Chapter 4

Polynomial and Rational Functions

4.4 Rational Functions II: Analyzing Graphs

In problems 1-38, we will use the terminology: $R(x) = \dfrac{p(x)}{q(x)}$*, where the degree of* $p(x) = n$ *and the degree of* $q(x) = m$*. The graphs in Step 6 are in dot mode.*

1. $R(x) = \dfrac{x+1}{x(x+4)}$ $p(x) = x + 1;$ $q(x) = x(x+4) = x^2 + 4x;$ $n = 1;$ $m = 2$

Step 1: Domain: $\{x \mid x \neq -4,\ x \neq 0\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: –1

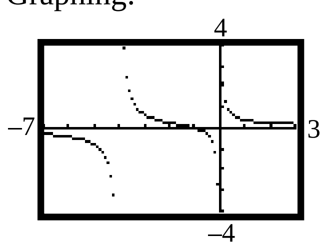
(b) There is no y-intercept; $R(0)$ is not defined, since $q(0) = 0$.

Step 3: $R(-x) = \dfrac{-x+1}{-x(-x+4)} = \dfrac{-x+1}{x^2 - 4x}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

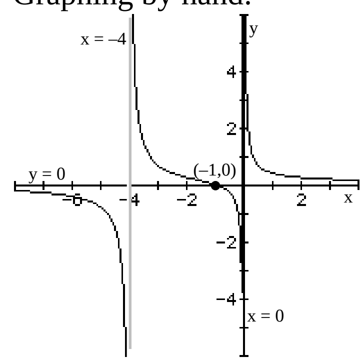
Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -4$ and $x = 0$

Step 5: Since $n < m$, the line $y = 0$ is the horizontal asymptote. $R(x)$ intersects $y = 0$ at (–1, 0).

Step 6: Graphing:



Step 7: Graphing by hand:



2. $R(x) = \dfrac{x}{(x-1)(x+2)}$ $p(x) = x;$ $q(x) = (x-1)(x+2) = x^2 + x - 2;$ $n = 1;$ $m = 2$

Step 1: Domain: $\{x \mid x \neq -2,\ x \neq 1\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: 0

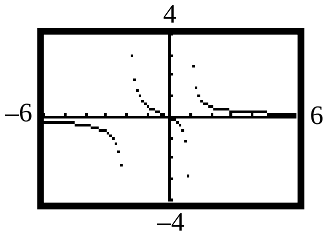
(b) The y-intercept; $R(0) = 0$

Step 3: $R(-x) = \dfrac{-x}{(-x-1)(-x+2)} = \dfrac{-x}{x^2 - x - 2}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

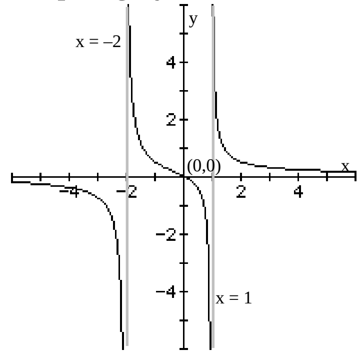
Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -2$ and $x = 1$

Step 5: Since $n < m$, the line $y = 0$ is the horizontal asymptote. $R(x)$ intersects $y = 0$ at (0, 0).

Step 6: Graphing:



Step 7: Graphing by hand:



3. $R(x) = \dfrac{3x+3}{2x+4}$ $p(x) = 3x + 3;$ $q(x) = 2x + 4;$ $n = 1;$ $m = 1$

Step 1: Domain: $\{x \mid x \neq -2\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: –1

(b) The y-intercept is $R(0) = \dfrac{3(0)+3}{2(0)+4} = \dfrac{3}{4}$.

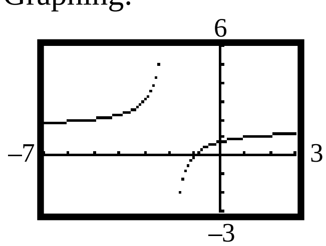
Step 3: $R(-x) = \dfrac{3(-x)+3}{2(-x)+4} = \dfrac{-3x+3}{-2x+4} = \dfrac{3x-3}{2x-4}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

Step 4: The vertical asymptote is the zero of $q(x)$: $x = -2$

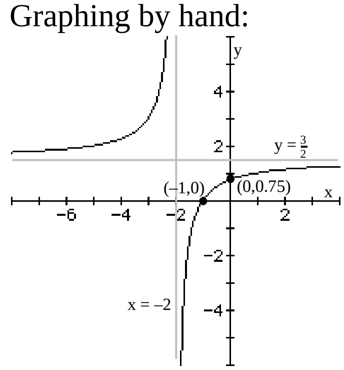
Step 5: Since $n = m$, the line $y = \dfrac{3}{2}$ is the horizontal asymptote.

$R(x)$ does not intersect $y = \dfrac{3}{2}$.

Step 6: Graphing:



Step 7: Graphing by hand:



4. $R(x) = \dfrac{2x+4}{x-1}$ $p(x) = 2x + 4;$ $q(x) = x - 1;$ $n = 1;$ $m = 1$

Step 1: Domain: $\{x \mid x \neq 1\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: –2

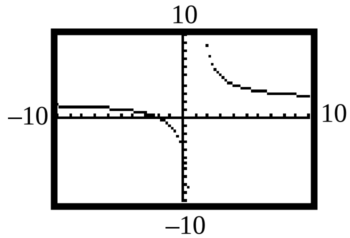
(b) The y-intercept is $R(0) = \dfrac{2(0)+4}{0-1} = \dfrac{4}{-1} = -4$.

Step 3: $R(-x) = \dfrac{2(-x)+4}{(-x)-1} = \dfrac{-2x+4}{-x-1} = \dfrac{2x-4}{x+1}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

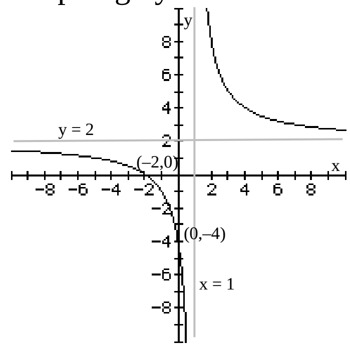
Step 4: The vertical asymptote is the zero of $q(x)$: $x = 1$

Step 5: Since $n = m$, the line $y = 2$ is the horizontal asymptote.
$R(x)$ does not intersect $y = 2$.

Step 6: Graphing:



Step 7: Graphing by hand:



5. $R(x) = \dfrac{3}{x^2 - 4}$ $p(x) = 3$; $q(x) = x^2 - 4$; $n = 0$; $m = 2$

Step 1: Domain: $\{x \mid x \neq -2,\ x \neq 2\}$

Step 2: (a) There is no x-intercept.

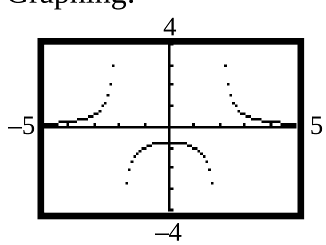
(b) The y-intercept is $R(0) = \dfrac{3}{0^2 - 4} = \dfrac{3}{-4} = \dfrac{-3}{4}$.

Step 3: $R(-x) = \dfrac{3}{(-x)^2 - 4} = \dfrac{3}{x^2 - 4} = R(x)$; $R(x)$ is symmetric to the y-axis.

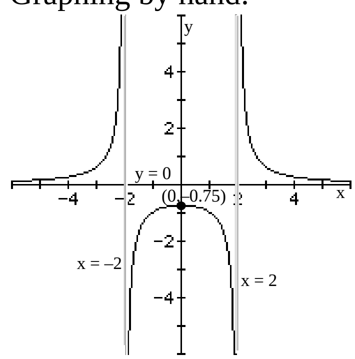
Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -2$ and $x = 2$

Step 5: Since $n < m$, the line $y = 0$ is the horizontal asymptote.
$R(x)$ does not intersect $y = 0$.

Step 6: Graphing:



Step 7: Graphing by hand:



6. $R(x) = \dfrac{6}{x^2 - x - 6} = \dfrac{6}{(x-3)(x+2)}$ $p(x) = 6$; $q(x) = x^2 - x - 6$; $n = 0$; $m = 2$

Step 1: Domain: $\{x \mid x \neq -2,\ x \neq 3\}$

Step 2: (a) There is no x-intercept.

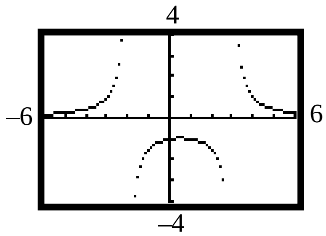
(b) The y-intercept is $R(0) = \dfrac{6}{0^2 - 0 - 6} = \dfrac{6}{-6} = -1$.

Step 3: $R(-x) = \dfrac{6}{(-x)^2 - (-x) - 6} = \dfrac{6}{x^2 + x - 6}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

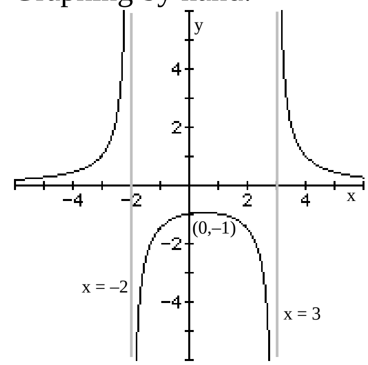
Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -2$ and $x = 3$

Step 5: Since $n < m$, the line $y = 0$ is the horizontal asymptote.
$R(x)$ does not intersect $y = 0$.

Step 6: Graphing:



Step 7: Graphing by hand:



7. $P(x) = \dfrac{x^4 + x^2 + 1}{x^2 - 1}$ $p(x) = x^4 + x^2 + 1$; $q(x) = x^2 - 1$; $n = 4$; $m = 2$

Step 1: Domain: $\{x \mid x \neq -1,\ x \neq 1\}$

Step 2: (a) There is no x-intercept.

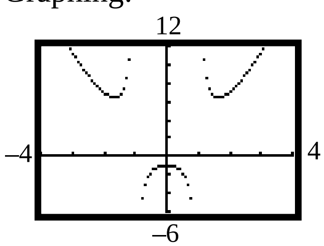
(b) The y-intercept is $P(0) = \dfrac{0^4 + 0^2 + 1}{0^2 - 1} = \dfrac{1}{-1} = -1$.

Step 3: $P(-x) = \dfrac{(-x)^4 + (-x)^2 + 1}{(-x)^2 - 1} = \dfrac{x^4 + x^2 + 1}{x^2 - 1} = P(x)$; $P(x)$ is symmetric to the y-axis.

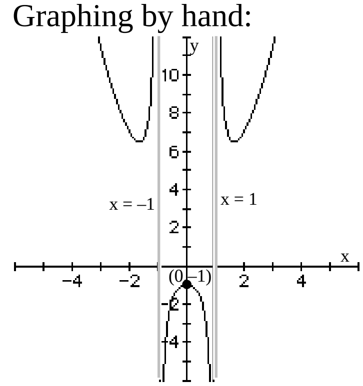
Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -1$ and $x = 1$

Step 5: Since $n > m + 1$, there is no horizontal asymptote and no oblique asymptote.

Step 6: Graphing:



Step 7: Graphing by hand:



8. $Q(x) = \dfrac{x^4 - 1}{x^2 - 4} = \dfrac{(x^2 + 1)(x + 1)(x - 1)}{(x + 2)(x - 2)}$ $p(x) = x^4 - 1$; $q(x) = x^2 - 4$; $n = 4$; $m = 2$

Step 1: Domain: $\{x \mid x \neq -2,\ x \neq 2\}$

Step 2: (a) The x-intercepts are the zeros of $p(x)$: –1 and 1.

