Exponential and Logarithmic Functions

6.R Chapter Review

1. $f(x) = \dfrac{2x+3}{5x-2}$

$$y = \frac{2x+3}{5x-2}$$

$$x = \frac{2y+3}{5y-2} \quad \text{Inverse}$$

$$x(5y-2) = 2y+3$$

$$5xy - 2x = 2y+3$$

$$5xy - 2y = 2x+3$$

$$y(5x-2) = 2x+3$$

$$y = \frac{2x+3}{5x-2}$$

$$f^{-1}(x) = \frac{2x+3}{5x-2}$$

Domain of f = range of f^{-1} = all real numbers except $\frac{2}{5}$

Range of f = domain of f^{-1} = all real numbers except $\frac{2}{5}$

2. $f(x) = \dfrac{2-x}{3+x}$

$$y = \frac{2-x}{3+x}$$

$$x = \frac{2-y}{3+y} \quad \text{Inverse}$$

$$x(3+y) = 2-y$$

$$3x + xy = 2-y$$

$$xy + y = 2-3x$$

$$y(x+1) = 2-3x$$

$$y = \frac{2-3x}{x+1}$$

$$f^{-1}(x) = \frac{2-3x}{x+1}$$

Domain of f = range of f^{-1} = all real numbers except –3

Range of f = domain of f^{-1} = all real numbers except –1

3. $f(x) = \dfrac{1}{x-1}$

$y = \dfrac{1}{x-1}$

$x = \dfrac{1}{y-1}$ Inverse

$x(y-1) = 1$

$xy - x = 1$

$xy = x + 1$

$y = \dfrac{x+1}{x}$

$f^{-1}(x) = \dfrac{x+1}{x}$

Domain of f = range of f^{-1} = all real numbers except 1

Range of f = domain of f^{-1} = all real numbers except 0

4. $f(x) = \sqrt{x-2}$

$y = \sqrt{x-2}$

$x = \sqrt{y-2}$ Inverse

$x^2 = y - 2 \qquad x \geq 0$

$y = x^2 + 2 \qquad x \geq 0$

$f^{-1}(x) = x^2 + 2 \qquad x \geq 0$

Domain of f = range of f^{-1} = all real numbers greater than or equal to 2

Range of f = domain of f^{-1} = all real numbers greater than or equal to 0

5. $f(x) = \dfrac{3}{x^{\frac{1}{3}}}$

$y = \dfrac{3}{x^{\frac{1}{3}}}$

$x = \dfrac{3}{y^{\frac{1}{3}}}$ Inverse

$xy^{\frac{1}{3}} = 3$

$y^{\frac{1}{3}} = \dfrac{3}{x}$

$y = \dfrac{27}{x^3}$

$f^{-1}(x) = \dfrac{27}{x^3}$

Domain of f = range of f^{-1} = all real numbers except 0

Range of f = domain of f^{-1} = all real numbers except 0

6. $f(x) = x^{\frac{1}{3}} + 1$
$y = x^{\frac{1}{3}} + 1$
$x = y^{\frac{1}{3}} + 1$ Inverse
$y^{\frac{1}{3}} = x - 1$
$y = (x-1)^3$
$f^{-1}(x) = (x-1)^3$

Domain of f = range of f^{-1} = all real numbers

Range of f = domain of f^{-1} = all real numbers

7. $\log_2\left(\frac{1}{8}\right) = \log_2 2^{-3} = -3\log_2 2 = -3$

8. $\log_3 81 = \log_3 3^4 = 4\log_3 3 = 4$

9. $\ln e^{\sqrt{2}} = \sqrt{2}$

10. $e^{\ln 0.1} = 0.1$

11. $2^{\log_2 0.4} = 0.4$

12. $\log_2 2^{\sqrt{3}} = \sqrt{3}\log_2 2 = \sqrt{3}$

13. $\log_3\left(\frac{uv^2}{w}\right) = \log_3 uv^2 - \log_3 w = \log_3 u + \log_3 v^2 - \log_3 w = \log_3 u + 2\log_3 v - \log_3 w$

14. $\log_2\left(a^2\sqrt{b}\right)^4 = 4\log_2\left(a^2\sqrt{b}\right) = 4\left(\log_2 a^2 + \log_2 b^{\frac{1}{2}}\right) = 4\left(2\log_2 a + \frac{1}{2}\log_2 b\right)$
$= 8\log_2 a + 2\log_2 b$

15. $\log\left(x^2\sqrt{x^3+1}\right) = \log x^2 + \log\left(x^3+1\right)^{\frac{1}{2}} = 2\log x + \frac{1}{2}\log\left(x^3+1\right)$

