

The Zeros of a Polynomial Function

5.1 Synthetic Division

1. Use synthetic division:
$$\begin{array}{r|rrrr} 2 & 1 & -1 & 2 & 4 \\ & & 2 & 2 & 8 \\ \hline & 1 & 1 & 4 & 12 \end{array}$$
 Quotient: $x^2 + x + 4$ Remainder: 12
2. Use synthetic division:
$$\begin{array}{r|rrrr} -1 & 1 & 2 & -3 & 1 \\ & & -1 & -1 & 4 \\ \hline & 1 & 1 & -4 & 5 \end{array}$$
 Quotient: $x^2 + x - 4$ Remainder: 5
3. Use synthetic division:
$$\begin{array}{r|rrrr} 3 & 3 & 2 & -1 & 3 \\ & & 9 & 33 & 96 \\ \hline & 3 & 11 & 32 & 99 \end{array}$$
 Quotient: $3x^2 + 11x + 32$ Remainder: 99
4. Use synthetic division:
$$\begin{array}{r|rrrr} -2 & -4 & 2 & -1 & 1 \\ & & 8 & -20 & 42 \\ \hline & -4 & 10 & -21 & 43 \end{array}$$
 Quotient: $-4x^2 + 10x - 21$ Remainder: 43
5. Use synthetic division:
$$\begin{array}{r|rrrrrr} -3 & 1 & 0 & -4 & 0 & 1 & 0 \\ & & -3 & 9 & -15 & 45 & -138 \\ \hline & 1 & -3 & 5 & -15 & 46 & -138 \end{array}$$
 Quotient: $x^4 - 3x^3 + 5x^2 - 15x + 46$
Remainder: -138
6. Use synthetic division:
$$\begin{array}{r|rrrr} 2 & 1 & 0 & 1 & 0 & 2 \\ & & 2 & 4 & 10 & 20 \\ \hline & 1 & 2 & 5 & 10 & 22 \end{array}$$
 Quotient: $x^3 + 2x^2 + 5x + 10$ Remainder: 22
7. Use synthetic division:
$$\begin{array}{r|rrrrrr} 1 & 4 & 0 & -3 & 0 & 1 & 0 & 5 \\ & & 4 & 4 & 1 & 1 & 2 & 2 \\ \hline & 4 & 4 & 1 & 1 & 2 & 2 & 7 \end{array}$$
 Quotient: $4x^5 + 4x^4 + x^3 + x^2 + 2x + 2$
Remainder: 7

8. Use synthetic division:

$$\begin{array}{r|rrrrrr} -1 & 1 & 0 & 5 & 0 & 0 & -10 \\ & & -1 & 1 & -6 & 6 & -6 \\ \hline & 1 & -1 & 6 & -6 & 6 & -16 \end{array}$$

Quotient: $x^4 - x^3 + 6x^2 - 6x + 6$ Remainder: -16

9. Use synthetic division:

$$\begin{array}{r|rrrr} -1.1 & 0.1 & 0 & 0.2 & 0 \\ & & -0.11 & 0.121 & -0.3531 \\ \hline & 0.1 & -0.11 & 0.321 & -0.3531 \end{array}$$

Quotient: $0.1x^2 - 0.11x + 0.321$ Remainder: -0.3531

10. Use synthetic division:

$$\begin{array}{r|rrr} -2.1 & 0.1 & 0 & -0.2 \\ & & -0.21 & 0.441 \\ \hline & 0.1 & -0.21 & 0.241 \end{array}$$

Quotient: $0.1x - 0.21$ Remainder: 0.241

11. Use synthetic division:

$$\begin{array}{r|rrrrrr} 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 0 \end{array}$$

Quotient: $x^4 + x^3 + x^2 + x + 1$ Remainder: 0

12. Use synthetic division:

$$\begin{array}{r|rrrrrr} -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ & & -1 & 1 & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & -1 & 1 & 0 \end{array}$$

Quotient: $x^4 - x^3 + x^2 - x + 1$ Remainder: 0

13. Use synthetic division:

$$\begin{array}{r|rrrr} 2 & 4 & -3 & -8 & 4 \\ & & 8 & 10 & 4 \\ \hline & 4 & 5 & 2 & 8 \end{array}$$

Remainder = $8 \neq 0$; therefore $x - 2$ is not a factor of $f(x)$.