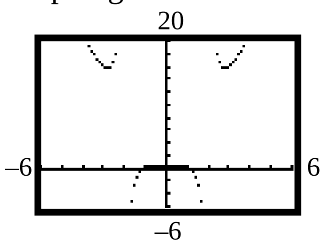
(b) The y-intercept is $Q(0) = \dfrac{0^4 - 1}{0^2 - 4} = \dfrac{-1}{-4} = \dfrac{1}{4}$.

Step 3: $Q(-x) = \dfrac{(-x)^4 - 1}{(-x)^2 - 4} = \dfrac{x^4 - 11}{x^2 - 4} = Q(x)$; $Q(x)$ is symmetric to the y-axis.

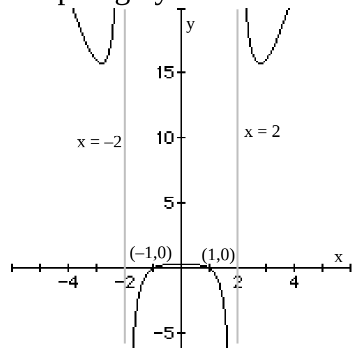
Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -2$ and $x = 2$

Step 5: Since $n > m + 1$, there is no horizontal asymptote and no oblique asymptote.

Step 6: Graphing:



Step 7: Graphing by hand:



9. $H(x) = \dfrac{x^3 - 1}{x^2 - 9}$ $p(x) = x^3 - 1$; $q(x) = x^2 - 9$; $n = 3$; $m = 2$

Step 1: Domain: $\{x \mid x \neq -3, x \neq 3\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: 1.

(b) The y-intercept is $H(0) = \dfrac{0^3 - 1}{0^2 - 9} = \dfrac{-1}{-9} = \dfrac{1}{9}$.

Step 3: $H(-x) = \dfrac{(-x)^3 - 1}{(-x)^2 - 9} = \dfrac{-x^3 - 1}{x^2 - 9}$; this is neither $H(x)$ nor $-H(x)$, so there is no symmetry.

Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -3$ and $x = 3$

Step 5: Since $n = m + 1$, there is an oblique asymptote. Dividing:

$$\begin{array}{r} x \\ x^2 - 9 \overline{\smash{\big)} x^3 + 0x^2 + 0x - 1} \\ \underline{x^3 - 9x } \\ 9x - 1 \end{array} \qquad H(x) = x + \frac{9x - 1}{x^2 - 9}$$

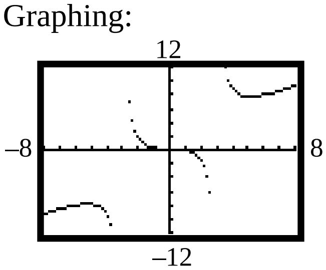
The oblique asymptote is $y = x$.

Solve to find intersection points:

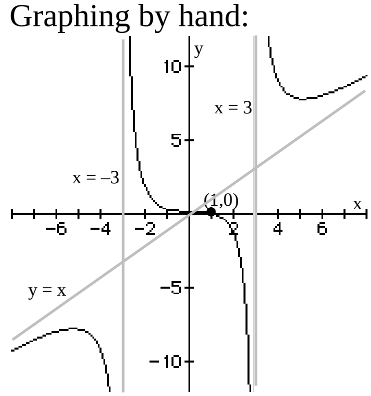
$$\frac{x^3 - 1}{x^2 - 9} = x$$
$$x^3 - 1 = x^3 - 9x$$
$$-1 = -9x$$
$$x = \frac{1}{9}$$

The oblique asymptote intersects $H(x)$ at $\left(\frac{1}{9}, \frac{1}{9}\right)$.

Step 6: Graphing:



Step 7: Graphing by hand:



10. $G(x)=\dfrac{x^3+1}{x^2+2x}=\dfrac{(x+1)(x^2-x+1)}{x(x+2)}$ $\quad p(x)=x^3+1;\ \ q(x)=x^2+2x;\ \ n=3;\ \ m=2$

Step 1: Domain: $\{x \mid x\neq -2,\ x\neq 0\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: –1.

(b) The y-intercept is $G(0)=\dfrac{0^3+1}{0^2+2(0)}=\dfrac{1}{0}$. No y-intercept.

Step 3: $G(-x)=\dfrac{(-x)^3+1}{(-x)^2+2(-x)}=\dfrac{-x^3+1}{x^2-2x}$; this is neither $G(x)$ nor $-G(x)$, so there is no symmetry.

Step 4: The vertical asymptotes are the zeros of $q(x)$: $x=-2$ and $x=0$

Step 5: Since $n=m+1$, there is an oblique asymptote. Dividing:

$$\begin{array}{r|l} & x-2 \\ \hline x^2+2x & x^3+0x^2+0x+1 \\ & \underline{x^3+2x^2} \\ & \quad -2x^2 \qquad +1 \\ & \quad \underline{-2x^2-4x} \\ & \qquad\quad 4x+1 \end{array} \qquad G(x)=x-2+\frac{4x+1}{x^2+2x}$$

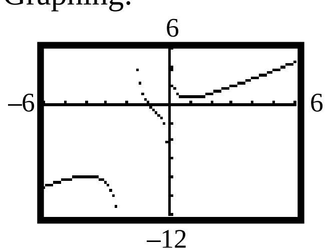
The oblique asymptote is $y=x-2$.

Solve to find intersection points:

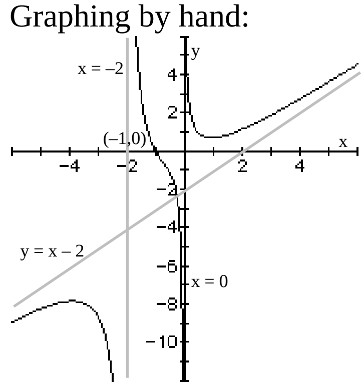
$$\begin{aligned} \frac{x^3+1}{x^2+2x}&=x-2\\ x^3+1&=x^3-4x\\ 1&=-4x\\ x&=-\frac{1}{4} \end{aligned}$$

The oblique asymptote intersects $G(x)$ at $\left(-\frac{1}{4},\,-\frac{9}{4}\right)$.

Step 6: Graphing:



Step 7: Graphing by hand:



11. $R(x)=\dfrac{x^2}{x^2+x-6}=\dfrac{x^2}{(x+3)(x-2)}$ $\quad p(x)=x^2;\ \ q(x)=x^2+x-6;\ \ n=2;\ \ m=2$

Step 1: Domain: $\{x \mid x\neq -3,\ x\neq 2\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: 0

(b) The y-intercept is $R(0)=\dfrac{0^2}{0^2+0-6}=\dfrac{0}{-6}=0$.

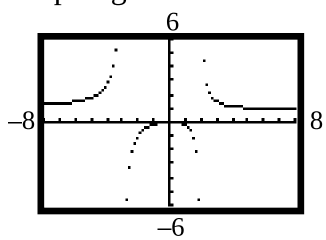
Step 3: $R(-x) = \dfrac{(-x)^2}{(-x)^2 + (-x) - 6} = \dfrac{x^2}{x^2 - x - 6}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -3$ and $x = 2$

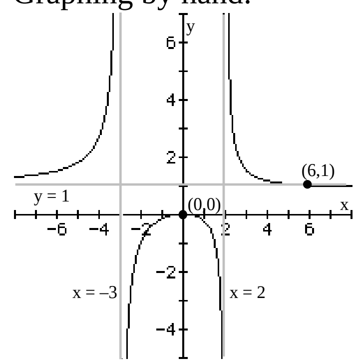
Step 5: Since $n = m$, the line $y = 1$ is the horizontal asymptote.
$R(x)$ intersects $y = 1$ at (6, 1), since:

$$\frac{x^2}{x^2 + x - 6} = 1 \rightarrow x^2 = x^2 + x - 6 \rightarrow 0 = x - 6 \rightarrow x = 6$$

Step 6: Graphing:



Step 7: Graphing by hand:



12. $R(x) = \dfrac{x^2 + x - 12}{x^2 - 4} = \dfrac{(x+4)(x-3)}{(x+2)(x-2)}$ $p(x) = x^2 + x - 12;\ q(x) = x^2 - 4;\ n = 2;\ m = 2$

Step 1: Domain: $\{x \mid x \neq -2,\ x \neq 2\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: –4 and 3

(b) The y-intercept is $R(0) = \dfrac{0^2 + 0 - 12}{0^2 - 4} = \dfrac{-12}{-4} = 3$.

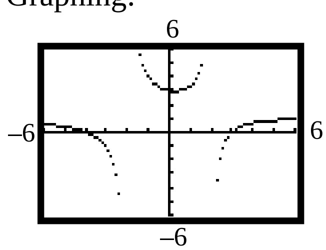
Step 3: $R(-x) = \dfrac{(-x)^2 + (-x) - 12}{(-x)^2 - 4} = \dfrac{x^2 - x - 12}{x^2 - 4}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -2$ and $x = 2$

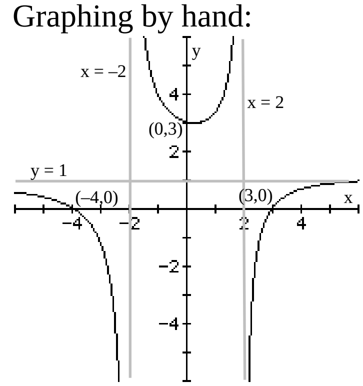
Step 5: Since $n = m$, the line $y = 1$ is the horizontal asymptote.
$R(x)$ intersects $y = 1$ at (8, 1), since:

$$\frac{x^2 + x - 12}{x^2 - 4} = 1 \rightarrow x^2 + x - 12 = x^2 - 4 \rightarrow x = 8$$

Step 6: Graphing:



Step 7: Graphing by hand:



13. $G(x)=\dfrac{x}{x^2-4}=\dfrac{x}{(x+2)(x-2)}$ $\quad p(x)=x;\ q(x)=x^2-4;\ n=1;\ m=2$

Step 1: Domain: $\{x \mid x\neq -2,\ x\neq 2\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: 0

(b) The y-intercept is $G(0)=\dfrac{0}{0^2-4}=\dfrac{0}{-4}=0$.

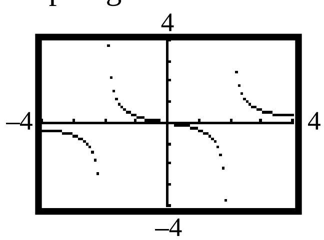
Step 3: $G(-x)=\dfrac{-x}{(-x)^2-4}=\dfrac{-x}{x^2-4}=-G(x)$; $G(x)$ is symmetric to the origin.

Step 4: The vertical asymptotes are the zeros of $q(x)$: $x=-2$ and $x=2$

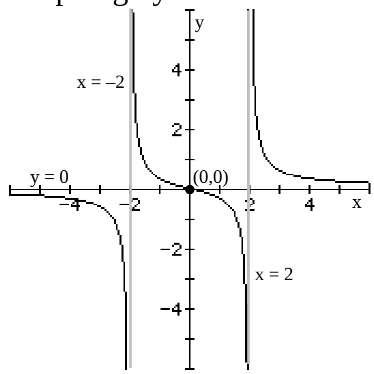
Step 5: Since $n<m$, the line $y=0$ is the horizontal asymptote.

