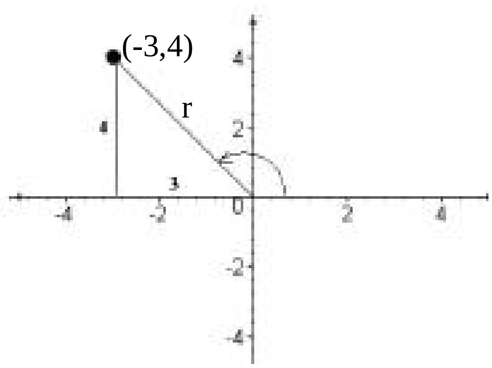
Chapter 7

Trigonometric Functions

7.4 Trigonometric Functions of General Angles

1. $(-3,4) \rightarrow a = -3, b = 4$ $\quad r = \sqrt{a^2 + b^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

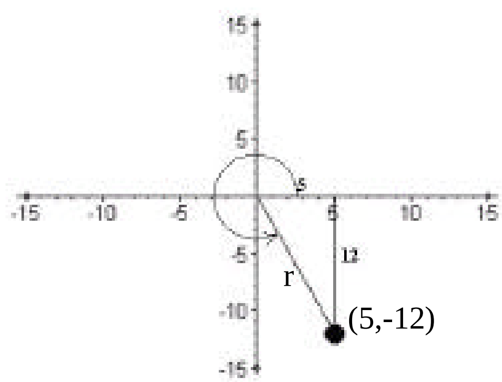


$$\sin\theta = \frac{b}{r} = \frac{4}{5} \qquad \cos\theta = \frac{a}{r} = \frac{-3}{5} = -\frac{3}{5}$$

$$\tan\theta = \frac{b}{a} = \frac{4}{-3} = -\frac{4}{3} \qquad \cot\theta = \frac{a}{b} = \frac{-3}{4} = -\frac{3}{4}$$

$$\sec\theta = \frac{r}{a} = \frac{5}{-3} = -\frac{5}{3} \qquad \csc\theta = \frac{r}{b} = \frac{5}{4}$$

2. $(5,-12)$ $\quad r = \sqrt{a^2 + b^2} = \sqrt{5^2 + 12^2} = \sqrt{25+144} = \sqrt{169} = 13$

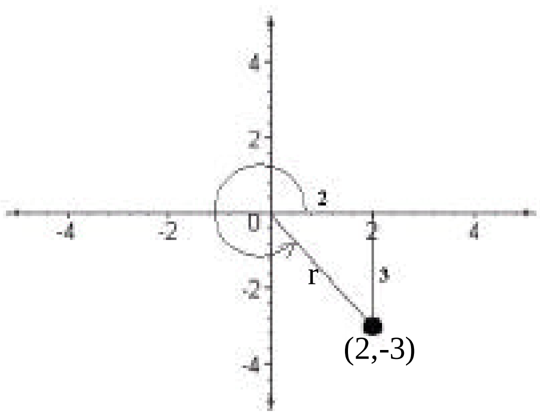


$$\sin\theta = \frac{b}{r} = \frac{-12}{13} = -\frac{12}{3} \qquad \cos\theta = \frac{a}{r} = \frac{5}{13}$$

$$\tan\theta = \frac{b}{a} = \frac{-12}{5} = -\frac{12}{5} \qquad \cot\theta = \frac{a}{b} = \frac{5}{-12} = -\frac{5}{12}$$

$$\sec\theta = \frac{r}{a} = \frac{13}{5} \qquad \csc\theta = \frac{r}{b} = \frac{13}{-12} = -\frac{13}{12}$$

3. $(2,-3)$ $\qquad r=\sqrt{a^2+b^2}=\sqrt{2^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$



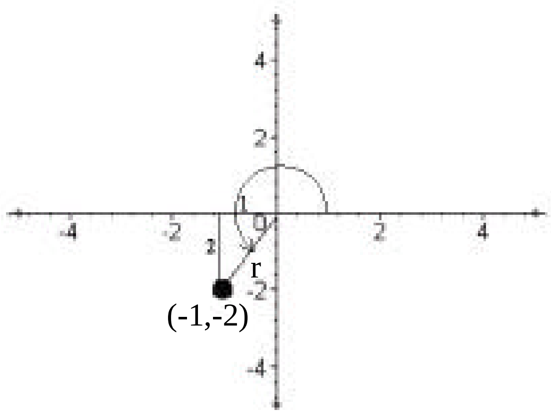
$$\sin\theta=\frac{b}{r}=\frac{-3}{\sqrt{13}}\frac{\sqrt{13}}{\sqrt{13}}=-\frac{3\sqrt{13}}{13}$$

$$\cos\theta=\frac{a}{r}=\frac{2}{\sqrt{13}}\frac{\sqrt{13}}{\sqrt{13}}=-\frac{2\sqrt{13}}{13}$$

$$\tan\theta=\frac{b}{a}=\frac{-3}{2}=-\frac{3}{2} \qquad \cot\theta=\frac{a}{b}=\frac{2}{-3}=-\frac{2}{3}$$

$$\sec\theta=\frac{r}{a}=\frac{\sqrt{13}}{2} \qquad \csc\theta=\frac{r}{b}=\frac{\sqrt{13}}{-3}=-\frac{\sqrt{13}}{3}$$

4. $(-1,-2)$ $\qquad r=\sqrt{a^2+b^2}=\sqrt{(-1)^2+(-2)^2}=\sqrt{1+4}=\sqrt{5}$



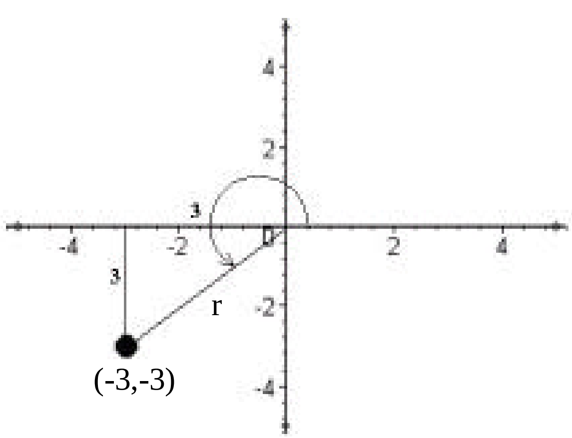
$$\sin\theta=\frac{b}{r}=\frac{-2}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}}=-\frac{2\sqrt{5}}{5}$$

$$\cos\theta=\frac{a}{r}=\frac{-1}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}}=-\frac{\sqrt{5}}{5}$$

$$\tan\theta=\frac{b}{a}=\frac{-2}{-1}=2 \qquad \cot\theta=\frac{a}{b}=\frac{-1}{-2}=\frac{1}{2}$$

$$\sec\theta=\frac{r}{a}=\frac{\sqrt{5}}{-1}=-\sqrt{5} \qquad \csc\theta=\frac{r}{b}=\frac{\sqrt{5}}{-2}=-\frac{\sqrt{5}}{2}$$

