Chapter 6

Exponential and Logarithmic Functions

6.4 Properties of Logarithms; Exponential and Logarithmic Models

1. $\log_3 3^{71} = 71$

2. $\log_2 2^{-13} = -13$

3. $\ln e^{-4} = -4$

4. $\ln e^{\sqrt{2}} = \sqrt{2}$

5. $2^{\log_2 7} = 7$

6. $e^{\ln 8} = 8$

7. $\log_8 2 + \log_8 4 = \log_8(4 \cdot 2) = \log_8(8) = 1$

8. $\log_6 9 + \log_6 4 = \log_6(9 \cdot 4) = \log_6(36) = \log_6(6^2) = 2$

9. $\log_6 18 - \log_6 3 = \log_6\left(\frac{18}{3}\right) = \log_6(6) = 1$

10. $\log_8 16 - \log_8 2 = \log_8\left(\frac{16}{2}\right) = \log_8(8) = 1$

11. $\log_2 6 \cdot \log_6 4$

$$= \log_6\left(4^{\log_2 6}\right) = \log_6\left(\left(2^2\right)^{\log_2 6}\right)$$

$$= \log_6\left((2)^{2\log_2 6}\right) = \log_6\left((2)^{\log_2 6^2}\right) = \log_6\left(6^2\right) = 2$$

12. $\log_3 8 \cdot \log_8 9$

$$= \log_8\left(9^{\log_3 8}\right) = \log_8\left(\left(3^2\right)^{\log_3 8}\right)$$

$$= \log_8\left((3)^{2\log_3 8}\right) = \log_8\left((3)^{\log_3 8^2}\right) = \log_8\left(8^2\right) = 2$$

13. $3^{\log_3 5 - \log_3 4} = 3^{\log_3\left(\frac{5}{4}\right)} = \frac{5}{4}$

14. $5^{\log_5 6 + \log_5 7} = 5^{\log_5(6 \cdot 7)} = 6 \cdot 7 = 42$

15. $e^{\log_{e^2} 16}$

Simplify the exponent:

$$\begin{aligned} \text{Let} \quad a &= \log_{e^2} 16 \\ \left(e^2\right)^a &= 16 \\ e^{2a} &= 16 = 4^2 \\ e^a &= 4 \\ a &= \ln 4 \end{aligned}$$

Thus, $e^{\log_{e^2} 16} = e^{\ln 4} = 4$

16. $e^{\log_{e^2} 9}$

Simplify the exponent:

$$\begin{aligned} \text{Let} \quad a &= \log_{e^2} 9 \\ \left(e^2\right)^a &= 9 \\ e^{2a} &= 9 = 3^2 \\ e^a &= 3 \\ a &= \ln 3 \end{aligned}$$

Thus, $e^{\log_{e^2} 9} = e^{\ln 3} = 3$

17. $\ln 6 = \ln(3 \cdot 2) = \ln 3 + \ln 2 = b + a$

18. $\ln \frac{2}{3} = \ln 2 - \ln 3 = a - b$

19. $\ln 1.5 = \ln \frac{3}{2} = \ln 3 - \ln 2 = b - a$

20. $\ln 0.5 = \ln \frac{1}{2} = \ln 1 - \ln 2 = 0 - a = -a$

21. $\ln 8 = \ln 2^3 = 3 \cdot \ln 2 = 3a$

22. $\ln 27 = \ln 3^3 = 3 \cdot \ln 3 = 3b$

23. $\ln \sqrt[5]{6} = \ln 6^{1/5} = \frac{1}{5} \cdot \ln 6 = \frac{1}{5} \cdot \ln(2 \cdot 3) = \frac{1}{5} \cdot (\ln 2 + \ln 3) = \frac{1}{5} \cdot (a + b)$

24. $\ln \sqrt[4]{\left(\frac{2}{3}\right)} = \ln\left(\frac{2}{3}\right)^{1/4} = \frac{1}{4} \cdot \ln\left(\frac{2}{3}\right) = \frac{1}{4} \cdot (\ln 2 - \ln 3) = \frac{1}{4} \cdot (a - b)$

