Polynomial and Rational Functions

4.2 Polynomial Functions

1. $f(x) = 4x + x^3$ is a polynomial function of degree 3.

2. $f(x) = 5x^2 + 4x^4$ is a polynomial function of degree 4.

3. $g(x) = \frac{1 - x^2}{2} = \frac{1}{2} - \frac{1}{2}x^2$ is a polynomial function of degree 2.

4. $h(x) = 3 - \frac{1}{2}x$ is a polynomial function of degree 1.

5. $f(x) = 1 - \frac{1}{x} = 1 - x^{-1}$ is not a polynomial function because it contains a negative exponent.

6. $f(x) = x(x - 1) = x^2 - x$ is a polynomial function of degree 2.

7. $g(x) = x^{\frac{3}{2}} - x^2 + 2$ is not a polynomial function because it contains a fractional exponent.

8. $h(x) = \sqrt{x}\left(\sqrt{x} - 1\right) = x - \sqrt{x}$ is not a polynomial function because it contains a square root.

9. $F(x) = 5x^4 - \pi x^3 + \frac{1}{2}$ is a polynomial function of degree 4.

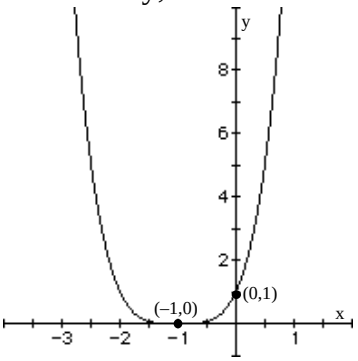
10. $F(x) = \frac{x^2 - 5}{x^3} = \frac{1}{x} - \frac{5}{x^3}$ is not a polynomial function because it contains a variable with a positive exponent in the denominator.

11. $G(x) = 2(x - 1)^2\left(x^2 + 1\right)$ is a polynomial function of degree 4.

12. $G(x) = -3x^2(x + 2)^3$ is a polynomial function of degree 5.

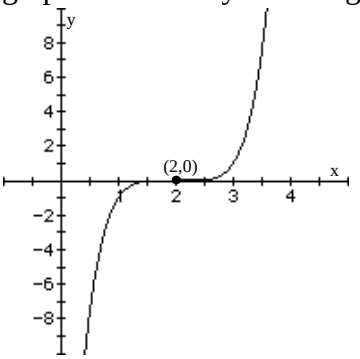
13. $f(x) = (x+1)^4$

Using the graph of $y = x^4$, shift the graph horizontally, 1 unit to the left.



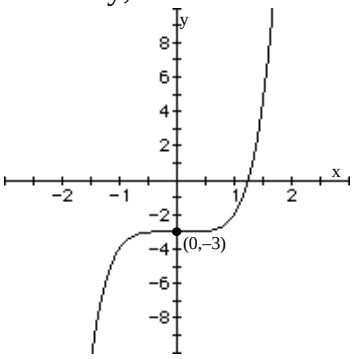
14. $f(x) = (x-2)^5$

Using the graph of $y = x^5$, shift the graph horizontally to the right 2 units.



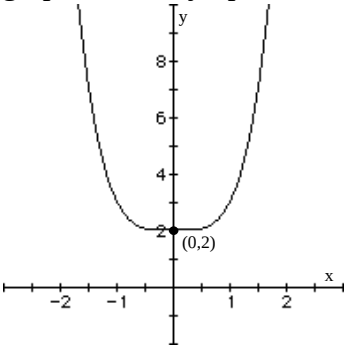
15. $f(x) = x^5 - 3$

Using the graph of $y = x^5$, shift the graph vertically, 3 units down.



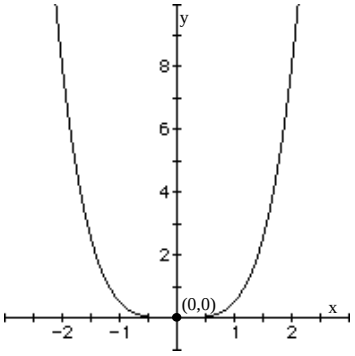
16. $f(x) = x^4 + 2$

Using the graph of $y = x^4$, shift the graph vertically up 2 units.



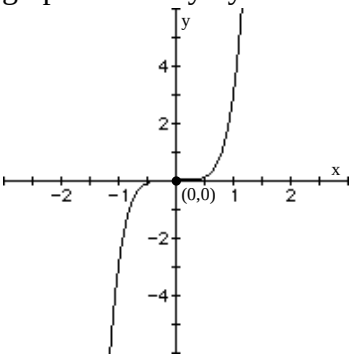
17. $f(x) = \frac{1}{2}x^4$

Using the graph of $y = x^4$, compress the graph vertically by a factor of $\frac{1}{2}$.



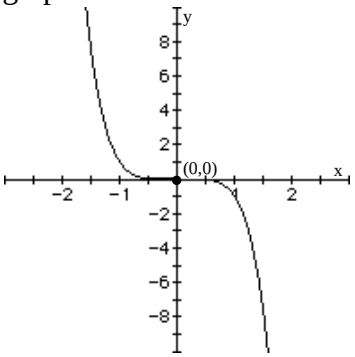
18. $f(x) = 3x^5$

Using the graph of $y = x^5$, stretch the graph vertically by a factor of 3.



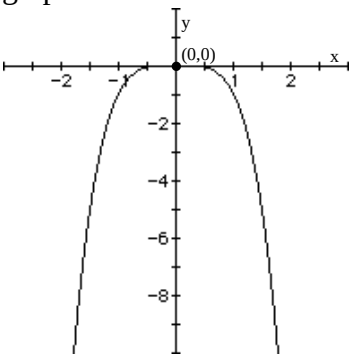
19. $f(x) = -x^5$

Using the graph of $y = x^5$, reflect the graph about the x-axis.



