Chapter 6

Exponential and Logarithmic Functions

6.2 Exponential Functions

1. (a) $3^{2.2} = 11.212$ (b) $3^{2.23} = 11.587$ (c) $3^{2.236} = 11.664$ (d) $3^{\sqrt{5}} = 11.665$

2. (a) $5^{1.7} = 15.426$ (b) $5^{1.73} = 16.189$ (c) $5^{1.732} = 16.241$ (d) $5^{\sqrt{3}} = 16.242$

3. (a) $2^{3.14} = 8.815$ (b) $2^{3.141} = 8.821$ (c) $2^{3.1415} = 8.824$ (d) $2^{\pi} = 8.825$

4. (a) $2^{2.7} = 6.498$ (b) $2^{2.71} = 6.543$ (c) $2^{2.718} = 6.580$ (d) $2^{e} = 6.581$

5. (a) $3.1^{2.7} = 21.217$ (b) $3.14^{2.71} = 22.217$
   (c) $3.141^{2.718} = 22.440$ (d) $\pi^{e} = 22.459$

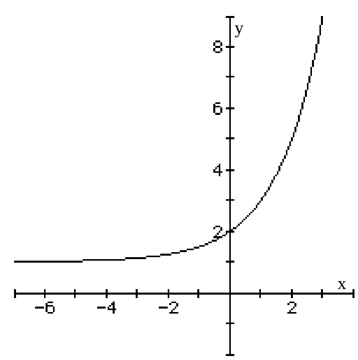
6. (a) $2.7^{3.1} = 21.738$ (b) $2.71^{3.14} = 22.884$
   (c) $2.718^{3.141} = 23.119$ (d) $e^{\pi} = 23.141$

7. $e^{1.2} = 3.320$ 8. $e^{-1.3} = 0.273$

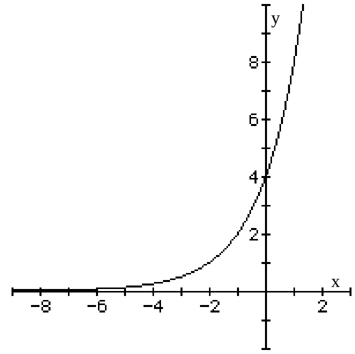
9. $e^{-0.85} = 0.427$ 10. $e^{2.1} = 8.166$

11. B 12. F 13. D 14. H 15. A 16. C 17. E 18. G

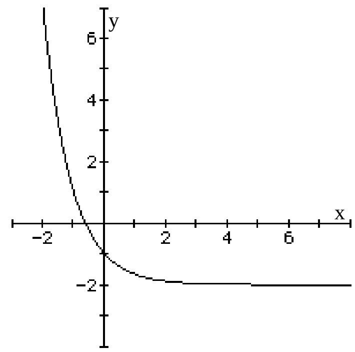
19. $f(x) = 2^x + 1$
    Using the graph of $y = 2^x$, shift the graph up 1 unit.
    Domain: $(-\infty, \infty)$
    Range: $(1, \infty)$
    Horizontal Asymptote: $y = 1$



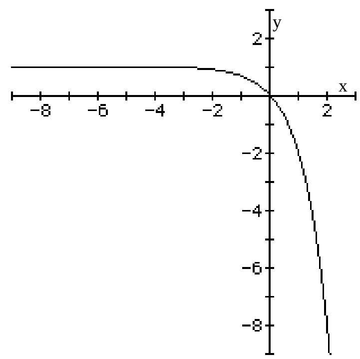
20. $f(x) = 2^{x+2}$
Using the graph of $y = 2^x$, shift the graph left 2 units.
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Horizontal Asymptote: $y = 0$



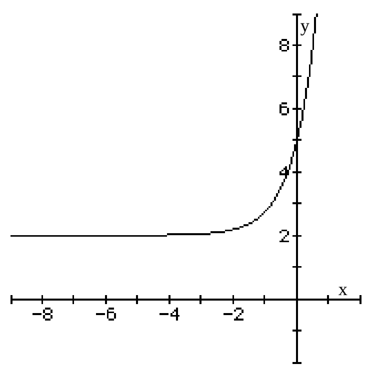
21. $f(x) = 3^{-x} - 2$
Using the graph of $y = 3^x$, reflect the graph about the y-axis, and shift down 2 units.
Domain: $(-\infty, \infty)$
Range: $(-2, \infty)$
Horizontal Asymptote: $y = -2$



22. $f(x) = -3^x + 1$
Using the graph of $y = 3^x$, reflect the graph about the x-axis, and shift up 1 unit.
Domain: $(-\infty, \infty)$
Range: $(-\infty, 1)$
Horizontal Asymptote: $y = 1$