25. $\log_a\left(u^2 v^3\right) = \log_a u^2 + \log_a v^3 = 2\log_a u + 3\log_a v$

26. $\log_2\left(\frac{a}{b^2}\right) = \log_2 a - \log_2 b^2 = \log_2 a - 2\log_2 b$

27. $\log \frac{1}{M^3} = \log M^{-3} = -3\log M$

28. $\log\left(10u^2\right) = \log 10 + \log u^2 = 1 + 2\log u$

29. $\log_5 \sqrt{\frac{a^3}{b}} = \log_5\left(\frac{a^3}{b}\right)^{\frac{1}{2}} = \log_5 \frac{a^{\frac{3}{2}}}{b^{\frac{1}{2}}} = \log_5 a^{\frac{3}{2}} - \log_5 b^{\frac{1}{2}} = \frac{3}{2}\log_5 a - \frac{1}{2}\log_5 b$

30. $\log_6\left(\frac{ab^4}{\sqrt[3]{c^2}}\right) = \log_6\left(ab^4\right) - \log_6\left(c^2\right)^{\frac{1}{3}} = \log_6 a + \log_6 b^4 - \log_6 c^{\frac{2}{3}}$

$= \log_6 a + 4\log_6 b - \frac{2}{3}\log_6 c$

31. $\ln\left(x^2\sqrt{1-x}\right) = \ln x^2 + \ln\sqrt{1-x} = \ln x^2 + \ln(1-x)^{\frac{1}{2}} = 2\ln x + \frac{1}{2}\ln(1-x)$

32. $\ln\left(x\sqrt{1+x^2}\right) = \ln x + \ln\sqrt{1+x^2} = \ln x + \ln(1+x^2)^{\frac{1}{2}} = \ln x + \frac{1}{2}\ln(1+x^2)$

33. $\log_2\left(\frac{x^3}{x-3}\right) = \log_2 x^3 - \log_2(x-3) = 3\log_2 x - \log_2(x-3)$

34. $\log_5\left(\frac{\sqrt[3]{x^2+1}}{x^2-1}\right) = \log_5\left(x^2+1\right)^{\frac{1}{3}} - \log_5(x^2-1) = \frac{1}{3}\log_5\left(x^2+1\right) - \log_5\left(x^2-1\right)$

35. $\log\left[\frac{x(x+2)}{(x+3)^2}\right] = \log x(x+2) - \log(x+3)^2 = \log x + \log(x+2) - 2\log(x+3)$

36. $\log\left[\frac{x^3\sqrt{x+1}}{(x-2)^2}\right] = \log x^3\sqrt{x+1} - \log(x-2)^2 = \log x^3 + \log(x+1)^{\frac{1}{2}} - 2\log(x-2)$

$= 3\log x + \frac{1}{2}\log(x+1) - 2\log(x-2)$

37. $\ln\left[\frac{x^2-x-2}{(x+4)^2}\right]^{\frac{1}{3}} = \frac{1}{3}\ln\left[\frac{(x-2)(x+1)}{(x+4)^2}\right] = \frac{1}{3}\left[\ln(x-2)(x+1) - \ln(x+4)^2\right]$

$= \frac{1}{3}\left[\ln(x-2) + \ln(x+1) - 2\ln(x+4)\right] = \frac{1}{3}\ln(x-2) + \frac{1}{3}\ln(x+1) - \frac{2}{3}\ln(x+4)$

38. $\ln\left[\frac{(x-4)^2}{x^2-1}\right]^{\frac{2}{3}} = \frac{2}{3}\ln\left[\frac{(x-4)^2}{x^2-1}\right] = \frac{2}{3}\left[\ln(x-4)^2 - \ln\left(x^2-1\right)\right]$

$= \frac{2}{3}\left[2\ln(x-4) - \ln(x-1)(x+1)\right] = \frac{2}{3}\left[2\ln(x-4) - \ln(x-1) - \ln(x+1)\right]$

39. $\ln\frac{5x\sqrt{1-3x}}{(x-4)^3} = \ln 5x\sqrt{1-3x} - \ln(x-4)^3 = \ln 5 + \ln x + \ln\sqrt{1-3x} - 3\ln(x-4)$

