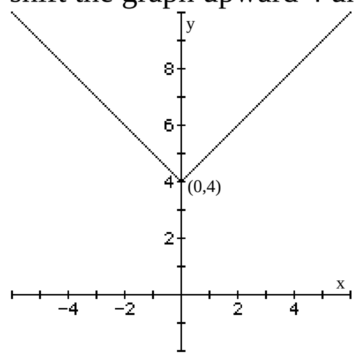
Using the graph of $y=|x|$, vertically shift the graph downward 4 units.



Intercepts: (–4,0), (4,0), (0,–4)
Domain: {Real Numbers}
Range: $\{y \mid y \geq -4\}$

38. $f(x)=|x|+4$

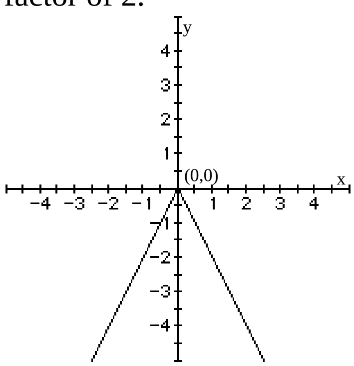
Using the graph of $y=|x|$, vertically shift the graph upward 4 units.



Intercepts: (0, 4)
Domain: {Real Numbers}
Range: $\{y \mid y \geq 4\}$

39. $g(x)=-2|x|$

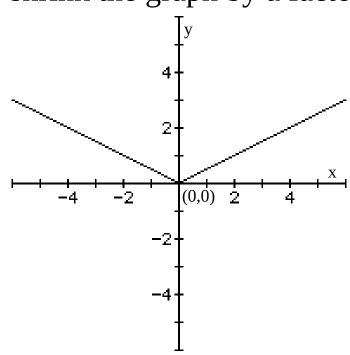
Reflect the graph of $y=|x|$ about the x-axis and vertically stretch the graph by a factor of 2.



Intercepts: (0,0)
Domain: {Real Numbers}
Range: $\{y \mid y \leq 0\}$

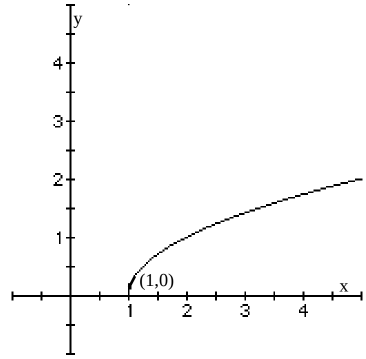
40. $g(x)=\frac{1}{2}|x|$

Using the graph of $y=|x|$, vertically shrink the graph by a factor of $\frac{1}{2}$.

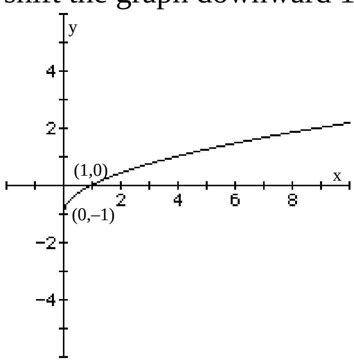


Intercepts: (0,0)
Domain: {Real Numbers}
Range: $\{y \mid y \geq 0\}$

41. $h(x) = \sqrt{x-1}$
Using the graph of $y = \sqrt{x}$, horizontally shift the graph to the right 1 unit.
Intercepts: (1,0)
Domain: $\{x \mid x \geq 1\}$
Range: $\{y \mid y \geq 0\}$

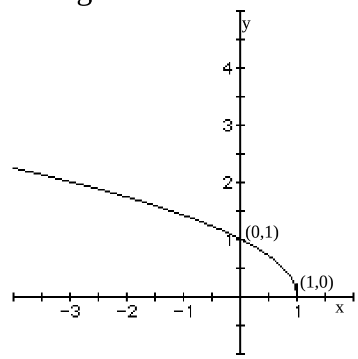


42. $h(x) = \sqrt{x} - 1$
Using the graph of $y = \sqrt{x}$, vertically shift the graph downward 1 unit.



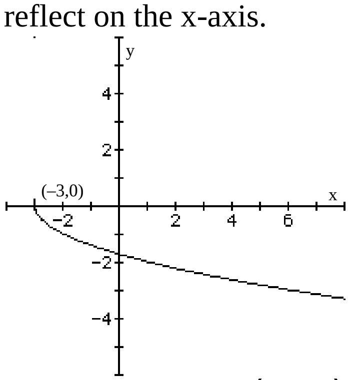
Intercepts: (1,0), (0, –1)
Domain: $\{x \mid x \geq 0\}$
Range: $\{y \mid y \geq -1\}$

43. $f(x) = \sqrt{1-x} = \sqrt{-1(x-1)}$
Reflect the graph of $y = \sqrt{x}$ about the y-axis and horizontally shift the graph to the right 1 unit..



