## Chapter 6

# Exponential and Logarithmic Functions

## 6.6 Compound Interest

1. $P = \$100,\ r = 0.04,\ n = 4,\ t = 2$

$$A = P\left[1 + \frac{r}{n}\right]^{nt} = 100\left[1 + \frac{0.04}{4}\right]^{(4)(2)} = \$108.29$$

2. $P = \$50,\ r = 0.06,\ n = 12,\ t = 3$

$$A = P\left[1 + \frac{r}{n}\right]^{nt} = 50\left[1 + \frac{0.06}{12}\right]^{(12)(3)} = \$59.83$$

3. $P = \$500,\ r = 0.08,\ n = 4,\ t = 2.5$

$$A = P\left[1 + \frac{r}{n}\right]^{nt} = 500\left[1 + \frac{0.08}{4}\right]^{(4)(2.5)} = \$609.50$$

4. $P = \$300,\ r = 0.12,\ n = 12,\ t = 1.5$

$$A = P\left[1 + \frac{r}{n}\right]^{nt} = 300\left[1 + \frac{0.12}{12}\right]^{(12)(1.5)} = \$358.84$$

5. $P = \$600,\ r = 0.05,\ n = 365,\ t = 3$

$$A = P\left[1 + \frac{r}{n}\right]^{nt} = 600\left[1 + \frac{0.05}{365}\right]^{(365)(3)} = \$697.09$$

6. $P = \$700,\ r = 0.06,\ n = 365,\ t = 2$

$$A = P\left[1 + \frac{r}{n}\right]^{nt} = 700\left[1 + \frac{0.06}{365}\right]^{(365)(2)} = \$789.24$$

7. $P = \$10,\ r = 0.11,\ t = 2$

$$A = Pe^{rt} = 10e^{(0.11)(2)} = \$12.46$$

8. $P = \$40,\ r = 0.07,\ t = 3$

$$A = Pe^{rt} = 40e^{(0.07)(3)} = \$49.35$$

9. $P = \$100,\ r = 0.10,\ t = 2.25$

$$A = Pe^{rt} = 100e^{(0.10)(2.25)} = \$125.23$$

10. $P = \$100,\ r = 0.12,\ t = 3.75$

$$A = Pe^{rt} = 100e^{(0.12)(3.75)} = \$156.83$$

11. $A = \$100,\ r = 0.06,\ n = 12,\ t = 2$

$$P = A\left[1+\frac{r}{n}\right]^{-nt} = 100\left[1+\frac{0.06}{12}\right]^{(-12)(2)} = \$88.72$$

12. $A = \$75,\ r = 0.08,\ n = 4,\ t = 3$

$$P = A\left[1+\frac{r}{n}\right]^{-nt} = 75\left[1+\frac{0.08}{4}\right]^{(-4)(3)} = \$59.14$$

13. $A = \$1000,\ r = 0.06,\ n = 365,\ t = 2.5$

$$P = A\left[1+\frac{r}{n}\right]^{-nt} = 1000\left[1+\frac{0.06}{365}\right]^{(-365)(2.5)} = \$860.72$$

14. $A = \$800,\ r = 0.07,\ n = 12,\ t = 3.5$

$$P = A\left[1+\frac{r}{n}\right]^{-nt} = 800\left[1+\frac{0.07}{12}\right]^{(-12)(3.5)} = \$626.61$$

15. $A = \$600,\ r = 0.04,\ n = 4,\ t = 2$

$$P = A\left[1+\frac{r}{n}\right]^{-nt} = 600\left[1+\frac{0.04}{4}\right]^{(-4)(2)} = \$554.09$$

16. $A = \$300,\ r = 0.03,\ n = 365,\ t = 4$

$$P = A\left[1+\frac{r}{n}\right]^{-nt} = 300\left[1+\frac{0.03}{365}\right]^{(-365)(4)} = \$266.08$$