16. $\log_5\left(\frac{x^2+2x+1}{x^2}\right) = \log_5(x+1)^2 - \log_5\left(x^2\right) = 2\log_5(x+1) - 2\log_5 x$

17. $\ln\left(\frac{x\sqrt[3]{x^2+1}}{x-3}\right) = \ln\left(x\sqrt[3]{x^2+1}\right) - \ln(x-3) = \ln x + \ln\left(x^2+1\right)^{\frac{1}{3}} - \ln(x-3)$
$= \ln x + \frac{1}{3}\ln\left(x^2+1\right) - \ln(x-3)$

18. $\ln\left(\frac{2x+3}{x^2-3x+2}\right)^2 = 2\ln\left(\frac{2x+3}{x^2-3x+2}\right) = 2\left(\ln(2x+3) - \ln(x-2)(x-1)\right)$
$= 2\left(\ln(2x+3) - \ln(x-2) - \ln(x-1)\right) = 2\ln(2x+3) - 2\ln(x-2) - 2\ln(x-1)$

19. $3\log_4 x^2 + \frac{1}{2}\log_4\sqrt{x} = \log_4\left(x^2\right)^3 + \log_4\left(x^{\frac{1}{2}}\right)^{\frac{1}{2}} = \log_4 x^6 + \log_4 x^{\frac{1}{4}} = \log_4 x^6 \cdot x^{\frac{1}{4}}$
$= \log_4 x^{\frac{25}{4}} = \frac{25}{4}\log_4 x$

20. $-2\log_3\left(\frac{1}{x}\right) + \frac{1}{3}\log_3\sqrt{x} = \log_3\left(x^{-1}\right)^{-2} + \log_3\left(x^{\frac{1}{2}}\right)^{\frac{1}{3}} = \log_3 x^2 + \log_3 x^{\frac{1}{6}}$
$= \log_3 x^2 \cdot x^{\frac{1}{6}} = \log_3 x^{\frac{13}{6}}$

21. $\ln\left(\frac{x-1}{x}\right)+\ln\left(\frac{x}{x+1}\right)-\ln\left(x^2-1\right)=\ln\frac{x-1}{x}\;\frac{x}{x+1}-\ln\left(x^2-1\right)=\ln\frac{\frac{x-1}{x+1}}{x^2-1}$

$=\ln\left(\frac{x-1}{x+1}\;\frac{1}{(x-1)(x+1)}\right)=\ln\frac{1}{(x+1)^2}=\ln(x+1)^{-2}=-2\ln(x+1)$

22. $\log\left(x^2-9\right)-\log\left(x^2+7x+12\right)=\log\left(\frac{(x-3)(x+3)}{(x+3)(x+4)}\right)=\log\frac{x-3}{x+4}$

23. $2\log 2+3\log x-\frac{1}{2}\left[\log(x+3)+\log(x-2)\right]=\log 2^2+\log x^3-\frac{1}{2}\log(x+3)(x-2)$

$=\log 4x^3-\log\left((x+3)(x-2)\right)^{\frac{1}{2}}=\log\frac{4x^3}{\left((x+3)(x-2)\right)^{\frac{1}{2}}}$

24. $\frac{1}{2}\ln\left(x^2+1\right)-4\ln\frac{1}{2}-\frac{1}{2}\left[\ln(x-4)+\ln x\right]=\ln\left(x^2+1\right)^{\frac{1}{2}}-\ln\left(\frac{1}{2}\right)^4-\frac{1}{2}\ln\left(x(x-4)\right)$

$=\ln\left(x^2+1\right)^{\frac{1}{2}}-\ln\left(\frac{1}{2}\right)^4-\ln\left(x(x-4)\right)^{\frac{1}{2}}=\ln\frac{\left(x^2+1\right)^{\frac{1}{2}}}{\frac{1}{16}\left(x(x-4)\right)^{\frac{1}{2}}}$

25. $\log_4 19=\frac{\log 19}{\log 4}=2.124$

26. $\log_2 21=\frac{\log 21}{\log 2}=4.392$

27.
$$\begin{aligned}\ln y&=2x^2+\ln C\\ \ln y&=\ln e^{2x^2}+\ln C\\ \ln y&=\ln\left(Ce^{2x^2}\right)\\ y&=Ce^{2x^2}\end{aligned}$$

28.
$$\begin{aligned}\ln(y-3)&=\ln(2x^2)+\ln C\\ \ln(y-3)&=\ln\left(2x^2\ C\right)\\ y-3&=2Cx^2\\ y&=2Cx^2+3\end{aligned}$$