23. $f(x) = 2 + 3\left(4^x\right)$
Using the graph of $y = 4^x$, stretch the graph vertically by a factor of 3, and shift up 2 units.
Domain: $(-\infty, \infty)$
Range: $(2, \infty)$
Horizontal Asymptote: $y = 2$



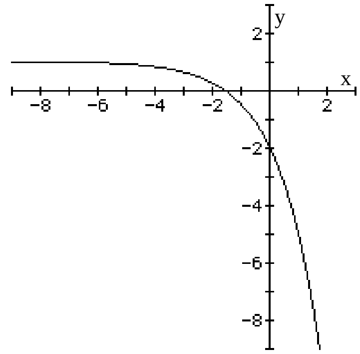
24. $f(x) = 1 - 3\left(2^x\right)$

Using the graph of $y = 2^x$, stretch the graph vertically by a factor of 3, reflect about the x-axis, and shift up 1 unit.

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1)$

Horizontal Asymptote: $y = 1$



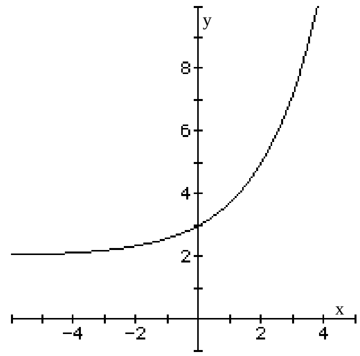
25. $f(x) = 2 + 3^{\frac{x}{2}}$

Using the graph of $y = 3^x$, stretch the graph horizontally by a factor of 2, and shift up 2 units.

Domain: $(-\infty, \infty)$

Range: $(2, \infty)$

Horizontal Asymptote: $y = 2$



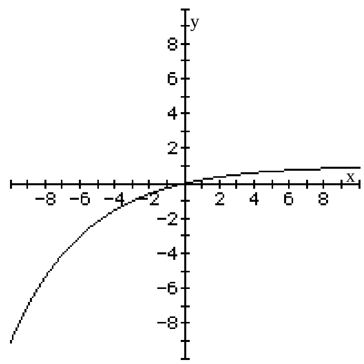
26. $f(x) = 1 - 2^{-\frac{x}{3}}$

Using the graph of $y = 2^x$, stretch the graph horizontally by a factor of 3, reflect about the y-axis, reflect about the x-axis, and shift up 1 unit.

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1)$

Horizontal Asymptote: $y = 1$



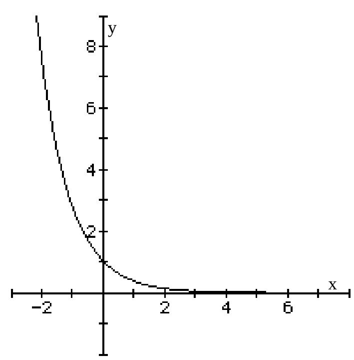
27. $f(x) = e^{-x}$

Using the graph of $y = e^x$, reflect the graph about the y-axis.

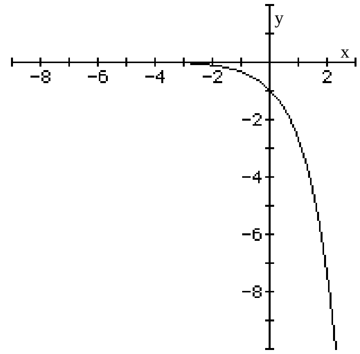
Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

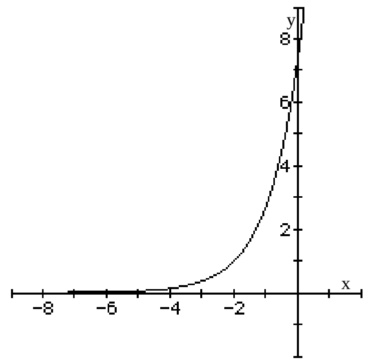
Horizontal Asymptote: $y = 0$



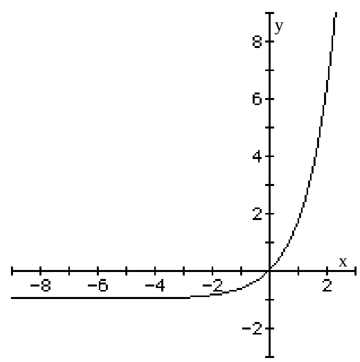
28. $f(x) = -e^x$
Using the graph of $y = e^x$, reflect the graph about the x-axis.
Domain: $(-\infty, \infty)$
Range: $(-\infty, 0)$
Horizontal Asymptote: $y = 0$