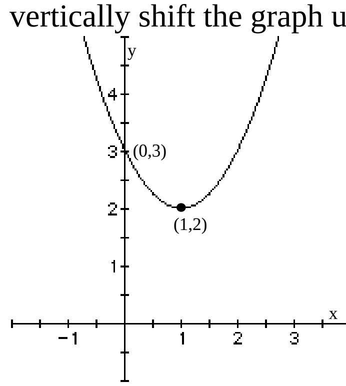
Intercepts: (1,0), (0,1)
Domain: $\{x \mid x \leq 1\}$
Range: $\{y \mid y \geq 0\}$

44. $f(x) = -\sqrt{x+3}$
Using the graph of $y = \sqrt{x}$, horizontally shift the graph to the left 3 units, and reflect on the x-axis.



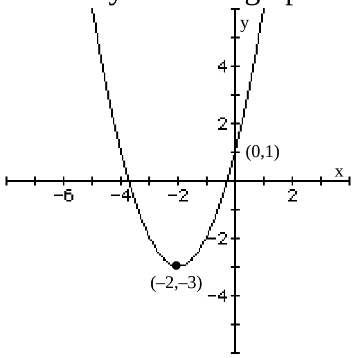
Intercepts: (–3,0), $\left(0, \sqrt{3}\right)$
Domain: $\{x \mid x \geq -3\}$
Range: $\{y \mid y \leq 0\}$

45. $h(x) = (x-1)^2 + 2$
Using the graph of $y = x^2$, horizontally shift the graph to the right 1 unit and vertically shift the graph up 2 units.



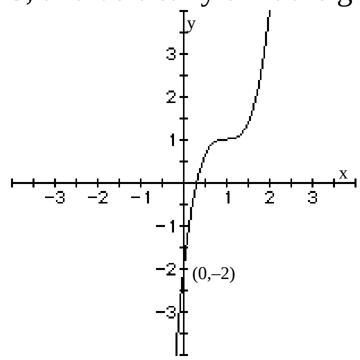
Intercepts: (0,3)
Domain: {Real Numbers}
Range: $\{y \mid y \geq 2\}$

46. $h(x) = (x+2)^2 - 3$
Using the graph of $y = x^2$, horizontally shift the graph to the left 2 units and vertically shift the graph down 3 units.



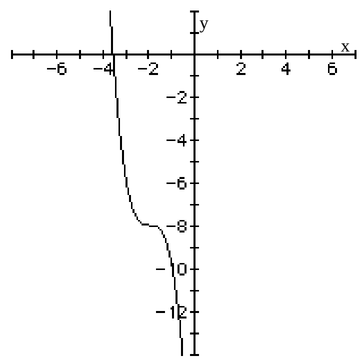
Intercepts: (0,1),
$\left(-2+\sqrt{3},\, 0\right), \left(-2-\sqrt{3},\, 0\right)$
Domain: {Real Numbers}
Range: $\{y \mid y \geq -3\}$

47. $g(x) = 3(x-1)^3 + 1$
Using the graph of $y = x^3$, horizontally shift the graph to the right 1 unit, vertically stretch the graph by a factor of 3, and vertically shift the graph up 1 unit.



Intercepts: (0,–2), $\left(1-\dfrac{\sqrt[3]{3}}{3},\, 0\right)$
Domain: {Real Numbers}
Range: {Real Numbers}

48. $g(x) = -2(x+2)^3 - 8$
Using the graph of $y = x^3$, horizontally shift the graph to the left 2 units, vertically stretch the graph by a factor of 2, reflect about the x-axis, and vertically shift the graph down 8 units.
Intercepts: (0,–24), $\left(-2-\sqrt[3]{4},\, 0\right)$
Domain: {Real Numbers}
Range: {Real Numbers}



49. $f(x) = 3x - 5 \qquad g(x) = 1 - 2x^2$
(a) $(f \circ g)(2) = f(g(2)) = f\left(1-2(2)^2\right) = f(-7) = 3(-7) - 5 = -26$
(b) $(g \circ f)(-2) = g(f(-2)) = g\left(3(-2)-5\right) = g(-11) = 1 - 2(-11)^2 = -241$
(c) $(f \circ f)(4) = f(f(4)) = f\left(3(4)-5\right) = f(7) = 3(7) - 5 = 16$
(d) $(g \circ g)(-1) = g(g(-1)) = g\left(1-2(-1)^2\right) = g(-1) = 1 - 2(-1)^2 = -1$

50. $f(x) = 4 - x \qquad g(x) = 1 + x^2$
(a) $(f \circ g)(2) = f(g(2)) = f\left(1+2^2\right) = f(5) = 4 - 5 = -1$
(b) $(g \circ f)(-2) = g(f(-2)) = g\left(4-(-2)\right) = g(6) = 1 + 6^2 = 37$
(c) $(f \circ f)(4) = f(f(4)) = f\left(4-4\right) = f(0) = 4 - 0 = 4$
(d) $(g \circ g)(-1) = g(g(-1)) = g\left(1+(-1)^2\right) = g(2) = 1 + 2^2 = 5$

51. $f(x)=\sqrt{x+2} \qquad g(x)=2x^2+1$

(a) $(f\circ g)(2)=f(g(2))=f\left(2(2)^2+1\right)=f(9)=\sqrt{9+2}=\sqrt{11}$

(b) $(g\circ f)(-2)=g(f(-2))=g\left(\sqrt{-2+2}\right)=g(0)=2(0)^2+1=1$

(c) $(f\circ f)(4)=f(f(4))=f\left(\sqrt{4+2}\right)=f\left(\sqrt{6}\right)=\sqrt{\sqrt{6}+2}$