$= \ln 5 + \ln x + \ln(1-3x)^{\frac{1}{2}} - 3\ln(x-4) = \ln 5 + \ln x + \frac{1}{2}\ln(1-3x) - 3\ln(x-4)$

40. $\ln\left[\frac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right] = \ln\left(5x^2\sqrt[3]{1-x}\right) - \ln\left(4(x+1)^2\right)$

$= \ln 5 + \ln x^2 + \ln(1-x)^{\frac{1}{3}} - \left[\ln 4 + \ln(x+1)^2\right]$

$= \ln 5 + 2\ln x + \frac{1}{3}\ln(1-x) - \ln 4 - 2\ln(x+1)$

41. $3\log_5 u + 4\log_5 v = \log_5 u^3 + \log_5 v^4 = \log_5(u^3v^4)$

42. $\log_3 u^2 - \log_3 v = \log_3\left(\frac{u^2}{v}\right)$

43. $\log_{\frac{1}{2}}\sqrt{x} - \log_{\frac{1}{2}}x^3 = \log_{\frac{1}{2}}\frac{\sqrt{x}}{x^3} = \log_{\frac{1}{2}}\frac{x^{\frac{1}{2}}}{x^3} = \log_{\frac{1}{2}}x^{\frac{-5}{2}} = \frac{-5}{2}\log_{\frac{1}{2}}x$

44. $\log_2\left(\frac{1}{x}\right)+\log_2\left(\frac{1}{x^2}\right)=\log_2\left(\frac{1}{x}\cdot\frac{1}{x^2}\right)=\log_2\left(\frac{1}{x^3}\right)=\log_2 x^{-3}=-3\log_2 x$

45. $\ln\frac{x}{x-1}+\ln\frac{x+1}{x}-\ln\left(x^2-1\right)=\ln\left[\frac{x}{x-1}\cdot\frac{x+1}{x}\right]-\ln\left(x^2-1\right)=\ln\left[\frac{x+1}{x-1}\div\left(x^2-1\right)\right]$

$=\ln\left[\frac{x+1}{(x-1)(x-1)(x+1)}\right]=\ln\frac{1}{(x-1)^2}=\ln(x-1)^{-2}=-2\ln(x-1)$

46. $\log\left(\frac{x^2+2x-3}{x^2-4}\right)-\log\left(\frac{x^2+7x+6}{x+2}\right)=\log\frac{\frac{x^2+2x-3}{x^2-4}}{\frac{x^2+7x+6}{x+2}}$

$=\log\left(\frac{(x+3)(x-1)}{(x-2)(x+2)}\cdot\frac{x+2}{(x+6)(x+1)}\right)=\log\left(\frac{(x+3)(x-1)}{(x-2)(x+6)(x+1)}\right)$

47. $8\log_2\sqrt{3x-2}-\log_2\frac{4}{x}+\log_2 4=\log_2\left(\sqrt{3x-2}\right)^8-\left(\log_2 4-\log_2 x\right)+\log_2 4$

$=\log_2(3x-2)^4-\log_2 4+\log_2 x+\log_2 4=\log_2\left[x(3x-2)^4\right]$

48. $21\log_3\sqrt[3]{x}+\log_3\left(9x^2\right)-\log_3 25=\log_3\left(x^{\frac{1}{3}}\right)^{21}+\log_3\left(9x^2\right)-\log_3 25$

$=\log_3\left(x^7\cdot 9x^2\right)-\log_3 25=\log_3\left(\frac{9x^9}{25}\right)$

49. $2\log_a 5x^3-\frac{1}{2}\log_a(2x+3)=\log_a\left(5x^3\right)^2-\log_a(2x-3)^{\frac{1}{2}}=\log_a\left[\frac{25x^6}{(2x-3)^{\frac{1}{2}}}\right]$

50. $\frac{1}{3}\log\left(x^3+1\right)+\frac{1}{2}\log\left(x^2+1\right)=\log\left(x^3+1\right)^{\frac{1}{3}}+\log\left(x^2+1\right)^{\frac{1}{2}}=\log\left[\sqrt[3]{x^3+1}\cdot\sqrt{x^2+1}\right]$