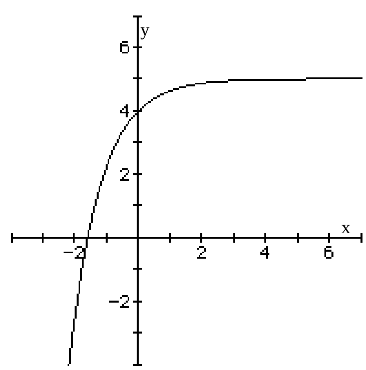
29. $f(x) = e^{x+2}$
Using the graph of $y = e^x$, shift the graph 2 units to the left.
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Horizontal Asymptote: $y = 0$



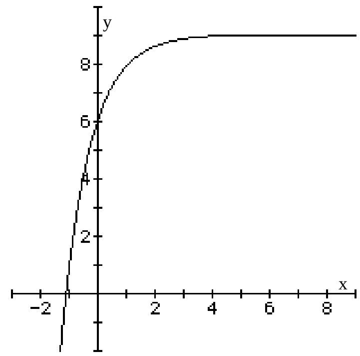
30. $f(x) = e^x - 1$
Using the graph of $y = e^x$, shift the graph down 1 unit.
Domain: $(-\infty, \infty)$
Range: $(-1, \infty)$
Horizontal Asymptote: $y = -1$



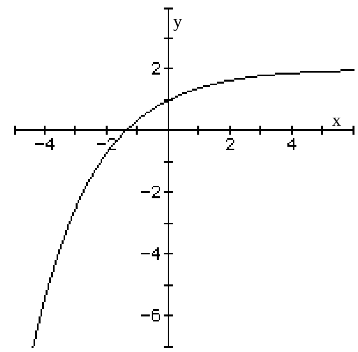
31. $f(x) = 5 - e^{-x}$
Using the graph of $y = e^x$, reflect the graph about the y-axis, reflect about the x-axis, and shift up 5 units.
Domain: $(-\infty, \infty)$
Range: $(-\infty, 5)$
Horizontal Asymptote: $y = 5$



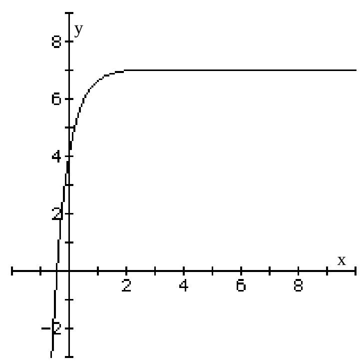
32. $f(x) = 9 - 3e^{-x}$
Using the graph of $y = e^x$, reflect the graph about the y-axis, stretch vertically by a factor of 3, reflect about the x-axis, and shift up 9 units.
Domain: $(-\infty, \infty)$
Range: $(-\infty, 9)$
Horizontal Asymptote: $y = 9$



33. $f(x) = 2 - e^{-\frac{x}{2}}$
Using the graph of $y = e^x$, reflect the graph about the y-axis, stretch horizontally by a factor of 2, reflect about the x-axis, and shift up 2 units.
Domain: $(-\infty, \infty)$
Range: $(-\infty, 2)$
Horizontal Asymptote: $y = 2$



34. $f(x) = 7 - 3e^{-2x}$
Using the graph of $y = e^x$, reflect the graph about the y-axis, shrink horizontally by a factor of $\frac{1}{2}$, stretch vertically by a factor of 3, reflect about the x-axis, and shift up 7 units.
Domain: $(-\infty, \infty)$
Range: $(-\infty, 7)$
Horizontal Asymptote: $y = 7$



35.
$$\begin{aligned} 2^{2x+1} &= 4 \\ 2^{2x+1} &= 2^2 \\ 2x+1 &= 2 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$
The solution is $\left\{\frac{1}{2}\right\}$.

36.
$$\begin{aligned} 5^{1-2x} &= \tfrac{1}{5} \\ 5^{1-2x} &= 5^{-1} \\ 1-2x &= -1 \\ -2x &= -2 \\ x &= 1 \end{aligned}$$
The solution is $\{1\}$.

37.
$$\begin{aligned}
3^{x^3} &= 9^x \\
3^{x^3} &= \left(3^2\right)^x \\
3^{x^3} &= 3^{2x} \\
x^3 &= 2x \\
x^3 - 2x &= 0 \\
x(x^2 - 2) &= 0 \\
x &= 0 \text{ or } x^2 = 2 \\
x &= 0 \text{ or } x = \pm\sqrt{2}
\end{aligned}$$
The solution is $\left\{-\sqrt{2},\ 0,\ \sqrt{2}\right\}$.