(d) $(g\circ g)(-1)=g(g(-1))=g\left(2(-1)^2+1\right)=g(3)=2(3)^2+1=19$

52. $f(x)=1-3x^2 \qquad g(x)=\sqrt{4-x}$

(a) $(f\circ g)(2)=f(g(2))=f\left(\sqrt{4-2}\right)=f\left(\sqrt{2}\right)=1-3\left(\sqrt{2}\right)^2=1-3\ \ 2=-5$

(b) $(g\circ f)(-2)=g(f(-2))=g\left(1-3(-2)^2\right)=g(-11)=\sqrt{4-(-11)}=\sqrt{15}$

(c) $(f\circ f)(4)=f(f(4))=f\left(1-3(4)^2\right)=f(-47)=1-3(-47)^2=-6626$

(d) $(g\circ g)(-1)=g(g(-1))=g\left(\sqrt{4-(-1)}\right)=g\left(\sqrt{5}\right)=\sqrt{4-\sqrt{5}}$

53. $f(x)=\dfrac{1}{x^2+4} \qquad g(x)=3x-2$

(a) $(f\circ g)(2)=f(g(2))=f\left(3(2)-2\right)=f(4)=\dfrac{1}{4^2+4}=\dfrac{1}{20}$

(b) $(g\circ f)(-2)=g(f(-2))=g\left(\dfrac{1}{(-2)^2+4}\right)=g\left(\dfrac{1}{8}\right)=3\left(\dfrac{1}{8}\right)-2=\dfrac{-13}{8}$

(c) $(f\circ f)(4)=f(f(4))=f\left(\dfrac{1}{4^2+4}\right)=f\left(\dfrac{1}{20}\right)=\dfrac{1}{\left(\dfrac{1}{20}\right)^2+4}=\dfrac{1}{\dfrac{1601}{400}}=\dfrac{400}{1601}$

(d) $(g\circ g)(-1)=g(g(-1))=g\left(3(-1)-2\right)=g(-5)=3(-5)-2=-17$

54. $f(x)=\dfrac{2}{1+2x^2} \qquad g(x)=3x$

(a) $(f\circ g)(2)=f(g(2))=f\left(3(2)\right)=f(6)=\dfrac{2}{1+2(6)^2}=\dfrac{2}{73}$

(b) $(g\circ f)(-2)=g(f(-2))=g\left(\dfrac{2}{1+2(-2)^2}\right)=g\left(\dfrac{2}{9}\right)=3\left(\dfrac{2}{9}\right)=\dfrac{2}{3}$

(c) $(f\circ f)(4)=f(f(4))=f\left(\dfrac{2}{1+2\ \ 4^2}\right)=f\left(\dfrac{2}{33}\right)=\dfrac{2}{1+2\left(\dfrac{2}{33}\right)^2}=\dfrac{2}{\dfrac{1097}{1089}}=\dfrac{2178}{1097}$

(d) $(g\circ g)(-1)=g(g(-1))=g\left(3(-1)\right)=g(-3)=3(-3)=-9$

55. $f(x)=2-x \qquad g(x)=3x+1$

The domain of f is all real numbers. The domain of g is all real numbers.

(a) $(f\circ g)(x)=f(g(x))=f(3x+1)=2-(3x+1)=2-3x-1=1-3x$

Domain: All real numbers.

(b) $(g\circ f)(x)=g(f(x))=g(2-x)=3(2-x)+1=6-3x+1=7-3x$

Domain: All real numbers.

(c) $(f\circ f)(x)=f(f(x))=f(2-x)=2-(2-x)=2-2+x=x$

Domain: All real numbers.

(d) $(g\circ g)(x)=g(g(x))=g(3x+1)=3(3x+1)+1=9x+3+1=9x+4$

Domain: All real numbers.

56. $f(x)=2x-1 \qquad g(x)=2x+1$

The domain of f is all real numbers. The domain of g is all real numbers.

(a) $(f\circ g)(x)=f(g(x))=f(2x+1)=2(2x+1)-1=4x+2-1=4x+1$

Domain: All real numbers.

(b) $(g\circ f)(x)=g(f(x))=g(2x-1)=2(2x-1)+1=4x-2+1=4x-1$

Domain: All real numbers.

(c) $(f\circ f)(x)=f(f(x))=f(2x-1)=2(2x-1)-1=4x-2-1=4x-3$

Domain: All real numbers.

(d) $(g\circ g)(x)=g(g(x))=g(2x+1)=2(2x+1)+1=4x+2+1=4x+3$

Domain: All real numbers.

57. $f(x)=3x^2+x+1 \qquad g(x)=|3x|$

The domain of f is all real numbers. The domain of g is all real numbers.

(a) $(f\circ g)(x)=f(g(x))=f(|3x|)=3(|3x|)^2+(|3x|)+1=27x^2+3|x|+1$

Domain: All real numbers.

(b) $(g\circ f)(x)=g(f(x))=g(3x^2+x+1)=|3(3x^2+x+1)|=|9x^2+3x+3|$

Domain: All real numbers.