5. $(-3,-3)$ $\qquad r=\sqrt{a^2+b^2}=\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$



$$\sin\theta=\frac{b}{r}=\frac{-3}{3\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$$

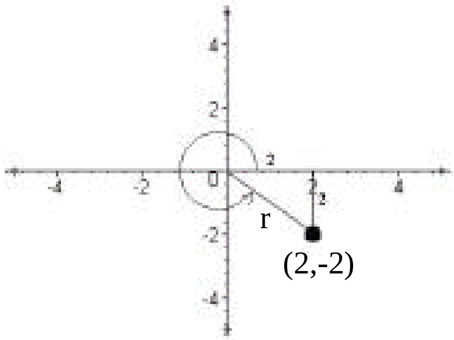
$$\cos\theta=\frac{a}{r}=\frac{-3}{3\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$$

$$\tan\theta=\frac{b}{a}=\frac{-3}{-3}=1 \qquad \cot\theta=\frac{a}{b}=\frac{-3}{-3}=1$$

$$\sec\theta=\frac{r}{a}=\frac{3\sqrt{2}}{-3}=-\sqrt{2} \qquad \csc\theta=\frac{r}{b}=\frac{3\sqrt{2}}{-3}=-\sqrt{2}$$

6. $(2,-2)$

$$r=\sqrt{a^2+b^2}=\sqrt{2^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$



$$\sin\theta=\frac{b}{r}=\frac{-2}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$$

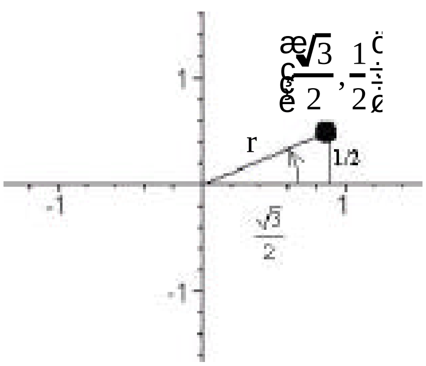
$$\cos\theta=\frac{a}{r}=\frac{2}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

$$\tan\theta=\frac{b}{a}=\frac{-2}{2}=-1 \qquad \cot\theta=\frac{a}{b}=\frac{2}{-2}=-1$$

$$\sec\theta=\frac{r}{a}=\frac{2\sqrt{2}}{2}=\sqrt{2} \qquad \csc\theta=\frac{r}{b}=\frac{2\sqrt{2}}{-2}=-\sqrt{2}$$

7. $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

$$r=\sqrt{a^2+b^2}=\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\sqrt{\frac{3}{4}+\frac{1}{4}}=\sqrt{1}=1$$



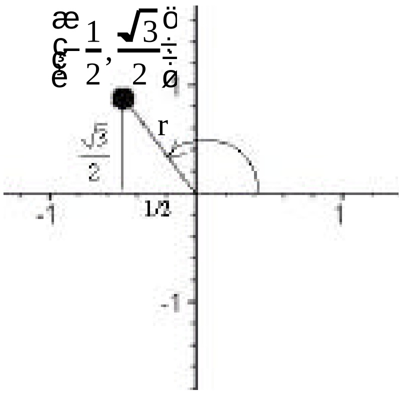
$$\sin\theta=\frac{b}{r}=\frac{\left(\frac{1}{2}\right)}{1}=\frac{1}{2} \qquad \cos\theta=\frac{\left(\frac{\sqrt{3}}{2}\right)}{1}=\frac{\sqrt{3}}{2}$$

$$\tan\theta=\frac{b}{a}=\frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3} \qquad \cot\theta=\frac{a}{b}=\frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)}=\sqrt{3}$$

$$\sec\theta=\frac{r}{a}=\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}=\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{3} \qquad \csc\theta=\frac{r}{b}=\frac{1}{\left(\frac{1}{2}\right)}=2$$

8. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$r = \sqrt{a^2 + b^2} = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$$



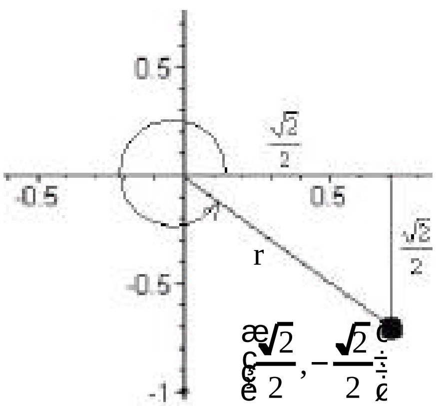
$$\sin\theta = \frac{b}{r} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{1} = \frac{\sqrt{3}}{2} \qquad \cos\theta = \frac{a}{r} = \frac{\left(-\frac{1}{2}\right)}{1} = -\frac{1}{2}$$

$$\tan\theta = \frac{b}{a} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = -\sqrt{3} \qquad \cot\theta = \frac{a}{b} = \frac{\left(-\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\sec\theta = \frac{r}{a} = \frac{1}{\left(-\frac{1}{2}\right)} = -2 \qquad \csc\theta = \frac{r}{b} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

9. $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$$r = \sqrt{a^2 + b^2} = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{1} = 1$$



$$\sin\theta = \frac{b}{r} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{1} = -\frac{\sqrt{2}}{2} \qquad \cos\theta = \frac{a}{r} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{1} = \frac{\sqrt{2}}{2}$$

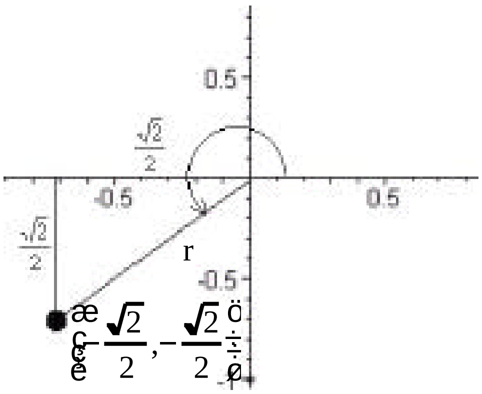
$$\tan\theta = \frac{b}{a} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = -1 \qquad \cot\theta = \frac{a}{b} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = -1$$

$$\sec\theta = \frac{r}{a} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$\csc\theta = \frac{r}{b} = \frac{1}{\left(-\frac{\sqrt{2}}{2}\right)} = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\sqrt{2}$$