38.
$$\begin{aligned}
4^{x^2} &= 2^x \\
\left(2^2\right)^{x^2} &= 2^x \\
2^{2x^2} &= 2^x \\
2x^2 &= x \\
2x^2 - x &= 0 \\
x(2x - 1) &= 0 \\
x &= 0 \text{ or } x = \tfrac{1}{2}
\end{aligned}$$
The solution is $\left\{0, \tfrac{1}{2}\right\}$.

39.
$$\begin{aligned}
8^{x^2 - 2x} &= \tfrac{1}{2} \\
\left(2^3\right)^{x^2 - 2x} &= 2^{-1} \\
2^{3x^2 - 6x} &= 2^{-1} \\
3x^2 - 6x &= -1 \\
3x^2 - 6x + 1 &= 0
\end{aligned}$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)} = \frac{6 \pm \sqrt{24}}{6} = \frac{6 \pm 2\sqrt{6}}{6} = \frac{3 \pm \sqrt{6}}{3}$$
The solution is $\left\{\dfrac{3 - \sqrt{6}}{3},\ \dfrac{3 + \sqrt{6}}{3}\right\}$.

40.
$$\begin{aligned}
9^{-x} &= \tfrac{1}{3} \\
\left(3^2\right)^{-x} &= 3^{-1} \\
3^{-2x} &= 3^{-1} \\
-2x &= -1 \\
x &= \tfrac{1}{2}
\end{aligned}$$
The solution is $\left\{\tfrac{1}{2}\right\}$.

41.
$$\begin{aligned}
2^x \ 8^{-x} &= 4^x \\
2^x \left(2^3\right)^{-x} &= \left(2^2\right)^x \\
2^x \ 2^{-3x} &= 2^{2x} \\
2^{-2x} &= 2^{2x} \\
-2x &= 2x \\
-4x &= 0 \\
x &= 0
\end{aligned}$$
The solution is $\{0\}$.

42.
$$\begin{aligned}
\left(\tfrac{1}{2}\right)^{1-x} &= 4 \\
\left(2^{-1}\right)^{1-x} &= 2^2 \\
2^{-1+x} &= 2^2 \\
-1 + x &= 2 \\
x &= 3
\end{aligned}$$
The solution is $\{3\}$.

43.
$$\begin{aligned}
\left(\tfrac{1}{5}\right)^{2-x} &= 25 \\
\left(5^{-1}\right)^{2-x} &= 5^2 \\
5^{x-2} &= 5^2 \\
x - 2 &= 2 \\
x &= 4
\end{aligned}$$
The solution is $\{4\}$.

44. $4^x - 2^x = 0$

$$\begin{aligned} \left(2^2\right)^x &= 2^x \\ 2^{2x} &= 2^x \\ 2x &= x \\ x &= 0 \end{aligned}$$

The solution is $\{0\}$.

45. $4^x = 8$

$$\begin{aligned} \left(2^2\right)^x &= 2^3 \\ 2^{2x} &= 2^3 \\ 2x &= 3 \\ x &= \frac{3}{2} \end{aligned}$$

The solution is $\left\{\frac{3}{2}\right\}$.

46. $9^{2x} = 27$

$$\begin{aligned} \left(3^2\right)^{2x} &= 3^3 \\ 3^{4x} &= 3^3 \\ 4x &= 3 \\ x &= \tfrac{3}{4} \end{aligned}$$

The solution is $\left\{\tfrac{3}{4}\right\}$.

47. $e^{x^2} = e^{3x}\,\frac{1}{e^2}$

$$\begin{aligned} e^{x^2} &= e^{3x-2} \\ x^2 &= 3x - 2 \\ x^2 - 3x + 2 &= 0 \\ (x-1)(x-2) &= 0 \\ x = 1 \text{ or } x &= 2 \end{aligned}$$

The solution is $\{1, 2\}$.

48. $\left(e^4\right)^x\ e^{x^2} = e^{12}$

$$\begin{aligned} e^{4x}\ e^{x^2} &= e^{12} \\ e^{4x+x^2} &= e^{12} \\ x^2 + 4x &= 12 \\ x^2 + 4x - 12 &= 0 \\ (x+6)(x-2) &= 0 \\ x = -6 \text{ or } x &= 2 \end{aligned}$$

The solution is $\{-6, 2\}$.