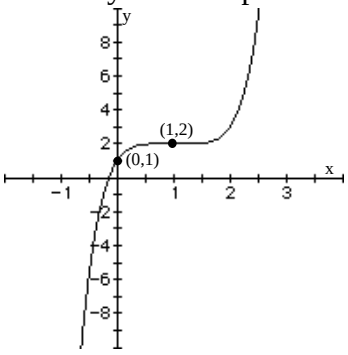
20. $f(x) = -x^4$

Using the graph of $y = x^4$, reflect the graph about the x-axis.



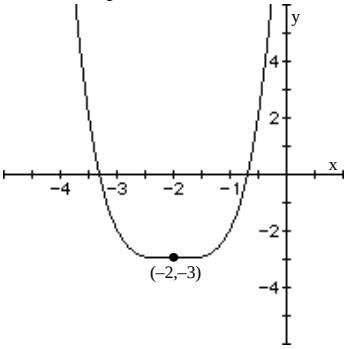
21. $f(x) = (x-1)^5 + 2$

Using the graph of $y = x^5$, shift the graph horizontally, 1 unit to the right, and shift vertically 2 units up.



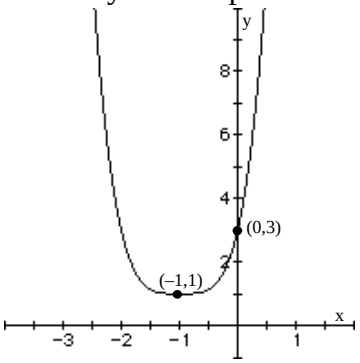
22. $f(x) = (x+2)^4 - 3$

Using the graph of $y = x^4$, shift the graph horizontally left 2 units, and shift vertically down 3 units.



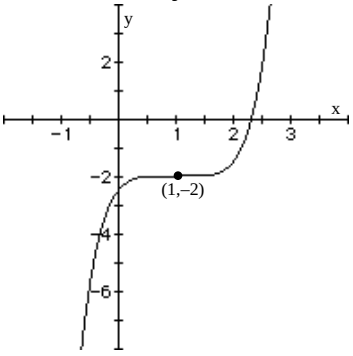
23. $f(x) = 2(x+1)^4 + 1$

Using the graph of $y = x^4$, shift the graph horizontally, 1 unit to the left, stretch vertically by a factor of 2, and shift vertically 1 unit up.



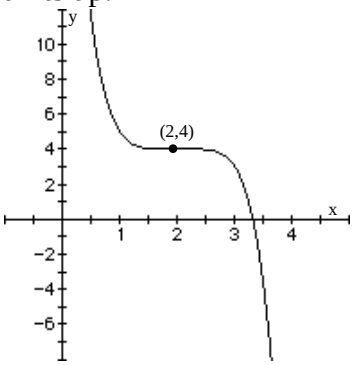
24. $f(x) = \frac{1}{2}(x-1)^5 - 2$

Using the graph of $y = x^5$, shift the graph horizontally 1 unit to the right, shrink vertically by a factor of $\frac{1}{2}$, and shift vertically down 2 units.



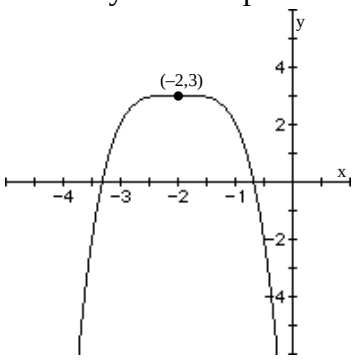
25. $f(x) = 4 - (x-2)^5 = -(x-2)^5 + 4$

Using the graph of $y = x^5$, shift the graph horizontally, 2 units to the right, reflect about the x-axis, and shift vertically 4 units up.



26. $f(x) = 3 - (x+2)^4 = -(x+2)^4 + 3$

Using the graph of $y = x^4$, shift the graph horizontally, 2 units to the left, reflect about the x-axis, and shift vertically 3 units up.



27. $f(x) = a\big(x-(-1)\big)(x-1)(x-3)$

For $a = 1$: $f(x) = (x+1)(x-1)(x-3)$

$f(x) = \left(x^2 - 1\right)(x-3)$

$f(x) = x^3 - 3x^2 - x + 3$

28. $f(x) = a\big(x-(-2)\big)(x-2)(x-3)$

For $a = 1$: $f(x) = (x+2)(x-2)(x-3)$

$f(x) = \left(x^2 - 4\right)(x-3)$

$f(x) = x^3 - 3x^2 - 4x + 12$

29. $f(x) = a\big(x-(-3)\big)(x-0)(x-4)$

For $a = 1$: $f(x) = (x+3)(x)(x-4)$

$f(x) = \left(x^2 + 3x\right)(x-4)$

$f(x) = x^3 - 4x^2 + 3x^2 - 12x$

$f(x) = x^3 - x^2 - 12x$

30. $f(x) = a\big(x-(-4)\big)(x-0)(x-2)$

For $a = 1$: $f(x) = (x+4)(x)(x-2)$

$f(x) = \left(x^2 + 4x\right)(x-2)$

$f(x) = x^3 - 2x^2 + 4x^2 - 8x$

$f(x) = x^3 + 2x^2 - 8x$

31. $f(x) = a\big(x-(-4)\big)\big(x-(-1)\big)(x-2)(x-3)$

For $a = 1$: $f(x) = (x+4)(x+1)(x-2)(x-3)$

$f(x) = \left(x^2 + 5x + 4\right)\left(x^2 - 5x + 6\right)$

$f(x) = x^4 - 5x^3 + 6x^2 + 5x^3 - 25x^2 + 30x + 4x^2 - 20x + 24$

$f(x) = x^4 - 15x^2 + 10x + 24$

32. $f(x) = a(x-(-3))(x-(-1))(x-2)(x-5)$
For $a = 1$: $f(x) = (x+3)(x+1)(x-2)(x-5)$
$f(x) = (x^2+4x+3)(x^2-7x+10)$
$f(x) = x^4 - 7x^3 + 10x^2 + 4x^3 - 28x^2 + 40x + 3x^2 - 21x + 30$
$f(x) = x^4 - 3x^3 - 15x^2 + 19x + 30$

33. The real zeros of $f(x) = 3(x-7)(x+3)^2$ are: 7, with multiplicity one; and –3, with multiplicity two. The graph crosses the x-axis at 7 and touches it at –3.
The function resembles $y = 3x^3$ for large values of $|x|$.