51. $\log_3 21=\frac{\log 21}{\log 3}=\frac{1.32222}{0.47712}=2.771$

52. $\log_5 18=\frac{\log 18}{\log 5}=\frac{1.25527}{0.69897}=1.796$

53. $\log_{\frac{1}{3}}71=\frac{\log 71}{\log\frac{1}{3}}=\frac{\log 71}{-\log 3}=\frac{1.85126}{-0.47712}=-3.880$

54. $\log_{\frac{1}{2}}15=\frac{\log 15}{\log\frac{1}{2}}=\frac{\log 15}{-\log 2}=\frac{1.17609}{-0.30103}=-3.907$

55. $\log_{\sqrt{2}}7=\frac{\log 7}{\log\sqrt{2}}=\frac{\log 7}{\log 2^{\frac{1}{2}}}=\frac{\log 7}{\frac{1}{2}\log 2}=\frac{0.84510}{0.5(0.30103)}=5.615$

56. $\log_{\sqrt{5}} 8 = \dfrac{\log 8}{\log \sqrt{5}} = \dfrac{\log 8}{\log 5^{\frac{1}{2}}} = \dfrac{\log 8}{\frac{1}{2}\log 5} = \dfrac{0.90309}{0.5(0.69897)} = 2.584$

57. $\log_{\pi} e = \dfrac{\ln e}{\ln \pi} = \dfrac{1}{1.14473} = 0.874$

58. $\log_{\pi} \sqrt{2} = \dfrac{\ln \sqrt{2}}{\ln \pi} = \dfrac{\ln 2^{\frac{1}{2}}}{\ln \pi} = \dfrac{\frac{1}{2}\ln 2}{\ln \pi}$
$= \dfrac{0.5(0.69315)}{1.14473} = 0.303$

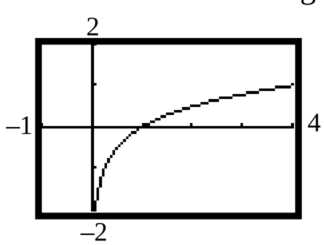
59. $\log_2 3 \;\; \log_3 4 \;\; \log_4 5 \;\; \log_5 6 \;\; \log_6 7 \;\; \log_7 8$
$= \dfrac{\log 3}{\log 2} \;\; \dfrac{\log 4}{\log 3} \dfrac{\log 5}{\log 4} \;\; \dfrac{\log 6}{\log 5} \;\; \dfrac{\log 7}{\log 6} \dfrac{\log 8}{\log 7} = \dfrac{\log 8}{\log 2} = \dfrac{\log 2^3}{\log 2} = \dfrac{3 \log 2}{\log 2} = 3$

60. $\log_2 4 \;\; \log_4 6 \;\; \log_6 8 = \dfrac{\log 4}{\log 2} \dfrac{\log 6}{\log 4} \;\; \dfrac{\log 8}{\log 6} = \dfrac{\log 8}{\log 2} = \dfrac{\log 2^3}{\log 2} = \dfrac{3 \log 2}{\log 2} = 3$

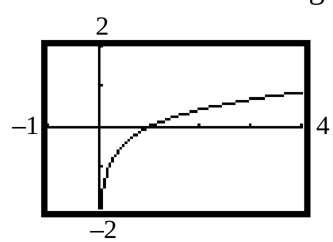
61. $\log_2 3 \;\; \log_3 4 \;\; . \; . \; . \log_n (n+1) \;\; \log_{n+1} 2$
$= \dfrac{\log 3}{\log 2} \;\; \dfrac{\log 4}{\log 3} \; . \; . \; . \dfrac{\log(n+1)}{\log n} \;\; \dfrac{\log 2}{\log(n+1)} = \dfrac{\log 2}{\log 2} = 1$

62. $\log_2 2 \;\; \log_2 4 \;\; . \; . \; . \log_2 2^n = \log_2 2 \;\; \log_2 2^2 \;\; . \; . \; . \;\; \log_2 2^n = 1 \;\; 2 \;\; 3 \;\; . \; . \; . \;\; n = n!$