49. $4^x = 7$

$$\left(4^x\right)^{-2} = 7^{-2} \quad \rightarrow \quad 4^{-2x} = \frac{1}{7^2} = \frac{1}{49}$$

50. $2^x = 3$

$$4^{-x} = \left(2^2\right)^{-x} = 2^{-2x} = \left(2^x\right)^{-2} = 3^{-2} = \frac{1}{9}$$

51. $3^{-x} = 2$

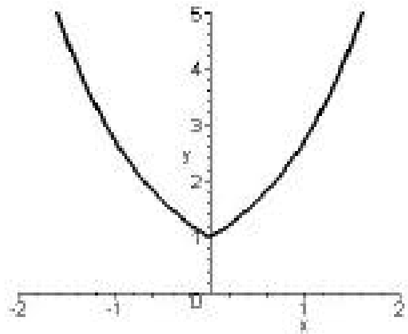
$$(3^{-x})^{-2} = 2^{-2} \quad \rightarrow \quad 3^{2x} = \frac{1}{2^2} = \frac{1}{4}$$

52. $5^{-x} = 3$

$$5^{3x} = \left(5^{-x}\right)^{-3} = 3^{-3} = \frac{1}{27}$$

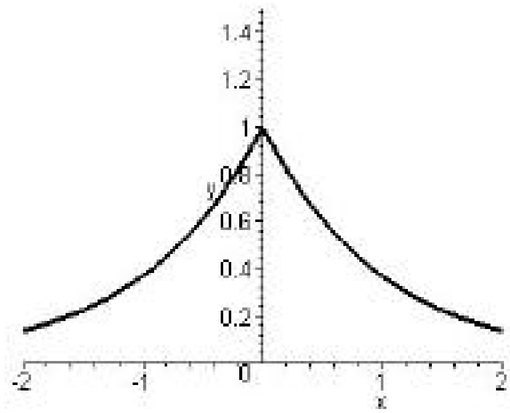
53. $f(x)=\begin{cases} e^{-x} & \text{if } x<0 \\ e^{x} & \text{if } x\geq 0 \end{cases}$

domain = $(-\infty,\infty)$
range = $[1,\infty)$
y-intercept (0, 1)



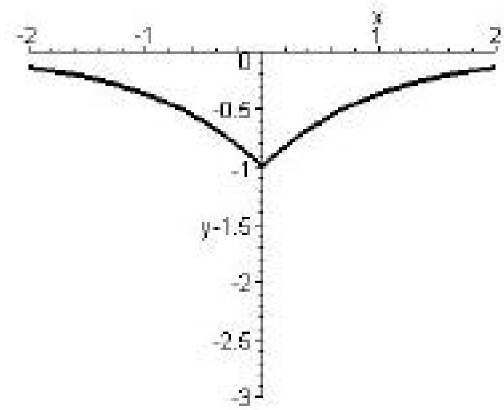
54. $f(x)=\begin{cases} e^{x} & \text{if } x<0 \\ e^{-x} & \text{if } x\geq 0 \end{cases}$

domain = $(-\infty,\infty)$
range = $(0,1]$
y-intercept (0, 1)
horizontal asymptote $y=0$



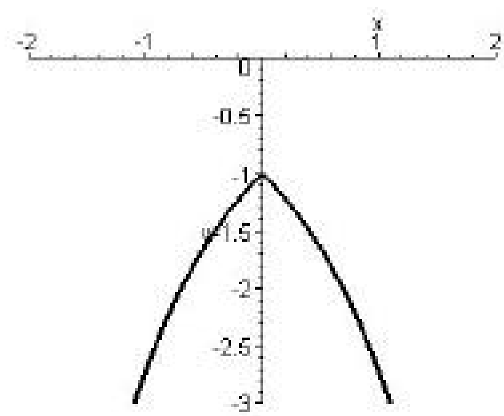
55. $f(x)=\begin{cases} -e^{x} & \text{if } x<0 \\ -e^{-x} & \text{if } x\geq 0 \end{cases}$

domain = $(-\infty,\infty)$
range = $[-1,0)$
y-intercept (0, - 1)
horizontal asymptote $y=0$



56. $f(x)=\begin{cases} -e^{-x} & \text{if } x<0 \\ -e^{x} & \text{if } x\geq 0 \end{cases}$

domain = $(-\infty,\infty)$
range = $(-\infty,-1]$
y-intercept (0, - 1)



57. $p = 100e^{-0.03n}$
 (a) $p = 100e^{-0.03(10)} = 100e^{-0.3} = 100(0.741) = 74.1\%$ of light
 (b) $p = 100e^{-0.03(25)} = 100e^{-0.75} = 100(0.472) = 47.2\%$ of light

58. $p(h) = 760e^{-0.145h}$
 (a) $p(2) = 760e^{-0.145(2)} = 760e^{-0.290} = 568.68$ mm of mercury
 (b) $p(10) = 760e^{-0.145(10)} = 760e^{-1.45} = 178.27$ mm of mercury