$G(x)$ intersects $y=0$ at (0, 0).

Step 6: Graphing:



Step 7: Graphing by hand:



14. $G(x)=\dfrac{3x}{x^2-1}=\dfrac{3x}{(x+1)(x-1)}$ $\quad p(x)=3x;\ q(x)=x^2-1;\ n=1;\ m=2$

Step 1: Domain: $\{x \mid x\neq -1,\ x\neq 1\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: 0

(b) The y-intercept is $G(0)=\dfrac{3(0)}{0^2-1}=\dfrac{0}{-1}=0$.

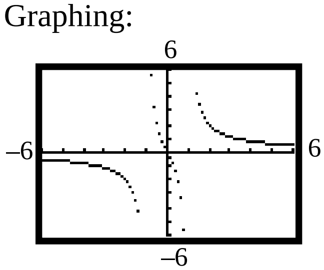
Step 3: $G(-x)=\dfrac{-3x}{(-x)^2-1}=\dfrac{-3x}{x^2-1}=-G(x)$; $G(x)$ is symmetric to the origin.

Step 4: The vertical asymptotes are the zeros of $q(x)$: $x=-1$ and $x=1$

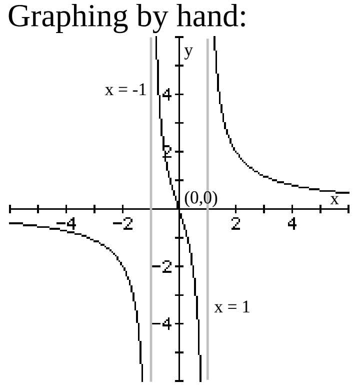
Step 5: Since $n<m$, the line $y=0$ is the horizontal asymptote.

$G(x)$ intersects $y=0$ at (0, 0).

Step 6: Graphing:



Step 7: Graphing by hand:



15. $R(x) = \dfrac{3}{(x-1)(x^2-4)} = \dfrac{3}{(x-1)(x+2)(x-2)}$ $p(x) = 3;$ $q(x) = (x-1)(x^2-4);$
$n = 0;$ $m = 3$

Step 1: Domain: $\{x \mid x \neq -2, x \neq 1, x \neq 2\}$

Step 2: (a) There is no x-intercept.

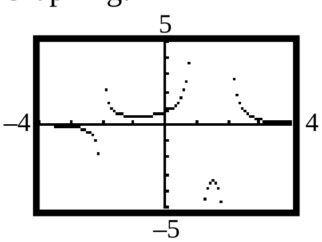
(b) The y-intercept is $R(0) = \dfrac{3}{(0-1)(0^2-4)} = \dfrac{3}{4}$.

Step 3: $R(-x) = \dfrac{3}{(-x-1)\left((-x)^2-4\right)} = \dfrac{3}{(-x-1)(x^2-4)}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

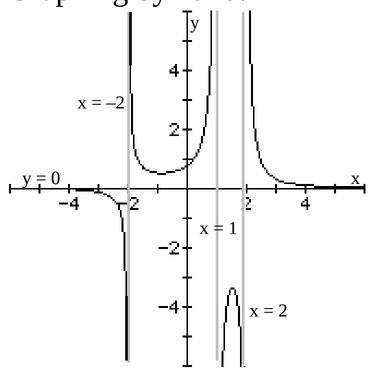
Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -2$, $x = 1$, and $x = 2$

Step 5: Since $n < m$, the line $y = 0$ is the horizontal asymptote. $R(x)$ does not intersect $y = 0$.

Step 6: Graphing:



Step 7: Graphing by hand:



16. $R(x) = \dfrac{-4}{(x+1)(x^2-9)} = \dfrac{-4}{(x+1)(x+3)(x-3)}$ $p(x) = -4;$ $q(x) = (x+1)(x^2-9);$
$n = 0;$ $m = 3$

Step 1: Domain: $\{x \mid x \neq -3, x \neq -1, x \neq 3\}$

Step 2: (a) There is no x-intercept.

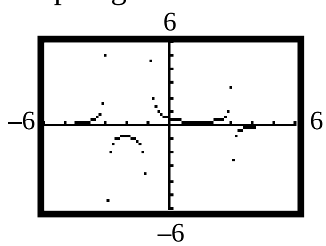
(b) The y-intercept is $R(0) = \dfrac{-4}{(0+1)(0^2-9)} = \dfrac{-4}{-9} = \dfrac{4}{9}$.

Step 3: $R(-x) = \dfrac{-4}{(-x+1)\left((-x)^2-9\right)} = \dfrac{-4}{(-x+1)(x^2-9)}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

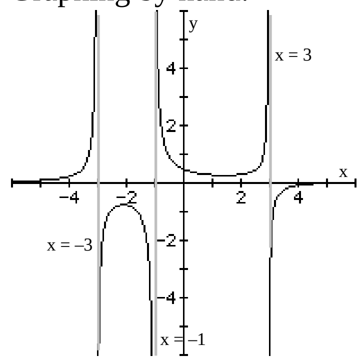
Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -3$, $x = -1$, and $x = 3$

Step 5: Since $n < m$, the line $y = 0$ is the horizontal asymptote. $R(x)$ does not intersect $y = 0$.

Step 6: Graphing:



Step 7: Graphing by hand:



17. $H(x) = \dfrac{4(x^2 - 1)}{x^4 - 16} = \dfrac{4(x-1)(x+1)}{(x^2+4)(x+2)(x-2)}$ $p(x) = 4(x^2 - 1);$ $q(x) = x^4 - 16;$

$n = 2;\ m = 4$

Step 1: Domain: $\{x \mid x \neq -2,\ x \neq 2\}$

Step 2: (a) The x-intercepts are the zeros of $p(x)$: –1 and 1

(b) The y-intercept is $H(0) = \dfrac{4(0^2 - 1)}{0^4 - 16} = \dfrac{-4}{-16} = \dfrac{1}{4}$.

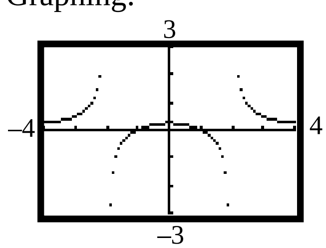
Step 3: $H(-x) = \dfrac{4\left((-x)^2 - 1\right)}{(-x)^4 - 16} = \dfrac{4\left(x^2 - 1\right)}{x^4 - 16} = H(x)$; $H(x)$ is symmetric to the y-axis.

Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -2,$ and $x = 2$

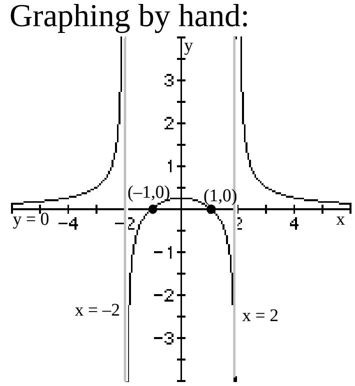
Step 5: Since $n < m$, the line $y = 0$ is the horizontal asymptote.

$H(x)$ intersects $y = 0$ at (–1, 0) and (1, 0).

Step 6: Graphing:



Step 7: Graphing by hand:



18. $H(x) = \dfrac{x^2 + 4}{x^4 - 1} = \dfrac{x^2 + 4}{(x^2+1)(x+1)(x-1)}$ $p(x) = x^2 + 4;$ $q(x) = x^4 - 1;$

$n = 2;\ m = 4$

Step 1: Domain: $\{x \mid x \neq -1,\ x \neq 1\}$

Step 2: (a) There are no x-intercepts.

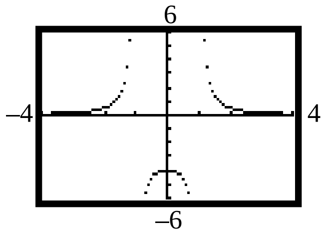
(b) The y-intercept is $H(0) = \dfrac{0^2 + 4}{0^4 - 1} = \dfrac{4}{-1} = -4$.

Step 3: $H(-x) = \dfrac{(-x)^2 + 4}{(-x)^4 - 1} = \dfrac{x^2 + 4}{x^4 - 1} = H(x)$; $H(x)$ is symmetric to the y-axis.

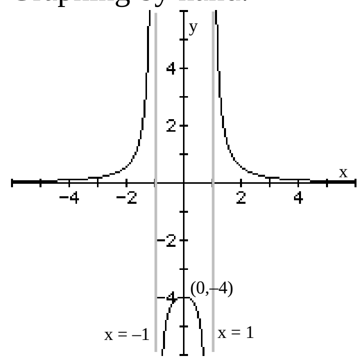
Step 4: The vertical asymptotes are the zeros of $q(x)$: $x = -1,$ and $x = 1$

Step 5: Since $n < m$, the line $y = 0$ is the horizontal asymptote.
$H(x)$ does not intersect $y = 0$.

Step 6: Graphing:



Step 7: Graphing by hand:



19. $F(x) = \dfrac{x^2 - 3x - 4}{x + 2} = \dfrac{(x+1)(x-4)}{x+2}$ $p(x) = x^2 - 3x - 4;$ $q(x) = x + 2;$ $n = 2;$ $m = 1$

Step 1: Domain: $\{x \mid x \neq -2\}$

Step 2: (a) The x-intercepts are the zeros of $p(x)$: –1 and 4.

(b) The y-intercept is $F(0) = \dfrac{0^2 - 3(0) - 4}{0 + 2} = \dfrac{-4}{2} = -2$.

Step 3: $F(-x) = \dfrac{(-x)^2 - 3(-x) - 4}{-x + 2} = \dfrac{x^2 + 3x - 4}{-x + 2}$; this is neither $F(x)$ nor $-F(x)$, so there is no symmetry.

Step 4: The vertical asymptote is the zero of $q(x)$: $x = -2$

Step 5: Since $n = m + 1$, there is an oblique asymptote. Dividing:

$$\begin{array}{r}
x - 5 \\
x + 2 \overline{\smash{)}\, x^2 - 3x - 4} \\
\underline{x^2 + 2x} \\
-5x - 4 \\
\underline{-5x - 10} \\
6
\end{array}
\qquad F(x) = x - 5 + \frac{6}{x + 2}$$

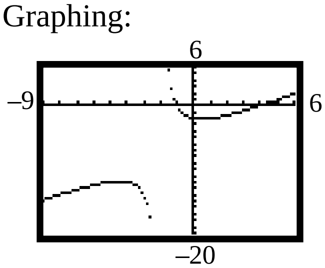
The oblique asymptote is $y = x - 5$.

Solve to find intersection points:

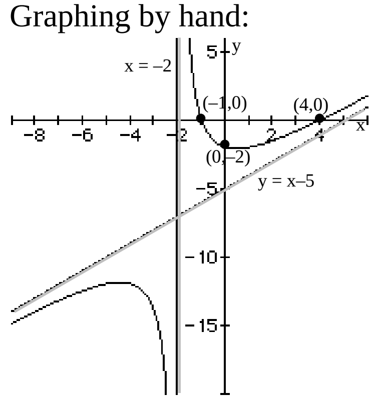
$$\frac{x^2 - 3x - 4}{x + 2} = x - 5 \rightarrow x^2 - 3x - 4 = x^2 - 3x - 10 \rightarrow -4 = -10$$

Since there is no solution, the oblique asymptote does not intersect $F(x)$.