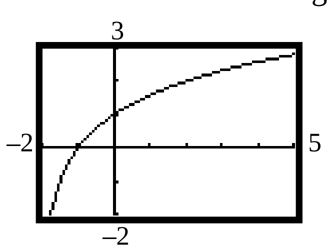
63. $y = \log_4 x = \dfrac{\ln x}{\ln 4}$ or $y = \dfrac{\log x}{\log 4}$



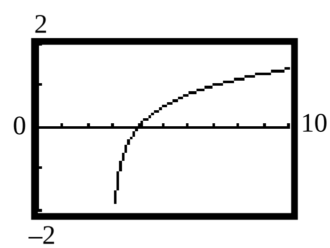
64. $y = \log_5 x = \dfrac{\ln x}{\ln 5}$ or $y = \dfrac{\log x}{\log 5}$



65. $y = \log_2(x+2) = \dfrac{\ln(x+2)}{\ln 2}$
or $y = \dfrac{\log(x+2)}{\log 2}$

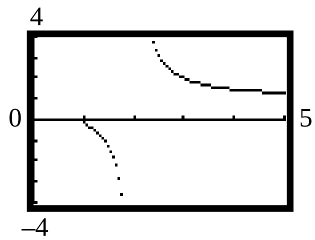


66. $y = \log_4(x-3) = \dfrac{\ln(x-3)}{\ln 4}$
or $y = \dfrac{\log(x-3)}{\log 4}$



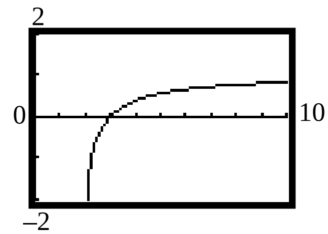
67. $y = \log_{x-1}(x+1) = \dfrac{\ln(x+1)}{\ln(x-1)}$

or $y = \dfrac{\log(x+1)}{\log(x-1)}$



68. $y = \log_{x+2}(x-2) = \dfrac{\ln(x-2)}{\ln(x+2)}$

or $y = \dfrac{\log(x-2)}{\log(x+2)}$



69. $\ln y = \ln x + \ln C$
$\ln y = \ln(xC)$
$y = Cx$

70. $\ln y = \ln(x+C)$
$y = x + C$

71. $\ln y = \ln x + \ln(x+1) + \ln C$
$\ln y = \ln\big(x(x+1)C\big)$
$y = Cx(x+1)$

72. $\ln y = 2\ln x - \ln(x+1) + \ln C$
$\ln y = \ln\left(\dfrac{x^2 C}{x+1}\right) \rightarrow y = \dfrac{Cx^2}{x+1}$

73. $\ln y = 3x + \ln C$
$\ln y = \ln e^{3x} + \ln C$
$\ln y = \ln(Ce^{3x})$
$y = Ce^{3x}$

74. $\ln y = -2x + \ln C$
$\ln y = \ln e^{-2x} + \ln C$
$\ln y = \ln(Ce^{-2x})$
$y = Ce^{-2x}$

75. $\ln(y-3) = -4x + \ln C$
$\ln(y-3) = \ln e^{-4x} + \ln C$
$\ln(y-3) = \ln\left(Ce^{-4x}\right)$
$y - 3 = Ce^{-4x} \rightarrow y = Ce^{-4x} + 3$

76. $\ln(y+4) = 5x + \ln C$
$\ln(y+4) = \ln e^{5x} + \ln C$
$\ln(y+4) = \ln\left(Ce^{5x}\right)$
$y + 4 = Ce^{5x} \rightarrow y = Ce^{5x} - 4$

77. $3\ln y = \tfrac{1}{2}\ln(2x+1) - \tfrac{1}{3}\ln(x+4) + \ln C$

$\ln y^3 = \ln(2x+1)^{\frac{1}{2}} - \ln(x+4)^{\frac{1}{3}} + \ln C$

$\ln y^3 = \ln\left[\dfrac{C(2x+1)^{\frac{1}{2}}}{(x+4)^{\frac{1}{3}}}\right]$

$y^3 = \dfrac{C(2x+1)^{\frac{1}{2}}}{(x+4)^{\frac{1}{3}}}$

$y = \left[\dfrac{C(2x+1)^{\frac{1}{2}}}{(x+4)^{\frac{1}{3}}}\right]^{\frac{1}{3}}$