14. Use synthetic division:

$$\begin{array}{r|rrrr} -3 & -4 & 5 & 0 & 8 \\ & & 12 & -51 & 153 \\ \hline & -4 & 17 & -51 & 161 \end{array}$$

Remainder = $161 \neq 0$; therefore $x + 3$ is not a factor of $f(x)$.

15. Use synthetic division:

$$\begin{array}{r|rrrrr} 2 & 3 & -6 & 0 & -5 & 10 \\ & & 6 & 0 & 0 & -10 \\ \hline & 3 & 0 & 0 & -5 & 0 \end{array}$$

Remainder = 0; therefore $x - 2$ is a factor of $f(x)$.

16. Use synthetic division:

$$\begin{array}{r|rrrrr} 2 & 4 & 0 & -15 & 0 & -4 \\ & & 8 & 16 & 2 & 4 \\ \hline & 4 & 8 & 1 & 2 & 0 \end{array}$$

Remainder = 0; therefore $x - 2$ is a factor of $f(x)$.

17. Use synthetic division:

$$\begin{array}{r|rrrrrrr} -3 & 3 & 0 & 0 & 82 & 0 & 0 & 27 \\ & & -9 & 27 & -81 & -3 & 9 & -27 \\ \hline & 3 & -9 & 27 & 1 & -3 & 9 & 0 \end{array}$$

Remainder = 0; therefore $x + 3$ is a factor of $f(x)$.

18. Use synthetic division:

$$\begin{array}{r|rrrrrrr} -3 & 2 & 0 & -18 & 0 & 1 & 0 & -9 \\ & & -6 & 18 & 0 & 0 & -3 & 9 \\ \hline & 2 & -6 & 0 & 0 & 1 & -3 & 0 \end{array}$$

Remainder = 0; therefore $x + 3$ is a factor of $f(x)$.

19. Use synthetic division:

$$\begin{array}{r|rrrrrrr} -4 & 4 & 0 & -64 & 0 & 1 & 0 & -15 \\ & & -16 & 64 & 0 & 0 & -4 & 16 \\ \hline & 4 & -16 & 0 & 0 & 1 & -4 & 1 \end{array}$$

Remainder = 1 \neq 0; therefore $x + 3$ is not a factor of $f(x)$.

20. Use synthetic division:

$$\begin{array}{r|rrrrrrr} -4 & 1 & 0 & -16 & 0 & 1 & 0 & -16 \\ & & -4 & 16 & 0 & 0 & -4 & 16 \\ \hline & 1 & -4 & 0 & 0 & 1 & -4 & 0 \end{array}$$

Remainder = 0; therefore $x + 4$ is a factor of $f(x)$.

21. Use synthetic division:

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -1 & 0 & 2 & -1 \\ & & 1 & 0 & 0 & 1 \\ \hline & 2 & 0 & 0 & 2 & 0 \end{array}$$

Remainder = 0; therefore $x - \frac{1}{2}$ is a factor of $f(x)$.

Chapter 5 The Zeros of a Polynomial Function

22. Use synthetic division:

$$\begin{array}{r|rrrrr} -\frac{1}{3} & 3 & 1 & 0 & -3 & 1 \\ & & -1 & 0 & 0 & 1 \\ \hline & 3 & 0 & 0 & -3 & 2 \end{array}$$

Remainder = 2 \neq 0; therefore $x + \frac{1}{3}$ is not a factor of $f(x)$.

23. Use synthetic division:

$$\begin{array}{r|rrrr} -2 & 1 & -2 & 3 & 5 \\ & & -2 & 8 & -22 \\ \hline & 1 & -4 & 11 & -17 \end{array}$$

$$\frac{x^3 - 2x^2 + 3x + 5}{x + 2} = x^2 - 4x + 11 + \frac{-17}{x + 2}$$

$$a + b + c + d = 1 - 4 + 11 - 17 = -9$$