Chapter 6

Exponential and Logarithmic Functions

6.8 Logarithmic Scales

1. $L\left(10^{-5}\right) = 10 \log \dfrac{10^{-5}}{10^{-12}} = 10 \log 10^{7} = 10 \cdot 7 = 70$ decibels

2. $L\left(10^{-3}\right) = 10 \log \dfrac{10^{-3}}{10^{-12}} = 10 \log 10^{9} = 10 \cdot 9 = 90$ decibels

3. $L(0.15) = 10 \log \dfrac{1.5 \times 10^{-1}}{10^{-12}} = 10 \log \left(1.5 \times 10^{11}\right)$

   $= 10\left(\log (1.5) + \log\left(10^{11}\right)\right) = 10\left(\log (1.5) + 11\right) \approx 111.76$ decibels

4. $L\left(10^{-9.8}\right) = 10 \log \dfrac{10^{-9.8}}{10^{-12}} = 10 \log 10^{2.2} = 10 \cdot 2.2 = 22$ decibels

5. $L(x) = 10 \log\left(\dfrac{x}{10^{-12}}\right) = 130$ decibels

   $10 \log\left(\dfrac{x}{10^{-12}}\right) = 130$

   $\log\left(\dfrac{x}{10^{-12}}\right) = 13$

   $\log\left(x \cdot 10^{12}\right) = 13$

   $\log(x) + \log\left(10^{12}\right) = 13$

   $\log(x) + 12 = 13$

   $\log(x) = 1$

   $10^{\log(x)} = 10^{1}$

   $x = 10^{1} = 10$ watts per square meter

6. We compute $L(50x)-L(x)$

$$L(50x)-L(x)=10\log\left(\frac{50x}{10^{-12}}\right)-10\log\left(\frac{x}{10^{-12}}\right)=10\left[\log\left(\frac{50x}{10^{-12}}\right)-\log\left(\frac{x}{10^{-12}}\right)\right]$$

$$=10\log\left(\frac{\frac{50x}{10^{-12}}}{\frac{x}{10^{-12}}}\right)=10\log\left(\frac{50x}{10^{-12}}\cdot\frac{10^{-12}}{x}\right)=10\log(50)\approx 16.99 \text{ decibels}$$

7. $M(10)=\log\dfrac{10}{10^{-3}}=\log 10^{4}=4$

8. $M(1210)=\log\dfrac{1210}{10^{-3}}=\log(1210000)\approx 6.08$

9. Mexico City:

$\log(x\ 10^{3})=8.1$

$\log(x)+\log(10^{3})=8.1$

$\log(x)+3=8.1$

$\log(x)=5.1$

$10^{\log(x)}=10^{5.1}$

$x=10^{5.1}\approx 125892.54$

San Francisco: $M(x)=\log\dfrac{x}{10^{-3}}=6.9$

$\log(x\ 10^{3})=6.9$

$\log(x)+\log(10^{3})=6.9$

$\log(x)+3=6.9$

$\log(x)=3.9$

$10^{\log(x)}=10^{3.9}$

$x=10^{3.9}\approx 7943.28$

10. We compute $M(x_1)-M(x_2)=1$

$$\log\left(\frac{x_1}{10^{-3}}\right)-\log\left(\frac{x_2}{10^{-3}}\right)=1\rightarrow\log\left(\frac{\frac{x_1}{10^{-3}}}{\frac{x_2}{10^{-3}}}\right)=1$$

$$\log\left(\frac{x_1}{10^{-3}}\cdot\frac{10^{-3}}{x_2}\right)=1\rightarrow\log\left(\frac{x_1}{x_2}\right)=1$$

$$10^{\log\left(\frac{x_1}{x_2}\right)}=10\rightarrow\frac{x_1}{x_2}=10\rightarrow x_1=10x_2\rightarrow x_1-x_2=9x_2$$

11. Delta Center

$L(x_1) = 10 \log\left(\frac{x_1}{10^{-12}}\right) = 110$ decibels

$10 \log\left(\frac{x_1}{10^{-12}}\right) = 110$

$\log\left(x_1 \cdot 10^{12}\right) = 11$

$\log(x_1) + \log\left(10^{12}\right) = 11$

$\log(x_1) + 12 = 11$

$\log(x_1) = -1$

$10^{\log(x_1)} = 10^{-1}$

$x_1 = 10^{-1} = 0.1$

NBA guidelines

$L(x_2) = 10 \log\left(\frac{x_2}{10^{-12}}\right) = 95$ decibels

$10 \log\left(\frac{x_2}{10^{-12}}\right) = 95$

$\log\left(x_2 \cdot 10^{12}\right) = 9.5$

$\log(x_2) + \log\left(10^{12}\right) = 9.5$

$\log(x_2) + 12 = 9.5$

$\log(x_2) = -2.5$

$10^{\log(x_2)} = 10^{-2.5}$

$x_2 = 10^{-2.5} \approx 0.0032$

Therefore $\frac{x_1}{x_2} = \frac{0.1}{0.0032} = 31.25$, which means that the crowd noise was approximately 31 times louder than NBA guidelines allow.