$y = \dfrac{\sqrt[3]{C}(2x+1)^{\frac{1}{6}}}{(x+4)^{\frac{1}{9}}}$

78. $2\ln y = -\tfrac{1}{2}\ln x + \tfrac{1}{3}\ln\left(x^2+1\right) + \ln C$

$\ln y^2 = -\ln x^{\frac{1}{2}} + \ln\left(x^2+1\right)^{\frac{1}{3}} + \ln C$

$\ln y^2 = \ln\left[\dfrac{C\left(x^2+1\right)^{\frac{1}{3}}}{x^{\frac{1}{2}}}\right]$

$y^2 = \dfrac{C\left(x^2+1\right)^{\frac{1}{3}}}{x^{\frac{1}{2}}}$

$y = \left[\dfrac{C\left(x^2+1\right)^{\frac{1}{3}}}{x^{\frac{1}{2}}}\right]^{\frac{1}{2}}$

$y = \dfrac{\sqrt{C}\left(x^2+1\right)^{\frac{1}{6}}}{x^{\frac{1}{4}}}$

79. Verifying:

$$\log_a\left(x+\sqrt{x^2-1}\right)+\log_a\left(x-\sqrt{x^2-1}\right)=\log_a\left[\left(x+\sqrt{x^2-1}\right)\left(x-\sqrt{x^2-1}\right)\right]$$
$$=\log_a\left[x^2-\left(x^2-1\right)\right]=\log_a\left[x^2-x^2+1\right]=\log_a 1=0$$

80. Verifying:

$$\log_a\left(\sqrt{x}+\sqrt{x-1}\right)+\log_a\left(\sqrt{x}-\sqrt{x-1}\right)=\log_a\left[\left(\sqrt{x}+\sqrt{x-1}\right)\left(\sqrt{x}-\sqrt{x-1}\right)\right]$$
$$=\log_a\left[x-(x-1)\right]=\log_a[x-x+1]=\log_a 1=0$$

81. Verifying:

$$2x+\ln\left(1+e^{-2x}\right)=\ln e^{2x}+\ln\left(1+e^{-2x}\right)=\ln\left(e^{2x}\left(1+e^{-2x}\right)\right)=\ln\left(e^{2x}+e^{0}\right)=\ln\left(e^{2x}+1\right)$$

82. Verifying:

$$\frac{f(x+h)-f(x)}{h}=\frac{\log_a(x+h)-\log_a x}{h}=\frac{\log_a\left(\frac{x+h}{x}\right)}{h}=\frac{1}{h}\log_a\left(1+\frac{h}{x}\right)=\log_a\left(1+\frac{h}{x}\right)^{\frac{1}{h}}$$

83. $f(x)=\log_a x$

$$x=a^{f(x)}\quad\rightarrow\quad x^{-1}=a^{-f(x)}=\left(a^{-1}\right)^{f(x)}=\left(\frac{1}{a}\right)^{f(x)}$$
$$\log_{\frac{1}{a}} x^{-1}=f(x)\quad\rightarrow\quad -\log_{\frac{1}{a}} x=f(x)\quad\rightarrow\quad -f(x)=\log_{\frac{1}{a}} x$$

84. $f(AB)=\log_a(AB)=\log_a A+\log_a B=f(A)+f(B)$

85. $f(x)=\log_a x$

$$a^{f(x)}=x\quad\rightarrow\quad\frac{1}{a^{f(x)}}=\frac{1}{x}\quad\rightarrow\quad a^{-f(x)}=\frac{1}{x}\quad\rightarrow\quad -f(x)=\log_a\frac{1}{x}=f\left(\frac{1}{x}\right)$$

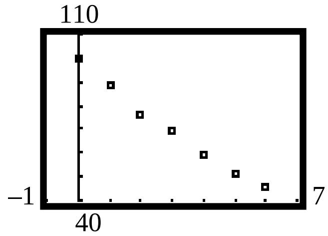
86. $f\left(x^{\alpha}\right)=\log_a x^{\alpha}=\alpha\log_a x=\alpha\, f(x)$