Step 6: Graphing:



Step 7: Graphing by hand:



20. $F(x)=\dfrac{x^2+3x+2}{x-1}=\dfrac{(x+2)(x+1)}{x-1}$ $p(x)=x^2+3x+2$; $q(x)=x-1$; $n=2$; $m=1$

Step 1: Domain: $\{x \mid x \neq 1\}$

Step 2: (a) The x-intercepts are the zeros of $p(x)$: –2 and –1.

(b) The y-intercept is $F(0)=\dfrac{0^2+3(0)+2}{0-1}=\dfrac{2}{-1}=-2$.

Step 3: $F(-x)=\dfrac{(-x)^2+3(-x)+2}{-x-1}=\dfrac{x^2-3x+2}{-x-1}$; this is neither $F(x)$ nor $-F(x)$, so there is no symmetry.

Step 4: The vertical asymptote is the zero of $q(x)$: $x=1$

Step 5: Since $n=m+1$, there is an oblique asymptote. Dividing:

$$\begin{array}{r|l} & x+4 \\ \hline x-1 & x^2+3x+2 \\ & \underline{x^2-x} \\ & \quad 4x+2 \\ & \quad \underline{4x-4} \\ & \qquad 6 \end{array} \qquad F(x)=x+4+\frac{6}{x-1}$$

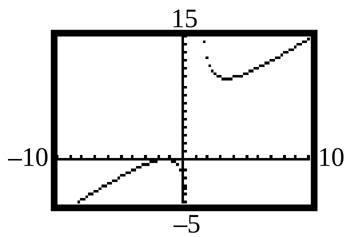
The oblique asymptote is $y=x+4$.

Solve to find intersection points:

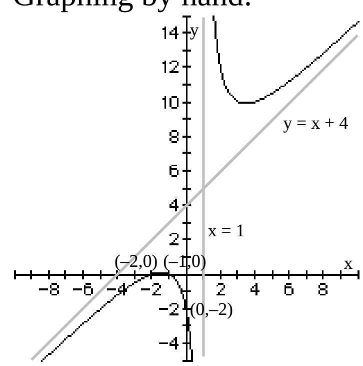
$$\frac{x^2+3x+2}{x-1}=x+4$$
$$x^2+3x+2=x^2+3x-4$$
$$2=-4$$

Since there is no solution, the oblique asymptote does not intersect $F(x)$.

Step 6: Graphing:



Step 7: Graphing by hand:



21. $R(x)=\dfrac{x^2+x-12}{x-4}=\dfrac{(x+4)(x-3)}{x-4}$ $p(x)=x^2+x-12$; $q(x)=x-4$; $n=2$; $m=1$

Step 1: Domain: $\{x \mid x \neq 4\}$

Step 2: (a) The x-intercepts are the zeros of $p(x)$: –4 and 3.

(b) The y-intercept is $R(0)=\dfrac{0^2+0-12}{0-4}=\dfrac{-12}{-4}=3$.

Step 3: $R(-x)=\dfrac{(-x)^2+(-x)-12}{-x-4}=\dfrac{x^2-x-12}{-x-4}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

Step 4: The vertical asymptote is the zero of $q(x)$: $x=4$

Step 5: Since $n = m+1$, there is an oblique asymptote. Dividing:

$$\begin{array}{r} x+5 \\ x-4\overline{\smash{)}\,x^2 + x - 12} \\ \underline{x^2 - 4x} \\ 5x - 12 \\ \underline{5x - 20} \\ 8 \end{array} \qquad R(x) = x + 5 + \frac{8}{x-4}$$

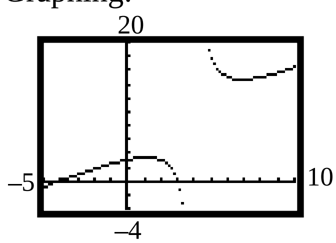
The oblique asymptote is $y = x+5$.

Solve to find intersection points:

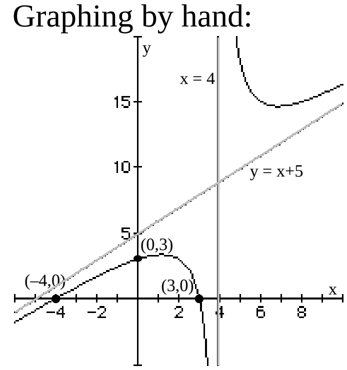
$$\frac{x^2 + x - 12}{x-4} = x + 5$$
$$x^2 + x - 12 = x^2 + x - 20$$
$$-12 = -20$$

Since there is no solution, the oblique asymptote does not intersect $R(x)$.

Step 6: Graphing:



Step 7: Graphing by hand:



22. $R(x) = \dfrac{x^2 - x - 12}{x+5} = \dfrac{(x-4)(x+3)}{x+5}$ $\quad p(x) = x^2 - x - 12; \quad q(x) = x+5; \quad n = 2; \quad m = 1$

Step 1: Domain: $\{x \mid x \neq -5\}$

Step 2: (a) The x-intercepts are the zeros of $p(x)$: –3 and 4.

(b) The y-intercept is $R(0) = \dfrac{0^2 - 0 - 12}{0+5} = \dfrac{-12}{5}$.

Step 3: $R(-x) = \dfrac{(-x)^2 - (-x) - 12}{-x+5} = \dfrac{x^2 + x - 12}{-x+5}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

Step 4: The vertical asymptote is the zero of $q(x)$: $x = -5$

Step 5: Since $n = m+1$, there is an oblique asymptote. Dividing:

$$\begin{array}{r} x-6 \\ x+5\overline{\smash{)}\,x^2 - x - 12} \\ \underline{x^2 + 5x} \\ -6x - 12 \\ \underline{-6x - 30} \\ 18 \end{array} \qquad R(x) = x - 6 + \frac{18}{x+5}$$

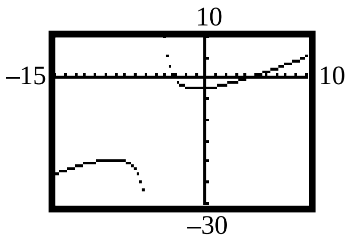
The oblique asymptote is $y = x-6$.

Solve to find intersection points:

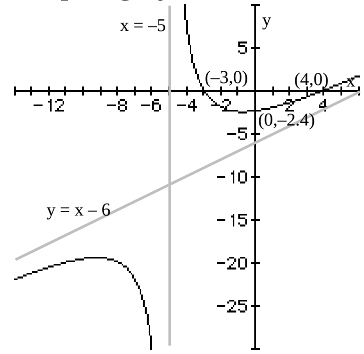
$$\frac{x^2 - x - 12}{x+5} = x - 6 \rightarrow x^2 - x - 12 = x^2 - x - 30 \rightarrow -12 = -30$$

Since there is no solution, the oblique asymptote does not intersect $R(x)$.

Step 6: Graphing:



Step 7: Graphing by hand:



23. $F(x) = \dfrac{x^2 + x - 12}{x+2} = \dfrac{(x+4)(x-3)}{x+2}$ $\quad p(x) = x^2 + x - 12;\ \ q(x) = x + 2;\ \ n = 2;\ \ m = 1$

Step 1: Domain: $\{x \mid x \neq -2\}$

Step 2: (a) The x-intercepts are the zeros of $p(x)$: –4 and 3.

(b) The y-intercept is $F(0) = \dfrac{0^2 + 0 - 12}{0+2} = \dfrac{-12}{2} = -6$.

Step 3: $F(-x) = \dfrac{(-x)^2 + (-x) - 12}{-x+2} = \dfrac{x^2 - x - 12}{-x+2}$; this is neither $F(x)$ nor $-F(x)$, so there is no symmetry.

Step 4: The vertical asymptote is the zero of $q(x)$: $x = -2$

Step 5: Since $n = m + 1$, there is an oblique asymptote. Dividing:

$$\begin{array}{r|l} & x - 1 \\ \hline x+2 & x^2 + x - 12 \\ & \underline{x^2 + 2x} \\ & \quad -x - 12 \\ & \quad \underline{-x - 2} \\ & \qquad -10 \end{array} \qquad F(x) = x - 1 + \frac{-10}{x+2}$$

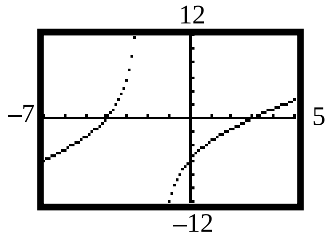
The oblique asymptote is $y = x - 1$.

Solve to find intersection points:

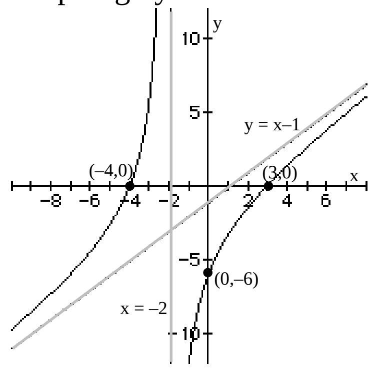
$$\frac{x^2 + x - 12}{x+2} = x - 1 \rightarrow x^2 + x - 12 = x^2 + x - 2 \rightarrow -12 = -2$$

Since there is no solution, the oblique asymptote does not intersect $F(x)$.

Step 6: Graphing:



Step 7: Graphing by hand:



24. $G(x)=\dfrac{x^2-x-12}{x+1}=\dfrac{(x+3)(x-4)}{x+1}$ $\quad p(x)=x^2-x-12;\;\; q(x)=x+1;\;\; n=2;\;\; m=1$

Step 1: Domain: $\{x \mid x \neq -1\}$

Step 2: (a) The x-intercepts are the zeros of $p(x)$: –3 and 4.

(b) The y-intercept is $F(0)=\dfrac{0^2-0-12}{0+1}=\dfrac{-12}{1}=-12$.

Step 3: $G(-x)=\dfrac{(-x)^2-(-x)-12}{-x+1}=\dfrac{x^2+x-12}{-x+1}$; this is neither $G(x)$ nor $-G(x)$, so there is no symmetry.

Step 4: The vertical asymptote is the zero of $q(x)$: $x=-1$

Step 5: Since $n=m+1$, there is an oblique asymptote. Dividing:

$$\begin{array}{r|l} & x-2 \\ \hline x+1 & x^2-x-12 \\ & \underline{x^2+x} \\ & -2x-12 \\ & \underline{-2x-2} \\ & -10 \end{array}$$

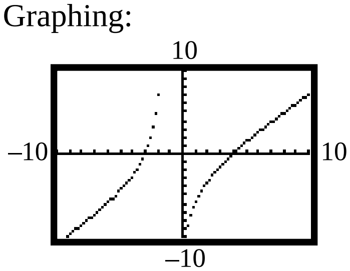
$G(x)=x-2+\dfrac{-10}{x+1}$

The oblique asymptote is $y=x-2$.

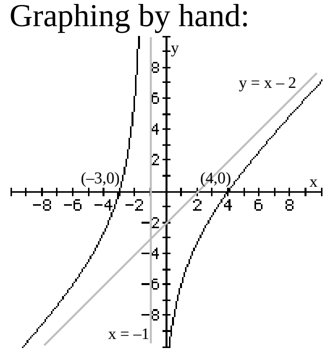
Solve to find intersection points: $\dfrac{x^2-x-12}{x+1}=x-2 \rightarrow x^2-x-12=x^2-x-2 \rightarrow -12=-2$

Since there is no solution, the oblique asymptote does not intersect $G(x)$.