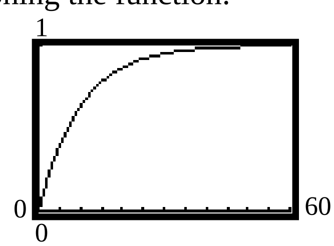
59. $w(d) = 50e^{-0.004d}$
 (a) $w(30) = 50e^{-0.004(30)} = 50e^{-0.12} = 50(0.887) = 44.35$ watts
 (b) $w(365) = 50e^{-0.004(365)} = 50e^{-1.46} = 50(0.232) = 11.61$ watts

60. $A(n) = A_0e^{-0.35n}$
 (a) $A(3) = 100e^{-0.35(3)} = 100e^{-1.05} = 100(0.350) = 35$ square millimeters
 (b) $A(10) = 100e^{-0.35(10)} = 100e^{-3.5} = 100(0.030) = 3$ square millimeters

61. $D(h) = 5e^{-0.4h}$
 $D(1) = 5e^{-0.4(1)} = 5e^{-0.4} = 5(0.670) = 3.35$ milligrams
 $D(6) = 5e^{-0.4(6)} = 5e^{-2.4} = 5(0.091) = 0.45$ milligrams

62. $N = P\left(1 - e^{-0.15d}\right)$
 $N = 1000\left(1 - e^{-0.15(3)}\right) = 1000\left(1 - e^{-0.45}\right) = 1000(1 - 0.638) = 1000(0.362) = 362$
 362 students will have heard the rumor after 3 days.

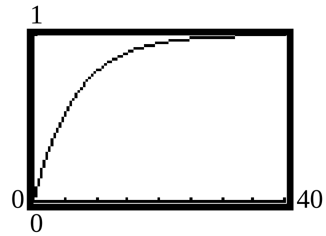
63. $F(t) = 1 - e^{-0.1t}$
 (a) $F(10) = 1 - e^{-0.1(10)} = 1 - e^{-1} = 1 - 0.368 = 0.632 = 63.2\%$
 (b) $F(40) = 1 - e^{-0.1(40)} = 1 - e^{-4} = 1 - 0.018 = 0.982 = 98.2\%$
 (c) $as\ t \to +\infty,\ F(t) = 1 - e^{-0.1t} \to 1 - 0 = 1$
 (d) Graphing the function:



 (e) $F(7) \approx 50$, so 7 minutes are needed for the probability to reach 50%.

64. $F(t) = 1 - e^{-0.15t}$
 (a) $F(15) = 1 - e^{-0.15(15)} = 1 - e^{-2.25} = 1 - 0.105 = 0.895 = 89.5\%$
 (b) $F(30) = 1 - e^{-0.15(30)} = 1 - e^{-4.5} = 1 - 0.011 = 0.989 = 98.9\%$
 (c) $as\ t \to +\infty,\ F(t) = 1 - e^{-0.15t} \to 1 - 0 = 1$

(d) Graphing the function:



(e) $F(6) \approx 60$, so 6 minutes are needed for the probability to reach 60%.

65. $P(x) = \dfrac{20^x e^{-20}}{x!}$

(a) $P(15) = \dfrac{20^{15} e^{-20}}{15!} = 0.0516 = 5.16\%$ The probability that 15 cars will arrive between 5:00 p.m. and 6:00 p.m. is 5.16%.

(b) $P(20) = \dfrac{20^{20} e^{-20}}{20!} = 0.0888 = 8.88\%$ The probability that 20 cars will arrive between 5:00 p.m. and 6:00 p.m. is 8.88%.

66. $P(x) = \dfrac{4^x e^{-4}}{x!}$

(a) $P(5) = \dfrac{4^5 e^{-4}}{5!} = 0.156 = 15.6\%$ The probability that 5 people will arrive within the next minute is 15.6%.

(b) $P(8) = \dfrac{4^8 e^{-4}}{8!} = 0.030 = 3.0\%$ The probability that 8 people will arrive within the next minute is 3.0%.

67. $R = 10^{\left(\frac{2345}{T} - \frac{2345}{D} + 2\right)}$

(a) $R = 10^{\left(\frac{2345}{283} - \frac{2345}{278} + 2\right)} = 10^{1.851} = 70.96\%$

(b) $R = 10^{\left(\frac{2345}{293} - \frac{2345}{288} + 2\right)} = 10^{1.861} = 72.61\%$

(c) $R = 10^{\left(\frac{2345}{x} - \frac{2345}{x} + 2\right)} = 10^{2} = 100\%$

68. $L(t) = 500\left(1 - e^{-0.0061\,t}\right)$

(a) $L(30) = 500\left(1 - e^{-0.0061(30)}\right) = 500\left(1 - e^{-0.183}\right) = 500\left(1 - 0.833\right) = 83.5$ words

(b) $L(60) = 500\left(1 - e^{-0.0061(60)}\right) = 500\left(1 - e^{-0.366}\right) = 500\left(1 - 0.694\right) = 153$ words

