Chapter 5

The Zeros of a Polynomial Function

5.4 Complex Zeros; Fundamental Theorem of Algebra

1. Since complex zeros appear in conjugate pairs, $4+i$, the conjugate of $4-i$, is the remaining zero of f.

2. Since complex zeros appear in conjugate pairs, $3-i$, the conjugate of $3+i$, is the remaining zero of f.

3. Since complex zeros appear in conjugate pairs, $-i$, the conjugate of i, and $1-i$, the conjugate of $1+i$, are the remaining zeros of f.

4. Since complex zeros appear in conjugate pairs, $2-i$, the conjugate of $2+i$, is the remaining zero of f.

5. Since complex zeros appear in conjugate pairs, $-i$, the conjugate of i, and $-2i$, the conjugate of $2i$, are the remaining zeros of f.

6. Since complex zeros appear in conjugate pairs, $-i$, the conjugate of i, is the remaining zero of f.

7. Since complex zeros appear in conjugate pairs, $-i$, the conjugate of i, is the remaining zero of f.

8. Since complex zeros appear in conjugate pairs, $2+i$, the conjugate of $2-i$, and i, the conjugate of $-i$, are the remaining zeros of f.

9. Since complex zeros appear in conjugate pairs, $2-i$, the conjugate of $2+i$, and $-3+i$, the conjugate of $-3-i$, are the remaining zeros of f.

10. Since complex zeros appear in conjugate pairs, $-i$, the conjugate of i, $3+2i$, the conjugate of $3-2i$, and $-2-i$, the conjugate of $-2+i$, are the remaining zeros of f.

11. Since $3+2i$ is a zero, its conjugate $3-2i$ is also a zero of f. Finding the function:

$$\begin{aligned}
f(x) &= (x-4)(x-4)\big(x-(3+2i)\big)\big(x-(3-2i)\big)\\
&= \left(x^2-8x+16\right)\big((x-3)-2i\big)\big((x-3)+2i\big)\\
&= \left(x^2-8x+16\right)\left(x^2-6x+9-4i^2\right)\\
&= \left(x^2-8x+16\right)\left(x^2-6x+13\right)\\
&= x^4-6x^3+13x^2-8x^3+48x^2-104x+16x^2-96x+208\\
&= x^4-14x^3+77x^2-200x+208
\end{aligned}$$

12. Since $1+2i$ and i are zeros, their conjugates $1-2i$ and $-i$ are also zeros of f. Finding the function:

$$\begin{aligned}
f(x) &= (x-i)(x-(-i))\big(x-(1+2i)\big)\big(x-(1-2i)\big)\\
&= (x-i)(x+i)\big((x-1)-2i\big)\big((x-1)+2i\big)\\
&= \left(x^2-i^2\right)\left(x^2-2x+1-4i^2\right) = \left(x^2+1\right)\left(x^2-2x+5\right)\\
&= x^4-2x^3+5x^2+1x^2-2x+5 = x^4-2x^3+6x^2-2x+5
\end{aligned}$$

13. Since $-i$ is a zero, its conjugate i is also a zero, and since $1+i$ is a zero, its conjugate $1-i$ is also a zero of f. Finding the function:

$$\begin{aligned}
f(x) &= (x-2)(x+i)(x-i)\big(x-(1+i)\big)\big(x-(1-i)\big)\\
&= (x-2)\left(x^2-i^2\right)\big((x-1)-i\big)\big((x-1)+i\big)\\
&= (x-2)\left(x^2+1\right)\left(x^2-2x+1-i^2\right)\\
&= \left(x^3-2x^2+x-2\right)\left(x^2-2x+2\right)\\
&= x^5-2x^4+2x^3-2x^4+4x^3-4x^2+x^3-2x^2+2x-2x^2+4x-4\\
&= x^5-4x^4+7x^3-8x^2+6x-4
\end{aligned}$$