(c) $(f\circ f)(x)=f(f(x))=f(3x^2+x+1)=3(3x^2+x+1)^2+(3x^2+x+1)+1$

$=3(9x^4+6x^3+7x^2+2x+1)+3x^2+x+1+1$

$=27x^4+18x^3+24x^2+7x+5$

Domain: All real numbers.

(d) $(g\circ g)(x)=g(g(x))=g(|3x|)=|3|3x||=9|x|$

Domain: All real numbers.

58. $f(x)=\sqrt{3x} \qquad g(x)=1+x$

The domain of f is $\{x \mid x\ge 0\}$. The domain of g is all real numbers.

(a) $(f\circ g)(x)=f(g(x))=f(1+x)=\sqrt{3(1+x)}=\sqrt{3+3x}$

$3+3x\ge 0$

$x\ge -1$ Domain: $\{x \mid x\ge -1\}$.

(b) $(g\circ f)(x)=g(f(x))=g(\sqrt{3x})=1+\sqrt{3x}$

Domain: $\{x \mid x\ge 0\}$.

(c) $(f\circ f)(x)=f(f(x))=f(\sqrt{3x})=\sqrt{3\sqrt{3x}}$ Domain: $\{x \mid x\ge 0\}$.

(d) $(g\circ g)(x)=g(g(x))=g(1+x)=1+(1+x)=2+x$ Domain: All real numbers.

59. $f(x)=\dfrac{x+1}{x-1} \qquad g(x)=\dfrac{1}{x}$

The domain of f is $\{x \mid x\ne 1\}$. The domain of g is $\{x \mid x\ne 0\}$.

(a) $(f\circ g)(x)=f(g(x))=f\left(\dfrac{1}{x}\right)=\dfrac{\frac{1}{x}+1}{\frac{1}{x}-1}=\dfrac{\frac{1+x}{x}}{\frac{1-x}{x}}=\dfrac{1+x}{1-x}$

Domain of $f\circ g$ is $\{x \mid x\ne 0,\ x\ne 1\}$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{x+1}{x-1}\right) = \frac{1}{\frac{x+1}{x-1}} = \frac{x-1}{x+1}$

Domain of $g \circ f$ is $\{x \mid x \neq -1, x \neq 1\}$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(\frac{x+1}{x-1}\right) = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} = \frac{\frac{x+1+x-1}{x-1}}{\frac{x+1-(x-1)}{x-1}} = \frac{2x}{2} = x$

Domain of $f \circ f$ is $\{x \mid x \neq 1\}$.

(d) $(g \circ g)(x) = g(g(x)) = g\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$ Domain of $g \circ g$ is $\{x \mid x \neq 0\}$.

60. $f(x) = \sqrt{x-3}$ $g(x) = 3x$

The domain of f is $\{x \mid x \geq 3\}$. The domain of g is $\{\text{Real numbers}\}$.

(a) $(f \circ g)(x) = f(g(x)) = f(3x) = \sqrt{3x-3}$

$3x - 3 \geq 0$

$\to x \geq 1$ Domain of $f \circ g$ is $\{x \mid x \geq 1\}$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x-3}\right) = 3\sqrt{x-3}$

Domain of $g \circ f$ is $\{x \mid x \geq 3\}$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(\sqrt{x-3}\right) = \sqrt{\sqrt{x-3}-3}$

$\sqrt{x-3} - 3 \geq 0 \quad \to \quad \sqrt{x-3} \geq 3 \quad \to \quad x - 3 \geq 9 \quad \to \quad x \geq 12$

Domain of $f \circ f$ is $\{x \mid x \geq 12\}$.

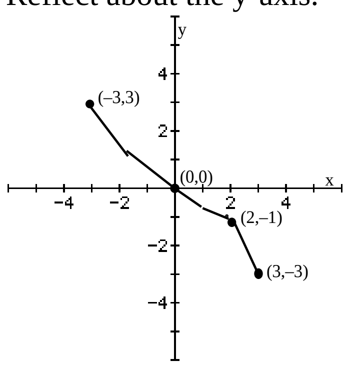
(d)

$(g \circ g)(x) = g(g(x)) = g(3x) = 3(3x) = 9x$

Domain of $g \circ g$ is $\{\text{Real numbers}\}$.

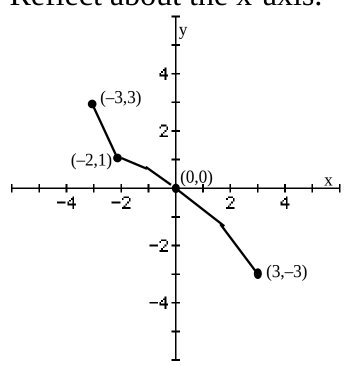
61. (a) $y = f(-x)$

Reflect about the y-axis.

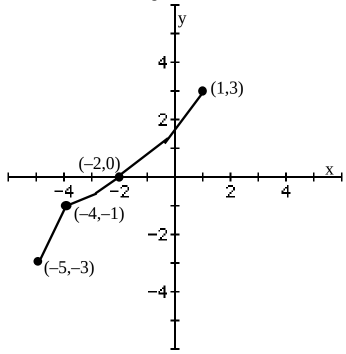


(b) $y = -f(x)$

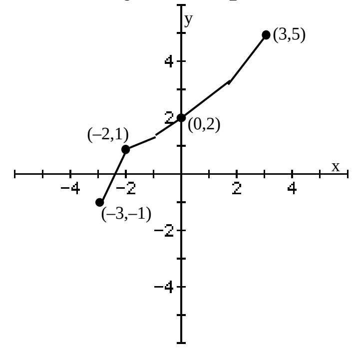
Reflect about the x-axis.