Step 6: Graphing:



Step 7: Graphing by hand:



25. $R(x)=\dfrac{x(x-1)^2}{(x+3)^3}$ $p(x)=x(x-1)^2$; $q(x)=(x+3)^3$; $n=3$; $m=3$

Step 1: Domain: $\{x \mid x \neq -3\}$

Step 2: (a) The x-intercepts are the zeros of $p(x)$: 0 and 1

(b) The y-intercept is $R(0)=\dfrac{0(0-1)^2}{(0+3)^3}=\dfrac{0}{27}=0$.

Step 3: $R(-x)=\dfrac{-x(-x-1)^2}{(-x+3)^3}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

Step 4: The vertical asymptote is the zero of $q(x)$: $x=-3$

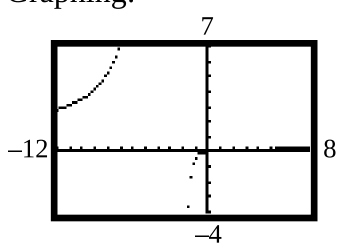
Step 5: Since $n=m$, the line $y=1$ is the horizontal asymptote.

Solve to find intersection points:

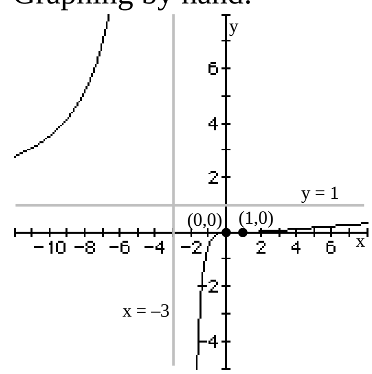
$$\frac{x(x-1)^2}{(x+3)^3}=1$$
$$x^3-2x^2+x=x^3+9x^2+27x+27$$
$$0=11x^2+26x+27$$

Since there is no real solution, $R(x)$ does not intersect $y=1$.

Step 6: Graphing:



Step 7: Graphing by hand:



26. $R(x)=\dfrac{(x-1)(x+2)(x-3)}{x(x-4)^2}$ $p(x)=(x-1)(x+2)(x-3)$; $q(x)=x(x-4)^2$;

$n=3$; $m=3$

Step 1: Domain: $\{x \mid x \neq 0,\ x \neq 4\}$

Step 2: (a) The x-intercepts are the zeros of $p(x)$: –2, 1, and 3

(b) The y-intercept is $R(0)=\dfrac{(0-1)(0+2)(0-3)}{0(0-4)^2}=\dfrac{6}{0}$. No y-intercept.

Step 3: $R(-x)=\dfrac{(-x-1)(-x+2)(-x-3)}{-x(-x-4)^2}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

Step 4: The vertical asymptotes are the zeros of $q(x)$: $x=0$ and $x=4$

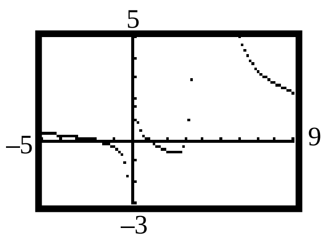
Step 5: Since $n=m$, the line $y=1$ is the horizontal asymptote.

Solve to find intersection points:

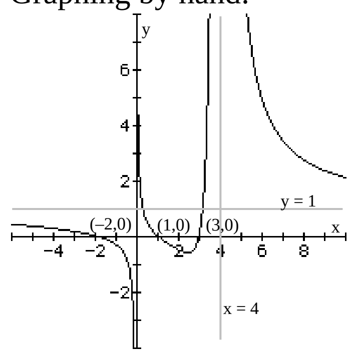
$$\frac{(x-1)(x+2)(x-3)}{x(x-4)^2} = 1$$
$$(x^2 + x - 2)(x-3) = x(x^2 - 8x + 16)$$
$$x^3 - 2x^2 - 5x + 6 = x^3 - 8x^2 + 16x$$
$$6x^2 - 21x + 6 = 0$$
$$2x^2 - 7x + 2 = 0$$
$$x = \frac{7 \pm \sqrt{49 - 4(2)(2)}}{2(2)} = \frac{7 \pm \sqrt{33}}{4}$$

$R(x)$ intersects $y = 1$ at $\left(\frac{7 - \sqrt{33}}{4}, 1 \right)$ and $\left(\frac{7 + \sqrt{33}}{4}, 1 \right)$

Step 6: Graphing:



Step 7: Graphing by hand:



27. $R(x) = \frac{x^2 + x - 12}{x^2 - x - 6} = \frac{(x+4)(x-3)}{(x-3)(x+2)} = \frac{x+4}{x+2}$ $\quad p(x) = x^2 + x - 12;\ q(x) = x^2 - x - 6;$
$n = 2;\ m = 2$

Step 1: Domain: $\{x \mid x \neq -2,\ x \neq 3\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: –4 (3 is not a zero because reduced form must be used to find the zeros.)

(b) The y-intercept is $R(0) = \frac{0^2 + 0 - 12}{0^2 - 0 - 6} = \frac{-12}{-6} = 2$.

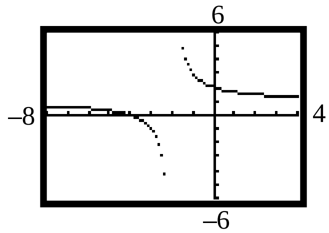
Step 3: $R(-x) = \frac{(-x)^2 + (-x) - 12}{(-x)^2 - (-x) - 6} = \frac{x^2 - x - 12}{x^2 + x - 6}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

Step 4: The vertical asymptote is the zero of $q(x)$: $x = -2$ ($x = 3$ is not a vertical asymptote because reduced form must be used to find the them.)

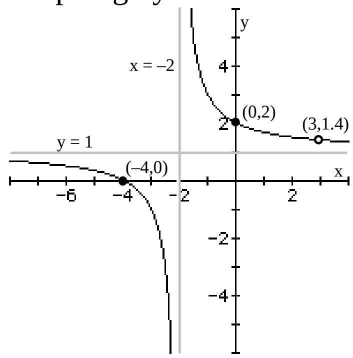
Step 5: Since $n = m$, the line $y = 1$ is the horizontal asymptote.
$R(x)$ does not intersect $y = 1$ because $R(x)$ is not defined at $x = 3$.

$$\frac{x^2 + x - 12}{x^2 - x - 6} = 1$$
$$x^2 + x - 12 = x^2 - x - 6$$
$$2x = 6$$
$$x = 3$$

Step 6: Graphing:



Step 7: Graphing by hand:



28. $R(x)=\dfrac{x^2+3x-10}{x^2+8x+15}=\dfrac{(x+5)(x-2)}{(x+5)(x+3)}=\dfrac{x-2}{x+3}\quad p(x)=x^2+3x-10;$

$q(x)=x^2+8x+15;\quad n=2;\ m=2$

Step 1: Domain: $\{x \mid x\neq -5,\ x\neq -3\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: 2 (–5 is not a zero because reduced form must be used to find the zeros.)

(b) The y-intercept is $R(0)=\dfrac{0^2+3(0)-10}{0^2+8(0)+15}=\dfrac{-10}{15}=-\dfrac{2}{3}$.

Step 3: $R(-x)=\dfrac{(-x)^2+3(-x)-10}{(-x)^2+8(-x)+15}=\dfrac{x^2-3x-10}{x^2-8x+15}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

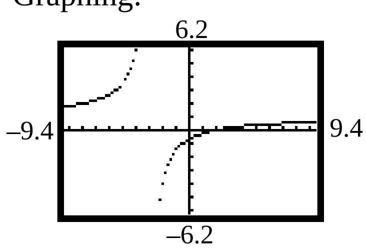
Step 4: The vertical asymptote is the zero of $q(x)$: $x=-3$ ($x=-5$ is not a vertical asymptote because reduced form must be used to find them.)

Step 5: Since $n=m$, the line $y=1$ is the horizontal asymptote.

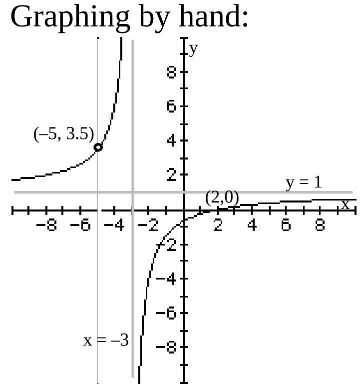
$R(x)$ does not intersect $y=1$ because $R(x)$ is not defined at $x=-5$.

$$\frac{x^2+3x-10}{x^2+8x+15}=1$$
$$x^2+3x-10=x^2+8x+15$$
$$-5x=25$$
$$x=-5$$

Step 6: Graphing:



Step 7: Graphing by hand:



29. $R(x)=\dfrac{6x^2-7x-3}{2x^2-7x+6}=\dfrac{(3x+1)(2x-3)}{(2x-3)(x-2)}=\dfrac{3x+1}{x-2}$ $p(x)=6x^2-7x-3$;
$q(x)=2x^2-7x+6$; $n=2$; $m=2$

Step 1: Domain: $\left\{x \,\middle|\, x\neq \frac{3}{2},\, x\neq 2\right\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: $\frac{-1}{3}$ ($\frac{3}{2}$ is not a zero because reduced form must be used to find the zeros.)

(b) The y-intercept is $R(0)=\dfrac{6(0)^2-7(0)-3}{2(0)^2-7(0)+6}=\dfrac{-3}{6}=\dfrac{-1}{2}$.

Step 3: $R(-x)=\dfrac{6(-x)^2-7(-x)-3}{2(-x)^2-7(-x)+6}=\dfrac{6x^2+7x-3}{2x^2+7x+6}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

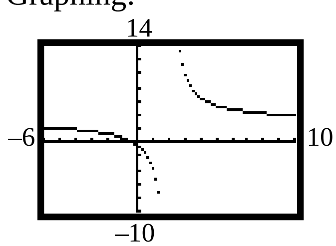
Step 4: The vertical asymptote is the zero of $q(x)$: $x=2$ ($x=\frac{3}{2}$ is not a vertical asymptote because reduced form must be used to find the them.)

Step 5: Since $n=m$, the line $y=3$ is the horizontal asymptote.

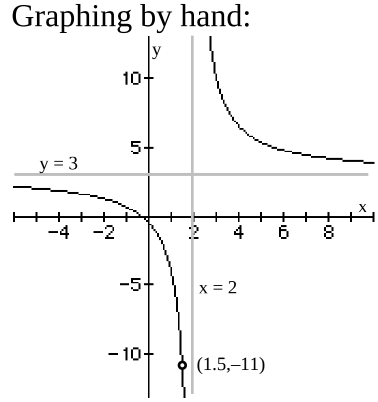
$R(x)$ does not intersect $y=3$ because $R(x)$ is not defined at $x=\frac{3}{2}$.

$$\frac{6x^2-7x-3}{2x^2-7x+6}=3$$
$$6x^2-7x-3=6x^2-21x+18$$
$$14x=21$$
$$x=\frac{3}{2}$$

Step 6: Graphing:



Step 7: Graphing by hand:



30. $R(x)=\dfrac{8x^2+26x+15}{2x^2-x-15}=\dfrac{(4x+3)(2x+5)}{(2x+5)(x-3)}=\dfrac{4x+3}{x-3}$ $p(x)=8x^2+26x+15$;
$q(x)=2x^2-x-15$; $n=2$; $m=2$

Step 1: Domain: $\left\{x \,\middle|\, x\neq -\frac{5}{2},\, x\neq 3\right\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: $-\frac{3}{4}$ ($-\frac{5}{2}$ is not a zero because reduced form must be used to find the zeros.)

(b) The y-intercept is $R(0)=\dfrac{8(0)^2+26(0)+15}{2(0)^2-(0)-15}=\dfrac{15}{-15}=-1$.