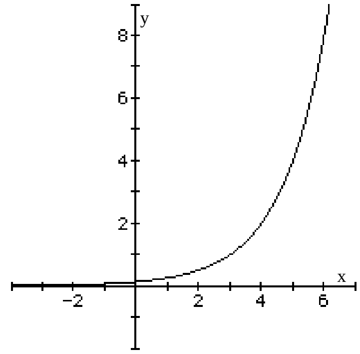
29.
$$\begin{aligned}\ln(y-3)+\ln(y+3)&=x+C\\ \ln(y-3)(y+3)&=x+C\\ (y-3)(y+3)&=e^{x+C}\\ y^2-9&=e^{x+C}\\ y^2&=9+e^{x+C}\\ y&=\sqrt{9+e^{x+C}}\end{aligned}$$

30.
$$\begin{aligned}\ln(y-1)+\ln(y+1)&=-x+C\\ \ln(y-1)(y+1)&=-x+C\\ (y-1)(y+1)&=e^{-x+C}\\ y^2-1&=e^{-x+C}\\ y^2&=1+e^{-x+C}\\ y&=\sqrt{1+e^{-x+C}}\end{aligned}$$

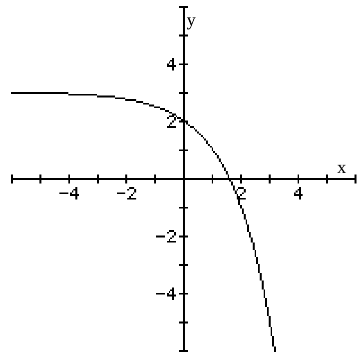
31.
$$\begin{aligned}e^{y+C}&=x^2+4\\ \ln e^{y+C}&=\ln(x^2+4)\\ y+C&=\ln(x^2+4)\\ y&=\ln(x^2+4)-C\end{aligned}$$

32.
$$\begin{aligned}e^{3y-C}&=(x+4)^2\\ \ln e^{3y-C}&=\ln(x+4)^2\\ 3y-C&=\ln(x+4)^2\\ 3y&=2\ln(x+4)+C\\ y&=\frac{2\ln(x+4)+C}{3}\end{aligned}$$

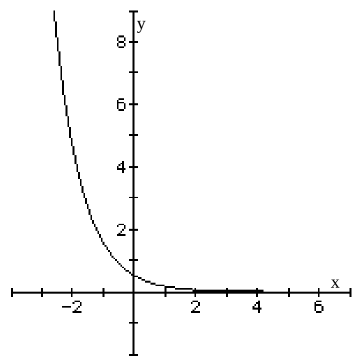
33. $f(x) = 2^{x-3}$
Using the graph of $y = 2^x$, shift the graph 3 units to the right.
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Horizontal Asymptote: $y = 0$



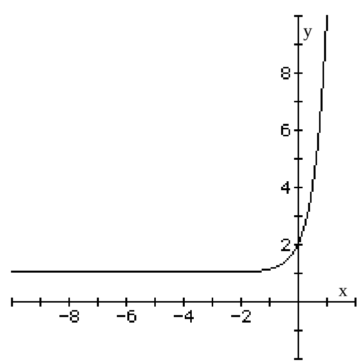
34. $f(x) = -2^x + 3$
Using the graph of $y = 2^x$, reflect the graph about the x-axis, and shift vertically 3 units up.
Domain: $(-\infty, \infty)$
Range: $(-\infty, 3)$
Horizontal Asymptote: $y = 3$



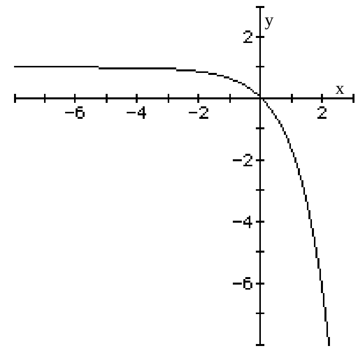
35. $f(x) = \frac{1}{2}\ 3^{-x}$
Using the graph of $y = 3^x$, reflect the graph about the y-axis, and shrink vertically by a factor of $\frac{1}{2}$.
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Horizontal Asymptote: $y = 0$