34. The real zeros of $f(x) = 4(x+4)(x+3)^3$ are: –4, with multiplicity one; and –3, with multiplicity three. The graph crosses the x-axis at –4 and at –3.
The function resembles $y = 4x^3$ for large values of $|x|$.

35. The real zeros of $f(x) = 4(x^2+1)(x-2)^3$ are: 2, with multiplicity three. $x^2 + 1 = 0$ has no real solution. The graph crosses the x-axis at 2.
The function resembles $y = 4x^5$ for large values of $|x|$.

36. The real zeros of $f(x) = 2(x-3)(x+4)^3$ are: 3, with multiplicity one; and –4, with multiplicity three. The graph crosses the x-axis at 3 and at –4.
The function resembles $y = 2x^4$ for large values of $|x|$.

37. The real zeros of $f(x) = -2\left(x + \frac{1}{2}\right)^2 (x^2+4)^2$ are: $\frac{-1}{2}$, with multiplicity two. $x^2 + 4 = 0$ has no real solution. The graph touches the x-axis at $\frac{-1}{2}$.
The function resembles $y = 2x^6$ for large values of $|x|$.

38. The real zeros of $f(x) = \left(x - \frac{1}{3}\right)^2 (x-1)^3$ are: $\frac{1}{3}$, with multiplicity two; and 1, with multiplicity 3. The graph touches the x-axis at $\frac{1}{3}$, and crosses the x-axis at 1.
The function resembles $y = x^5$ for large values of $|x|$.

39. The real zeros of $f(x) = (x-5)^3(x+4)^2$ are: 5, with multiplicity three; and –4, with multiplicity two. The graph crosses the x-axis at 5 and touches it at –4.
The function resembles $y = x^5$ for large values of $|x|$.

40. The real zeros of $f(x) = \left(x + \sqrt{3}\right)^2 (x-2)^4$ are: $-\sqrt{3}$, with multiplicity two; and 2, with multiplicity four. The graph touches the x-axis at $-\sqrt{3}$ and at 2.The function resembles $y = x^6$ for large values of $|x|$.

41. $f(x) = 3\left(x^2 + 8\right)\left(x^2 + 9\right)^2$ has no real zeros. $x^2 + 8 = 0$ and $x^2 + 9 = 0$ have no real solutions. The graph neither touches nor crosses the x-axis. The function resembles $y = 3x^6$ for large values of $|x|$.

42. $f(x) = -2\left(x^2 + 3\right)^3$ has no real zeros. $x^2 + 3 = 0$ has no real solutions.
The graph neither touches nor crosses the x-axis. The function resembles $y = -2x^6$ for large values of $|x|$.

43. The real zeros of $f(x) = -2x^2(x^2 - 2)$ are: $-\sqrt{2}$ and $\sqrt{2}$ with multiplicity one; and 0, with multiplicity two. The graph touches the x-axis at $-\sqrt{2}$ and $\sqrt{2}$ and crosses the x-axis at 2.The function resembles $y = -2x^4$ for large values of $|x|$.

44. The real zeros of $f(x) = 4x(x^2 - 3)$ are: $-\sqrt{3}, \sqrt{3}$ and 0, with multiplicity one.
The graph crosses the x-axis at $-\sqrt{3}, \sqrt{3}$ and 0.The function resembles $y = 4x^3$ for large values of $|x|$.

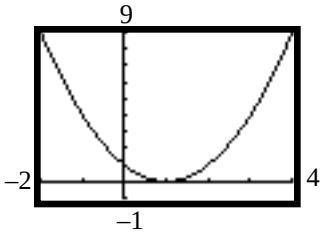
45. $f(x) = (x-1)^2$

(a) x-intercept: 1; y-intercept: 1
(b) touches x-axis at x = 1
(c) $y = x^2$
(d) 1
(e)

	$x < 1$	$x > 1$
f	+	+
Above or below x-axis	above	above

f is above the x-axis for $(-\infty, 1)$ $(1, \infty)$

(f)



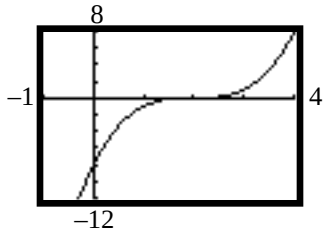
46. $f(x) = (x-2)^3$

(a) x-intercept: 2; y-intercept: –8
(b) crosses x-axis at x = 2
(d) $y = x^3$
(c) 2
(e)

	$x < 2$	$x > 2$
f	-	+
Above or below x-axis	below	above

f is below the x-axis for $(-\infty, 2)$
f is above the x-axis for $(2, \infty)$

(f)



47. $f(x) = x^2(x-3)$

(a) x-intercepts: 0, 3; y-intercept: 0

(b) touches x-axis at x = 0 ; crosses x-axis at x = 3

(c) $y = x^3$ (d) 2

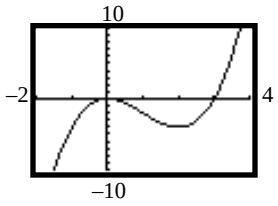
(e)

	$x < 0$	$0 < x < 3$	$x > 3$
f	-	-	+
Above or below x-axis	below	below	above

f is below the x-axis for $(-\infty,0)$ $(0,3)$

f is above the x-axis for $(3,\infty)$

(f)



48. $f(x) = x(x+2)^2$

(a) x-intercepts: –2, 0; y-intercept: 0

(b) touches x-axis at x = –2; crosses x-axis at x = 0

(c) $y = x^3$ (d) 2

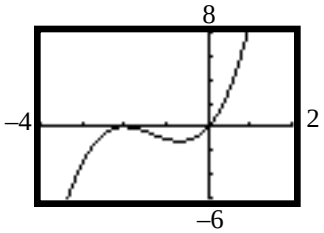
(e)