14. Since i is a zero, its conjugate $-i$ is also a zero; since $4-i$ is a zero, its conjugate $4+i$ is also a zero; and since $2+i$ is a zero, its conjugate $2-i$ is also a zero of f. Finding the function:

$$\begin{aligned}
f(x) &= (x+i)(x-i)\big(x-(4+i)\big)\big(x-(4-i)\big)\big(x-(2+i)\big)\big(x-(2-i)\big)\\
&= \left(x^2-i^2\right)\big((x-4)-i\big)\big((x-4)+i\big)\big((x-2)-i\big)\big((x-2)+i\big)\\
&= \left(x^2+1\right)\left(x^2-8x+16-i^2\right)\left(x^2-4x+4-i^2\right)\\
&= \left(x^2+1\right)\left(x^2-8x+17\right)\left(x^2-4x+5\right)\\
&= \left(x^4-8x^3+17x^2+x^2-8x+17\right)\left(x^2-4x+5\right)\\
&= \left(x^4-8x^3+18x^2-8x+17\right)\left(x^2-4x+5\right)\\
&= x^6-4x^5+5x^4-8x^5+32x^4-40x^3+18x^4-72x^3+90x^2-8x^3\\
&\qquad +32x^2-40x+17x^2-68x+85\\
&= x^6-12x^5+55x^4-120x^3+139x^2-108x+85
\end{aligned}$$

15. Since $-i$ is a zero, its conjugate i is also a zero of f. Finding the function:

$$
\begin{aligned}
f(x) &= (x-3)(x-3)(x+i)(x-i) \\
&= \left(x^2-6x+9\right)\left(x^2-i^2\right) \\
&= \left(x^2-6x+9\right)\left(x^2+1\right) \\
&= x^4+x^2-6x^3-6x+9x^2+9 \\
&= x^4-6x^3+10x^2-6x+9
\end{aligned}
$$

16. Since $1+i$ is a zero, its conjugate $1-i$ is also a zero of f. Finding the function:

$$
\begin{aligned}
f(x) &= (x-1)^3\left(x-(1+i)\right)\left(x-(1-i)\right) \\
&= \left(x^3-3x^2+3x-1\right)\left((x-1)-i\right)\left((x-1)+i\right) \\
&= \left(x^3-3x^2+3x-1\right)\left(x^2-2x+1-i^2\right) \\
&= \left(x^3-3x^2+3x-1\right)\left(x^2-2x+2\right) \\
&= x^5-2x^4+2x^3-3x^4+6x^3-6x^2+3x^3-6x^2+6x-x^2+2x-2 \\
&= x^5-5x^4+11x^3-13x^2+8x-2
\end{aligned}
$$

17. Since $2i$ is a zero, its conjugate $-2i$ is also a zero of f. $x-2i$ and $x+2i$ are factors of f. Thus, $(x-2i)(x+2i)=x^2+4$ is a factor of f. Using division to find the other factor:

$$
\begin{array}{r@{}l}
 & \;x-4 \\
x^2+4 \,\big) & \overline{x^3-4x^2+4x-16} \\
 & \underline{x^3\qquad\;\;\,+4x} \\
 & \quad -4x^2\qquad-16 \\
 & \quad \underline{-4x^2\qquad-16}
\end{array}
$$

$x-4$ is a factor and the remaining zero is 4. The zeros of f are $4, 2i, -2i$.

18. Since $-5i$ is a zero, its conjugate $5i$ is also a zero of g. $x+5i$ and $x-5i$ are factors of g. Thus, $(x+5i)(x-5i)=x^2+25$ is a factor of g. Using division to find the other factor:

$$
\begin{array}{r@{}l}
 & \;x+3 \\
x^2+25 \,\big) & \overline{x^3+3x^2+25x+75} \\
 & \underline{x^3\qquad\;\;+25x} \\
 & \quad 3x^2\qquad+75 \\
 & \quad \underline{3x^2\qquad+75}
\end{array}
$$

$x+3$ is a factor and the remaining zero is –3. The zeros of g are $-3, 5i, -5i$.

19. Since $-2i$ is a zero, its conjugate $2i$ is also a zero of f. $x-2i$ and $x+2i$ are factors of f. Thus, $(x-2i)(x+2i)=x^2+4$ is a factor of f. Using division to find the other factor:

$$
\begin{array}{r|l}
 & 2x^2+5x-3 \\
\hline
x^2+4 & 2x^4+5x^3+5x^2+20x-12 \\
 & \underline{2x^4 \qquad +8x^2} \\
 & \qquad 5x^3-3x^2+20x \\
 & \qquad \underline{5x^3 \qquad +20x} \\
 & \qquad\quad -3x^2 \qquad -12 \\
 & \qquad\quad \underline{-3x^2 \qquad -12}
\end{array}
$$

$2x^2+5x-3=(2x-1)(x+3)$ are factors and the remaining zeros are $\frac{1}{2}$ and -3. The zeros of f are $2i, -2i, -3, \frac{1}{2}$.