69. $I = \dfrac{E}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$

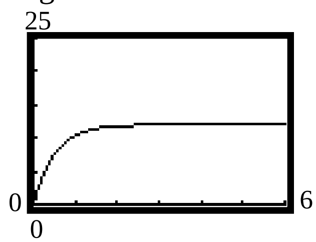
(a) $I = \dfrac{120}{10}\left[1 - e^{-\left(\frac{10}{5}\right)0.3}\right] = 12\left[1 - e^{-0.6}\right] = 5.414$ amperes after 0.3 second

$I = \dfrac{120}{10}\left[1 - e^{-\left(\frac{10}{5}\right)0.5}\right] = 12\left[1 - e^{-1}\right] = 7.585$ amperes after 0.5 second

$$I = \frac{120}{10}\left[1 - e^{-\left(\frac{10}{5}\right)1}\right] = 12\left[1 - e^{-2}\right] = 10.376 \text{ amperes after 1 second}$$

(b) As $t \to \infty$, $e^{-\left(\frac{10}{5}\right)t} \to 0$. Therefore, the maximum current is 12 amperes.

(c) Graphing the function:



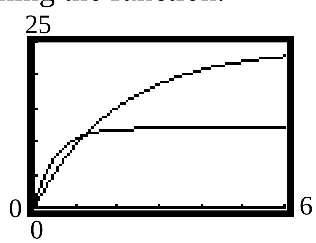
(d) $I = \frac{120}{5}\left[1 - e^{-\left(\frac{5}{10}\right)0.3}\right] = 24\left[1 - e^{-0.15}\right] = 3.343$ amperes after 0.3 second

$I = \frac{120}{5}\left[1 - e^{-\left(\frac{5}{10}\right)0.5}\right] = 24\left[1 - e^{-0.25}\right] = 5.309$ amperes after 0.5 second

$I = \frac{120}{5}\left[1 - e^{-\left(\frac{5}{10}\right)1}\right] = 24\left[1 - e^{-0.5}\right] = 9.443$ amperes after 1 second

(e) As $t \to \infty$, $e^{-\left(\frac{5}{10}\right)t} \to 0$. Therefore, the maximum current is 24 amperes.

(f) Graphing the function:



70. $I = \frac{E}{R}\, e^{\left(\frac{-t}{RC}\right)}$

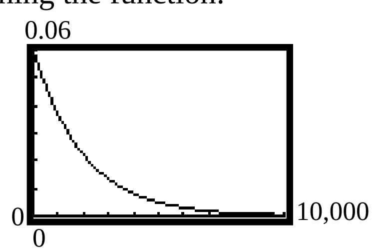
(a) $I = \frac{120}{2000}\, e^{\left(\frac{-0}{2000 \cdot 1}\right)} = \frac{120}{2000} e^{0} = 0.06$ amperes initially.

$I = \frac{120}{2000}\, e^{\left(\frac{-1000}{2000 \cdot 1}\right)} = \frac{120}{2000} e^{-\frac{1}{2}} = 0.0364$ amperes after 1000 microseconds

$I = \frac{120}{2000}\, e^{\left(\frac{-3000}{2000 \cdot 1}\right)} = \frac{120}{2000} e^{-1.5} = 0.0134$ amperes after 3000 microseconds

(b) The maximum current occurs at $t = 0$. Therefore, the maximum current is 0.06 amperes.

(c) Graphing the function:



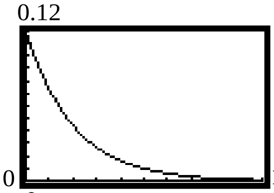
(d) $I = \dfrac{120}{1000}\ e^{\left(\frac{-0}{1000\ 2}\right)} = \dfrac{120}{1000}e^{0} = 0.12$ amperes initially.

$I = \dfrac{120}{1000}\ e^{\left(\frac{-1000}{1000\ 2}\right)} = \dfrac{120}{1000}e^{-\frac{1}{2}} = 0.0728$ amperes after 1000 microseconds

$I = \dfrac{120}{1000}\ e^{\left(\frac{-3000}{1000\ 2}\right)} = \dfrac{120}{1000}e^{-1.5} = 0.0268$ amperes after 3000 microseconds

(e) The maximum current occurs at $t = 0$. Therefore, the maximum current is 0.12 amperes.