Step 3: $R(-x) = \dfrac{8(-x)^2 + 26(-x) + 15}{2(-x)^2 - (-x) - 15} = \dfrac{8x^2 - 26x + 15}{2x^2 + x - 15}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

Step 4: The vertical asymptote is the zero of $q(x)$: $x = 3$ ($x = -\frac{5}{2}$ is not a vertical asymptote because reduced form must be used to find them.)

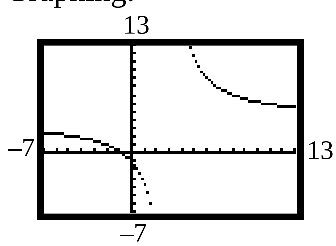
Step 5: Since $n = m$, the line $y = 4$ is the horizontal asymptote.

$R(x)$ does not intersect $y = 4$ because $R(x)$ is not defined at $x = -\dfrac{5}{2}$.

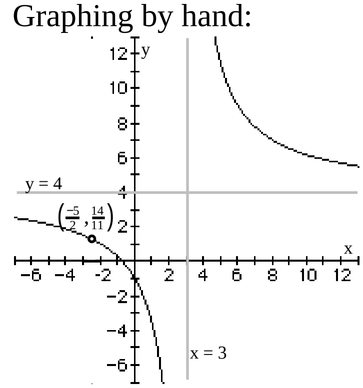
$$\frac{8x^2 + 26x + 15}{2x^2 - x - 15} = 4$$

$$8x^2 + 26x + 15 = 8x^2 - 4x - 60 \rightarrow 30x = -75 \rightarrow x = -\frac{5}{2}$$

Step 6: Graphing:



Step 7: Graphing by hand:



31. $R(x) = \dfrac{x^2 + 5x + 6}{x + 3} = \dfrac{(x+2)(x+3)}{x+3} = x + 2$ $p(x) = x^2 + 5x + 6$; $q(x) = x + 3$; $n = 2$; $m = 1$

Step 1: Domain: $\{x \mid x \neq -3\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: –2 (–3 is not a zero because reduced form must be used to find the zeros.)

(b) The y-intercept is $R(0) = \dfrac{0^2 + 5(0) + 6}{0 + 3} = \dfrac{6}{3} = 2$.

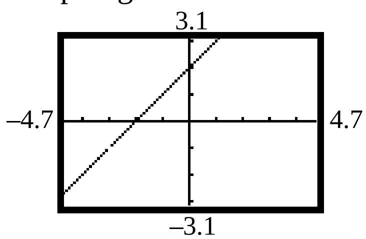
Step 3: $R(-x) = \dfrac{(-x)^2 + 5(-x) + 6}{-x + 3} = \dfrac{x^2 - 5x + 6}{-x + 3}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

Step 4: There are no vertical asymptotes. ($x = -3$ is not a vertical asymptote because reduced form must be used to find the them.)

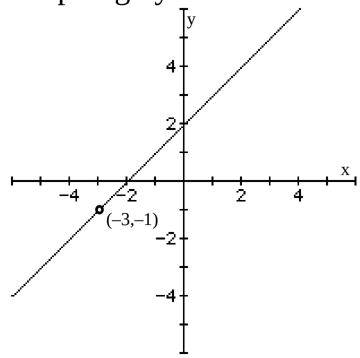
Step 5: Since $n = m + 1$ there is a oblique asymptote. The line $y = x + 2$ is the oblique asymptote.

The oblique asymptote does not intersect $R(x)$.

Step 6: Graphing:



Step 7: Graphing by hand:



32. $R(x)=\dfrac{x^2+x-30}{x+6}=\dfrac{(x+6)(x-5)}{x+6}=x-5 \quad p(x)=x^2+x-30; \quad q(x)=x+6;$
$n=2;\ m=1$

Step 1: Domain: $\{x \mid x \neq -6\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: 5 (–6 is not a zero because reduced form must be used to find the zeros.)

(b) The y-intercept is $R(0)=\dfrac{0^2+(0)-30}{0+6}=\dfrac{-30}{6}=-5$.

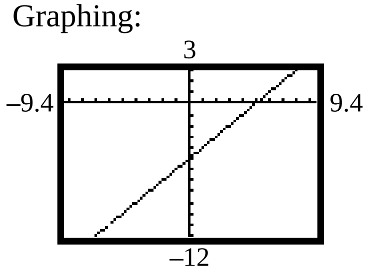
Step 3: $R(-x)=\dfrac{(-x)^2+(-x)-30}{-x+6}=\dfrac{x^2-x-30}{-x+6}$; this is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

Step 4: There are no vertical asymptotes. ($x=-6$ is not a vertical asymptote because reduced form must be used to find them.)

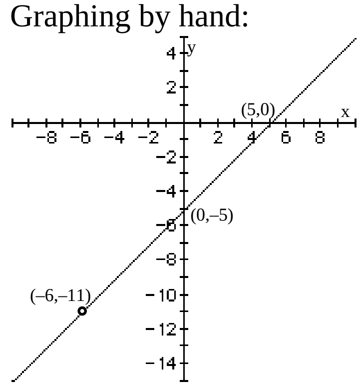
Step 5: Since $n=m+1$ there is a oblique asymptote. The line $y=x-5$ is the oblique asymptote.

The oblique asymptote intersects $R(x)$ at every point of the form $(x, x-5)$ except (–6, –11).

Step 6: Graphing:



Step 7: Graphing by hand:



33. $f(x)=x+\dfrac{1}{x}=\dfrac{x^2+1}{x} \quad p(x)=x^2+1; \quad q(x)=x; \quad n=2; \quad m=1$

Step 1: Domain: $\{x \mid x \neq 0\}$

Step 2: (a) There are no x-intercepts.

(b) There is no y-intercept because 0 is not in the domain.

Step 3: $f(-x)=\dfrac{(-x)^2+1}{-x}=\dfrac{x^2+1}{-x}=-f(x)$; The graph of $f(x)$ is symmetric to the origin.

Step 4: The vertical asymptote is the zero of $q(x)$: $x = 0$

Step 5: Since $n = m+1$, there is an oblique asymptote. Dividing:

$$\begin{array}{r} x \\ x\overline{)\,x^2+1} \\ \underline{x^2} \\ 1 \end{array} \qquad f(x) = x + \frac{1}{x}$$

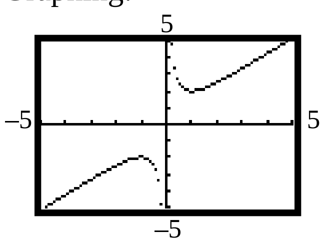
The oblique asymptote is $y = x$.

Solve to find intersection points:

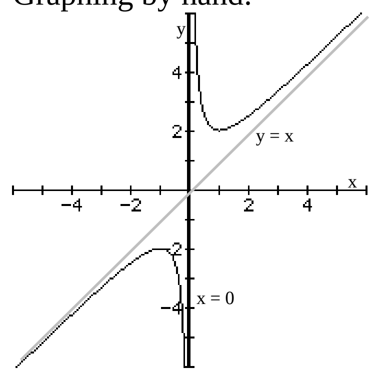
$$\frac{x^2+1}{x} = x \rightarrow x^2+1 = x^2 \rightarrow 1 = 0$$

Since there is no solution, the oblique asymptote does not intersect $f(x)$.

Step 6: Graphing:



Step 7: Graphing by hand:



34. $f(x) = 2x + \frac{9}{x} = \frac{2x^2+9}{x}$ $p(x) = 2x^2+9$; $q(x) = x$; $n = 2$; $m = 1$

Step 1: Domain: $\{x \mid x \neq 0\}$

Step 2: (a) There are no x-intercepts.

(b) There is no y-intercept because 0 is not in the domain.

Step 3: $f(-x) = \frac{2(-x)^2+9}{-x} = \frac{2x^2+9}{-x} = -f(x)$; The graph of $f(x)$ is symmetric to the origin.

Step 4: The vertical asymptote is the zero of $q(x)$: $x = 0$

Step 5: Since $n = m+1$, there is an oblique asymptote. Dividing:

$$\begin{array}{r} 2x \\ x\overline{)\,2x^2+9} \\ \underline{2x^2} \\ 9 \end{array} \qquad f(x) = 2x + \frac{9}{x}$$

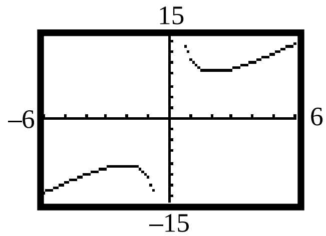
The oblique asymptote is $y = 2x$.

Solve to find intersection points:

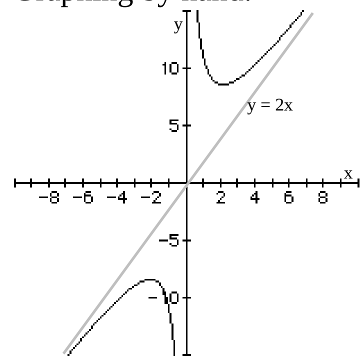
$$\frac{2x^2+9}{x} = 2x \rightarrow 2x^2+9 = 2x^2 \rightarrow 9 = 0$$

Since there is no solution, the oblique asymptote does not intersect $f(x)$.

Step 6: Graphing:



Step 7: Graphing by hand:



35. $f(x) = x^2 + \frac{1}{x} = \frac{x^3 + 1}{x}$ $\quad p(x) = x^3 + 1; \;\; q(x) = x; \;\; n = 3; \;\; m = 1$

Step 1: Domain: $\{x \mid x \neq 0\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: -1

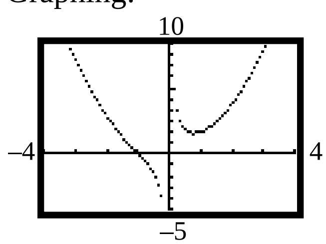
(b) There is no y-intercept because 0 is not in the domain.

Step 3: $f(-x) = \frac{(-x)^3 + 1}{-x} = \frac{-x^3 + 1}{-x}$; this is neither $f(x)$ nor $-f(x)$, so there is no symmetry.

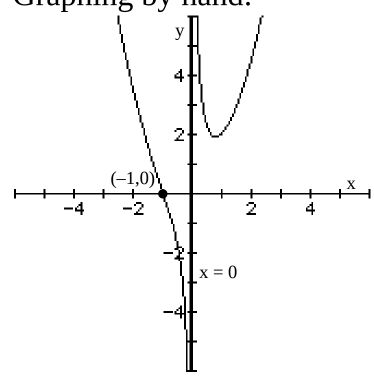
Step 4: The vertical asymptote is the zero of $q(x)$: $x = 0$

Step 5: Since $n > m + 1$, there is no horizontal or oblique asymptote.

Step 6: Graphing:



Step 7: Graphing by hand:



36. $f(x) = 2x^2 + \frac{9}{x} = \frac{2x^3 + 9}{x}$ $\quad p(x) = 2x^3 + 9; \;\; q(x) = x; \;\; n = 3; \;\; m = 1$

Step 1: Domain: $\{x \mid x \neq 0\}$

Step 2: (a) The x-intercept is the zero of $p(x)$: $-\sqrt[3]{\frac{9}{2}} \approx -1.65$

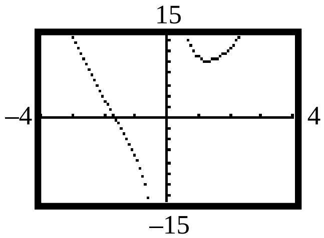
(b) There is no y-intercept because 0 is not in the domain.