20. Since $3i$ is a zero, its conjugate $-3i$ is also a zero of h. $x-3i$ and $x+3i$ are factors of h. Thus, $(x-3i)(x+3i)=x^2+9$ is a factor of h. Using division to find the other factor:

$$
\begin{array}{r|l}
 & 3x^2+5x-2 \\
\hline
x^2+9 & 3x^4+5x^3+25x^2+45x-18 \\
 & \underline{3x^4 \qquad +27x^2} \\
 & \qquad 5x^3-\ 2x^2+45x \\
 & \qquad \underline{5x^3 \qquad +45x} \\
 & \qquad\quad -2x^2 \qquad -18 \\
 & \qquad\quad \underline{-2x^2 \qquad -18}
\end{array}
$$

$3x^2+5x-2=(3x-1)(x+2)$ are factors and the remaining zeros are $\frac{1}{3}$ and -2. The zeros of h are $3i, -3i, -2, \frac{1}{3}$.

21. Since $3-2i$ is a zero, its conjugate $3+2i$ is also a zero of h. $x-(3-2i)$ and $x-(3+2i)$ are factors of h. Thus,
$(x-(3-2i))(x-(3+2i))=((x-3)+2i)((x-3)-2i)=x^2-6x+9-4i^2=x^2-6x+13$ is a factor of h. Using division to find the other factor:

$$
\begin{array}{r|l}
 & x^2-3x-10 \\
\hline
x^2-6x+13 & x^4-9x^3+21x^2+21x-130 \\
 & \underline{x^4-6x^3+13x^2} \\
 & \quad -3x^3+\ 8x^2+21x \\
 & \quad \underline{-3x^3+18x^2-39x} \\
 & \qquad\quad -10x^2+60x-130 \\
 & \qquad\quad \underline{-10x^2+60x-130}
\end{array}
$$

$x^2-3x-10=(x+2)(x-5)$ are factors and the remaining zeros are –2 and 5. The zeros of h are $3-2i, 3+2i, -2, 5$.

22. Since $1+3i$ is a zero, its conjugate $1-3i$ is also a zero of f. $x-(1+3i)$ and $x-(1-3i)$ are factors of f. Thus,
$(x-(1+3i))(x-(1-3i)) = ((x-1)-3i)((x-1)+3i) = x^2-2x+1-9i^2 = x^2-2x+10$ is a factor of f. Using division to find the other factor:

$$\begin{array}{r}
x^2-5x-6 \\
x^2-2x+10\overline{)\,x^4-7x^3+14x^2-38x-60} \\
\underline{x^4-2x^3+10x^2} \qquad\qquad \\
-5x^3+4x^2-38x \qquad \\
\underline{-5x^3+10x^2-50x} \qquad \\
-6x^2+12x-60 \\
\underline{-6x^2+12x-60}
\end{array}$$

$x^2-5x-6=(x+1)(x-6)$ are factors and the remaining zeros are –1 and 6. The zeros of f are $1+3i, 1-3i, -1, 6$.

23. Since $-4i$ is a zero, its conjugate $4i$ is also a zero of h. $x-4i$ and $x+4i$ are factors of h. Thus, $(x-4i)(x+4i)=x^2+16$ is a factor of h. Using division to find the other factor:

$$\begin{array}{r}
3x^3+2x^2-33x-22 \qquad\qquad \\
x^2+16\overline{)\,3x^5+2x^4+15x^3+10x^2-528x-352} \\
\underline{3x^5 \qquad +48x^3} \qquad\qquad\qquad\qquad \\
2x^4-33x^3+10x^2 \qquad\qquad \\
\underline{2x^4 \qquad +32x^2} \qquad\qquad \\
-33x^3-22x^2-528x \qquad \\
\underline{-33x^3 \qquad -528x} \qquad \\
-22x^2 \qquad -352 \\
\underline{-22x^2 \qquad -352}
\end{array}$$

$3x^3+2x^2-33x-22 = x^2(3x+2)-11(3x+2) = (3x+2)(x^2-11)$
$=(3x+2)(x-\sqrt{11})(x+\sqrt{11})$ are factors and the remaining zeros are $\frac{-2}{3}, \sqrt{11}$, and $-\sqrt{11}$.
The zeros of h are $4i, -4i, -\sqrt{11}, \sqrt{11}, \frac{-2}{3}$.