17. $A = \$80,\ r = 0.09,\ t = 3.25$

$$P = Ae^{-rt} = 80e^{(-0.09)(3.25)} = \$59.71$$

18. $A = \$800,\ r = 0.08,\ t = 2.5$

$$P = Ae^{-rt} = 800e^{(-0.08)(2.5)} = \$654.98$$

19. $A = \$400,\ r = 0.10,\ t = 1$

$$P = Ae^{-rt} = 400e^{(-0.10)(1)} = \$361.93$$

20. $A = \$1000,\ r = 0.12,\ t = 1$

$$P = Ae^{-rt} = 1000e^{(-0.12)(1)} = \$886.92$$

21. $r_e = \left[1+\frac{r}{n}\right]^{n} - 1 = \left[1+\frac{0.0525}{4}\right]^{4} - 1 = 1.0535 - 1 = 0.0535 = 5.35\%$

22. $0.07 = \left[1+\frac{r}{4}\right]^{4} - 1$

$$1.07 = \left(1+\frac{r}{4}\right)^{4} \rightarrow 1+\frac{r}{4} = 1.0170585 \rightarrow \frac{r}{4} = 0.0170585 \rightarrow r \approx 6.82\%$$

23. $2P = P(1+r)^3$
$2 = (1+r)^3$
$\sqrt[3]{2} = 1+r$
$r = \sqrt[3]{2} - 1 \approx 1.26 - 1 = 0.26 = 26\%$

24. $2P = P(1+r)^{10}$
$2 = (1+r)^{10} \rightarrow \sqrt[10]{2} = 1+r \rightarrow r = \sqrt[10]{2} - 1 \approx 1.0718 - 1 = 0.0718 = 7.18\%$

25. 6% compounded quarterly:
$$A = 10,000\left[1 + \frac{0.06}{4}\right]^{(4)(1)} = \$10,613.64$$
$6\frac{1}{4}\%$ compounded annually:
$$A = 10,000[1 + 0.0625]^1 = \$10,625$$
$6\frac{1}{4}\%$ compounded annually yields the larger amount.

26. 9% compounded quarterly:
$$A = 10,000\left[1 + \frac{0.09}{4}\right]^{(4)(1)} = \$10,930.83$$
$9\frac{1}{4}\%$ compounded annually:
$$A = 10,000[1 + 0.0925]^1 = \$10,925$$
9% compounded quarterly yields the larger amount.

27. 9% compounded monthly:
$$A = 10,000\left[1 + \frac{0.09}{12}\right]^{(12)(1)} = \$10,938.07$$
8.8% compounded daily:
$$A = 10,000\left[1 + \frac{0.088}{365}\right]^{365} = \$10,919.77$$
9% compounded monthly yields the larger amount.

28. 8% compounded semiannually:
$$A = 10,000\left[1 + \frac{0.08}{2}\right]^{(2)(1)} = \$10,816$$
7.9% compounded daily:
$$A = 10,000\left[1 + \frac{0.079}{365}\right]^{365} = \$10,821.95$$
7.9% compounded daily yields the larger amount.

29. Compounded monthly:
$$2P = P\left[1 + \frac{0.08}{12}\right]^{12t}$$
$$2 = (1.00667)^{12t}$$
$$\ln 2 = 12t \ln(1.00667)$$
$$t = \frac{\ln 2}{12\ln(1.00667)} \approx 8.69 \text{ years}$$

Compounded continuously:
$$2P = Pe^{0.08t}$$
$$2 = e^{0.08t}$$
$$\ln 2 = 0.08t$$
$$t = \frac{\ln 2}{0.08} \approx 8.66 \text{ years}$$

30. Compounded monthly:

$$2P = P\left[1+\frac{0.10}{12}\right]^{12t}$$
$$2 = (1.00833)^{12t}$$
$$\ln 2 = 12t\ln(1.00833)$$
$$t = \frac{\ln 2}{12\ln(1.00833)} \approx 6.96 \text{ years}$$