10. $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$$r = \sqrt{a^2 + b^2} = \sqrt{\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{1} = 1$$



$$\sin\theta = \frac{b}{r} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{1} = -\frac{\sqrt{2}}{2} \qquad \cos\theta = \frac{a}{r} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{1} = -\frac{\sqrt{2}}{2}$$

$$\tan\theta = \frac{b}{a} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = 1 \qquad \cot\theta = \frac{a}{b} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = 1$$

$$\sec\theta = \frac{r}{a} = \frac{1}{\left(-\frac{\sqrt{2}}{2}\right)} = -\frac{2}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = -\sqrt{2}$$

$$\csc\theta = \frac{r}{b} = \frac{1}{\left(-\frac{\sqrt{2}}{2}\right)} = -\frac{2}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = -\sqrt{2}$$

11. Since $\sin\theta > 0$ for points in quadrants I and II, and $\cos\theta < 0$ for points in quadrants II and III, the angle θ lies in quadrant II.

12. Since $\sin\theta < 0$ for points in quadrants III and IV, and $\cos\theta > 0$ for points in quadrants I and IV, the angle θ lies in quadrant IV.

13. Since $\sin\theta < 0$ for points in quadrants III and IV, and $\tan\theta < 0$ for points in quadrants II and IV, the angle θ lies in quadrant IV.

14. Since $\cos\theta > 0$ for points in quadrants I and IV, and $\tan\theta > 0$ for points in quadrants I and III, the angle θ lies in quadrant I.

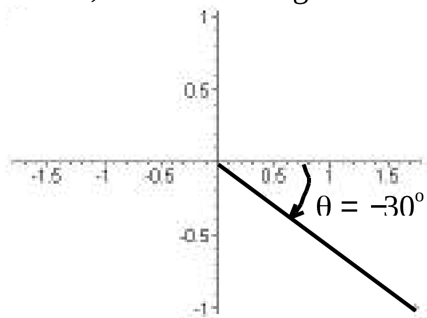
15. Since $\cos\theta > 0$ for points in quadrants I and IV, and $\cot\theta < 0$ for points in quadrants II and IV, the angle θ lies in quadrant IV.

16. Since $\sin\theta < 0$ for points in quadrants II and III, and $\cot\theta > 0$ for points in quadrants I and III, the angle θ lies in quadrant III.

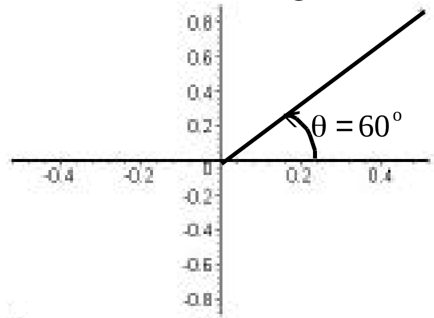
17. Since $\sec\theta < 0$ for points in quadrants II and III, and $\tan\theta > 0$ for points in quadrants I and III, the angle θ lies in quadrant III.

18. Since $\csc\theta > 0$ for points in quadrants I and II, and $\cot\theta < 0$ for points in quadrants II and IV, the angle θ lies in quadrant II.

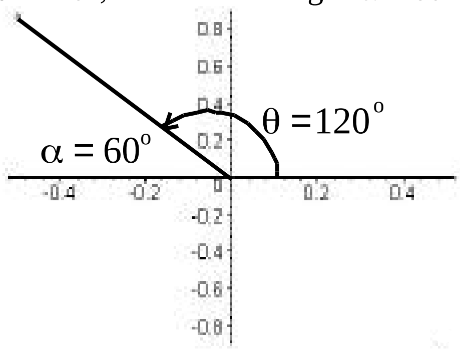
19. $\theta = -30^\circ$; reference angle $\alpha = 30^\circ$



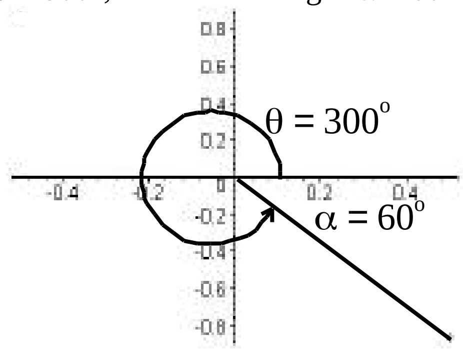
20. $\theta = 60^\circ$; reference angle $\alpha = 60^\circ$



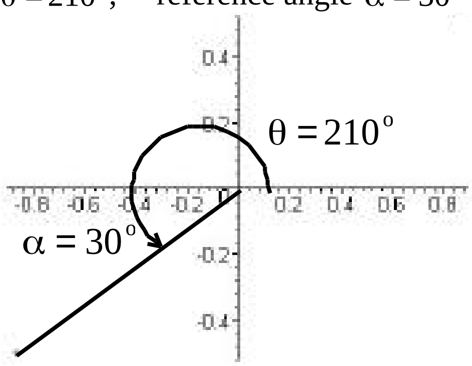
21. $\theta = 120^\circ$; reference angle $\alpha = 60^\circ$



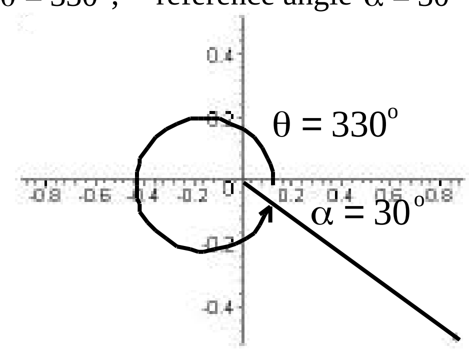
22. $\theta = 300^\circ$; reference angle $\alpha = 60^\circ$



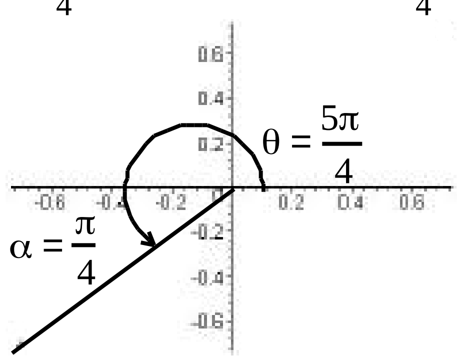
23. $\theta = 210^\circ$; reference angle $\alpha = 30^\circ$