	$x < -2$	$-2 < x < 0$	$x > 0$
f	-	-	+
Above or below x-axis	below	below	above

f is below the x-axis for $(-\infty,-2)$ $(-2,0)$

f is above the x-axis for $(0,\infty)$

(f)



49. $f(x) = 6x^3(x + 4)$

(a) x-intercepts: –4, 0; y-intercept: 0

(b) crosses x-axis at x = –4 and x = 0

(c) $y = 6x^4$ (d) 3

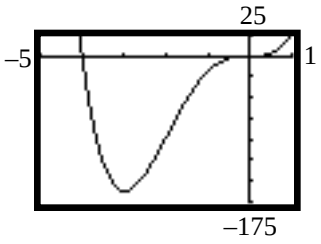
(e)

	$x < -4$	$-4 < x < 0$	$x > 0$
f	+	-	+
Above or below x-axis	above	below	above

f is above the x-axis for $(-\infty,-4)$ $(0,\infty)$

f is below the x-axis for $(-4,0)$

(f)



50. $f(x) = 5x(x - 1)^3$

(a) x-intercepts: 0, 1; y-intercept: 0

(b) crosses x-axis at x = 0 and x = 1

(c) $y = 5x^4$ (d) 3

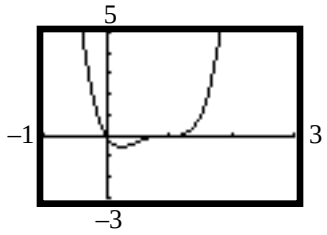
(e)

	$x < 0$	$0 < x < 1$	$x > 1$
f	+	-	+
Above or below x-axis	above	below	above

f is above the x-axis for $(-\infty,0)$ $(1,\infty)$

f is below the x-axis for $(0,1)$

(f)



51. $f(x) = -4x^2(x+2)$

(a) x-intercepts: 0, - 2; y-intercept: 0

(b) crosses x-axis at x = - 2 ; touches x-axis at x = 0

(c) $y = -4x^3$ (d) 2

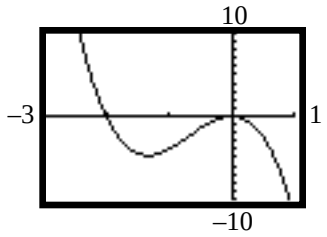
(e)

	$x < -2$	$-2 < x < 0$	$x > 0$
f	+	-	-
Above or below x-axis	above	below	below

f is above the x-axis for $(-\infty,-2)$

f is below the x-axis for $(-2,0)$ $(0,\infty)$

(f)



52. $f(x) = -\frac{1}{2}x^3(x+4)$

(a) x-intercepts: 0, - 4; y-intercept: 0

(b) crosses x-axis at x = - 4 and x = 0

(c) $y = -\frac{1}{2}x^4$ (d) 3

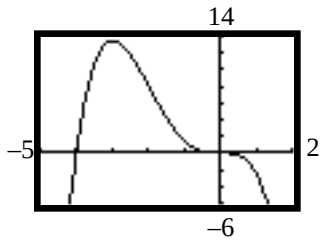
(e)

	$x < -4$	$-4 < x < 0$	$x > 0$
f	-	+	-
Above or below x-axis	below	above	below

f is above the x-axis for $(-4,0)$

f is below the x-axis for $(-\infty,-4)$ $(0,\infty)$

(f)



53. $f(x) = x(x-2)(x+4)$

(a) x-intercepts: 0, - 4, 2; y-intercept: 0

(b) crosses x-axis at x = 0, x = - 4 and x = 2

(c) $y = x^3$ (d) 2

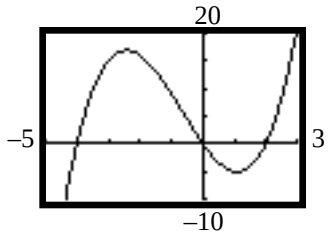
(e)

	$x < -4$	$-4 < x < 0$	$0 < x < 2$	$x > 2$
f	-	+	-	+
Above or below x-axis	below	above	below	above

f is above the x-axis for $(-4,0)$ $(2,\infty)$

f is below the x-axis for $(-\infty,-4)$ $(0,2)$

(f)



54. $f(x) = x(x+4)(x-3)$

(a) x-intercepts: 0, - 4, 3; y-intercept: 0

(b) crosses x-axis at x = 0, x = - 4 and x = 3

(c) $y = x^3$ (d) 2

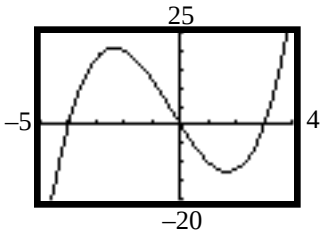
(e)

	$x < -4$	$-4 < x < 0$	$0 < x < 3$	$x > 3$
f	-	+	-	+
Above or below x-axis	below	above	below	above

f is above the x-axis for $(-4,0)$ $(3,\infty)$

f is below the x-axis for $(-\infty,-4)$ $(0,3)$

(f)



55. $f(x) = 4x - x^3 = x(4 - x^2) = x(2 + x)(2 - x)$

(a) x-intercepts: 0, - 2, 2; y-intercept: 0

(b) crosses x-axis at x = 0, x = - 2 and x = 2

(c) $y = -x^3$ (d) 2

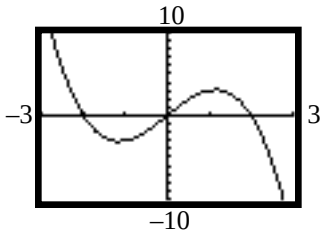
(e)

	$x < -2$	$-2 < x < 0$	$0 < x < 2$	$x > 2$
f	+	-	+	-
Above or below x-axis	above	below	above	below

f is above the x-axis for $(-\infty, -2)$ $(0,2)$

f is below the x-axis for $(-2,0)$ $(2,\infty)$

(f)