Compounded continuously:

$$2P = Pe^{0.10t}$$
$$2 = e^{0.10t}$$
$$\ln 2 = 0.10t$$
$$t = \frac{\ln 2}{0.10} \approx 6.93 \text{ years}$$

31. Compounded monthly:

$$150 = 100\left[1+\frac{0.08}{12}\right]^{12t}$$
$$1.5 = (1.00667)^{12t}$$
$$\ln 1.5 = 12t\ln(1.00667)$$
$$t = \frac{\ln 1.5}{12\ln(1.00667)} \approx 5.083 \text{ years}$$

Compounded continuously:

$$150 = 100e^{0.08t}$$
$$1.5 = e^{0.08t}$$
$$\ln 1.5 = 0.08t$$
$$t = \frac{\ln 1.5}{0.08} \approx 5.068 \text{ years}$$

32. Compounded monthly:

$$175 = 100\left[1+\frac{0.10}{12}\right]^{12t}$$
$$1.75 = (1.00833)^{12t}$$
$$\ln 1.75 = 12t\ln(1.00833)$$
$$t = \frac{\ln 1.75}{12\ln(1.00833)} \approx 5.62 \text{ years}$$

Compounded continuously:

$$175 = 100e^{0.10t}$$
$$1.75 = e^{0.10t}$$
$$\ln 1.75 = 0.10t$$
$$t = \frac{\ln 1.75}{0.10} \approx 5.60 \text{ years}$$

33.

$$25{,}000 = 10{,}000e^{0.06t}$$
$$2.5 = e^{0.06t}$$
$$\ln 2.5 = 0.06t$$
$$t = \frac{\ln 2.5}{0.06} \approx 15.27 \text{ years}$$

34.

$$80{,}000 = 25{,}000e^{0.07t}$$
$$3.2 = e^{0.07t}$$
$$\ln 3.2 = 0.07t$$
$$t = \frac{\ln 3.2}{0.07} \approx 16.62 \text{ year}$$

35. $A = 90{,}000(1+0.03)^5 = \$104{,}335$

36. $A = 200(1+0.0125)^5 = \$212.82$ (You get a 1 month grace period.)

37. $P = 15{,}000e^{(-0.05)(3)} = \$12{,}910.6$

38. $P = 3{,}000\left(1+\frac{0.03}{12}\right)^{(-12)(0.5)} = \$2{,}955.39$

39. $A = 1500(1+0.15)^5 = 1500(1.15)^5 = \$3017$

40. $20 = 15(1+r)^2 \rightarrow 1.333 = (1+r)^2 \rightarrow 1.155 = 1+r \rightarrow r = 0.155 \approx 15.5\%$

41. $850,000 = 650,000(1+r)^3$

$1.3077 = (1+r)^3$

$\sqrt[3]{1.3077} = 1 + r \rightarrow r = \sqrt[3]{1.3077} - 1 \approx 0.0935 = 9.35\%$

42. $A = 5000(1+0.08)^{-10} = \$2,315.97$

43. 5.6% compounded continuously:

$A = 1000e^{(0.056)(1)} = \$1057.60$

Jim does not have enough money to buy the computer.

5.9% compounded monthly:

$A = 1000\left[1 + \frac{0.059}{12}\right]^{12} = \$1060.62$

The second bank offers the better deal.

44. 6.8% compounded continuously for 3 months:

$A = 1000e^{(0.068)(0.25)} = \$1017.15$ -- Amount on April 1.

5.25% compounded monthly for 1 month:

$A = 1017.15\left[1 + \frac{0.0525}{12}\right]^{(12)(1/12)} = \$1021.60$ -- Amount on May 1.

45. Will - 9% compounded semiannually:

$A = 2000\left[1 + \frac{0.09}{2}\right]^{(2)(20)} = \$11,632.73$

Henry - 8.5% compounded continuously:

$A = 2000e^{(0.085)(20)} = \$10,947.89$

Will has more money after 20 years.