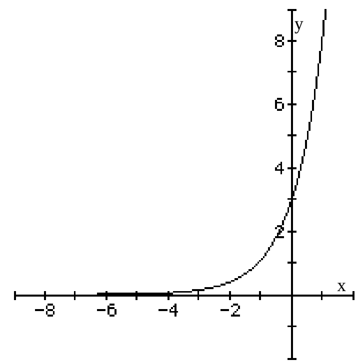
36. $f(x) = 1 + 3^{2x}$
Using the graph of $y = 3^x$, shrink the graph horizontally by a factor of $\frac{1}{2}$, and shift vertically 1 unit up..
Domain: $(-\infty, \infty)$
Range: $(1, \infty)$
Horizontal Asymptote: $y = 1$



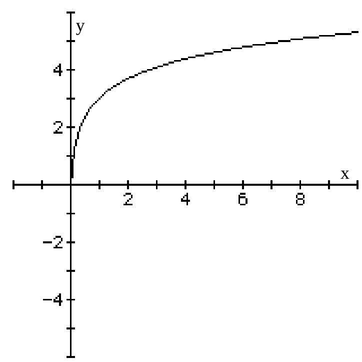
37. $f(x) = 1 - e^x$
Using the graph of $y = e^x$, reflect about the x-axis, and shift up 1 unit.
Domain: $(-\infty, \infty)$
Range: $(-\infty, 1)$
Horizontal Asymptote: $y = 1$



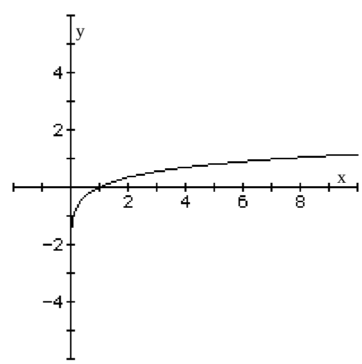
38. $f(x) = 3e^x$
Using the graph of $y = e^x$, stretch vertically by a factor of 3.
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Horizontal Asymptote: $y = 0$



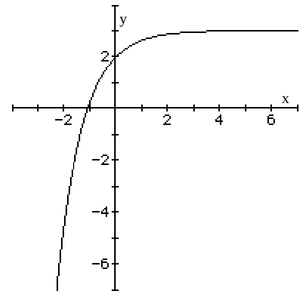
39. $f(x) = 3 + \ln x$
Using the graph of $y = \ln x$, shift the graph up 3 units.
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$



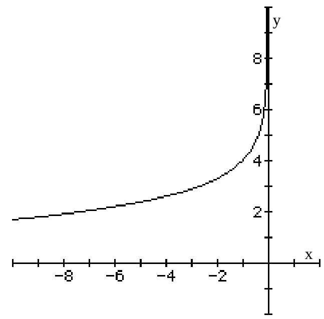
40. $f(x) = \frac{1}{2}\ln x$
Using the graph of $y = \ln x$, shrink vertically by a factor of $\frac{1}{2}$.
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$



41. $f(x) = 3 - e^{-x}$
Using the graph of $y = e^x$, reflect the graph about the y-axis, reflect about the x-axis, and shift up 3 units.
Domain: $(-\infty, \infty)$
Range: $(-\infty, 3)$
Horizontal Asymptote: $y = 3$



42. $f(x) = 4 - \ln(-x)$
Using the graph of $y = \ln x$, reflect the graph about the y-axis, reflect about the x-axis, and shift up 4 units.
Domain: $(-\infty, 0)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$



43.
$$4^{1-2x} = 2$$
$$\left(2^2\right)^{1-2x} = 2$$
$$2^{2-4x} = 2^1$$
$$2 - 4x = 1 \rightarrow -4x = -1 \rightarrow x = \tfrac{1}{4}$$

44.
$$8^{6+3x} = 4$$
$$\left(2^3\right)^{6+3x} = 2^2$$
$$2^{18+9x} = 2^2$$
$$18 + 9x = 2 \rightarrow 9x = -16 \rightarrow x = -\tfrac{16}{9}$$

45.
$$3^{x^2+x} = \sqrt{3}$$
$$3^{x^2+x} = 3^{\frac{1}{2}}$$
$$x^2 + x = \tfrac{1}{2} \rightarrow 2x^2 + 2x - 1 = 0 \rightarrow x = \frac{-2 \pm \sqrt{4 - 4(2)(-1)}}{2(2)} = \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$
$$x = \frac{-1 - \sqrt{3}}{2} \text{ or } x = \frac{-1 + \sqrt{3}}{2}$$

