Polynomial and Rational Functions

4.5 Polynomial and Rational Inequalities

1. $(x-5)(x+2)<0$ $\quad f(x)=(x-5)(x+2)$

$x=5,\ x=-2$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty<x<-2$	–3	8	Positive
$-2<x<5$	0	–10	Negative
$5<x<\infty$	6	8	Positive

The solution set is $\{x \mid -2<x<5\}$

2. $(x-5)(x+2)>0$ $\quad f(x)=(x-5)(x+2)$

$x=5,\ x=-2$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty<x<-2$	–3	8	Positive
$-2<x<5$	0	–10	Negative
$5<x<\infty$	6	8	Positive

The solution set is $\{x \mid x<-2 \text{ or } x>5\}$.

3. $x^2-4x\geq 0$ $\quad f(x)=x^2-4x$

$x(x-4)\geq 0$

$x=0,\ x=4$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty<x<0$	–1	5	Positive
$0<x<4$	1	–3	Negative
$4<x<\infty$	5	5	Positive

The solution set is $\{x \mid x\leq 0 \text{ or } x\geq 4\}$.

4. $x^2+8x\geq 0$ $\quad f(x)=x^2+8x$

$x(x+8)\geq 0$

$x=-8,\ x=0$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty<x<-8$	–9	9	Positive
$-8<x<0$	–1	–7	Negative
$0<x<\infty$	1	9	Positive

The solution set is $\{x \mid x\leq -8 \text{ or } x\geq 0\}$.

5. $x^2 - 9 < 0$ $\quad f(x) = x^2 - 9$

$(x+3)(x-3) < 0$

$x = -3,\ x = 3$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -3$	–4	7	Positive
$-3 < x < 3$	0	–9	Negative
$3 < x < \infty$	4	7	Positive

The solution set is $\{x \mid -3 < x < 3\}$.

6. $x^2 - 1 < 0$ $\quad f(x) = x^2 - 1$

$(x+1)(x-1) < 0$

$x = -1,\ x = 1$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -1$	–2	3	Positive
$-1 < x < 1$	0	–1	Negative
$1 < x < \infty$	2	3	Positive

The solution set is $\{x \mid -1 < x < 1\}$.

7. $x^2 + x \geq 2$ $\quad f(x) = x^2 + x - 2$

$x^2 + x - 2 \geq 0$

$(x+2)(x-1) \geq 0$

$x = -2, x = 1$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -2$	–5	18	Positive
$-2 < x < 1$	0	–2	Negative
$1 < x < \infty$	4	18	Positive

The solution set is $\{x \mid x \leq -2 \text{ or } x \geq 1\}$.

8. $x^2 + 7x \leq -12$ $\quad f(x) = x^2 + 7x + 12$

$x^2 + 7x + 12 \leq 0$

$(x+4)(x+3) \leq 0$

$x = -4,\ x = -3$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -4$	–5	2	Positive
$-4 < x < -3$	–3.5	–0.25	Negative
$-3 < x < \infty$	0	12	Positive

The solution set is $\{x \mid -4 \leq x \leq -3\}$.

9. $2x^2 \le 5x + 3 \qquad f(x) = 2x^2 - 5x - 3$

$2x^2 - 5x - 3 \le 0$

$(2x+1)(x-3) \le 0$

$x = \frac{-1}{2}$, $x = 3$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -1/2$	–1	4	Positive
$-1/2 < x < 3$	0	–3	Negative
$3 < x < \infty$	4	9	Positive

The solution set is $\left\{ x \middle| -\frac{1}{2} \le x \le 3 \right\}$.

10. $6x^2 \le 6 + 5x \qquad f(x) = 6x^2 - 5x - 6$

$6x^2 - 5x - 6 \le 0$

$(3x+2)(2x-3) \le 0$

$x = \frac{-2}{3}$, $x = \frac{3}{2}$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -2/3$	–1	5	Positive
$-2/3 < x < 3/2$	0	–6	Negative
$3/2 < x < \infty$	2	8	Positive

The solution set is $\left\{ x \middle| \frac{-2}{3} \le x \le \frac{3}{2} \right\}$.