56. $f(x) = x - x^3 = x(1 - x^2) = x(1 + x)(1 - x)$

(a) x-intercepts: 0, - 1, 1; y-intercept: 0

(b) crosses x-axis at x = 0, x = - 2 and x = 2

(c) $y = -x^3$ (d) 2

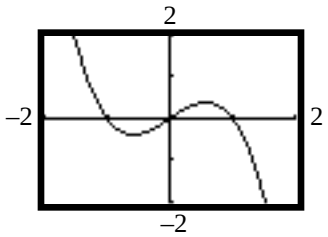
(e)

	$x < -1$	$-1 < x < 0$	$0 < x < 1$	$x > 1$
f	+	-	+	-
Above or below x-axis	above	below	above	below

f is above the x-axis for $(-\infty, -1)$ $(0,1)$

f is below the x-axis for $(-1,0)$ $(1,\infty)$

(f)



57. $f(x) = x^2(x-2)(x+2)$

(a) x-intercepts: 0, - 2, 2; y-intercept: 0

(b) crosses x-axis at x = - 2 and x = 2; touches x-axis at x = 0

(c) $y = x^4$ (d) 3

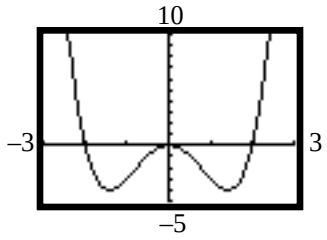
(e)

	$x < -2$	$-2 < x < 0$	$0 < x < 2$	$x > 2$
f	+	-	-	+
Above or below x-axis	above	below	below	above

f is above the x-axis for $(-\infty,-2)$ $(2,\infty)$

f is below the x-axis for $(-2,0)$ $(0,2)$

(f)



58. $f(x) = x^2(x-3)(x+4)$

(a) x-intercepts: 0, - 4, 3; y-intercept: 0

(b) crosses x-axis at x = - 4 and x = 3; touches x-axis at x = 0

(c) $y = x^4$ (d) 3

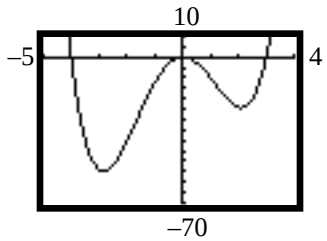
(e)

	$x < -4$	$-4 < x < 0$	$0 < x < 3$	$x > 3$
f	+	-	-	+
Above or below x-axis	above	below	below	above

f is above the x-axis for $(-\infty,-2)$ $(2,\infty)$

f is below the x-axis for $(-2,0)$ $(0,2)$

(f)



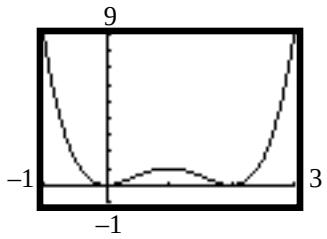
59. $f(x) = x^2(x-2)^2$

(a) x-intercepts: 0, 2; y-intercept: 0
(b) touches x-axis at x = 0 and x = 2
(c) $y = x^4$ (d) 3
(e)

	$x < 0$	$0 < x < 2$	$x > 2$
f	+	+	+
Above or below x-axis	above	above	above

f is above the x-axis for $(-\infty,0)$ $(0,2)$ $(2,\infty)$

(f)



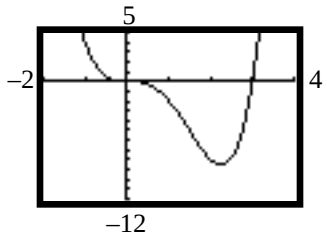
60. $f(x) = x^3(x-3)$

(a) x-intercepts: 0,3; y-intercept: 0
(b) crosses x-axis at x = 0 and x = 3
(c) $y = x^4$ (d) 3
(e)

	$x < 0$	$0 < x < 3$	$x > 3$
f	+	-	+
Above or below x-axis	above	below	above

f is above the x-axis for $(-\infty,0)$ $(3,\infty)$
f is below the x-axis for $(0,3)$

(f)

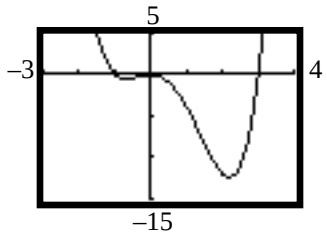


61. $f(x) = x^2(x-3)(x+1)$

(a) x-intercepts: 0, - 1, 3; y-intercept: 0
(b) crosses x-axis at x = - 1 and x = 3; touches x-axis at x = 0
(c) $y = x^4$ (d) 3
(e)

	$x < -1$	$-1 < x < 0$	$0 < x < 3$	$x > 3$
f	+	-	-	+
Above or below x-axis	above	below	below	above

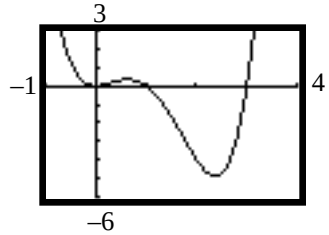
f is above the x-axis for $(-\infty,-1)$ $(3,\infty)$
f is below the x-axis for $(-1,0)$ $(0,3)$
(f)



62. $f(x) = x^2(x-3)(x-1)$
(a) x-intercepts: 0, 1, 3; y-intercept: 0
(b) crosses x-axis at x = 1 and x = 3; touches x-axis at x = 0
(c) $y = x^4$ (d) 3
(e)

	$x < 0$	$0 < x < 1$	$1 < x < 3$	$x > 3$
f	+	+	-	+
Above or below x-axis	above	above	below	above

f is above the x-axis for $(-\infty,0)$ $(0,1)$ $(3,\infty)$
f is below the x-axis for $(1,3)$
(f)