46. Value of $1000 compounded continuously at 10% for 3 years:

$A = 1000e^{(0.10)(3)} = \$1349.86$

April will have more money if she takes the $1000 now and invests it.

47. $P = 50,000; t = 5$

(a) Simple interest at 12% per annum:

$A = 50,000 + 50,000(0.12)(5) = \$80,000$

(b) 11.5% compounded monthly:

$A = 50,000\left[1 + \frac{0.115}{12}\right]^{(12)(5)} = \$88,613.59$

(c) 11.25% compounded continuously:

$A = 50,000e^{(0.1125)(5)} = \$87,752.73$

Subtract $50,000 from each to get the amount of interest:

(a) $30,000

(b) $38,613.59

(c) $37.752.73

Option (a) results in the least interest.

48. (a) 360 day year:

$$r_e = \left(1 + \frac{0.0425}{360}\right)^{360} - 1 = 1.043413439 - 1 = 0.043413439 = 4.3413439\%$$

(b) 365 day year:

$$r_e = \left(1 + \frac{0.0425}{365}\right)^{365} - 1 = 1.043413475 - 1 = 0.043413475 = 4.3413475\%$$

49. (a) $A = \$10{,}000,\ r = 0.10,\ n = 12,\ t = 20$ (compounded monthly)

$$P = 10{,}000\left[1 + \frac{0.10}{12}\right]^{(-12)(20)} = \$1364.62$$

(b) $A = \$10{,}000,\ r = 0.10,\ t = 20$ (compounded continuously)

$$P = 10{,}000e^{(-0.10)(20)} = \$1353.35$$

50. $A = \$40{,}000,\ r = 0.08,\ n = 1,\ t = 17$ (compounded annually)

$$P = 40{,}000[1 + 0.08]^{-17} = \$10{,}810.76$$

51. $A = \$10{,}000,\ r = 0.08,\ n = 1,\ t = 10$ (compounded annually)

$$P = 10{,}000\left[1 + \frac{0.08}{1}\right]^{(-1)(10)} = \$4631.93$$

52. $A = \$25{,}000,\ P = 12{,}485.52,\ n = 1,\ t = 8$ (compounded annually)

$$25{,}000 = 12{,}485.52(1 + r)^8$$
$$2.002319487 = (1 + r)^8$$
$$1.090665741 = 1 + r$$
$$r = 0.090665741 \approx 9.07\%$$

53. (a) $y = \dfrac{\ln 2}{1\ \ln\left(1 + \frac{0.12}{1}\right)} = \dfrac{\ln 2}{\ln 1.12} = 6.12$ years

(b) $y = \dfrac{\ln 3}{4\ \ln\left(1 + \frac{0.06}{4}\right)} = \dfrac{\ln 3}{4 \ln 1.015} = 18.45$ years

(c) $mP = P\left[1 + \dfrac{r}{n}\right]^{nt}$

$$m = \left[1 + \frac{r}{n}\right]^{nt} \quad \to \quad \ln m = n\,t\,\ln\left[1 + \frac{r}{n}\right] \quad \to \quad t = \frac{\ln m}{n\ \ln\left[1 + \frac{r}{n}\right]}$$

54. (a) $y = \dfrac{\ln 8000 - \ln 1000}{0.10} = 20.79$ years

(b) $35 = \dfrac{\ln 30000 - \ln 2000}{r} \quad \to \quad r = \dfrac{\ln 30000 - \ln 2000}{35} = 0.0774 \approx 7.74\%$

(c)

$$A = Pe^{rt}$$
$$\frac{A}{P} = e^{rt}$$
$$\ln\left(\frac{A}{P}\right) = rt$$
$$\ln A - \ln P = rt$$
$$t = \frac{\ln A - \ln P}{r}$$

55. Answers will vary.

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