11. $x(x-7) > 8 \qquad f(x) = x^2 - 7x - 8$

$x^2 - 7x > 8$

$x^2 - 7x - 8 > 0$

$(x+1)(x-8) > 0$

$x = -1$, $x = 8$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -1$	–2	10	Positive
$-1 < x < 8$	0	–8	Negative
$8 < x < \infty$	9	10	Positive

The solution set is $\{ x \mid x < -1 \text{ or } x > 8 \}$.

12. $x(x+1) > 20 \qquad f(x) = x^2 + x - 20$

$x^2 + x > 20$

$x^2 + x - 20 > 0$

$(x+5)(x-4) > 0$

$x = -5$, $x = 4$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -5$	–6	10	Positive
$-5 < x < 4$	0	–20	Negative
$4 < x < \infty$	5	10	Positive

The solution set is $\{ x \mid x < -5 \text{ or } x > 4 \}$.

13. $4x^2 + 9 < 6x \qquad f(x) = 4x^2 - 6x + 9$

$4x^2 - 6x + 9 < 0$

$b^2 - 4ac = (-6)^2 - 4(4)(9) = 36 - 144 = -108$

Since the discriminant is negative, there are no real zeros.
There is only one interval, the entire number line; choose any value and test.
For $x = 0$, $4x^2 - 6x + 9 = 9 > 0$. Thus, there are no real zeros.

14. $25x^2 + 16 < 40x \qquad f(x) = 25x^2 - 40x + 16$

$25x^2 - 40x + 16 < 0$

$(5x - 4)^2 < 0$

$x = \frac{4}{5}$ is the zero.
Since the square of a quantity is always positive or zero, there are no values of x for which the expression is negative. Thus, there is no solution.

15. $6(x^2 - 1) > 5x \qquad f(x) = 6x^2 - 5x - 6$

$6x^2 - 6 > 5x$

$6x^2 - 5x - 6 > 0$

$(3x + 2)(2x - 3) > 0$

$x = \frac{-2}{3}$, $x = \frac{3}{2}$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -2/3$	–1	5	Positive
$-2/3 < x < 3/2$	0	–6	Negative
$3/2 < x < \infty$	2	8	Positive

The solution set is $\left\{ x \mid x < \frac{-2}{3} \text{ or } x > \frac{3}{2} \right\}$.

16. $2(2x^2 - 3x) > -9 \qquad f(x) = 4x^2 - 6x + 9$

$4x^2 - 6x > -9$

$4x^2 - 6x + 9 > 0$

$b^2 - 4ac = (-6)^2 - 4(4)(9) = 36 - 144 = -108$

$4x^2 - 6x + 9 = 0$ has no real solutions. Therefore it is always positive or always negative. Test at 0: $4(0)^2 - 6(0) + 9 = 9 > 0$ Since it is positive at 0, it will be positive for all real numbers.

17. $(x - 1)(x^2 + x + 4) \geq 0 \qquad f(x) = (x - 1)(x^2 + x + 4)$

$x = 1$ is the zero. $x^2 + x + 4 = 0$ has no real zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 1$	0	–4	Negative
$1 < x < \infty$	2	10	Positive

The solution set is $\{ x \mid x \geq 1 \}$.

18. $(x+2)\left(x^2-x+1\right) \geq 0$ $\quad f(x) = (x+2)\left(x^2-x+1\right)$

$x = -2$ is the zero. $x^2 - x + 1 = 0$ has no real zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -2$	–3	–13	Negative
$-2 < x < \infty$	0	2	Positive

The solution set is $\left\{ x \mid x \geq -2 \right\}$.

19. $(x-1)(x-2)(x-3) \leq 0$ $\quad f(x) = (x-1)(x-2)(x-3)$

$x = 1$, $x = 2$, $x = 3$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 1$	0	–6	Negative
$1 < x < 2$	1.5	0.375	Positive
$2 < x < 3$	2.5	–0.375	Negative
$3 < x < \infty$	4	6	Positive

The solution set is $\left\{ x \mid x \leq 1 \text{ or } 2 \leq x \leq 3 \right\}$.