24. Since $3i$ is a zero, its conjugate $-3i$ is also a zero of g. $x-3i$ and $x+3i$ are factors of g. Thus, $(x-3i)(x+3i)=x^2+9$ is a factor of g. Using division to find the other factor:

$$\begin{array}{r|l} & 2x^3-3x^2-23x+12 \\ \hline x^2+9 & 2x^5-3x^4-5x^3-15x^2-207x+108 \\ & \underline{2x^5 \quad\quad +18x^3} \\ & -3x^4-23x^3-15x^2 \\ & \underline{-3x^4 \quad\quad -27x^2} \\ & -23x^3+12x^2-207x \\ & \underline{-23x^3 \quad\quad -207x} \\ & 12x^2 \quad\quad +108 \\ & \underline{12x^2 \quad\quad +108} \end{array}$$

Graph: $y=2x^3-3x^2-23x+12$ It appears that –3 is a zero.

$$\begin{array}{r|rrrr} -3 & 2 & -3 & -23 & 12 \\ & & -6 & 27 & -12 \\ \hline & 2 & -9 & 4 & 0 \end{array}$$

$x+3$ is a factor. The remaining factor is $2x^2-9x+4=(2x-1)(x-4)$.
The zeros of g are $3i, -3i, -3, \frac{1}{2}, 4$.

25. $f(x)=x^3-1=(x-1)\left(x^2+x+1\right)$ The zeros of $x^2+x+1=0$ are:

$$x=\frac{-1\pm\sqrt{1^2-4(1)(1)}}{2(1)}=\frac{-1\pm\sqrt{-3}}{2}=\frac{-1}{2}+\frac{\sqrt{3}}{2}i \text{ or } \frac{-1}{2}-\frac{\sqrt{3}}{2}i$$

The zeros are: $1, \ \frac{-1}{2}+\frac{\sqrt{3}}{2}i, \ \frac{-1}{2}-\frac{\sqrt{3}}{2}i$.

26. $f(x)=x^4-1=\left(x^2-1\right)\left(x^2+1\right)=(x-1)(x+1)\left(x^2+1\right)$
The zeros of $x^2+1=0$ are $x=\pm i$ are:
The zeros are: $-1, \ 1, \ -i, \ i$.

27. $f(x)=x^3-8x^2+25x-26$
Step 1: $f(x)$ has 3 complex zeros.
Step 2: By Descartes Rule of Signs, there are 3 or 1 positive real zeros.
$f(-x)=(-x)^3-8(-x)^2+25(-x)-26=-x^3-8x^2-25x-26$; thus, there are no negative real zeros.
Step 3: Possible rational zeros:

$$p=\pm 1, \pm 2, \pm 13, \pm 26; \quad q=\pm 1; \quad \frac{p}{q}=\pm 1, \pm 2, \pm 13, \pm 26$$

Step 4: Using synthetic division:

$$\begin{array}{r|rrrr} 2 & 1 & -8 & 25 & -26 \\ & & 2 & -12 & 26 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

Since the remainder is 0, $x-2$ is a factor. The other factor is the quotient: $x^2-6x+13$.

Using the quadratic formula to find the zeros of $x^2 - 6x + 13 = 0$:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)} = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i.$$

The complex zeros are $2,\ 3-2i,\ 3+2i$.

28. $f(x) = x^3 + 13x^2 + 57x + 85$

Step 1: $f(x)$ has 3 complex zeros.

Step 2: By Descartes Rule of Signs, there are no positive real zeros.
$f(-x) = (-x)^3 + 13(-x)^2 + 57(-x) + 85 = -x^3 + 13x^2 - 57x + 85$; thus, there are 3 or 1 negative real zeros.