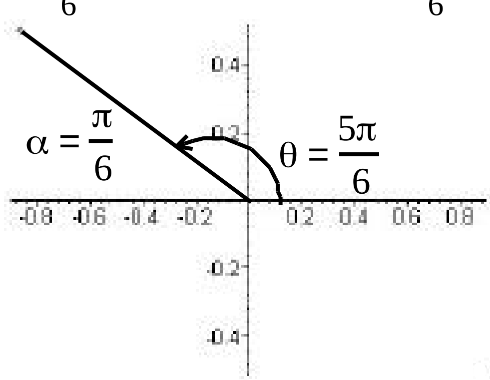
24. $\theta = 330^\circ$; reference angle $\alpha = 30^\circ$



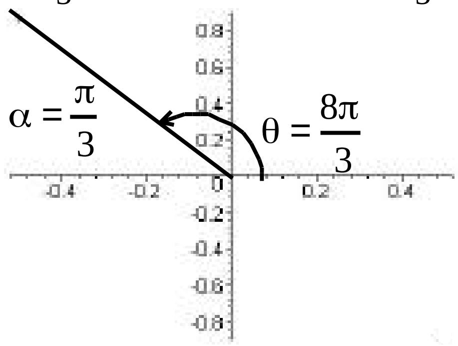
25. $\theta = \frac{5\pi}{4}$; reference angle $\alpha = \frac{\pi}{4}$



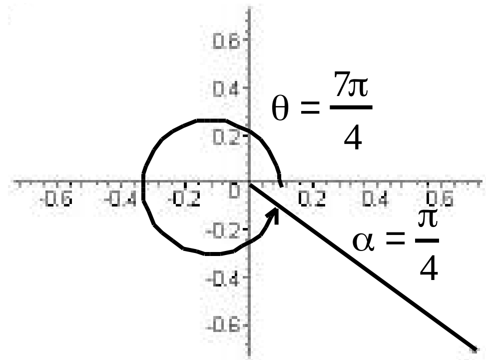
26. $\theta = \frac{5\pi}{6}$; reference angle $\alpha = \frac{\pi}{6}$



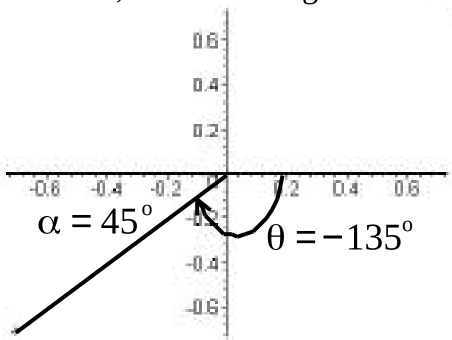
27. $\theta = \dfrac{8\pi}{3}$; reference angle $\alpha = \dfrac{\pi}{3}$



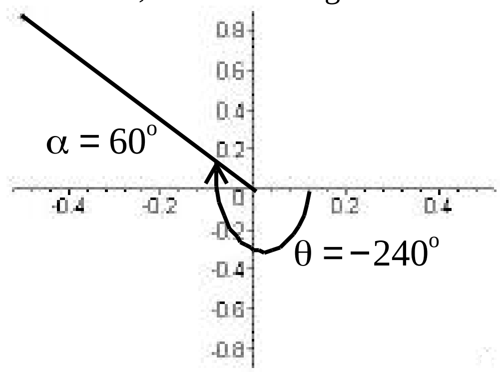
28. $\theta = \dfrac{7\pi}{4}$; reference angle $\alpha = \dfrac{\pi}{4}$



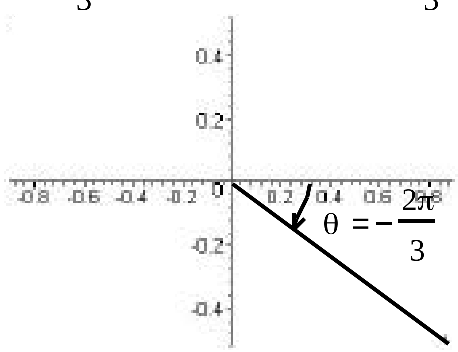
29. $\theta = -135^{o}$; reference angle $\alpha = 45^{o}$



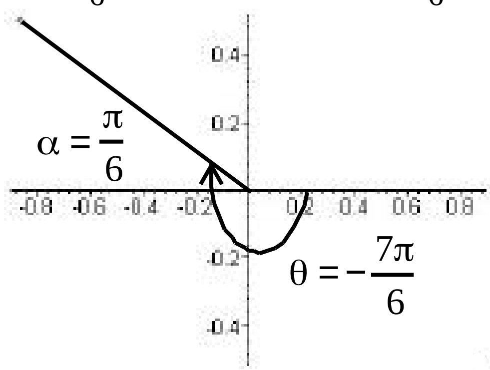
30. $\theta = -240^{o}$; reference angle $\alpha = 60^{o}$



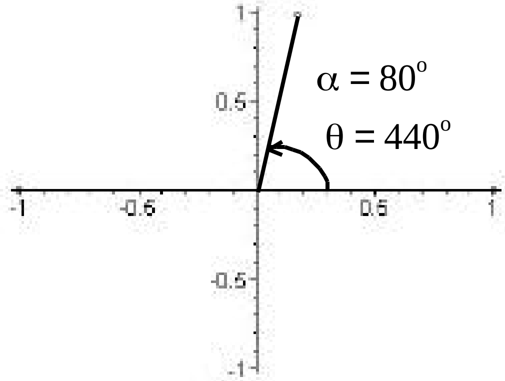
31. $\theta = -\dfrac{2\pi}{3}$; reference angle $\alpha = \dfrac{2\pi}{3}$



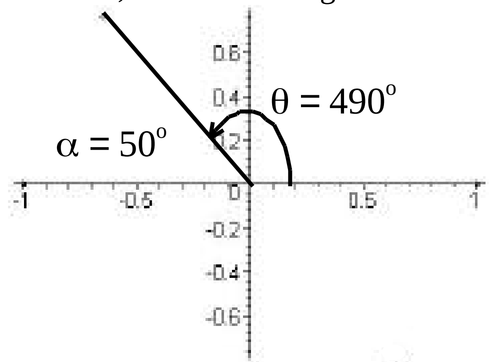
32. $\theta = -\dfrac{7\pi}{6}$; reference angle $\alpha = \dfrac{\pi}{6}$