20. $(x+1)(x+2)(x+3) \leq 0$ $\quad f(x) = (x+1)(x+2)(x+3)$

$x = -1$, $x = -2$, $x = -3$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -3$	–4	–6	Negative
$-3 < x < -2$	–2.5	0.375	Positive
$-2 < x < -1$	–1.5	–0.375	Negative
$-1 < x < \infty$	0	6	Positive

The solution set is $\left\{ x \mid x \leq -3 \text{ or } -2 \leq x \leq -1 \right\}$.

21. $x^3 - 2x^2 - 3x > 0$ $\quad f(x) = x^3 - 2x^2 - 3x$

$x\left(x^2 - 2x - 3\right) > 0$

$x(x+1)(x-3) > 0$

$x = -1$, $x = 0$, $x = 3$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -1$	–2	–10	Negative
$-1 < x < 0$	–0.5	0.875	Positive
$0 < x < 3$	1	–4	Negative
$3 < x < \infty$	4	20	Positive

The solution set is $\left\{ x \mid -1 < x < 0 \text{ or } x > 3 \right\}$.

22. $x^3 + 2x^2 - 3x > 0$ $\quad f(x) = x^3 + 2x^2 - 3x$

$x\left(x^2 + 2x - 3\right) > 0 \rightarrow x(x-1)(x+3) > 0$

$x = -3$, $x = 0$, $x = 1$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -3$	–4	–20	Negative
$-3 < x < 0$	–1	4	Positive
$0 < x < 1$	0.5	–0.875	Negative
$1 < x < \infty$	2	10	Positive

The solution set is $\left\{ x \mid -3 < x < 0 \text{ or } x > 1 \right\}$.

23. $x^4 > x^2 \qquad f(x) = x^4 - x^2$

$x^4 - x^2 > 0$

$x^2\left(x^2 - 1\right) > 0$

$x^2(x+1)(x-1) > 0$

$x = -1,\ x = 0,\ x = 1$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -1$	–2	12	Positive
$-1 < x < 0$	–0.5	–0.1875	Negative
$0 < x < 1$	0.5	–0.1875	Negative
$1 < x < \infty$	2	12	Positive

The solution set is $\left\{ x \mid x < -1 \text{ or } x > 1 \right\}$.

24. $x^4 < 4x^2 \qquad f(x) = x^4 - 4x^2$

$x^4 - 4x^2 < 0$

$x^2\left(x^2 - 4\right) < 0$

$x^2(x+2)(x-2) < 0$

$x = -2,\ x = 0,\ x = 2$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -2$	–3	45	Positive
$-2 < x < 0$	–1	–3	Negative
$0 < x < 2$	1	–3	Negative
$2 < x < \infty$	3	45	Positive

The solution set is $\left\{ x \mid -2 < x < 0 \text{ or } 0 < x < 2 \right\}$.

25. $x^3 \geq 4x^2 \qquad f(x) = x^3 - 4x^2$

$x^3 - 4x^2 \geq 0$

$x^2\left(x - 4\right) \geq 0$

$x = 0, x = 4$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 0$	–1	–3	Negative
$0 < x < 4$	0.5	–0.875	Negative
$4 < x < \infty$	5	25	Positive

The solution set is $\left\{ x \mid x \geq 4 \right\}$.

26. $x^3 \leq 9x^2 \qquad f(x) = x^3 - 9x^2$

$x^3 - 9x^2 \leq 0 \rightarrow x^2\left(x - 9\right) \leq 0$

$x = 0, x = 9$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 0$	–1	–10	Negative
$0 < x < 9$	1	–8	Negative
$9 < x < \infty$	10	100	Positive

The solution set is $\left\{ x \mid x < 0 \text{ or } 0 < x < 9 \right\}$.