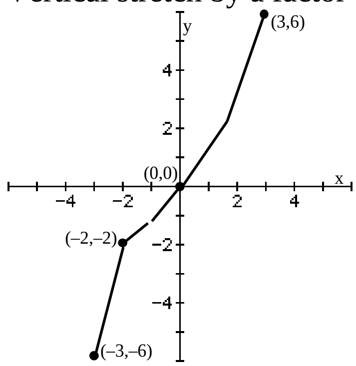
(c) $y = f(x+2)$
Horizontally shift left 2 units.



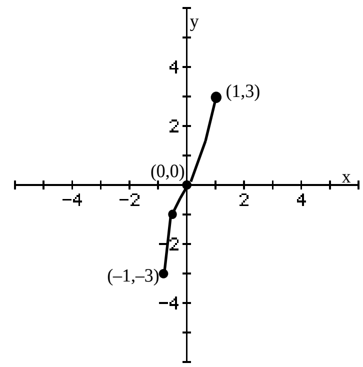
(d) $y = f(x)+2$
Vertically shift up 2 units.



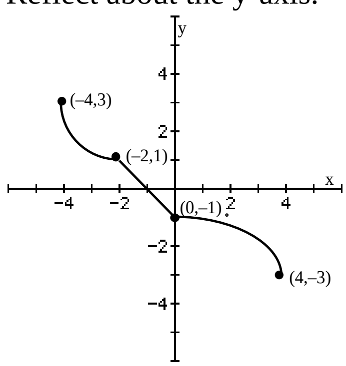
(e) $y = 2f(x)$
Vertical stretch by a factor of 2.



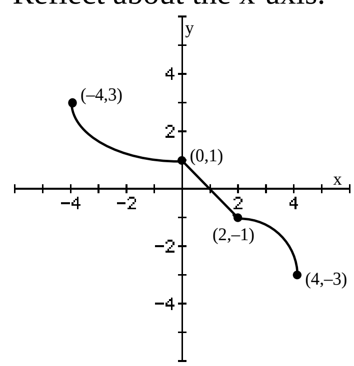
(f) $y = f(3x)$
Horizontal compression by $\frac{1}{3}$.



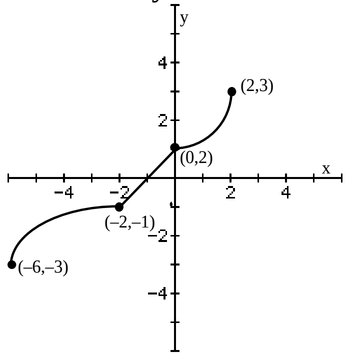
62. (a) $y = f(-x)$
Reflect about the y-axis.



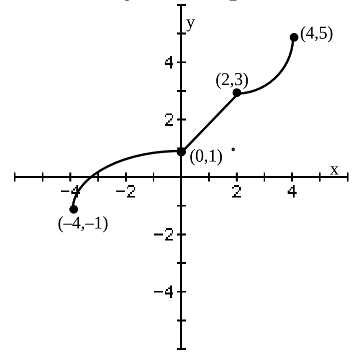
(b) $y = -f(x)$
Reflect about the x-axis.



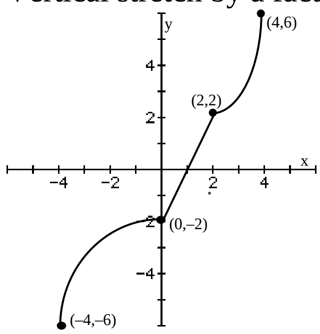
(c) $y = f(x+2)$
Horizontally shift left 2 units.



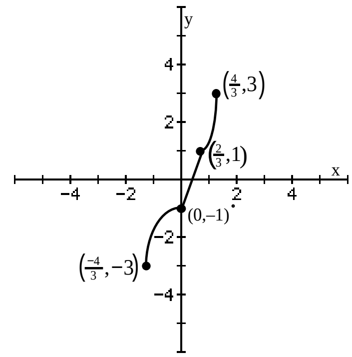
(d) $y = f(x)+2$
Vertically shift up 2 units.



(e) $y = 2f(x)$
Vertical stretch by a factor of 2.



(f) $y = f(3x)$
Horizontal compression by $\frac{1}{3}$.