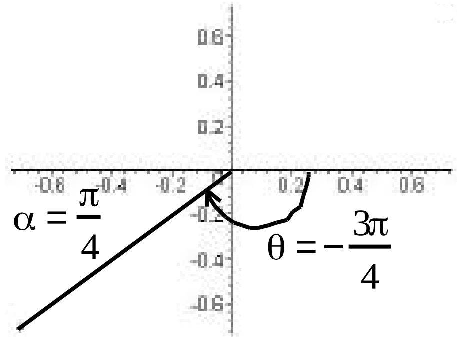
33. $\theta = 440^{\circ}$; reference angle $\alpha = 80^{\circ}$



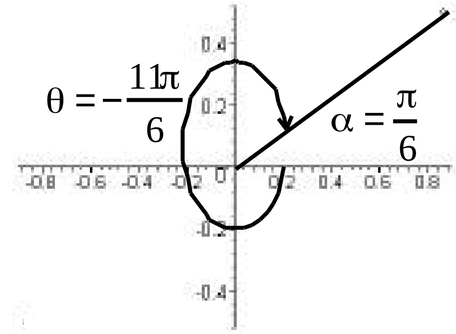
34. $\theta = 490^{\circ}$; reference angle $\alpha = 50^{\circ}$



35. $\theta = -\dfrac{3\pi}{4}$; reference angle $\alpha = \dfrac{\pi}{4}$



36. $\theta = -\dfrac{11\pi}{6}$; reference angle $\alpha = \dfrac{\pi}{6}$



37. $\sin(405^{\circ}) = \sin(360^{\circ} + 45^{\circ}) = \sin(45^{\circ}) = \dfrac{\sqrt{2}}{2}$

38. $\cos(420^{\circ}) = \cos(360^{\circ} + 60^{\circ}) = \cos(60^{\circ}) = \dfrac{1}{2}$

39. $\tan(405^{\circ}) = \tan(180^{\circ} + 180^{\circ} + 45^{\circ}) = \tan(45^{\circ}) = 1$

40. $\sin(390^{\circ}) = \sin(360^{\circ} + 30^{\circ}) = \sin(30^{\circ}) = \dfrac{1}{2}$

41. $\csc(450^{\circ}) = \csc(360^{\circ} + 90^{\circ}) = \csc(90^{\circ}) = 1$

42. $\sec(540^{\circ}) = \sec(360^{\circ} + 180^{\circ}) = \sec(180^{\circ}) = -1$

43. $\cot(390^{\circ}) = \cot(180^{\circ} + 180^{\circ} + 30^{\circ}) = \cot(30^{\circ}) = \sqrt{3}$

44. $\sec(420^{\circ}) = \sec(360^{\circ} + 60^{\circ}) = \sec(60^{\circ}) = 2$

45. $\cos\left(\dfrac{33\pi}{4}\right) = \cos\left(\dfrac{\pi}{4} + \dfrac{32\pi}{4}\right) = \cos\left(\dfrac{\pi}{4} + 8\pi\right) = \cos\left(\dfrac{\pi}{4} + 4 \cdot 2\pi\right) = \cos\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

46. $\sin\left(\frac{9\pi}{4}\right)=\sin\left(\frac{\pi}{4}+\frac{8\pi}{4}\right)=\sin\left(\frac{\pi}{4}+2\pi\right)=\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

47. $\tan(21\pi)=\tan(0+21\pi)=\tan(0)=0$

48. $\csc\left(\frac{9\pi}{2}\right)=\csc\left(\frac{\pi}{2}+\frac{8\pi}{2}\right)=\csc\left(\frac{\pi}{2}+4\pi\right)=\csc\left(\frac{\pi}{2}+2\cdot 2\pi\right)=\csc\left(\frac{\pi}{2}\right)=1$

49. $\sec\left(\frac{17\pi}{4}\right)=\sec\left(\frac{\pi}{4}+\frac{16\pi}{4}\right)=\sec\left(\frac{\pi}{4}+4\pi\right)=\sec\left(\frac{\pi}{4}+2\cdot 2\pi\right)=\sec\left(\frac{\pi}{4}\right)=\sqrt{2}$

50. $\cot\left(\frac{17\pi}{4}\right)=\cot\left(\frac{\pi}{4}+\frac{16\pi}{4}\right)=\cot\left(\frac{\pi}{4}+4\pi\right)=\cot\left(\frac{\pi}{4}+2\cdot 2\pi\right)=\cot\left(\frac{\pi}{4}\right)=1$

51. $\tan\left(\frac{19\pi}{6}\right)=\tan\left(\frac{\pi}{6}+\frac{18\pi}{6}\right)=\tan\left(\frac{\pi}{6}+3\pi\right)=\tan\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}$

52. $\sec\left(\frac{25\pi}{6}\right)=\sec\left(\frac{\pi}{6}+\frac{24\pi}{6}\right)=\sec\left(\frac{\pi}{6}+4\pi\right)=\sec\left(\frac{\pi}{6}+2\cdot 2\pi\right)=\sec\left(\frac{\pi}{6}\right)=\frac{2\sqrt{3}}{3}$

53. $\sin(150^\circ)=\sin(30^\circ)=\frac{1}{2}$, since $\theta=150^\circ$ has reference angle $\alpha=30^\circ$ in quadrant II.

54. $\cos(210^\circ)=-\cos(30^\circ)=-\frac{\sqrt{3}}{2}$, since $\theta=210^\circ$ has reference angle $\alpha=30^\circ$ in quadrant III.

55. $\cos(315^\circ)=\cos(45^\circ)=\frac{\sqrt{2}}{2}$, since $\theta=315^\circ$ has reference angle $\alpha=45^\circ$ in quadrant IV.

56. $\sin(120^\circ)=\sin(60^\circ)=\frac{\sqrt{3}}{2}$, since $\theta=120^\circ$ has reference angle $\alpha=60^\circ$ in quadrant II.

57. $\sec(240^\circ)=-\sec(60^\circ)=-2$, since $\theta=240^\circ$ has reference angle $\alpha=60^\circ$ in quadrant III.

58. $\csc(300^\circ)=-\csc(60^\circ)=-\frac{2\sqrt{3}}{3}$, since $\theta=300^\circ$ has reference angle $\alpha=60^\circ$ in quadrant IV.

59. $\cot(330^\circ)=-\cot(30^\circ)=-\sqrt{3}$, since $\theta=330^\circ$ has reference angle $\alpha=30^\circ$ in quadrant IV.

60. $\tan(225^\circ)=\tan(45^\circ)=1$, since $\theta=225^\circ$ has reference angle $\alpha=45^\circ$ in quadrant III.