27. $x^4 > 1$ $\qquad f(x) = x^4 - 1$

$x^4 - 1 > 0 \rightarrow (x^2 + 1)(x^2 - 1) > 0 \rightarrow (x^2 + 1)(x + 1)(x - 1) > 0$

$x = -1,\ x = 1$ are the zeros.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -1$	–2	15	Positive
$-1 < x < 1$	0	–1	Negative
$1 < x < \infty$	2	15	Positive

The solution set is $\{ x \mid x < -1 \text{ or } x > 1 \}$.

28. $x^3 > 1$ $\qquad f(x) = x^3 - 1$

$x^3 - 1 > 0$

$(x - 1)(x^2 + x + 1) > 0$

$x = 1$ is the zero. ($x^2 + x + 1 = 0$ has no real zeros.)

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 1$	0	–1	Negative
$1 < x < \infty$	2	7	Positive

The solution set is $\{ x \mid x > 1 \}$

29. $\dfrac{x+1}{x-1} > 0$ $\qquad f(x) = \dfrac{x+1}{x-1}$

The zeros and values where the expression is undefined are $x = -1$ and $x = 1$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -1$	–2	1/3	Positive
$-1 < x < 1$	0	–1	Negative
$1 < x < \infty$	2	3	Positive

The solution set is $\{ x \mid x < -1 \text{ or } x > 1 \}$.

30. $\dfrac{x-3}{x+1} > 0$ $\qquad f(x) = \dfrac{x-3}{x+1}$

The zeros and values where the expression is undefined are $x = -1$ and $x = 3$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -1$	–2	5	Positive
$-1 < x < 3$	0	–3	Negative
$3 < x < \infty$	4	1/5	Positive

The solution set is $\{ x \mid x < -1 \text{ or } x > 3 \}$.

31. $\dfrac{(x-1)(x+1)}{x} < 0$ $\qquad f(x) = \dfrac{(x-1)(x+1)}{x}$

The zeros and values where the expression is undefined are $x = -1,\ x = 0$, and $x = 1$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -1$	–2	–1.5	Negative
$-1 < x < 0$	–0.5	1.5	Positive
$0 < x < 1$	0.5	–1.5	Negative
$1 < x < \infty$	2	1.5	Positive

The solution set is $\{ x \mid x < -1 \text{ or } 0 < x < 1 \}$.

32. $\dfrac{(x-3)(x+2)}{x-1} \le 0$ $\qquad f(x) = \dfrac{(x-3)(x+2)}{x-1}$

The zeros and values where the expression is undefined are $x = -2$, $x = 1$, and $x = 3$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -2$	–3	–1.5	Negative
$-2 < x < 1$	0	6	Positive
$1 < x < 3$	2	–4	Negative
$3 < x < \infty$	4	2	Positive

The solution set is $\{ x \mid x \le -2 \text{ or } 1 < x \le 3 \}$.

33. $\dfrac{(x-2)^2}{x^2-1} \ge 0$ $\qquad f(x) = \dfrac{(x-2)^2}{x^2-1}$

$\dfrac{(x-2)^2}{(x+1)(x-1)} \ge 0$

The zeros and values where the expression is undefined are $x = -1$, $x = 1$, and $x = 2$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -1$	–2	16/3	Positive
$-1 < x < 1$	0	–4	Negative
$1 < x < 2$	1.5	0.2	Positive
$2 < x < \infty$	3	0.125	Positive

The solution set is $\{ x \mid x < -1 \text{ or } x > 1 \}$.

34. $\dfrac{(x+5)^2}{x^2-4} \ge 0$ $\qquad f(x) = \dfrac{(x+5)^2}{x^2-4}$

$\dfrac{(x+5)^2}{(x+2)(x-2)} \ge 0$

The zeros and values where the expression is undefined are $x = -5$, $x = -2$, and $x = 2$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -5$	–6	1/32	Positive
$-5 < x < -2$	–3	4/5	Positive
$-2 < x < 2$	0	–25/4	Negative
$2 < x < \infty$	3	64/5	Positive

The solution set is $\{ x \mid x < -2 \text{ or } x > 2 \}$.