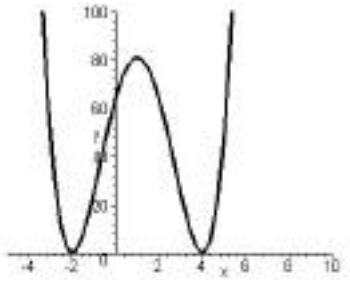
63. $f(x) = (x+2)^2(x-4)^2$

(a) x-intercepts: - 2, 4; y-intercept: 64
(b) touches x-axis at x = - 2 and x = 4
(c) $y = x^4$ (d) 3
(e)

	$x < -2$	$-2 < x < 4$	$x > 4$
f	+	+	+
Above or below x-axis	above	above	above

f is above the x-axis for $(-\infty,-2)$ $(-2,4)$ $(4,\infty)$

(f)



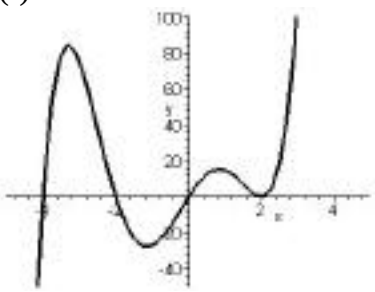
64. $f(x) = x(x-2)^2(x+2)(x+4)$

(a) x-intercepts: 0, - 4, - 2,2; y-intercept: 0
(b) crosses x-axis at x = - 4, x = - 2 and x = 0; touches x-axis at x = 2
(c) $y = x^5$ (d) 4
(e)

	$x < -4$	$-4 < x < -2$	$-2 < x < 0$	$0 < x < 2$	$x > 2$
f	-	+	-	+	+
Above or below x-axis	below	above	below	above	above

f is above the x-axis for $(-4,-2)$ $(0,2)$ $(2,\infty)$
f is below the x-axis for $(-\infty,-4)$ $(-2,0)$

(f)



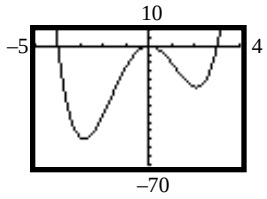
65. $f(x) = x^2(x-3)(x+4)$

(a) x-intercepts: 0, - 4, 3; y-intercept: 0
(b) crosses x-axis at x = - 4 and x = 3; touches x-axis at x = 0
(c) $y = x^4$ (d) 3
(e)

	$x < -4$	$-4 < x < 0$	$0 < x < 3$	$x > 3$
f	+	-	-	+
Above or below x-axis	above	below	below	above

f is above the x-axis for $(-\infty,-2)$ $(2,\infty)$
f is below the x-axis for $(-2,0)$ $(0,2)$

(f)



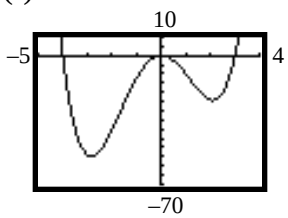
66. $f(x) = x^2(x-3)(x+4)$

(a) x-intercepts: 0, - 4, 3; y-intercept: 0
(b) crosses x-axis at x = - 4 and x = 3; touches x-axis at x = 0
(c) $y = x^4$ (d) 3
(e)

	$x < -4$	$-4 < x < 0$	$0 < x < 3$	$x > 3$
f	+	-	-	+
Above or below x-axis	above	below	below	above

f is above the x-axis for $(-\infty,-2)$ $(2,\infty)$
f is below the x-axis for $(-2,0)$ $(0,2)$

(f)



67. $f(x) = -x^2(x^2-1)(x+1) = -x^2(x-1)(x+1)(x+1) = -x^2(x-1)(x+1)^2$

(a) x-intercepts: 0, - 1, 1; y-intercept: 0

(b) crosses x-axis at x = 1; touches x-axis at x = 0 and x = - 1

(c) $y = -x^5$ (d) 4

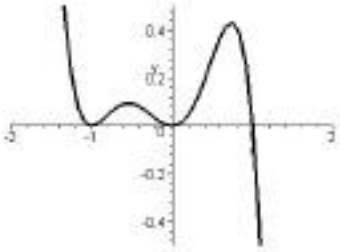
(e)

	$x < -1$	$-1 < x < 0$	$0 < x < 1$	$x > 1$
f	+	+	+	-
Above or below x-axis	above	above	above	below

f is above the x-axis for $(-\infty,-1)$ $(-1,0)$ $(0,1)$

f is below the x-axis for $(1,\infty)$

(f)



68. $f(x) = -x^2(x^2-4)(x-5)$

(a) x-intercepts: 0, - 2, 2,5; y-intercept: 0

(b) crosses x-axis at x = - 2, x = 2 and x = 5; touches x-axis at x = 0

(c) $y = -x^5$ (d) 4

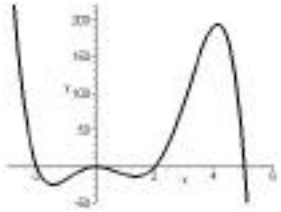
(e)

	$x < -2$	$-2 < x < 0$	$0 < x < 2$	$2 < x < 5$	$x > 5$
f	+	-	-	+	-
Above or below x-axis	above	below	below	above	below

f is above the x-axis for $(-\infty,-2)$ $(2,5)$

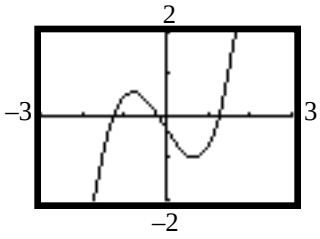
f is below the x-axis for $(-2,0)$ $(0,2)$ $(5,\infty)$

(f)



69. c and e 70. c, e and f 71. c and e 72. d and f

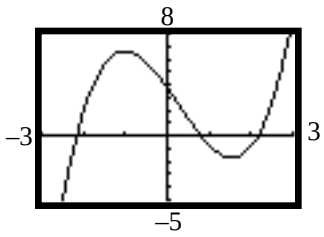
73. $f(x) = x^3 + 0.2x^2 - 1.5876x - 0.31752$



x-intercepts: –1.26, –0.2, 1.26

turning points: (–0.80, 0.57);
(0.66, –0.99)