63. we have the points $(h_1,T_1) = (0,30)$ and $(h_2,T_2) = (10000,5)$

$$slope = \frac{\Delta T}{\Delta h} = \frac{5-30}{10000-0} = \frac{-25}{10000} = -0.0025$$

using the point-slope formula yields

$$T - T_1 = m(h-h_1) \rightarrow T - 30 = -0.0025(h-0)$$
$$T - 30 = -0.0025h \rightarrow T = -0.0025h + 30$$
$$T(h) = -0.0025h + 30$$

64. we have the point $(t_1,v_1) = (20,80)$ and $slope = m = 5$
using the point-slope formula yields

$$v - v_1 = m(t-t_1) \rightarrow v - 80 = 5(t-20)$$
$$v - 80 = 5t - 100 \rightarrow v = 5t - 20$$
$$v(t) = 5t - 20$$

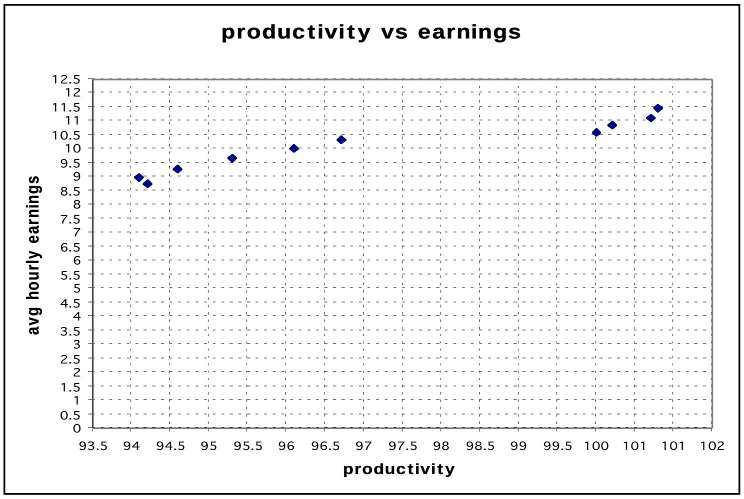
after 30 seconds, $v(30) = 5(30) - 20 = 150 - 20 = 130$ feet per second.

65. $S = kxd^3$, $x =$ width; $d =$ depth

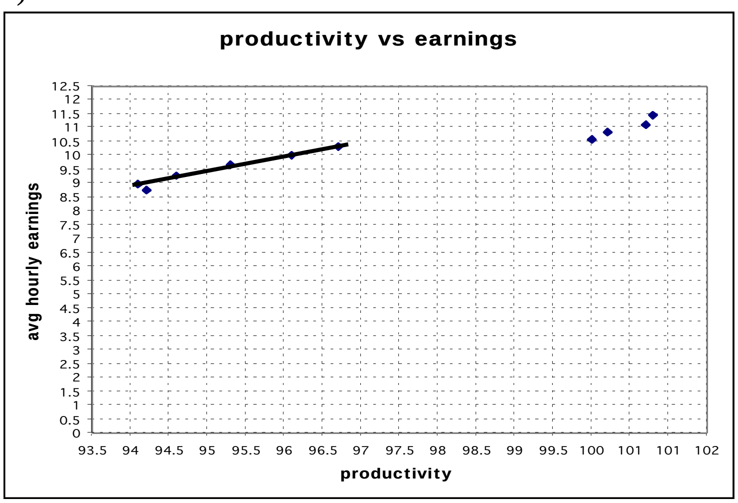
in the diagram, depth = diameter of the log = 6

$S(x) = kx(6)^3 = 216kx$ domain is $\{x \mid 0 \le x \le 6\}$

66. (a)



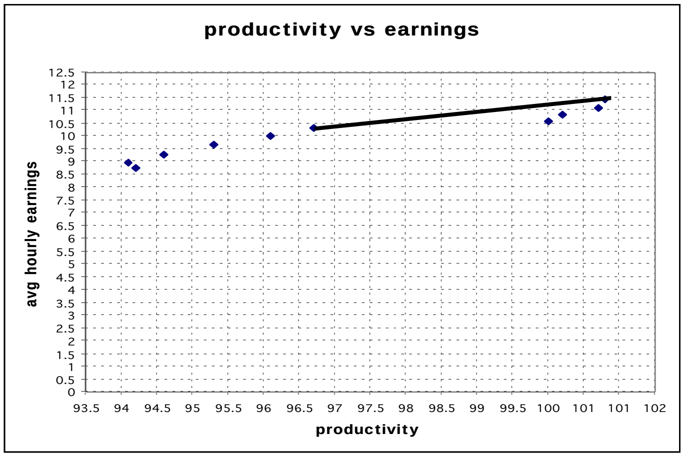
(b)



(c) average rate of change $= \dfrac{10.32 - 8.76}{96.7 - 94.2} = \dfrac{1.56}{2.5} = 0.624$ dollars per unit of productivity

(d) for each 1 unit increase in productivity, the avg. hourly earning increases by 0.624 dollars.

(e)

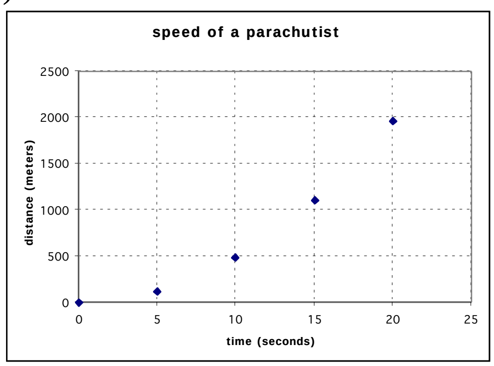


(f) average rate of change $= \dfrac{11.44 - 10.32}{100.8 - 96.7} = \dfrac{1.12}{4.1} = 0.273$ dollars per unit of productivity