35. $6x - 5 < \dfrac{6}{x}$ $\qquad f(x) = 6x - 5 - \dfrac{6}{x}$

$6x - 5 - \dfrac{6}{x} < 0 \rightarrow \dfrac{6x^2 - 5x - 6}{x} < 0 \rightarrow \dfrac{(2x-3)(3x+2)}{x} < 0$

The zeros and values where the expression is undefined are $x = \frac{-2}{3}$, $x = 0$, and $x = \frac{3}{2}$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -2/3$	–1	–5	Negative
$-2/3 < x < 0$	–0.5	4	Positive
$0 < x < 3/2$	1	–5	Negative
$3/2 < x < \infty$	2	4	Positive

The solution set is $\{ x \mid x < \frac{-2}{3} \text{ or } 0 < x < \frac{3}{2} \}$.

36. $x + \frac{12}{x} < 7 \qquad f(x) = x + \frac{12}{x} - 7$

$$x + \frac{12}{x} - 7 < 0 \rightarrow \frac{x^2 - 7x + 12}{x} < 0 \rightarrow \frac{(x-3)(x-4)}{x} < 0$$

The zeros and values where the expression is undefined are $x = 0$, $x = 3$, and $x = 4$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 0$	–1	–20	Negative
$0 < x < 3$	1	6	Positive
$3 < x < 4$	3.5	–0.07	Negative
$4 < x < \infty$	5	0.4	Positive

The solution set is $\{x \mid x < 0 \text{ or } 3 < x < 4\}$.

37. $\frac{x+4}{x-2} \le 1 \qquad f(x) = \frac{x+4}{x-2} - 1$

$$\frac{x+4}{x-2} - 1 \le 0 \rightarrow \frac{x+4-(x-2)}{x-2} \le 0 \rightarrow \frac{6}{x-2} \le 0$$

The value where the expression is undefined is $x = 2$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 2$	0	–3	Negative
$2 < x < \infty$	3	6	Positive

The solution set is $\{x \mid x < 2\}$.

38. $\frac{x+2}{x-4} \ge 1 \qquad f(x) = \frac{x+2}{x-4} - 1$

$$\frac{x+2}{x-4} - 1 \ge 0 \rightarrow \frac{x+2-(x-4)}{x-4} \ge 0 \rightarrow \frac{6}{x-4} \ge 0$$

The value where the expression is undefined is $x = 4$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 4$	0	–3/2	Negative
$4 < x < \infty$	5	6	Positive

The solution set is $\{x \mid x > 4\}$.

39. $\frac{3x-5}{x+2} \le 2 \qquad f(x) = \frac{3x-5}{x+2} - 2$

$$\frac{3x-5}{x+2} - 2 \le 0 \rightarrow \frac{3x-5-2(x+2)}{x+2} \le 0 \rightarrow \frac{x-9}{x+2} \le 0$$

The zeros and values where the expression is undefined are $x = -2$, and $x = 9$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -2$	–3	12	Positive
$-2 < x < 9$	0	–4.5	Negative
$9 < x < \infty$	10	1/12	Positive

The solution set is $\{x \mid -2 < x \le 9\}$.

40. $\frac{x-4}{2x+4} \ge 1 \qquad f(x) = \frac{x-4}{2x+4} - 1$

$$\frac{x-4}{2x+4} - 1 \ge 0 \rightarrow \frac{x-4-2x-4}{2x+4} \ge 0 \rightarrow \frac{-x-8}{2x+4} \ge 0$$

The zeros and values where the expression is undefined are $x = -8,$ and $x = -2.$

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -8$	–9	–1/14	Negative
$-8 < x < -2$	–3	5/2	Positive
$-2 < x < \infty$	0	–2	Negative

The solution set is $\{ x \mid -8 \le x < -2 \}$.