Step 3: $f(-x) = \frac{2(-x)^3 + 9}{-x} = \frac{-2x^3 + 9}{-x}$; this is neither $f(x)$ nor $-f(x)$, so there is no symmetry.

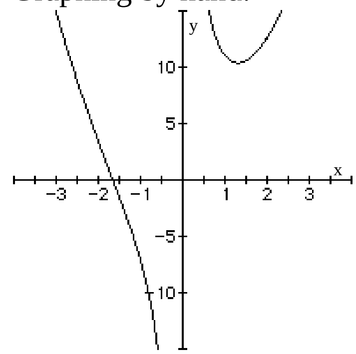
Step 4: The vertical asymptote is the zero of $q(x)$: $x = 0$

Step 5: Since $n > m + 1$, there is no horizontal or oblique asymptote.

Step 6: Graphing:



Step 7: Graphing by hand:



37. $f(x) = x + \dfrac{1}{x^3} = \dfrac{x^4+1}{x^3}$ $\quad p(x) = x^4 + 1; \quad q(x) = x^3; \quad n = 4; \quad m = 3$

Step 1: Domain: $\{x \mid x \neq 0\}$

Step 2: (a) There are no x-intercepts.

(b) There is no y-intercept because 0 is not in the domain.

Step 3: $f(-x) = \dfrac{(-x)^4+1}{(-x)^3} = \dfrac{x^4+1}{-x^3} = -f(x)$; The graph of $f(x)$ is symmetric to the origin.

Step 4: The vertical asymptote is the zero of $q(x)$: $x = 0$

Step 5: Since $n = m + 1$, there is an oblique asymptote. Dividing:

$$f(x) = x + \frac{1}{x^3}$$

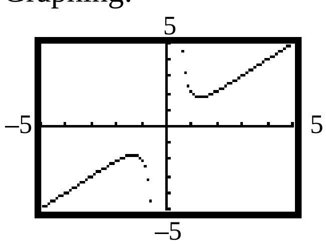
The oblique asymptote is $y = x$.

Solve to find intersection points:

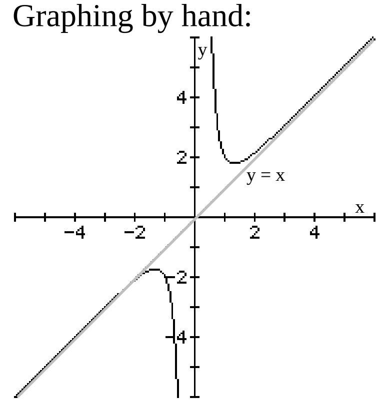
$$\frac{x^4+1}{x^3} = x \rightarrow x^4 + 1 = x^4 \rightarrow 1 = 0$$

Since there is no solution, the oblique asymptote does not intersect $f(x)$.

Step 6: Graphing:



Step 7: Graphing by hand:



38. $f(x) = 2x + \dfrac{9}{x^3} = \dfrac{2x^4+9}{x^3}$ $\quad p(x) = 2x^4 + 9; \quad q(x) = x^3; \quad n = 4; \quad m = 3$

Step 1: Domain: $\{x \mid x \neq 0\}$

Step 2: (a) There are no x-intercepts.

(b) There is no y-intercept because 0 is not in the domain.

Step 3: $f(-x)=\dfrac{2(-x)^4+9}{(-x)^3}=\dfrac{2x^4+9}{-x^3}=-f(x)$; The graph of $f(x)$ is symmetric to the origin.

Step 4: The vertical asymptote is the zero of $q(x)$: $x=0$

Step 5: Since $n=m+1$, there is an oblique asymptote. Dividing:

$$f(x)=2x+\frac{9}{x^3}$$

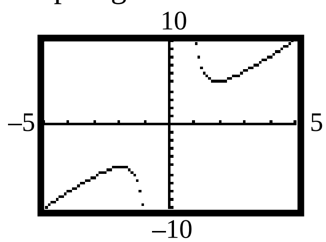
The oblique asymptote is $y=2x$.

Solve to find intersection points:

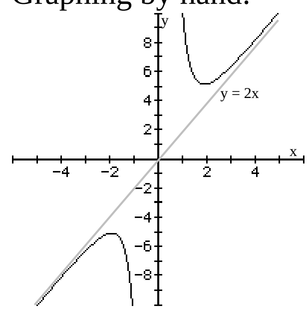
$$\frac{2x^4+9}{x^3}=2x \rightarrow 2x^4+9=2x^4 \rightarrow 9=0$$

Since there is no solution, the oblique asymptote does not intersect $f(x)$.

Step 6: Graphing:



Step 7: Graphing by hand:



39. $f(x)=\dfrac{x^2}{x^2-4}$

40. $f(x)=\dfrac{-3x}{x^2-1}$

41. $f(x)=\dfrac{(x-1)^3(x-3)}{(x+1)^2(x-2)^2}$

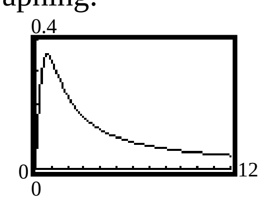
42. $f(x)=\dfrac{3(x+2)(x-1)^2}{(x+3)(x-4)^2}$

43. (a) $C(t)=\dfrac{t}{2t^2+1}\approx\dfrac{t}{2t^2}=\dfrac{1}{2t}\rightarrow 0$ as $t\rightarrow\pm\propto$

therefore the horizontal asymptote is .

The concentration of the drug decreases to 0 as time increases.

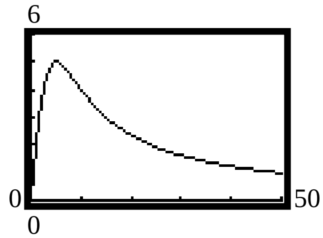
(b) Graphing:



(c) Using MAXIMUM, the concentration is highest when $t=0.71$ hours.

44. (a) The degree of the numerator is 1 and the degree of the denominator is 2. Thus, the horizontal asymptote is $C(t)=0$. The concentration of the drug decreases to 0 as time increases.

(b) Graphing:



(c) Using MAXIMUM, the concentration is highest when $t = 5$ minutes.

45. (a) The average cost function is: $\bar{C}(x) = \dfrac{0.2x^3 - 2.3x^2 + 14.3x + 10.2}{x}$

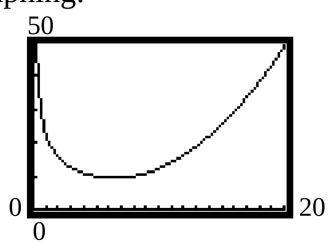
(b) $\bar{C}(6) = \dfrac{0.2(6)^3 - 2.3(6)^2 + 14.3(6) + 10.2}{6} = \dfrac{56.4}{6} = 9.4$

The average cost of producing 6 Cavaliers per hour is \$9400.

(c) $\bar{C}(9) = \dfrac{0.2(9)^3 - 2.3(9)^2 + 14.3(9) + 10.2}{9} = \dfrac{98.4}{9} = 10.933$

The average cost of producing 9 Cavaliers per hour is \$10,933.

(d) Graphing:



(e) Using MINIMUM, the number of Cavaliers that should be produced per hour to minimize cost is 6.38.

(f) The minimum average cost is \$9,366.

46. (a) The average cost function is: $\bar{C}(x) = \dfrac{0.015x^3 - 0.595x^2 + 9.15x + 98.43}{x}$

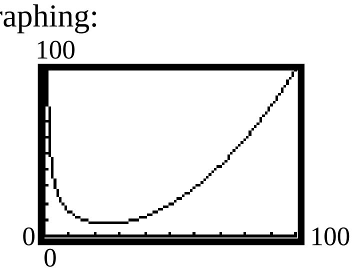
(b) $\bar{C}(13) = \dfrac{0.015(13)^3 - 0.595(13)^2 + 9.15(13) + 98.43}{13} = \dfrac{149.78}{13} \approx 11.52$

The average cost of producing 13,000 textbooks per week is \$11.52.

(c) $\bar{C}(25) = \dfrac{0.015(25)^3 - 0.595(25)^2 + 9.15(25) + 98.43}{25} = \dfrac{189.68}{25} \approx 7.59$

The average cost of producing 25,000 textbooks per week is \$7.59.

(d) Graphing:



(e) Using MINIMUM, the number of textbooks that should be produced per week to minimize cost is 25.058 thousand or 25,058 textbooks.

(f) The minimum average cost is \$7.59.

47. (a) The surface area is the sum of the areas of the six sides.

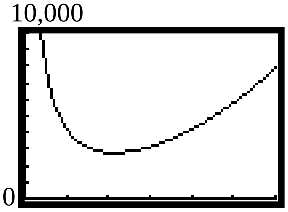
$$S = xy + xy + xy + xy + x^2 + x^2 = 4xy + 2x^2$$

The volume is $x\ x\ y = x^2 y = 10{,}000 \rightarrow y = \dfrac{10000}{x^2}$

Thus, $S(x) = 4x\left(\dfrac{10000}{x^2}\right) + 2x^2 = 2x^2 + \dfrac{40000}{x} = \dfrac{2x^3 + 40000}{x}$

(b) Graphing:

10,000



0 60

0

(c) The minimum surface area (amount of cardboard) is 2,785 square inches.

(d) The surface area is a minimum when $x = 21.544$.

$$y = \frac{10000}{21.544^2} = 21.545$$

The dimensions of the box are: 21.544 in. by 21.544 in. by 21.545 in.

48. (a) The surface area is the sum of the areas of the six sides.

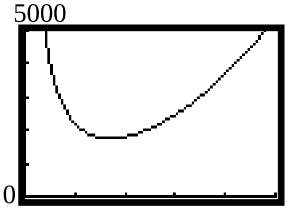
$$S = xy + xy + xy + xy + x^2 + x^2 = 4xy + 2x^2$$

The volume is $x\ x\ y = x^2 y = 5{,}000 \rightarrow y = \dfrac{5000}{x^2}$

Thus, $S(x) = 4x\left(\dfrac{5000}{x^2}\right) + 2x^2 = 2x^2 + \dfrac{20000}{x} = \dfrac{2x^3 + 20000}{x}$

(b) Graphing:

5000



0 50

0

(c) The minimum surface area (amount of cardboard) is 1,754.41 square inches.

(d) The surface area is a minimum when $x = 17.1$.

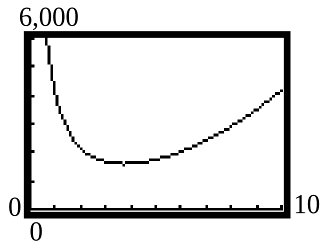
$$y = \frac{5000}{17.1^2} \approx 17.1$$

The dimensions of the box are: 17.1 in. by 17.1 in. by 17.1 in.

49. (a) $500 = \pi r^2 h \rightarrow h = \dfrac{500}{\pi r^2}$

$$C(r) = 6(2\pi r^2) + 4(2\pi r h) = 12\pi r^2 + 8\pi r\left(\frac{500}{\pi r^2}\right) = 12\pi r^2 + \frac{4000}{r}$$

(b) Graphing:



The cost is least for $r = 3.76$ cm.