61. $\sin\left(\frac{3\pi}{4}\right)=\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$, since $\theta=\frac{3\pi}{4}$ has reference angle $\alpha=\frac{\pi}{4}$ in quadrant III.

62. $\cos\left(\frac{2\pi}{3}\right)=-\cos\left(\frac{\pi}{3}\right)=-\frac{1}{2}$, since $\theta=\frac{2\pi}{3}$ has reference angle $\alpha=\frac{\pi}{3}$ in quadrant II.

63. $\cot\left(\frac{7\pi}{6}\right)=\cot\left(\frac{\pi}{6}\right)=\sqrt{3}$, since $\theta=\frac{7\pi}{6}$ has reference angle $\alpha=\frac{\pi}{6}$ in quadrant III.

64. $\csc\left(\frac{7\pi}{4}\right)=-\csc\left(\frac{\pi}{4}\right)=-\sqrt{2}$, since $\theta=\frac{7\pi}{4}$ has reference angle $\alpha=\frac{\pi}{4}$ in quadrant IV.

65. $\cos\left(-60^\circ\right)=\cos\left(60^\circ\right)=\frac{1}{2}$, since $\theta=-60^\circ$ has reference angle $\alpha=60^\circ$ in quadrant IV.

66. $\tan\left(-120^\circ\right)=\tan\left(60^\circ\right)=\sqrt{3}$, since $\theta=-120^\circ$ has reference angle $\alpha=60^\circ$ in quadrant III.

67. $\sin\left(-\frac{2\pi}{3}\right)=-\sin\left(\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{2}$, since $\theta=-\frac{2\pi}{3}$ has reference angle $\alpha=\frac{\pi}{3}$ in quadrant III.

68. $\cot\left(-\frac{\pi}{6}\right)=-\cot\left(\frac{\pi}{6}\right)=-\sqrt{3}$, since $\theta=-\frac{\pi}{6}$ has reference angle $\alpha=\frac{\pi}{6}$ in quadrant IV.

69. $\tan\left(\frac{14\pi}{3}\right)=-\tan\left(\frac{\pi}{3}\right)=-\sqrt{3}$, since $\theta=\frac{14\pi}{3}$ has reference angle $\alpha=\frac{\pi}{3}$ in quadrant II.

70. $\sec\left(\frac{11\pi}{4}\right)=-\sec\left(\frac{\pi}{4}\right)=-\sqrt{2}$, since $\theta=\frac{11\pi}{4}$ has reference angle $\alpha=\frac{\pi}{4}$ in quadrant II.

71. $\csc\left(-315^\circ\right)=\csc\left(45^\circ\right)=\sqrt{2}$, since $\theta=-315^\circ$ has reference angle $\alpha=45^\circ$ in quadrant I.

72. $\sec\left(-225^\circ\right)=-\sec\left(45^\circ\right)=-\sqrt{2}$, since $\theta=-225^\circ$ has reference angle $\alpha=45^\circ$ in quadrant II.

73. $\sin(8\pi)=\sin(0+8\pi)=\sin(0)=0$

74. $\cos(-2\pi)=\cos(0-2\pi)=\cos(0)=1$

75. $\tan(7\pi)=\tan(\pi+6\pi)=\tan(\pi)=0$

76. $\cot(5\pi)=\cot(\pi+4\pi)=\cot(\pi)$, which is undefined

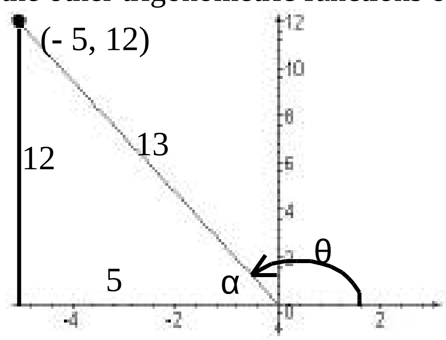
77. $\sec(-3\pi) = \sec(\pi) = -1$, since $\theta = -3\pi$ is coterminal with $\alpha = \pi$.

78. $\csc\left(-\frac{5\pi}{2}\right) = \csc\left(\frac{3\pi}{2}\right) = -1$, since $\theta = -\frac{5\pi}{2}$ is coterminal with $\alpha = \frac{3\pi}{2}$.

79. $\sin\theta = \frac{12}{13}$, θ in quadrant II

Since θ is in quadrant II, we know that $\sin\theta > 0$ and $\csc\theta > 0$;
while $\cos\theta < 0$, $\sec\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$.

If α is the reference angle for θ, then $\sin\alpha = \frac{12}{13}$.
Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{5}{13}$ $\tan\alpha = \frac{12}{5}$ $\sec\alpha = \frac{13}{5}$

$\csc\alpha = \frac{13}{12}$ $\cot\alpha = \frac{5}{12}$

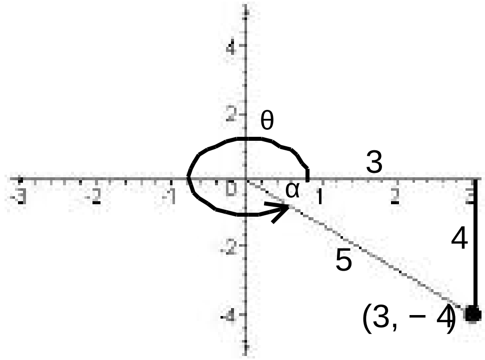
Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{5}{13}$ $\tan\theta = -\frac{12}{5}$ $\sec\theta = -\frac{13}{5}$

$\csc\theta = \frac{13}{12}$ $\cot\theta = -\frac{5}{12}$

80. $\cos\theta = \frac{3}{5}$, θ in quadrant IV

Since θ is in quadrant IV, we know that $\cos\theta > 0$ and $\sec\theta > 0$;
while $\sin\theta < 0$, $\csc\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$.
If α is the reference angle for θ, then $\cos\alpha = \frac{3}{5}$.
Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{4}{5}$ $\tan\alpha = \frac{4}{3}$ $\sec\alpha = \frac{5}{3}$

$\csc\alpha = \frac{5}{4}$ $\cot\alpha = \frac{3}{4}$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{4}{5}$ $\tan\theta = -\frac{4}{3}$ $\sec\theta = \frac{5}{3}$

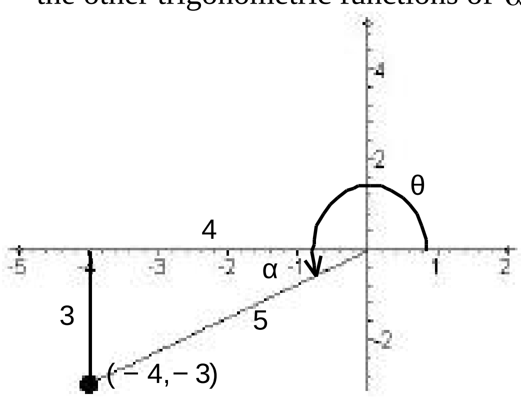
$\csc\theta = -\frac{5}{4}$ $\cot\theta = -\frac{3}{4}$

81. $\cos\theta = -\frac{4}{5}$, θ in quadrant III

Since θ is in quadrant III, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$; while $\tan\theta > 0$ and $\cot\theta > 0$.