41. $\frac{1}{x-2} < \frac{2}{3x-9} \qquad f(x) = \frac{1}{x+2} - \frac{2}{3x-9}$

$$\frac{1}{x-2} - \frac{2}{3x-9} < 0 \rightarrow \frac{3x-9-2(x-2)}{(x-2)(3x-9)} < 0 \rightarrow \frac{x-5}{(x-2)(3x-9)} < 0$$

The zeros and values where the expression is undefined are $x = 2,$ $x = 3,$ and $x = 5.$

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 2$	0	–5/18	Negative
$2 < x < 3$	2.5	10/3	Positive
$3 < x < 5$	4	–1/6	Negative
$5 < x < \infty$	6	1/36	Positive

The solution set is $\{ x \mid x < 2 \text{ or } 3 < x < 5 \}$.

42. $\frac{5}{x-3} > \frac{3}{x+1} \qquad f(x) = \frac{5}{x-3} - \frac{3}{x+1}$

$$\frac{5}{x-3} - \frac{3}{x+1} > 0 \rightarrow \frac{5x+5-3x+9}{(x-3)(x+1)} > 0 \rightarrow \frac{2x+14}{(x-3)(x+1)} > 0$$

The zeros and values where the expression is undefined are $x = -7,$ $x = -1,$ and $x = 3.$

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -7$	–8	–2/77	Negative
$-7 < x < -1$	–2	2	Positive
$-1 < x < 3$	0	–14/3	Negative
$3 < x < \infty$	4	22/5	Positive

The solution set is $\{ x \mid -7 < x < -1 \text{ or } x > 3 \}$.

43. $\frac{2x+5}{x+1} > \frac{x+1}{x-1} \qquad f(x) = \frac{2x+5}{x+1} - \frac{x+1}{x-1}$

$$\frac{2x+5}{x+1} - \frac{x+1}{x-1} > 0 \rightarrow \frac{(2x+5)(x-1)-(x+1)(x+1)}{(x+1)(x-1)} > 0$$

$$\frac{2x^2+3x-5-\left(x^2+2x+1\right)}{(x+1)(x-1)} > 0 \rightarrow \frac{x^2+x-6}{(x+1)(x-1)} > 0 \rightarrow \frac{(x+3)(x-2)}{(x+1)(x-1)} > 0$$

The zeros and values where the expression is undefined are $x = -3, x = -1, x = 1, x = 2$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -3$	–4	2/5	Positive
$-3 < x < -1$	–2	–4/3	Negative
$-1 < x < 1$	0	6	Positive
$1 < x < 2$	1.5	–9/5	Negative
$2 < x < \infty$	3	3/4	Positive

The solution set is $\{x \mid x < -3,\ -1 < x < 1,\ x > 2\}$.

44. $\dfrac{1}{x+2} > \dfrac{3}{x+1}$ $\qquad f(x) = \dfrac{1}{x+2} - \dfrac{3}{x+1}$

$$\frac{1}{x+2} - \frac{3}{x+1} > 0 \rightarrow \frac{x+1-3(x+2)}{(x+2)(x+1)} > 0 \rightarrow \frac{x+1-3x-6}{(x+2)(x+1)} > 0 \rightarrow \frac{-2x-5}{(x+2)(x+1)} > 0$$

The zeros and values where the expression is undefined are $x = -\frac{5}{2}$, $x = -2$, and $x = -1$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -5/2$	–3	1/2	Positive
$-5/2 < x < -2$	–2.1	–7.27	Negative
$-2 < x < -1$	–1.5	8	Positive
$-1 < x < \infty$	0	–5/2	Negative

The solution set is $\{x \mid x < -\frac{5}{2} \text{ or } -2 < x < -1\}$.

45. $\dfrac{x^2(3+x)(x+4)}{(x+5)(x-1)} \geq 0$ $\qquad f(x) = \dfrac{x^2(3+x)(x+4)}{(x+5)(x-1)}$

The zeros and values where the expression is undefined are
$x = -5$, $x = -4$, $x = -3$, $x = 0$ and $x = 1$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -5$	–6	216/7	Positive
$-5 < x < -4$	–4.5	–243/44	Negative
$-4 < x < -3$	–3.5	49/108	Positive
$-3 < x < 0$	–1	–3/4	Negative
$0 < x < 1$	0.5	–63/44	Negative
$1 < x < \infty$	2	120/7	Positive

The solution set is $\{x \mid x < -5, -4 \leq x \leq -3,\ x > 1\}$.