87. If $A=\log_a M$ and $B=\log_a N$, then $a^A=M$ and $a^B=N$.

$$\log_a\left(\frac{M}{N}\right)=\log_a\left(\frac{a^A}{a^B}\right)=\log_a a^{A-B}=A-B=\log_a M-\log_a N$$

88. $\log_a\left(\frac{1}{N}\right)=\log_a N^{-1}=-1\ \log_a N=-\log_a N,\quad a\neq 1$

89. (a) Graphing:



(b) $y = 100(0.88)^x$

$0.88 = e^{\ln(0.88)}$

$\rightarrow y = 100\left(e^{\ln(0.88)}\right)^x = 100e^{\ln(0.88)x}$

$A = A_0 e^{-0.1278t}, A_0 = 100$

(c)

$100e^{-0.1278t} = 50$

$e^{-0.1278t} = \frac{50}{100}$

$e^{-0.1278t} = 0.5$

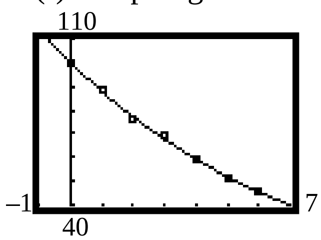
$\ln\left(e^{-0.1278t}\right) = \ln(0.5)$

$-0.1278t = \ln(0.5)$

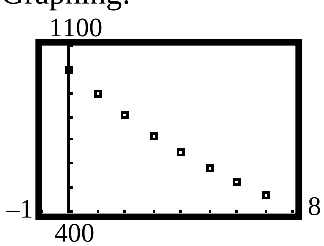
$t = \frac{\ln(0.5)}{-0.1278} \approx 5.42$ weeks

(d) $A = 100e^{-0.1278(50)}$
≈ 0.1678 grams

(e) and (f) Graphing:



90. (a) Graphing:



(b) $y = 999(0.898)^t$

$0.898 = e^{\ln(0.898)}$

$\rightarrow y = 999\left(e^{\ln(0.898)}\right)^x = 999e^{\ln(0.898)x}$

$A = A_0 e^{-0.1076t}, A_0 = 999$

(c)

$999e^{-0.1076t} = 500$

$e^{-0.1076t} = \frac{500}{999}$

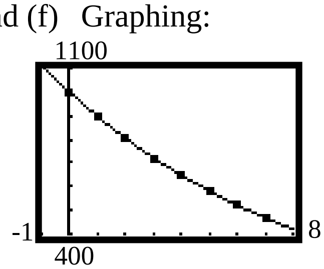
$\ln(e^{-0.1076t}) = \ln\left(\frac{500}{999}\right)$

$-0.1076t = \ln\left(\frac{500}{999}\right)$

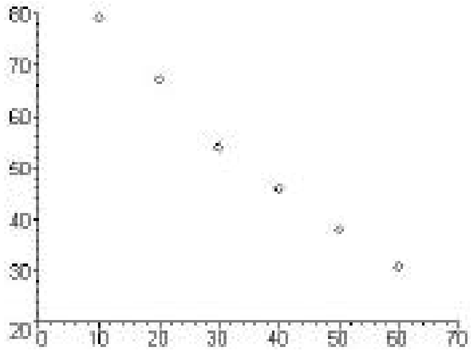
$t = \frac{\ln\left(\frac{500}{999}\right)}{-0.1076} \approx 6.43$ days

(d) $A = 999e^{-0.1076(20)}$
≈ 116.14 grams

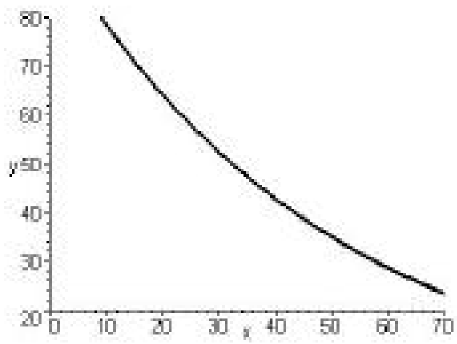
(e) and (f) Graphing:



91. (a) Graphing:



(d) and (e) Graphing



(b) $y = 96(0.98)^x$

$0.98 = e^{\ln(0.98)}$

$\to y = 96\left(e^{\ln(0.98)}\right)^x = 96e^{\ln(0.98)x}$

$A = A_0 e^{-0.0202t}, A_0 = 96$

(c)

$96e^{-0.0202t} = 60$

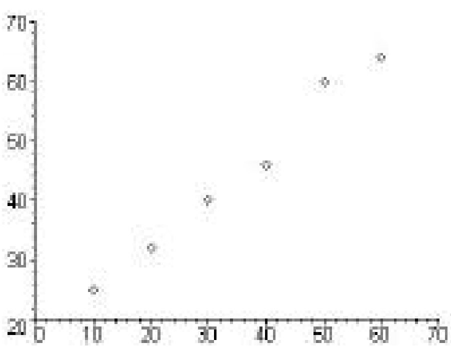
$e^{-0.0202t} = \frac{60}{96}$

$\ln(e^{-0.0202t}) = \ln\left(\frac{60}{96}\right)$

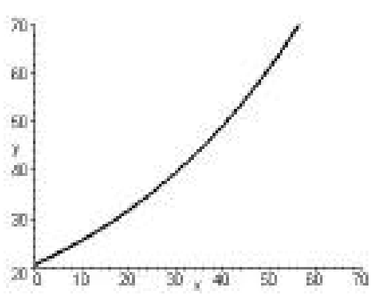
$-0.0202t = \ln\left(\frac{60}{96}\right)$

$t = \frac{\ln\left(\frac{60}{96}\right)}{-0.0202} \approx 23$ shoes

92. (a) Graphing:



(d) and (e) Graphing:



(b) $y = 20.524(1.022)^x$

$1.022 = e^{\ln(1.022)}$

$\to y = 20.524\left(e^{\ln(1.022)}\right)^x = 20.524e^{\ln(1.022)x}$

$A = A_0 e^{0.0218t}, A_0 = 20.524$

(c)

$20.524e^{0.0218t} = 45$

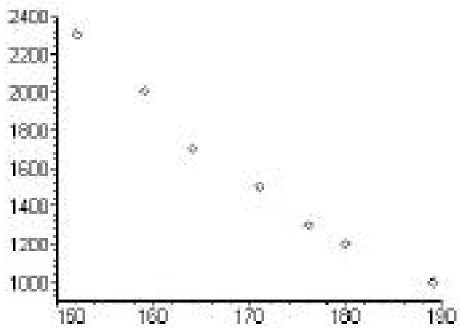
$e^{0.0218t} = \frac{45}{20.524}$

$\ln(e^{0.0218t}) = \ln\left(\frac{45}{20.524}\right)$

$0.0218t = \ln\left(\frac{45}{20.524}\right)$

$t = \frac{\ln\left(\frac{45}{20.524}\right)}{0.0218} \approx 36$ dresses

93. (a) Graphing:



(b)

$y = 32741 - 6071 \ln x$

$1650 = 32741 - 6071 \ln x$

$-31091 = -6071 \ln x$

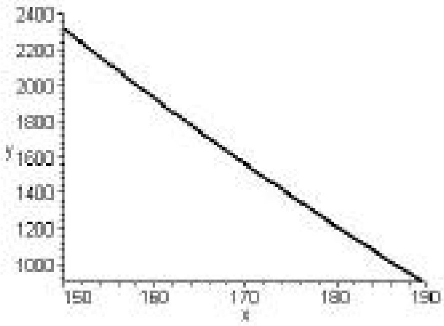
$\dfrac{-31091}{-6071} = \ln x$

$\dfrac{31091}{6071} = \ln x$

$e^{\left(\frac{31091}{6071}\right)} = e^{\ln x} = x$

$x \approx 168$ computers

(c) and (d) Graphing:



94.

domain of $f(x) = \log_a x^2$ is all real numbers $\neq 0$

domain of $g(x) = 2\log_a x$ is all real numbers > 0

These two domains are different because the logarithm property $\log_a x^n = n \log_a x$ holds only when $\log_a x$ exists