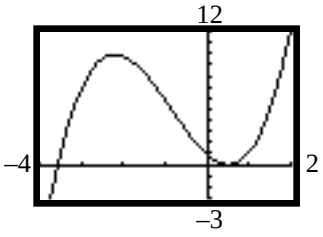
74. $f(x) = x^3 - 0.8x^2 - 4.6656x + 3.73248$



x-intercepts: –2.16, 0.8, 2.16

turning points: (–1.01, 6.60);
(1.54, –1.70)

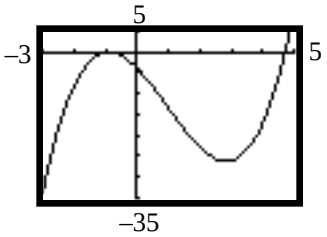
75. $f(x) = x^3 + 2.56x^2 - 3.31x + 0.89$



x-intercepts: –3.56, 0.50

turning points: (–2.21, 9.91);
(0.50, 0)

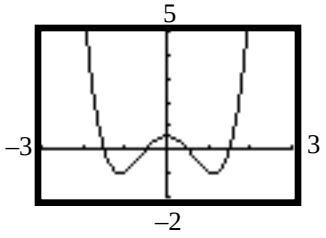
76. $f(x) = x^3 - 2.91x^2 - 7.668x - 3.8151$



x-intercepts: –0.90, 4.71

turning points: (–0.90, 0);
(2.84, –26.16)

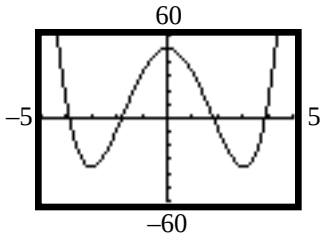
77. $f(x) = x^4 - 2.5x^2 + 0.5625$



x-intercepts: –1.50, –0.50, 0.50, 1.50

turning points: (0, 0.5625);
(–1.12, –1);
(1.12, –1)

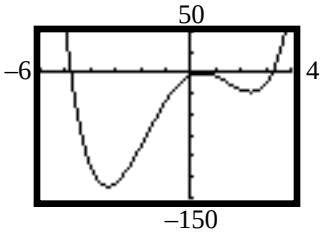
78. $f(x) = x^4 - 18.5x^2 + 50.2619$



x-intercepts: –3.90, –1.82, 1.82, 3.90

turning points: (0, 50.26);
(–3.04, –35.30);
(3.04, –35.30)

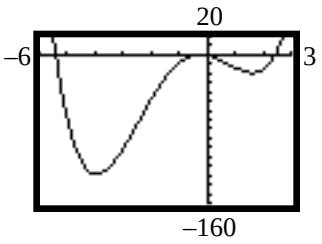
79. $f(x) = x^4 + 0.65x^3 - 16.6319x^2 + 14.209335x - 3.1264785$



x-intercepts: –4.78, 0.45, 3.23

turning points: (0.45, 0)
(–3.32, –135.92);
(2.38, –22.67)

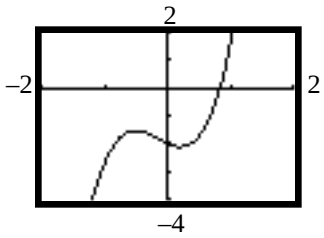
80. $f(x) = x^4 + 3.45x^3 - 11.6639x^2 - 5.864241x - 0.69257738$



x-intercepts: –5.41, –0.23, 2.42

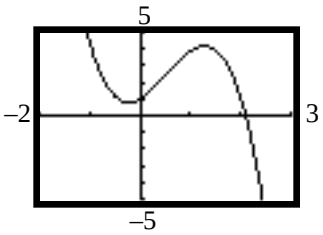
turning points: (–0.23, 0);
(–3.97, –128.71);
(1.61, –19.25)

81. $f(x) = \pi x^3 + \sqrt{2}x^2 - x - 2$



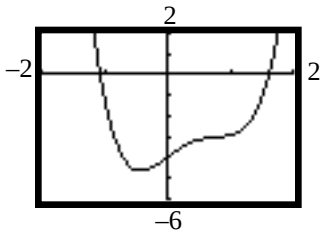
x-intercept: 0.84
turning points: (–0.51, –1.54);
(0.21, –2.12)

82. $f(x) = -2x^3 + \pi x^2 + \sqrt{3}x + 1$



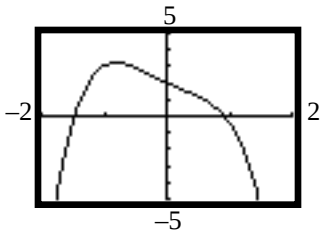
x-intercept: 2.10
turning points: (1.27, 4.17);
(–0.23, 0.79)

83. $f(x) = 2x^4 - \pi x^3 + \sqrt{5}x - 4$



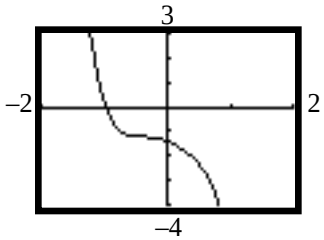
x-intercepts: –1.07, 1.62
turning point: (–0.42, –4.64)

84. $f(x) = -1.2x^4 + 0.5x^2 - \sqrt{3}x + 2$



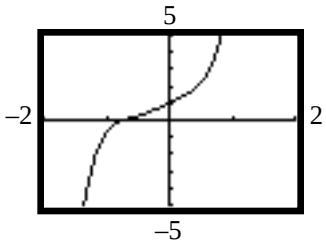
x-intercepts: –1.47, 0.91
turning point: (–0.81, 3.21)

85. $f(x) = -2x^5 - \sqrt{2}x^2 - x - \sqrt{2}$



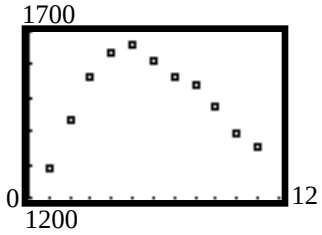
x-intercept: –0.98
no turning points

86. $f(x) = \pi x^5 + \pi x^4 + \sqrt{3}x + 1$



x-intercept: –0.71
no turning points

87. (a) Graphing:



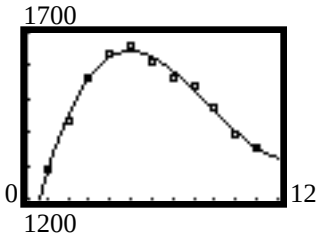
The graph may be a cubic relation.