46. $\dfrac{x(x^2+1)(x-2)}{(x-1)(x+1)} \geq 0$ $\qquad f(x) = \dfrac{x(x^2+1)(x-2)}{(x-1)(x+1)}$

The zeros and values where the expression is undefined are $x = -1, x = 0, x = 1, x = 2$.

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -1$	–2	40/3	Positive
$-1 < x < 0$	–0.5	–2.08	Negative
$0 < x < 1$	0.5	1.25	Positive
$1 < x < 2$	1.5	–1.95	Negative
$2 < x < \infty$	3	3.75	Positive

The solution set is $\{x \mid x < -1, 0 < x < 1, x \geq 2\}$.

47. Let x be the positive number. Then

$$x^3 > 4x^2 \rightarrow x^3 - 4x^2 > 0 \rightarrow x^2(x-4) > 0$$

The zeros are $x = 0$ and $x = 4$. $f(x) = x^3 - 4x^2$

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 0$	–1	–5	Negative
$0 < x < 4$	1	–3	Negative
$4 < x < \infty$	5	25	Positive

The solution set is $\{x \mid x > 4\}$. All real number larger than 4 satisfy the condition.

48. Let x be the positive number. Then

$$x^2 > 2x \rightarrow x^2 - 2x > 0 \rightarrow x(x-2) > 0$$

The zeros are $x = 0$ and $x = 2$. $f(x) = x^2 - 2x$

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 0$	–1	3	Positive
$0 < x < 2$	1	–1	Negative
$2 < x < \infty$	3	3	Positive

The solution set is $\{x \mid x < 0 \text{ or } x > 2\}$. Since x is positive, all real numbers larger than 2 satisfy the condition.

49. The domain of the expression includes all values for which

$$x^2 - 16 \geq 0 \rightarrow (x+4)(x-4) \geq 0$$

The zeros are $x = -4$ and $x = 4$. $f(x) = x^2 - 16$

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -4$	–5	9	Positive
$-4 < x < 4$	0	–16	Negative
$4 < x < \infty$	5	9	Positive

The solution or domain is $\{x \mid x \leq -4 \text{ or } x \geq 4\}$.

50. The domain of the expression includes all values for which

$$x^3 - 3x^2 \geq 0 \rightarrow x^2(x-3) \geq 0$$

The zeros are $x = 0$ and $x = 3$. $f(x) = x^3 - 3x^2$

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < 0$	–1	–4	Negative
$0 < x < 3$	1	–2	Negative
$3 < x < \infty$	4	16	Positive

The solution or domain is $\{x \mid x = 0 \text{ or } x \geq 3\}$.

51. The domain of the expression includes all values for which
$\frac{x-2}{x+4} \geq 0$
The zeros and values where the expression is undefined are $x = -4$ and $x = 2$.

$$f(x) = \frac{x-2}{x+4}$$

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -4$	–5	7	Positive
$-4 < x < 2$	0	–1/2	Negative
$2 < x < \infty$	3	1/7	Positive

The solution or domain is $\{x \mid x < -4 \text{ or } x \geq 2\}$.

52. The domain of the expression includes all values for which
$\frac{x-1}{x+4} \geq 0$
The zeros and values where the expression is undefined are $x = -4$ and $x = 1$.

$$f(x) = \frac{x-1}{x+4}$$

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < x < -4$	–5	6	Positive
$-4 < x < 1$	0	–1/4	Negative
$1 < x < \infty$	2	1/6	Positive

The solution or domain is $\{x \mid x < -4 \text{ or } x \geq 1\}$.