46.
$$4^{x-x^2} = \tfrac{1}{2}$$
$$\left(2^2\right)^{x-x^2} = 2^{-1}$$
$$2^{2x-2x^2} = 2^{-1}$$
$$2x - 2x^2 = -1 \rightarrow 2x^2 - 2x - 1 = 0 \rightarrow x = \frac{-(-2) \pm \sqrt{4 - 4(2)(-1)}}{2(2)} = \frac{2 \pm \sqrt{12}}{4} = \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$$
$$x = \frac{1 - \sqrt{3}}{2} \text{ or } x = \frac{1 + \sqrt{3}}{2}$$

47.
$$\begin{aligned}\log_x 64 &= -3\\ x^{-3} &= 64\\ \left(x^{-3}\right)^{\frac{-1}{3}} &= 64^{\frac{-1}{3}}\\ x &= \frac{1}{\sqrt[3]{64}} = \frac{1}{4}\end{aligned}$$

48.
$$\begin{aligned}\log_{\sqrt{2}} x &= -6\\ x &= \left(\sqrt{2}\right)^{-6} = \left(2^{\frac{1}{2}}\right)^{-6} = 2^{-3} = \tfrac{1}{8}\end{aligned}$$

49.
$$\begin{aligned}5^x &= 3^{x+2}\\ \log\left(5^x\right) &= \log\left(3^{x+2}\right)\\ x\log 5 &= (x+2)\log 3\\ x\log 5 &= x\log 3 + 2\log 3\\ x\log 5 - x\log 3 &= 2\log 3\\ x(\log 5 - \log 3) &= 2\log 3\\ x &= \frac{2\log 3}{\log 5 - \log 3}\\ x &\approx 4.301\end{aligned}$$

50.
$$\begin{aligned}5^{x+2} &= 7^{x-2}\\ \log\left(5^{x+2}\right) &= \log\left(7^{x-2}\right)\\ (x+2)\log 5 &= (x-2)\log 7\\ x\log 5 + 2\log 5 &= x\log 7 - 2\log 7\\ x\log 5 - x\log 7 &= -2\log 7 - 2\log 5\\ x(\log 5 - \log 7) &= -2\log 7 - 2\log 5\\ x &= \frac{-2\log 7 - 2\log 5}{\log 5 - \log 7}\\ x &\approx 21.133\end{aligned}$$

51.
$$\begin{aligned}9^{2x} &= 27^{3x-4}\\ \left(3^2\right)^{2x} &= \left(3^3\right)^{3x-4}\\ 3^{4x} &= 3^{9x-12}\\ 4x &= 9x-12\\ -5x &= -12\\ x &= \frac{12}{5}\end{aligned}$$

52.
$$\begin{aligned}25^{2x} &= 5^{x^2-12}\\ \left(5^2\right)^{2x} &= 5^{x^2-12}\\ 5^{4x} &= 5^{x^2-12}\\ 4x &= x^2-12\\ x^2-4x-12 &= 0\\ (x-6)(x+2) &= 0\\ x = 6 \quad &\text{or}\quad x = -2\end{aligned}$$

53.
$$\begin{aligned}\log_3 \sqrt{x-2} &= 2\\ \sqrt{x-2} &= 3^2\\ x-2 &= 9^2\\ x-2 &= 81\\ x &= 83\end{aligned}$$

54.
$$\begin{aligned}2^{x+1}\ \ 8^{-x} &= 4\\ 2^{x+1}\ \left(2^3\right)^{-x} &= 2^2\\ 2^{x+1}\ \ 2^{-3x} &= 2^2\\ 2^{-2x+1} &= 2^2\\ -2x+1 &= 2\\ -2x &= 1\\ x &= -\tfrac{1}{2}\end{aligned}$$

55.
$$\begin{aligned}8 &= 4^{x^2}\ \ 2^{5x}\\ 2^3 &= \left(2^2\right)^{x^2}\ \ 2^{5x}\\ 2^3 &= 2^{2x^2+5x}\\ 3 &= 2x^2+5x\\ 0 &= 2x^2+5x-3\\ 0 &= (2x-1)(x+3)\\ x = \tfrac{1}{2} \quad &\text{or}\quad x = -3\end{aligned}$$

56.
$$\begin{aligned}2^x\ \ 5 &= 10^x\\ \ln\left(2^x\ \ 5\right) &= \ln 10^x\\ \ln 2^x + \ln 5 &= \ln 10^x\\ x\ln 2 + \ln 5 &= x\ln 10\\ x(\ln 2 - \ln 10) &= -\ln 5\\ x &= \frac{-\ln 5}{\ln 2 - \ln 10} = 1\end{aligned}$$

57. $\log_6(x+3)+\log_6(x+4)=1$

$$\begin{aligned}\log_6(x+3)(x+4)&=1\\(x+3)(x+4)&=6^1\\x^2+7x+12&=6\\x^2+7x+6&=0\\(x+6)(x+1)&=0\\x=-6 \text{ or } x&=-1\end{aligned}$$

The logarithms are undefined when $x=-6$, so $x=-1$ is the only solution.