Step 3: Possible rational zeros:

$$p = \pm 1, \pm 5, \pm 17, \pm 85; \quad q = \pm 1; \quad \frac{p}{q} = \pm 1, \pm 5, \pm 17, \pm 85$$

Step 4: Using synthetic division:

$$\begin{array}{r|rrrr} -5 & 1 & 13 & 57 & 85 \\ & & -5 & -40 & -85 \\ \hline & 1 & 8 & 17 & 0 \end{array}$$

Since the remainder is 0, $x+5$ is a factor. The other factor is the quotient: $x^2 + 8x + 17$.

Using the quadratic formula to find the zeros of $x^2 + 8x + 17 = 0$:

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(17)}}{2(1)} = \frac{-8 \pm \sqrt{-4}}{2} = \frac{-8 \pm 2i}{2} = -4 \pm i.$$

The complex zeros are $-5,\ -4-i,\ -4+i$.

29. $f(x) = x^4 + 5x^2 + 4 = \left(x^2+4\right)\left(x^2+1\right) = (x+2i)(x-2i)(x+i)(x-i)$

The zeros are: $-2i,\ -i,\ i,\ 2i$.

30. $f(x) = x^4 + 13x^2 + 36 = \left(x^2+4\right)\left(x^2+9\right) = (x+2i)(x-2i)(x+3i)(x-3i)$

The zeros are: $-3i,\ -2i,\ 2i,\ 3i$.

31. $f(x) = x^4 + 2x^3 + 22x^2 + 50x - 75$

Step 1: $f(x)$ has 4 complex zeros.

Step 2: By Descartes Rule of Signs, there is 1 positive real zero.

$$\begin{aligned} f(-x) &= (-x)^4 + 2(-x)^3 + 22(-x)^2 + 50(-x) - 75 \\ &= x^4 - 2x^3 + 22x^2 - 50x - 75 \end{aligned}$$

thus, there are 3 or 1 negative real zeros.

Step 3: Possible rational zeros:

$$p = \pm 1, \pm 3, \pm 5, \pm 15, \pm 25, \pm 75; \quad q = \pm 1;$$

$$\frac{p}{q} = \pm 1, \pm 3, \pm 5, \pm 15, \pm 25, \pm 75$$

Step 4: Using synthetic division:

$$\begin{array}{r|rrrrr} -3 & 1 & 2 & 22 & 50 & -75 \\ & & -3 & 3 & -75 & 75 \\ \hline & 1 & -1 & 25 & -25 & 0 \end{array}$$

Since the remainder is 0, $x+3$ is a factor. The other factor is the quotient:

$$x^3 - x^2 + 25x - 25 = x^2(x-1) + 25(x-1) = (x-1)\left(x^2+25\right)$$
$$= (x-1)(x+5i)(x-5i)$$

The complex zeros are $-3,\ 1,\ -5i,\ 5i$.

32. $f(x) = x^4 + 3x^3 - 19x^2 + 27x - 252$

Step 1: $f(x)$ has 4 complex zeros.

Step 2: By Descartes Rule of Signs, there are 3 or 1 positive real zeros.

$$f(-x) = (-x)^4 + 3(-x)^3 - 19(-x)^2 + 27(-x) - 252$$
$$= x^4 - 3x^3 - 19x^2 - 27x - 252$$

thus, there is 1 negative real zero.

Step 3: Possible rational zeros:

$$p = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 7, \pm 9, \pm 12, \pm 14, \pm 18, \pm 21, \pm 28, \pm 36,$$
$$\pm 42, \pm 63, \pm 84, \pm 126, \pm 252; \quad q = \pm 1;$$

The possible rational zeros are the same as the values of p.

Step 4: Using synthetic division:

$$\begin{array}{r|rrrrr} -7 & 1 & 3 & -19 & 27 & -252 \\ & & -7 & 28 & -63 & 252 \\ \hline & 1 & -4 & 9 & -36 & 0 \end{array}$$

Since the remainder is 0, $x+7$ is a factor. The other factor is the quotient:

$$x^3 - 4x^2 + 9x - 36 = x^2(x-4) + 9(x-4) = (x-4)\left(x^2+9\right)$$
$$= (x-4)(x+3i)(x-3i)$$

The complex zeros are $-7,\ 4,\ -3i,\ 3i$.

33. $f(x) = 3x^4 - x^3 - 9x^2 + 159x - 52$

Step 1: $f(x)$ has 4 complex zeros.