53. Find the values of t for which
$80t - 16t^2 > 96$
$-16t^2 + 80t - 96 > 0 \rightarrow 16t^2 - 80t + 96 < 0 \rightarrow 16(t^2 - 5t + 6) < 0 \rightarrow 16(t-2)(t-3) < 0$
The zeros are $t = 2$ and $t = 3$. $s(t) = 16t^2 - 80t + 96$

Interval	Test Number	$s(t)$	Positive/Negative
$-\infty < t < 2$	1	32	Positive
$2 < t < 3$	2.5	–4	Negative
$3 < t < \infty$	4	32	Positive

The solution set is $\{t \mid 2 < t < 3\}$. The ball is more than 96 feet above the ground for times between 2 and 3 seconds.

54. Find the values of t for which
$96t - 16t^2 > 112$
$-16t^2 + 96t - 112 > 0$
Graph $s(t) = -16t^2 + 96t - 112$.
The zeros are $t = 1.59$ and $t = 4.41$. $s(t) = -16t^2 + 96t - 112$

Interval	Test Number	$s(t)$	Positive/Negative
$-\infty < t < 1.59$	1	–32	Negative
$1.59 < t < 4.41$	2	16	Positive
$4.41 < t < \infty$	5	–32	Negative

The solution set is $\{t \mid 1.59 < t < 4.41\}$. The ball is more than 112 feet above the ground for times between 1.59 and 4.41 seconds.

55. Profit = Revenue – Cost

The zeros are approximately $x = 7.75$ and $x = 32.25$.

$$f(x) = x^2 - 40x + 250$$

Interval	Test Number	$f(x)$	Positive/Negative
$0 < x < 7.75$	7	19	Positive
$7.75 < x < 32.25$	10	–50	Negative
$32.25 < x < \infty$	40	250	Positive

The profit is at least \$50 when at least 8 and no more than 32 watches are sold.

56. Profit = Revenue – Cost

$$x(5 - 0.05x) - 1.50x \geq 60$$
$$5x - 0.05x^2 - 1.5x \geq 60$$
$$-0.05x^2 + 3.5x - 60 \geq 0$$
$$x^2 - 70x + 1200 \leq 0$$
$$(x - 40)(x - 30) \leq 0$$

The zeros are $x = 30$ and $x = 40$.

$$f(x) = x^2 - 70x + 1200$$

Interval	Test Number	$f(x)$	Positive/Negative
$0 < x < 30$	10	600	Positive
$30 < x < 40$	35	–25	Negative
$40 < x < \infty$	50	200	Positive

The profit is at least \$60 when at least 30 and no more than 40 boxes of candy are sold.

57. The equation $x^2 + kx + 1 = 0$ has no real solutions whenever the discriminant is less than zero.

Solving $b^2 - 4ac = k^2 - 4 < 0 \rightarrow (k + 4)(k - 4) < 0$

The zeros are $k = -4$ and $k = 4$.

$$f(k) = k^2 - 4$$

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < k < -4$	-5	21	Positive
$-4 < k < 4$	0	–4	Negative
$4 < k < \infty$	5	21	Positive

Therefore the equation $x^2 + kx + 1 = 0$ has no real solutions whenever $-4 < k < 4$.

58. The equation $kx^2 + 2x + 1 = 0$ has two distinct real solutions whenever the discriminant is greater than zero.

Solving $b^2 - 4ac = 4 - 4k > 0 \rightarrow 4(1 - k) > 0$

The zero is $k = 1$.

$$f(k) = 4 - 4k$$

Interval	Test Number	$f(x)$	Positive/Negative
$-\infty < k < 1$	0	4	Positive
$1 < k < \infty$	2	–4	Negative

Therefore the equation $x^2 + kx + 1 = 0$ has two distinct real solutions whenever $-\infty < k < 1$.

59. Answers will vary, for example,

$x^2 < 0$ has no real solution $x^2 \le 0$ has exactly one real solution

60. $x^2 + 1 < -5$ has no real solution since $x^2 + 1$ always yields a positive number and therefore can never be less than – 5.