58. $\log_{10}(7x-12)=2\log_{10}x$

$$\begin{aligned}\log_{10}(7x-12)&=\log_{10}x^2\\7x-12&=x^2\\x^2-7x+12&=0\\(x-4)(x-3)&=0\\x=4 \text{ or } x&=3\end{aligned}$$

59. $e^{1-x}=5$

$$\begin{aligned}1-x&=\ln 5\\-x&=-1+\ln 5\\x&=1-\ln 5\approx-0.609\end{aligned}$$

60. $e^{1-2x}=4$

$$\begin{aligned}1-2x&=\ln 4\\-2x&=-1+\ln 4\\x&=\frac{1-\ln 4}{2}\approx-0.193\end{aligned}$$

61.

$$\begin{aligned}2^{3x}&=3^{2x+1}\\\ln 2^{3x}&=\ln 3^{2x+1}\\3x\ln 2&=(2x+1)\ln 3\\3x\ln 2&=2x\ln 3+\ln 3\\3x\ln 2-2x\ln 3&=\ln 3\\x(3\ln 2-2\ln 3)&=\ln 3\\x&=\frac{\ln 3}{3\ln 2-2\ln 3}\\x&\approx-9.327\end{aligned}$$

62.

$$\begin{aligned}2^{x^3}&=3^{x^2}\\\ln 2^{x^3}&=\ln 3^{x^2}\\x^3\ln 2&=x^2\ln 3\\x^3\ln 2-x^2\ln 3&=0\\x^2(x\ln 2-\ln 3)&=0\\x^2=0 \text{ or } x&=\frac{\ln 3}{\ln 2}\\x=0 \text{ or } x&\approx 1.585\end{aligned}$$

63. $h(300)=\left(30(0)+8000\right)\log\left(\frac{760}{300}\right)=8000\log 2.5333\approx 3229.5$ meters

64. $h(500)=\left(30(5)+8000\right)\log\left(\frac{760}{500}\right)=8150\log 1.52=1482$ meters

65. $h(x) = 10000$

$$10000 = \left(30(-100) + 8000\right)\log\left(\frac{760}{x}\right)$$

$$10000 = \left(5000\right)\log\left(\frac{760}{x}\right)$$

$$2 = \log\left(\frac{760}{x}\right)$$

$$10^2 = 10^{\log\left(\frac{760}{x}\right)}$$

$$100 = \frac{760}{x} \rightarrow x = \frac{760}{100} = 7.6 \text{ mm}$$

66. $h(x) = 10000$

$$8900 = \left(30(5) + 8000\right)\log\left(\frac{760}{x}\right)$$

$$8900 = \left(8150\right)\log\left(\frac{760}{x}\right)$$

$$\frac{8900}{8150} = \log\left(\frac{760}{x}\right)$$

$$10^{\left(\frac{8900}{8150}\right)} = 10^{\log\left(\frac{760}{x}\right)}$$

$$\frac{8900}{8150} = \frac{760}{x}$$

$$\rightarrow x = 760\left(\frac{8150}{8900}\right) = 695.96 \text{ mm}$$

67. $P = 25e^{0.1d}$

(a) $P = 25e^{0.1(4)}$

$= 25e^{0.4}$

$= 37.3$ watts

(b) $50 = 25e^{0.1d}$

$2 = e^{0.1d}$

$\ln 2 = 0.1d$

$d = \dfrac{\ln 2}{0.1} = 6.9$ decibels

68. $L = 9 + 5.1\log d$

(a) $L = 9 + 5.1\log 3.5 \approx 11.8$

(b) $14 = 9 + 5.1\log d$

$5 = 5.1\log d$

$\log d = \dfrac{5}{5.1} \approx 0.9804$

$d = 10^{0.9804} \approx 9.56$ inches

69. (a) $P = 90 - 80\left(\frac{3}{4}\right)^5 \approx 71.02\%$

(b) $P = 90 - 80\left(\frac{3}{4}\right)^{10} \approx 85.5\%$

(c) as $t \rightarrow \infty$, $P = 90 - 80\left(\frac{3}{4}\right)^t \rightarrow 90 - 0 = 90\%$