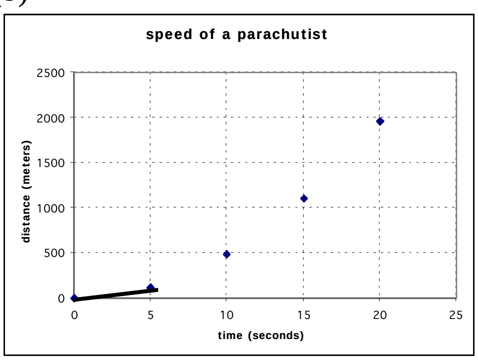
(g) for each 1 unit increase in productivity, the avg. hourly earning increases by 0.273 dollars

(h) the average rate of change of hourly earnings is decreasing as the productivity increases

67. (a)



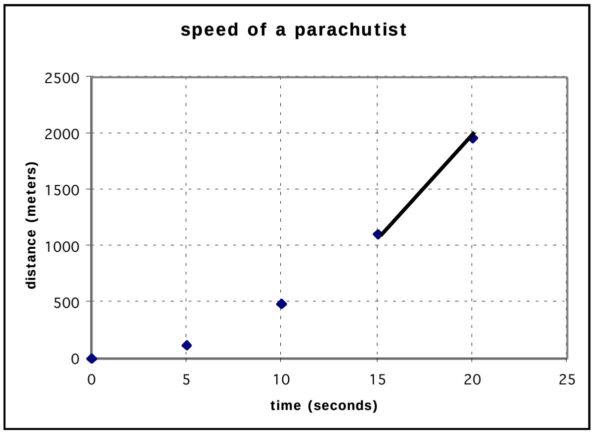
(b)



(c) average rate of change $= \dfrac{112.5 - 0}{5 - 0} = \dfrac{112.5}{5} = 22.5$ feet per second

(d) for each 1 second increase in time, the distance fallen increases by 22.5 feet

(e)



(f) average rate of change $= \dfrac{1960 - 1102.5}{20 - 15} = \dfrac{857.5}{5} = 171.5$ feet per second

(g) for each 1 second increase in time, the distance fallen increases by 171.5 feet

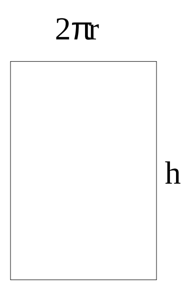
(h) the average rate of change of distance is increasing as time passes

68. (a) We are given that the volume = 100 cubic feet, so we have

$$V = \pi r^2 h = 100 \rightarrow h = \frac{100}{\pi r^2}$$

The amount of material needed to construct the barrel = the surface area of the barrel.

The cylindrical body of the barrel can be viewed as a rectangle whose dimensions are given by



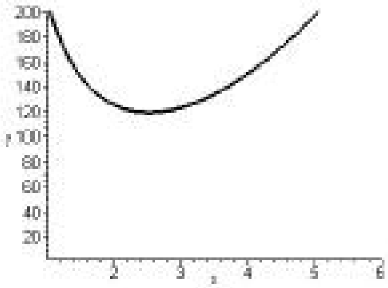
$$\begin{aligned} A &= \text{area of top} + \text{area of bottom} + \text{area of body} \\ &= \pi r^2 + \pi r^2 + 2\pi r h \\ &= 2\pi r^2 + 2\pi r h, \qquad A(r) = 2\pi r^2 + 2\pi r\left(\frac{100}{\pi r^2}\right) = 2\pi r^2 + \frac{200}{r} \end{aligned}$$

(b) $A(3) = 2\pi(3)^2 + \dfrac{200}{3} = 18\pi + \dfrac{200}{3} \approx 123.22$ square feet

(c) $A(4) = 2\pi(4)^2 + \dfrac{200}{4} = 32\pi + 50 \approx 150.53$ square feet

(d) $A(5) = 2\pi(5)^2 + \frac{200}{5} = 50\pi + 40 \approx 197.08$ square feet

(e)



The minimum value occurs at $x \approx 2.51 \to A(2.51) \approx 119.27$ square feet

69. (a) We are given that the volume = 500 cubic feet, so we have

$V = \pi r^2 h = 100 \to h = \frac{500}{\pi r^2}$

Total Cost = cost of top + cost of bottom + cost of body

= 2(cost of top) + cost of body

= 2(area of top)(cost per area of top) + (area of body)(cost per area of body)

$= 2(\pi r^2)(.06) + (2\pi r h)(.04)$

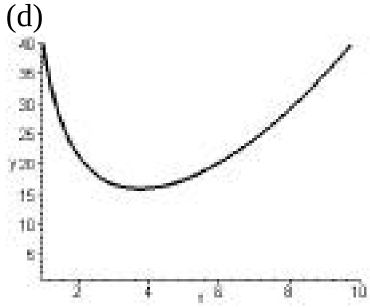
$= 0.12\pi r^2 + .08\pi r h = .12\pi r^2 + .08\pi r\left(\frac{500}{\pi r^2}\right)$