If α is the reference angle for θ, then $\cos\alpha = \frac{4}{5}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{3}{5}$ $\tan\alpha = \frac{3}{4}$ $\sec\alpha = \frac{5}{4}$

$\csc\alpha = \frac{5}{3}$ $\cot\alpha = \frac{4}{3}$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{3}{5}$ $\tan\theta = \frac{3}{4}$ $\sec\theta = -\frac{5}{4}$

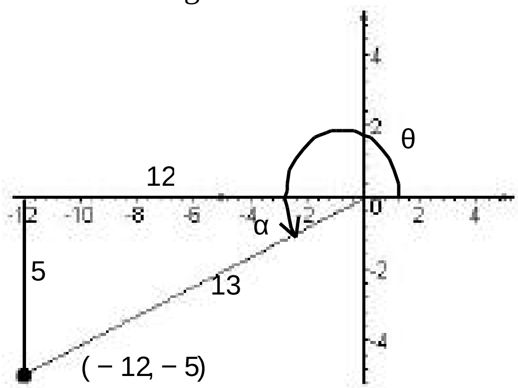
$\csc\theta = -\frac{5}{3}$ $\cot\theta = \frac{4}{3}$

82. $\sin\theta = -\frac{5}{13}$, θ in quadrant III

Since θ is in quadrant III, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$; while $\tan\theta > 0$ and $\cot\theta > 0$.

If α is the reference angle for θ, then $\sin\alpha = \frac{5}{13}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{12}{13}$ $\tan\alpha = \frac{5}{12}$ $\sec\alpha = \frac{13}{12}$

$\csc\alpha = \frac{13}{5}$ $\cot\alpha = \frac{12}{5}$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{12}{13}$ $\tan\theta = \frac{5}{12}$ $\sec\theta = -\frac{13}{12}$

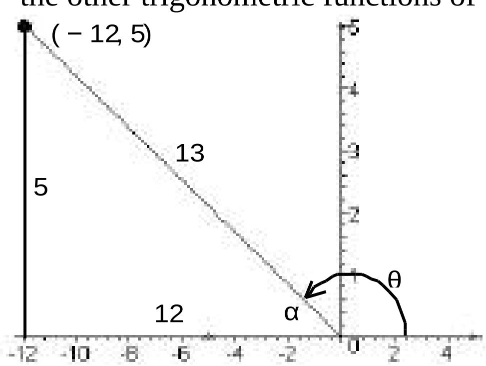
$\csc\theta = -\frac{13}{5}$ $\cot\theta = \frac{12}{5}$

83. $\sin\theta = \frac{5}{13}$, $90° < \theta < 180°$ θ in quadrant II

Since θ is in quadrant II, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$; while $\sin\theta > 0$ and $\csc\theta > 0$.

If α is the reference angle for θ, then $\sin\alpha = \frac{5}{13}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{12}{13}$ $\tan\alpha = \frac{5}{12}$ $\sec\alpha = \frac{13}{12}$

$\csc\alpha = \frac{13}{5}$ $\cot\alpha = \frac{12}{5}$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{12}{13}$ $\tan\theta = -\frac{5}{12}$ $\sec\theta = -\frac{13}{12}$

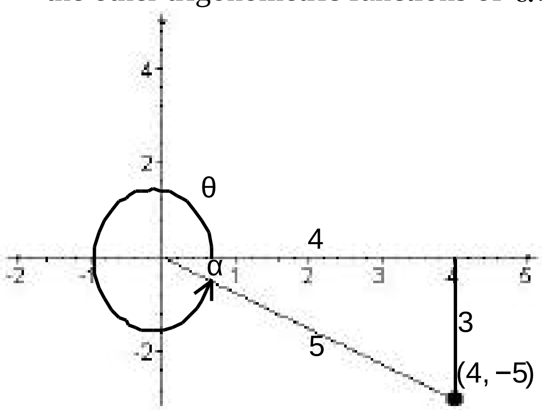
$\csc\theta = \frac{13}{5}$ $\cot\theta = -\frac{12}{5}$

84. $\cos\theta = \frac{4}{5}$, $270^\circ < \theta < 360^\circ$ θ in quadrant IV

Since θ is in quadrant IV, we know that $\sin\theta < 0$, $\csc\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$; while $\cos\theta > 0$ and $\sec\theta > 0$.

If α is the reference angle for θ, then $\cos\alpha = \frac{4}{5}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{3}{5}$ $\tan\alpha = \frac{3}{4}$ $\sec\alpha = \frac{5}{4}$

$\csc\alpha = \frac{5}{3}$ $\cot\alpha = \frac{4}{3}$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{3}{5}$ $\tan\theta = -\frac{3}{4}$ $\sec\theta = \frac{5}{4}$

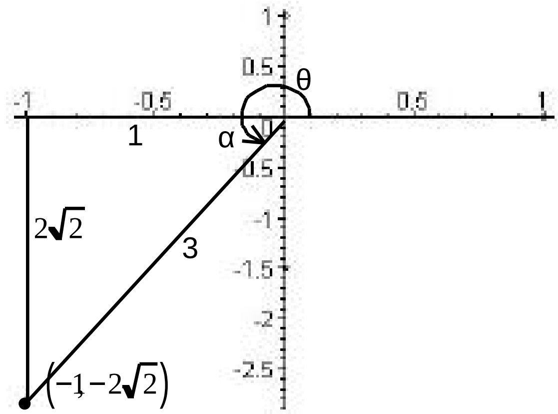
$\csc\theta = -\frac{5}{3}$ $\cot\theta = -\frac{4}{3}$

85. $\cos\theta = -\frac{1}{3}$, $180^\circ < \theta < 270^\circ$ θ in quadrant III

Since θ is in quadrant III, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$; while $\tan\theta > 0$ and $\cot\theta > 0$.