(d) $40 = 90 - 80\left(\frac{3}{4}\right)^t$

$$-50 = -80\left(\frac{3}{4}\right)^t \rightarrow \frac{5}{8} = \left(\frac{3}{4}\right)^t \rightarrow \ln\left(\frac{5}{8}\right) = \ln\left(\frac{3}{4}\right)^t$$

$$\rightarrow \ln\left(\frac{5}{8}\right) = t\ln\left(\frac{3}{4}\right) \rightarrow t = \frac{\ln\left(\frac{5}{8}\right)}{\ln\left(\frac{3}{4}\right)} \approx 1.63 \text{ months}$$

(e) $70 = 90 - 80\left(\frac{3}{4}\right)^t$

$-20 = -80\left(\frac{3}{4}\right)^t \rightarrow 0.25 = \left(\frac{3}{4}\right)^t \rightarrow \ln(0.25) = \ln\left(\frac{3}{4}\right)^t$

$\rightarrow \ln(0.25) = t\,\ln\left(\frac{3}{4}\right) \rightarrow t = \dfrac{\ln(0.25)}{\ln\left(\frac{3}{4}\right)} \approx 4.82$ months

70. $m = 55.3 - 6\ln(10000 - 5000) = 55.3 - 6\ln(5000) \approx 4.2$ months

71. (a) $n = \dfrac{\log 10000 - \log 90000}{\log(1 - 0.20)} = 9.85$ years

(b) $n = \dfrac{\log 0.5i - \log i}{\log(1 - 0.15)} = \dfrac{\log \dfrac{0.5i}{i}}{\log 0.85} = \dfrac{\log 0.5}{\log 0.85} = 4.27$ years

72. $A = 10000\left(1 + \dfrac{0.04}{2}\right)^{(2)(18)} = 10000(1.02)^{36} = \$20{,}398.87$

73. $P = A\left(1 + \dfrac{r}{n}\right)^{-nt} = 85000\left(1 + \dfrac{0.04}{2}\right)^{-2(18)} = \$41{,}668.97$

74. (a) $5000 = 620.17e^{r(20)}$

$8.0623 = e^{20r}$

$\ln 8.0623 = 20r$

$r = \dfrac{\ln 8.0623}{20} \approx 0.10436 \approx 10.436\%$

(b) $A = 4000e^{0.10436(20)} = \$32{,}249.24$

The banks claim is correct.

75. $L\left(10^{-4}\right) = 10\log\left(\dfrac{10^{-4}}{10^{-12}}\right) = 10\log\left(10^{8}\right) = 10 \cdot 8 = 80$ decibels

76. Chicago: $M(x) = \log \dfrac{x}{10^{-3}} = 3.0$

$\log\left(x \cdot 10^{3}\right) = 3.0$

$\log(x) + \log\left(10^{3}\right) = 3$

$\log(x) + 3 = 3$

$\log(x) = 0$

$10^{\log(x)} = 10^{0}$

$x = 10^{0} = 1$

San Francisco: $M(x) = \log \dfrac{x}{10^{-3}} = 6.9$

$\log\left(x \cdot 10^{3}\right) = 6.9$

$\log(x) + \log\left(10^{3}\right) = 6.9$

$\log(x) + 3 = 6.9$

$\log(x) = 3.9$

$10^{\log(x)} = 10^{3.9}$

$x = 10^{3.9} \approx 7943.28$

77. $A = A_0 e^{kt}$

$\frac{1}{2} A_0 = A_0 e^{k(5600)}$

$0.5 = e^{5600k}$

$\ln 0.5 = 5600k$

$k = \dfrac{\ln 0.5}{5600} \approx -0.000124$

$0.05 A_0 = A_0 e^{-0.000124t}$

$0.05 = e^{-0.000124t}$

$\ln 0.05 = -0.000124t$

$t = \dfrac{\ln 0.05}{-0.000124} \approx 24{,}159$ years ago

78. Using $u = T + (u_0 - T)e^{kt}$ where $t = 5$,

$T = 70,\ u_0 = 450,\ u = 400$:

$400 = 70 + (450 - 70)e^{k(5)}$

$330 = 380e^{5k}$

$0.86842 = e^{5k}$

$5k = \ln 0.86842$

$k = \dfrac{\ln 0.86842}{5} \approx -0.028216$

Find time for temperature of 150˚F:

$150 = 70 + (450 - 70)e^{-0.028216t}$

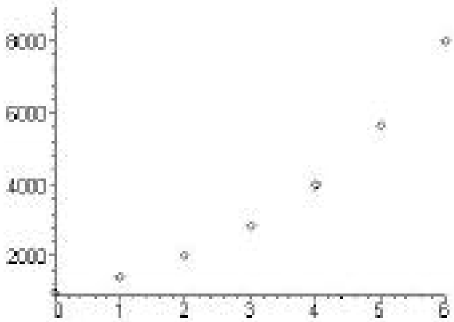
$80 = 380e^{-0.028216t}$

$0.210526 = e^{-0.028216t}$

$\ln 0.210526 = -0.028216t$

$t = \dfrac{\ln 0.210526}{-0.028216} \approx 55.2$ minutes

79. (a) Graphing:



(b) $y = 1000\left(\sqrt{2}\right)^x$

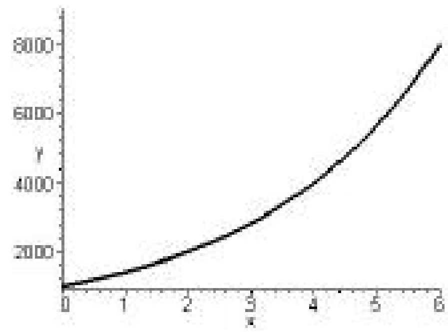
$\sqrt{2} = e^{\ln\sqrt{2}}$

$\rightarrow y = 1000\left(e^{\ln\sqrt{2}}\right)^x = 1000e^{(\ln\sqrt{2})t}$

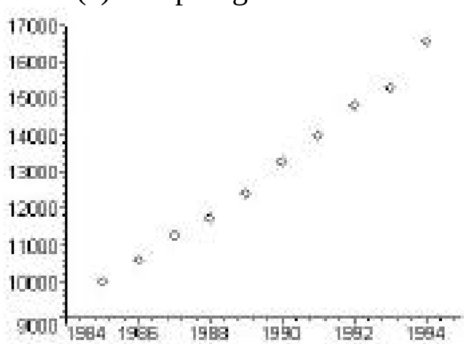
$N = N_o e^{0.3466t}, N_0 = 1000, k = 0.3466$

(c) $N = 1000e^{0.3466(7)} \approx 11314$ bacteria

(d) and (e) Graphing:



80. (a) Graphing:



(b) $y = 10014(1.057)^x$

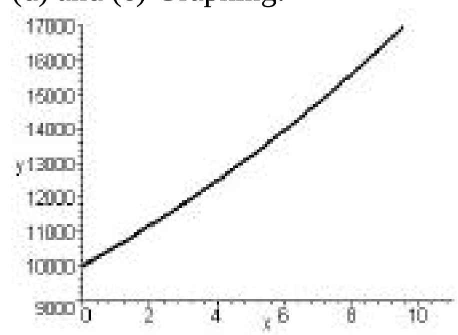
$1.057 = e^{\ln 1.057}$

$\rightarrow y = 10014\left(e^{\ln 1.057}\right)^x = 1000e^{(0.0554)x}$

$A = A_0 e^{0.0554t}, A_0 = 10014, k = 0.0554$

(c) $A = 10014e^{0.0554(35)} \approx \$69699.90$

(d) and (e) Graphing:



81. $P = P_0 e^{kt} = 5{,}840{,}445{,}216\, e^{0.0133(3)} = 6{,}078{,}190{,}457$

82. $A = A_0 e^{kt}$

$\frac{1}{2}A_0 = A_0 e^{k(5.27)}$

$\frac{1}{2} = e^{5.27k}$

$\ln 0.5 = 5.27k$

$k = \dfrac{\ln 0.5}{5.27} \approx -0.13153$

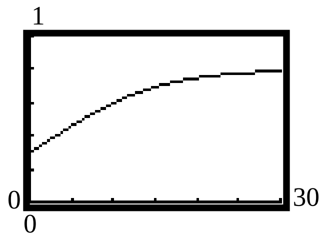
In 20 years: $A = 100e^{-0.13153(20)} = 7.2$ grams

In 40 years: $A = 100e^{-0.13153(40)} = 0.52$ grams

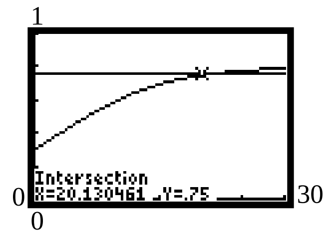
83. (a) $P(0) = \dfrac{0.8}{1+1.67e^{-0.16(0)}} = \dfrac{0.8}{1+1.67} = 0.2996$

(b) 0.8

(c) Graphing:



(d) Using INTERSECT we have:



75% use Windows 98 in 2018.