$= .12\pi r^2 + \frac{40}{r}, \qquad C(r) = .12\pi r^2 + \frac{40}{r}$

(b) $C(4) = .12\pi(4)^2 + \frac{40}{4} = 1.92\pi + 10 \approx 16.03$ dollars

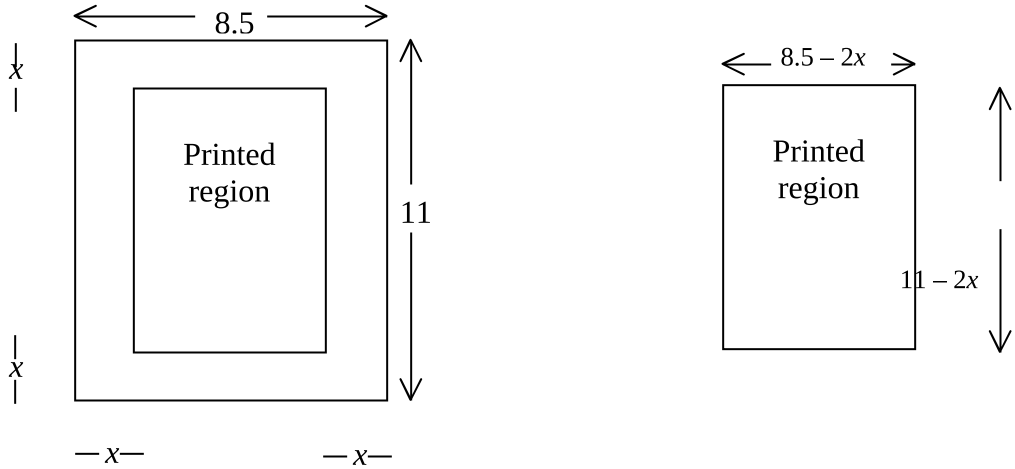
(c) $C(8) = .12\pi(8)^2 + \frac{40}{8} = 7.68\pi + 5 \approx 29.13$ dollars

(d)



The minimum value occurs at $r \approx 3.79 \to C(3.79) \approx \$15.97$

70. we can consider the following diagram



(a) The printed region is a rectangle, so its area is given by

$A = (length)(width) = (11-2x)(8.5-2x)$

$A(x) = (11-2x)(8.5-2x)$

(b) To find the domain of $A(x) = (11-2x)(8.5-2x)$ we need to remember that the dimensions of a rectangle must be non-negative.

That is, we need

$x \geq 0 \quad and \quad 11-2x \geq 0 \quad and \quad 8.5-2x \geq 0$

$\rightarrow x \geq 0 \quad and \quad x \leq 5.5 \quad and \quad x \leq 4.25$

domain is $\{x \mid 0 \leq x \leq 4.25\}$

The range of $A(x) = (11-2x)(8.5-2x)$ is given by

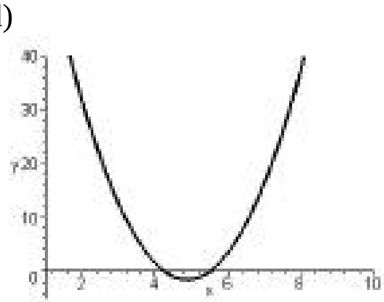
$A(4.25) \leq A \leq A(0) \rightarrow 0 \leq A \leq 93.5$

(c) $A(1) = (11-2(1))(8.5-2(1)) = 9 \;\; 6.5 = 58.5$ square inches

$A(1.2) = (11-2(1.2))(8.5-2(1.2)) = 8.6 \;\; 5.6 = 48.16$ square inches

$A(1.5) = (11-2(1.5))(8.5-2(1.5)) = 8 \;\; 5.5 = 44$ square inches

(d)



(e)

$A = 70$ when $x \approx 0.643$ inches $\qquad A = 50$ when $x \approx 1.29$ inches

71. $S = 4\pi r^2 \to r = \sqrt{\frac{S}{4\pi}}$ $\quad V(S) = \frac{4}{3}\pi r^3 = \frac{4\pi}{3}\left(\sqrt{\frac{S}{4\pi}}\right)^3 = \frac{4\pi}{3}\cdot\frac{S}{4\pi}\sqrt{\frac{S}{4\pi}} = \frac{S}{6}\sqrt{\frac{S}{\pi}}$

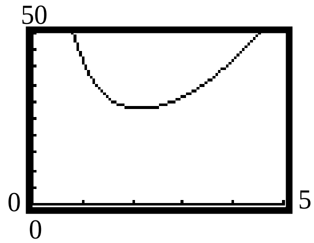
$V(2S) = \frac{2S}{6}\sqrt{\frac{2S}{\pi}} = 2\sqrt{2}\left(\frac{S}{6}\sqrt{\frac{S}{\pi}}\right)$ The volume is $2\sqrt{2}$ times as large.

72. (a) $x^2h = 10 \to h = \frac{10}{x^2}$ $\quad A(x) = 2x^2 + 4xh = 2x^2 + 4x\left(\frac{10}{x^2}\right) = 2x^2 + \frac{40}{x}$

(b) $A(1) = 2(1)^2 + \frac{40}{1} = 2 + 40 = 42 \text{ ft}^2$

(c) $A(2) = 2(2)^2 + \frac{40}{2} = 8 + 20 = 28 \text{ ft}^2$

(d) Graphing:



The area is smallest when $x = 2.15$.