50. (a) $100 = \pi r^2 h \quad \rightarrow \quad h = \dfrac{100}{\pi r^2}$

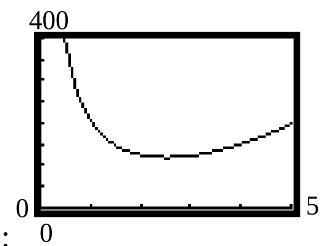
$A(r) = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r\left(\dfrac{100}{\pi r^2}\right) = 2\pi r^2 + \dfrac{200}{r}$

(b) $A(3) = 2\pi\ 3^2 + \dfrac{200}{3} = 18\pi + \dfrac{200}{3} \approx 123.22$ square feet

(c) $A(2) = 2\pi\ 2^2 + \dfrac{200}{2} = 8\pi + 100 \approx 125.13$ square feet

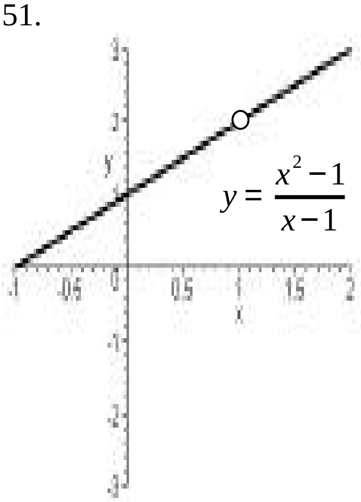
(d) $A(4) = 2\pi\ 4^2 + \dfrac{200}{4} = 32\pi + 50 \approx 150.53$ square feet

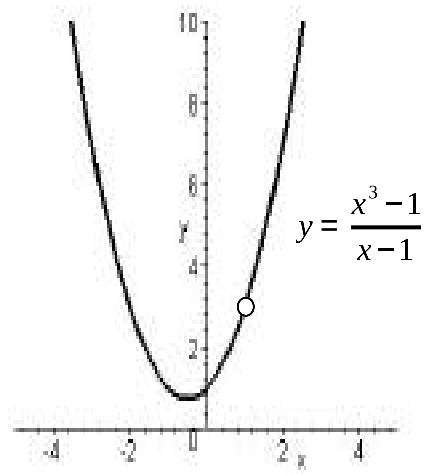
(e) Graphing:

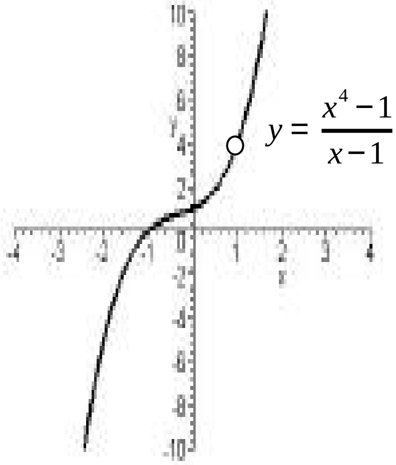


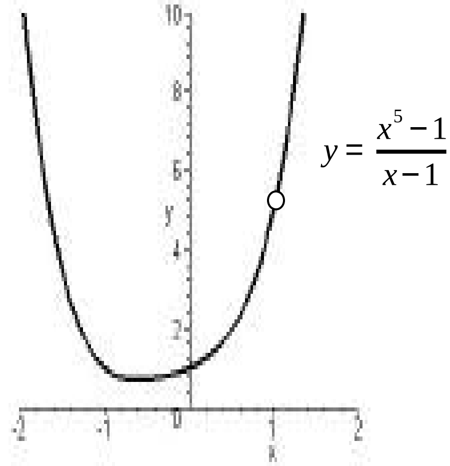
The area is smallest when $r = 2.52$ feet.

51.









x = 1 is not a vertical asymptote because of the following behavior:

$$y = \frac{x^2 - 1}{x - 1} = \frac{(x+1)(x-1)}{x-1} = x + 1 \text{ when } x \neq 1$$

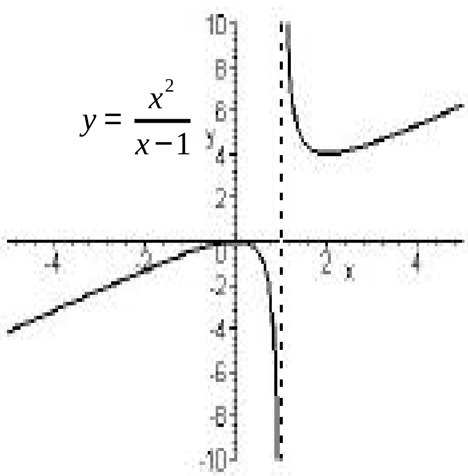
$$y = \frac{x^3 - 1}{x - 1} = \frac{(x-1)(x^2 + x + 1)}{x-1} = x^2 + x + 1 \text{ when } x \neq 1$$

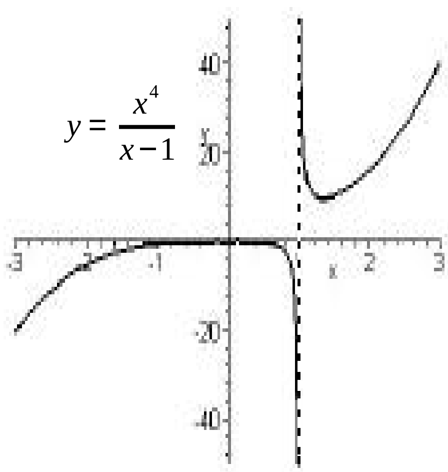
$$y = \frac{x^4 - 1}{x - 1} = \frac{(x^2+1)(x^2-1)}{x-1} = \frac{(x^2+1)(x-1)(x+1)}{x-1} = x^3 + x^2 + x + 1 \text{ when } x \neq 1$$

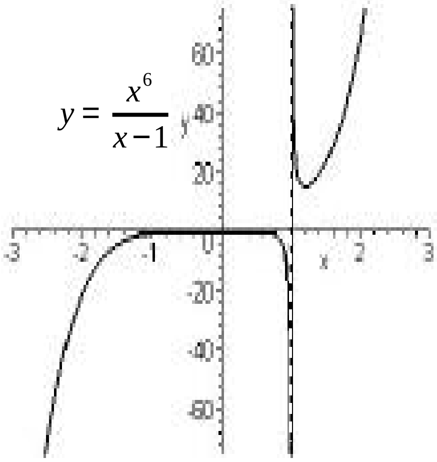
$$y = \frac{x^5 - 1}{x - 1} = \frac{(x^4 + x^3 + x^2 + x + 1)(x-1)}{x-1} = x^4 + x^3 + x^2 + x + 1 \text{ when } x \neq 1$$

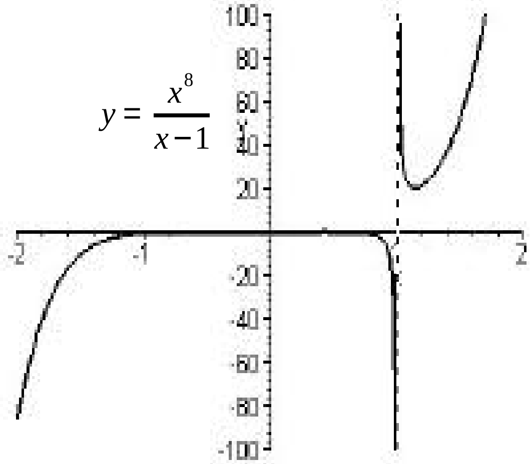
In general, the graph of $y = \dfrac{x^n - 1}{x - 1}, n \geq 1$ an integer will have a “hole” with coordinates $(1, n)$.

52.

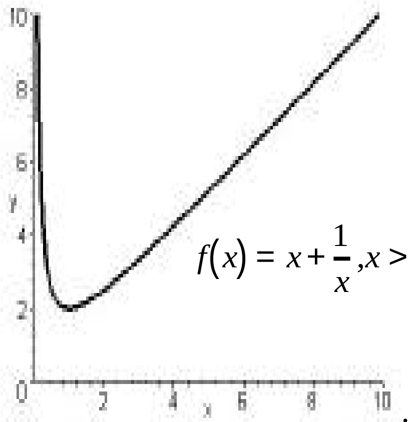






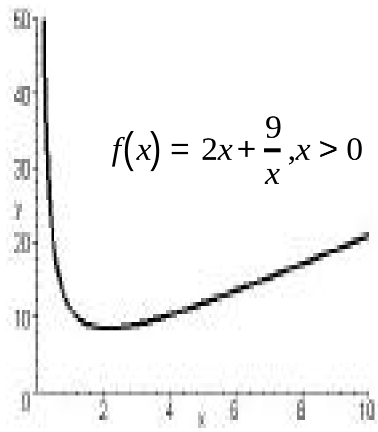


53.



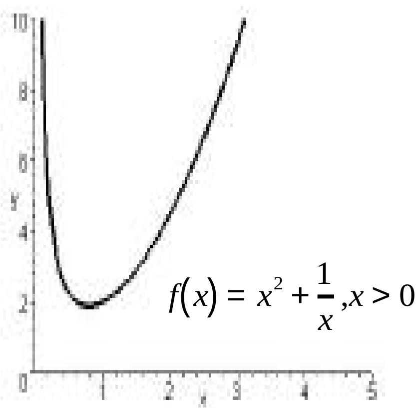
minimum value: $f(1) = 2$

54.



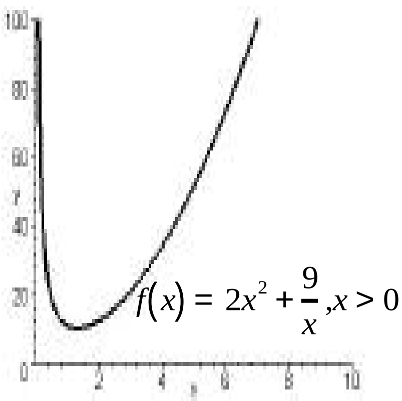
minimum value: $f\left(\sqrt{\frac{9}{2}}\right) \approx 6.24$

55.



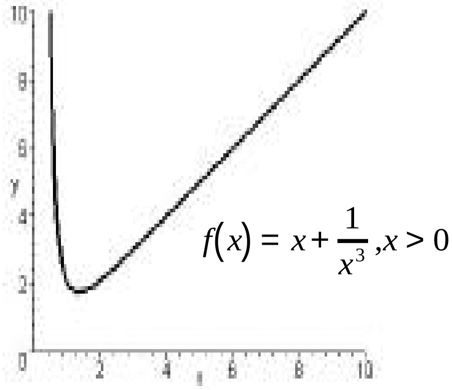
minimum value: $f\left(\sqrt[3]{\frac{1}{2}}\right) \approx 1.89$

56.



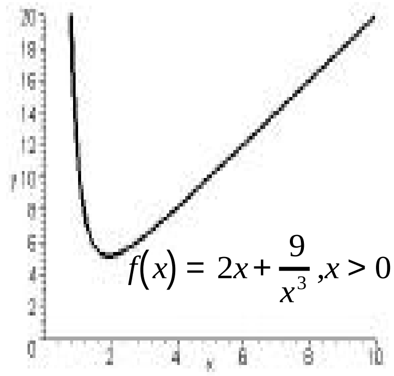
minimum value: $f\left(\sqrt[3]{\frac{9}{4}}\right) \approx 10.30$

57.



minimum value: $f\left(\sqrt[4]{3}\right) \approx 1.76$

58.



minimum value: $f\left(\sqrt[4]{\frac{27}{2}}\right) \approx 5.11$

59. Answers will vary.

60. Answers will vary, one example is $R(x) = \dfrac{3(x-2)(x+1)^2}{(x+5)(x-6)^2}$

61. Answers will vary, one example is $R(x) = \dfrac{2(x-3)(x+2)^2}{(x-1)^3}$