(b) $M(x) = 1.52x^3 - 39.81x^2 + 282.29x + 1035.5$

$M(8) = 1.52(8)^3 - 39.81(8)^2 + 282.29(8) + 1035.5 = 1580.22$

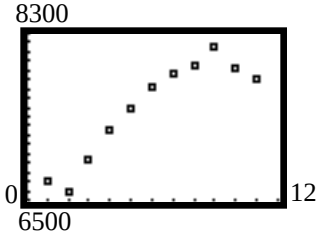
According to the function there would be approximately 1,580,220 motor vehicle thefts in 1994.

(c) and (d) Graphing the cubic function of best fit:



(e) answers will vary

88. (a) Graphing:



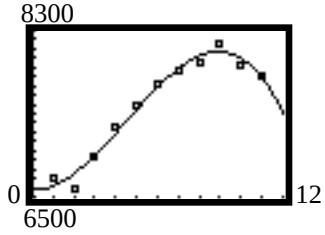
The graph may be a cubic relation.

(b) $L(x) = -4.372x^3 + 59.29x^2 - 14.02x + 6578$

$L(12) = -4.37(12)^3 + 59.29(12)^2 - 14.02(12) + 6578 \approx 7396.13$

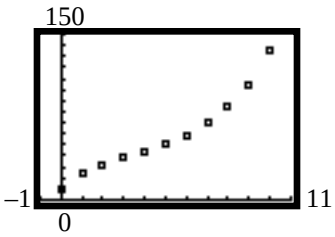
The predicted number of larceny thefts in 1994 is 7,396,130.

(c) and (d) Graphing the cubic function of best fit:



(e) answers will vary

89. (a) Graphing: The graph may be a cubic relation.



(b) Average rate of change $= \frac{50-43}{5-4} = \frac{7}{1} = 7$

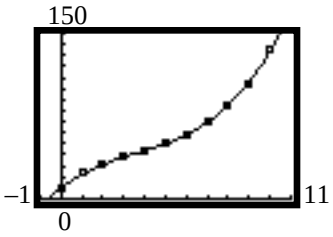
(c) Average rate of change $= \frac{105-85}{9-8} = \frac{20}{1} = 20$

(d) $C(x) = 0.2x^3 - 2.3x^2 + 14.3x + 10.2$

$C(11) = 0.2(11)^3 - 2.3(11)^2 + 14.3(11) + 10.2 \approx 155.4$

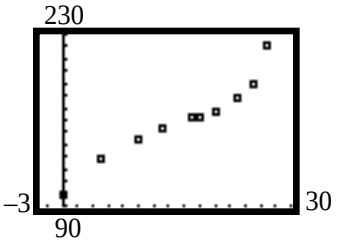
The cost of manufacturing 11 Cavaliers in 1 hour would be approximately \$155,400.

(e) and (f) Graphing the cubic function of best fit:



(g) The y-intercept would indicate the fixed costs before any cars are made.

90. (a) Graphing: The graph may be a cubic relation.



(b) Average rate of change $= \frac{153.5-144}{13-10} = \frac{9.5}{3} \approx 3.167$

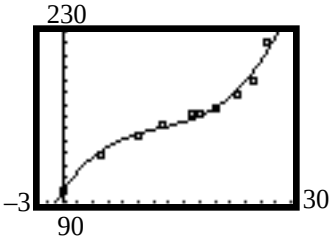
(c) Average rate of change $= \frac{166.3-162.6}{20-18} = \frac{3.7}{2} \approx 1.85$

(d) $C(x) = 0.015x^3 - 0.595x^2 + 9.15x + 98.43$

$C(22) = 0.015(22)^3 - 0.595(22)^2 + 9.15(22) + 98.43 \approx 176.3$

The cost of producing 22,000 texts per week would be about \$176,300.

(e) and (f) Graphing the cubic function of best fit:



(g) The y-intercept represents the fixed costs.

91. The graph of a polynomial function will always have a y-intercept since the domain of every polynomial function is the set of real numbers. Therefore $f(0)$ will always produce a y-coordinate on the graph.
A polynomial function might have no x-intercepts. For example $f(x) = x^2 + 1$ has no x-intercepts since the equation $x^2 + 1 = 0$ has no real solutions.

92. Answers will vary

93. Answers will vary, one such polynomial is $f(x) = x^2(x+1)(4-x)(x-2)^2$

94. Answers will vary , $f(x) = (x+2)(x-1)^2$ and $g(x) = (x+2)^3(x-1)^2$ are two such polynomials

95. $f(x) = \frac{1}{x}$ is smooth and not continuous; $g(x) = |x|$ is continuous but not smooth

96. $f(x) = x^3 + bx^2 + cx + d$
(a) true since every polynomial function has exactly one y-intercept, in this case (0,d)
(b) true, a third degree polynomial will have at most 3 x-intercepts since the equation $x^3 + bx^2 + cx + d = 0$ will have at most 3 real solutions
(c) true, a third degree polynomial will have at least 1 x-intercept since the equation $x^3 + bx^2 + cx + d = 0$ will have at least 1 real solution
(d) true, since f has degree = 3
(e) false, since $f(-x) = (-x)^3 + b(-x)^2 + c(-x) + d = -x^3 + bx^2 - cx + d \neq -f(x)$
(f) true only if d = 0, otherwise the statement is false.