Step 2: By Descartes Rule of Signs, there are 3 or 1 positive real zeros.

$$f(-x) = 3(-x)^4 - (-x)^3 - 9(-x)^2 + 159(-x) - 52$$
$$= 3x^4 + x^3 - 9x^2 - 159x - 52$$

thus, there is 1 negative real zero.

Step 3: Possible rational zeros:

$$p = \pm 1, \pm 2, \pm 4, \pm 13, \pm 26, \pm 52; \quad q = \pm 1, \pm 3;$$
$$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 13, \pm 26, \pm 52, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{13}{3}, \pm\frac{26}{3}, \pm\frac{52}{3}$$

Step 4: Using synthetic division:

$$\begin{array}{r|rrrrr} -4 & 3 & -1 & -9 & 159 & -52 \\ & & -12 & 52 & -172 & 52 \\ \hline & 3 & -13 & 43 & -13 & 0 \end{array} \qquad \begin{array}{r|rrrr} \frac{1}{3} & 3 & -13 & 43 & -13 \\ & & 1 & -4 & 13 \\ \hline & 3 & -12 & 39 & 0 \end{array}$$

Since the remainder is 0, $x+4$ and $x-\frac{1}{3}$ are factors. The other factor is the quotient: $3x^2-12x+39=3\left(x^2-4x+13\right)$.

Using the quadratic formula to find the zeros of $x^2-4x+13=0$:

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(13)}}{2(1)}=\frac{4\pm\sqrt{-36}}{2}=\frac{4\pm 6i}{2}=2\pm 3i.$$

The complex zeros are $-4,\ \frac{1}{3},\ 2-3i,\ 2+3i$.

34. $f(x)=2x^4+x^3-35x^2-113x+65$

Step 1: $f(x)$ has 4 complex zeros.

Step 2: By Descartes Rule of Signs, there are 2 or 0 positive real zeros.

$$f(-x)=2(-x)^4+(-x)^3-35(-x)^2-113(-x)+65=2x^4-x^3-35x^2+113x+65$$

thus, there are 2 or 0 negative real zeros.

Step 3: Possible rational zeros:

$p=\pm 1,\ \pm 5,\ \pm 13,\ \pm 65;\quad q=\pm 1,\ \pm 2;$

$$\frac{p}{q}=\pm 1,\ \pm 5,\ \pm 13,\ \pm 65,\ \pm\frac{1}{2},\pm\frac{5}{2},\pm\frac{13}{2},\pm\frac{65}{2}$$

Step 4: Using synthetic division:

$$\begin{array}{r|rrrrr} 5 & 2 & 1 & -35 & -113 & 65 \\ & & 10 & 55 & 100 & -65 \\ \hline & 2 & 11 & 20 & -13 & 0 \end{array} \qquad \begin{array}{r|rrrr} \frac{1}{2} & 2 & 11 & 20 & -13 \\ & & 1 & 6 & 13 \\ \hline & 2 & 12 & 26 & 0 \end{array}$$

Since the remainder is 0, $x-5$ and $x-\frac{1}{2}$ are factors. The other factor is the quotient: $2x^2+12x+26=2\left(x^2+6x+13\right)$.

Using the quadratic formula to find the zeros of $x^2+6x+13=0$:

$$x=\frac{-6\pm\sqrt{6^2-4(1)(13)}}{2(1)}=\frac{-6\pm\sqrt{-16}}{2}=\frac{-6\pm 4i}{2}=-3\pm 2i.$$

The complex zeros are $5,\ \frac{1}{2},\ -3-2i,\ -3+2i$.

35. If the coefficients are real numbers and $2+i$ is a zero, then $2-i$ would also be a zero. This would then require a polynomial of degree 4.

36. Three zeros are given. If the coefficients are real numbers, then the complex zeros would also have their conjugates as zeros. This would mean that there are 5 zeros which would require a polynomial of degree 5.

37. If the coefficients are real numbers, then complex zeros must appear in conjugate pairs. We have a conjugate pair and one real zero. Thus, there is only one remaining zero and it must be real because a complex zero would require a pair and the polynomial would then have to be of degree 5.

38. One of the remaining zeros must be $4+i$, the conjugate of $4-i$. The third zero is a real number. Thus the fourth zero must also be a real number in order to have a degree 4 polynomial.