(f) Graphing the function:



71. $2 + \dfrac{1}{2!} + \dfrac{1}{3!} + \dfrac{1}{4!} + \ldots + \dfrac{1}{n!}$

$n = 4;\quad 2 + \dfrac{1}{2!} + \dfrac{1}{3!} + \dfrac{1}{4!} = 2.7083$

$n = 6;\quad 2 + \dfrac{1}{2!} + \dfrac{1}{3!} + \dfrac{1}{4!} + \dfrac{1}{5!} + \dfrac{1}{6!} = 2.7181$

$n = 8;\quad 2 + \dfrac{1}{2!} + \dfrac{1}{3!} + \dfrac{1}{4!} + \dfrac{1}{5!} + \dfrac{1}{6!} + \dfrac{1}{7!} + \dfrac{1}{8!} = 2.7182788$

$n = 10;\quad 2 + \dfrac{1}{2!} + \dfrac{1}{3!} + \dfrac{1}{4!} + \dfrac{1}{5!} + \dfrac{1}{6!} + \dfrac{1}{7!} + \dfrac{1}{8!} + \dfrac{1}{9!} + \dfrac{1}{10!} = 2.7182818$

$e = 2.718281828$

72. For $n = 2$ $\quad 2 + \dfrac{1}{1 + \frac{1}{2}} = 2.66667$

$n = 3$ $\quad 2 + \dfrac{1}{1 + \dfrac{1}{2 + \frac{2}{3}}} = 2.72727$

$n = 4 \qquad \approx 2.71698$

$n = 5 \qquad \approx 2.71845$

$n = 6 \qquad \approx 2.71826$

$e = 2.718281828$

73. $f(x) = a^x$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{a^{x+h} - a^x}{h} = \dfrac{a^x a^h - a^x}{h} = \dfrac{a^x\left(a^h - 1\right)}{h} = a^x\left(\dfrac{a^h - 1}{h}\right)$

74. $f(x) = a^x$

$f(A + B) = a^{A+B} = a^A\ \ a^B = f(A)\ \ f(B)$

75. $f(x) = a^x$

$f(-x) = a^{-x} = \dfrac{1}{a^x} = \dfrac{1}{f(x)}$

76. $f(x) = a^x$

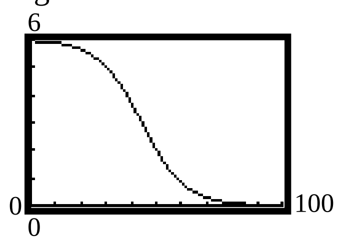
$f(\alpha x) = a^{\alpha x} = \left(a^x\right)^{\alpha} = [f(x)]^{\alpha}$

77. (a) $y = \dfrac{6}{1 + e^{-(5.085 - 0.1156(100))}} = 0.0092$ O-rings

(b) $y = \dfrac{6}{1 + e^{-(5.085 - 0.1156(60))}} = 0.8145$ O-rings

(c) $y = \dfrac{6}{1 + e^{-(5.085 - 0.1156(30))}} = 5.0063$ O-rings

(d) Graphing:



At 58˚F, there would be 1 leaky O-ring.
At 44˚F, there would be 3 leaky O-rings.
At 30˚F, there would be 5 leaky O-rings.

78. $f(x) = 2^{\left(2^x\right)} + 1$

$f(1) = 2^{\left(2^1\right)} + 1 = 2^2 + 1 = 4 + 1 = 5$

$f(2) = 2^{\left(2^2\right)} + 1 = 2^4 + 1 = 16 + 1 = 17$

$f(3) = 2^{\left(2^3\right)} + 1 = 2^8 + 1 = 256 + 1 = 257$

$f(4) = 2^{\left(2^4\right)} + 1 = 2^{16} + 1 = 65536 + 1 = 65537$

$f(5) = 2^{\left(2^5\right)} + 1 = 2^{32} + 1 = 4,294,967,296 + 1 = 4,294,967,297$

$4,294,967,297 = 641 \times 6,700,417$

79. We can use the function $f(t) = f(0)e^{kt}$

the number of bacteria doubles every minute means

$f(1) = 2f(0)$

$f(0)e^{k(1)} = 2f(0)$

$e^k = 2$

the container is full after 60 minutes means

$f(60) = 4$

$f(0)e^{k(60)} = 4$

$$f(0)\left(e^{k}\right)^{(60)} = 4$$

$$f(0)(2)^{(60)} = 4$$

$$\rightarrow\ f(0) = \frac{4}{2^{60}} = \frac{2^{2}}{2^{60}} = \frac{1}{2^{58}}$$

We want to find t so that $f(t) = 2$.

$$f(t) = f(0)e^{kt} = 2$$

$$f(0)\left(e^{k}\right)^{t} = 2$$

$$f(0)(2)^{t} = 2$$

$$\left(\frac{1}{2^{58}}\right)(2)^{t} = 2 \rightarrow 2^{t} = 2\ \ 2^{58} = 2^{59} \rightarrow t = 59 \text{ minutes}$$

80. Answers will vary.
81. Answers will vary.