If α is the reference angle for θ, then $\cos\alpha = \frac{1}{3}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \frac{2\sqrt{2}}{3}$ $\tan\alpha = 2\sqrt{2}$ $\sec\alpha = 3$

$\csc\alpha = \frac{3\sqrt{2}}{4}$ $\cot\alpha = \frac{\sqrt{2}}{4}$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{2\sqrt{2}}{3}$ $\tan\theta = 2\sqrt{2}$ $\sec\theta = -3$

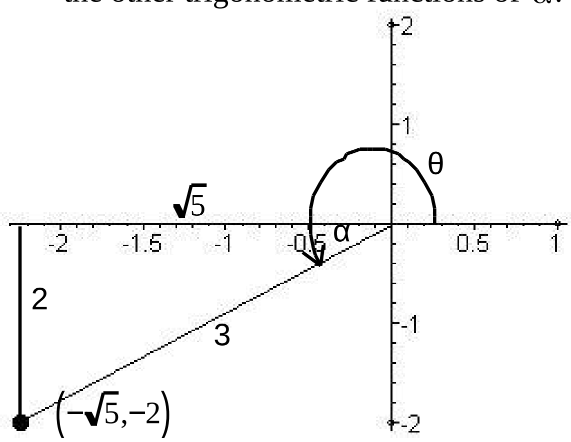
$\csc\theta = -\frac{3\sqrt{2}}{4}$ $\cot\theta = \frac{\sqrt{2}}{4}$

86. $\sin\theta = -\frac{2}{3}$, $180^\circ < \theta < 270^\circ$ θ in quadrant III

Since θ is in quadrant III, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$; while $\tan\theta > 0$ and $\cot\theta > 0$.

If α is the reference angle for θ, then $\sin\alpha = \frac{2}{3}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{\sqrt{5}}{3}$ $\tan\alpha = \frac{2\sqrt{5}}{5}$ $\sec\alpha = \frac{3\sqrt{5}}{5}$

$\csc\alpha = \frac{3}{2}$ $\cot\alpha = \frac{\sqrt{5}}{2}$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{\sqrt{5}}{3}$ $\tan\theta = \frac{2\sqrt{5}}{5}$ $\sec\theta = -\frac{3\sqrt{5}}{5}$

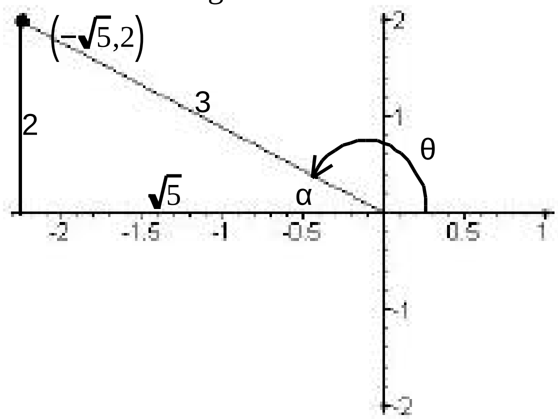
$\csc\theta = -\frac{3}{2}$ $\cot\theta = \frac{\sqrt{5}}{2}$

87. $\sin\theta = \frac{2}{3}$, $\tan\theta < 0$ $\quad\theta$ in quadrant II

Since θ is in quadrant II, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$; while $\sin\theta > 0$ and $\csc\theta > 0$.

If α is the reference angle for θ, then $\sin\alpha = \frac{2}{3}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{\sqrt{5}}{3}$ $\quad\tan\alpha = \frac{2\sqrt{5}}{5}$ $\quad\sec\alpha = \frac{3\sqrt{5}}{5}$

$\csc\alpha = \frac{3}{2}$ $\quad\cot\alpha = \frac{\sqrt{5}}{2}$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{\sqrt{5}}{3}$ $\quad\tan\theta = -\frac{2\sqrt{5}}{5}$ $\quad\sec\theta = -\frac{3\sqrt{5}}{5}$

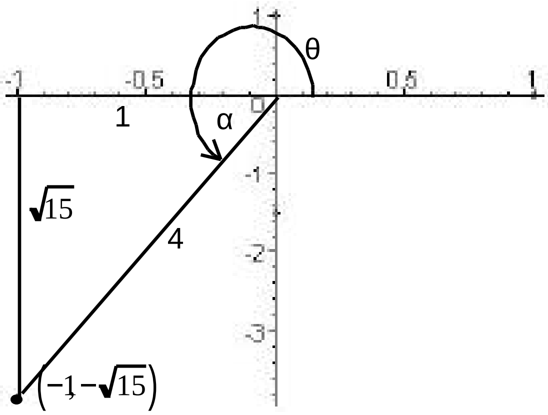
$\csc\theta = \frac{3}{2}$ $\quad\cot\theta = -\frac{\sqrt{5}}{2}$

88. $\cos\theta = -\frac{1}{4}$, $\tan\theta > 0$ $\quad\theta$ in quadrant III

Since θ is in quadrant III, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$; while $\tan\theta > 0$ and $\cot\theta > 0$.

If α is the reference angle for θ, then $\cos\alpha = \frac{1}{4}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \dfrac{\sqrt{15}}{4}$ $\tan\alpha = \sqrt{15}$ $\sec\alpha = 4$

$\csc\alpha = \dfrac{4\sqrt{15}}{15}$ $\cot\alpha = \dfrac{\sqrt{15}}{15}$

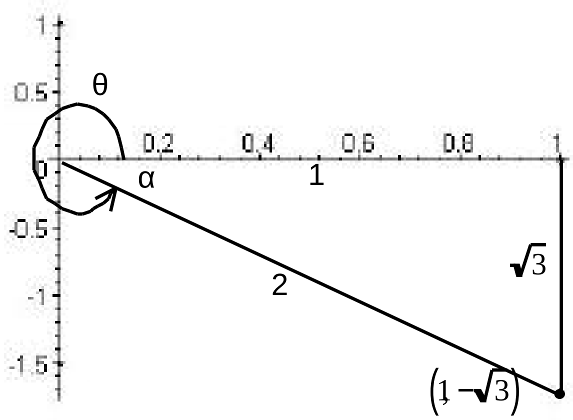
Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\sin\theta = -\dfrac{\sqrt{15}}{4}$ $\tan\theta = \sqrt{15}$ $\sec\theta = -4$

$\csc\theta = -\dfrac{4\sqrt{15}}{15}$ $\cot\theta = \dfrac{\sqrt{15}}{15}$

89. $\sec\theta = 2$, $\sin\theta < 0$, θ in quadrant IV

Since θ is in quadrant IV, we know that $\sin\theta < 0$, $\csc\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$; while $\cos\theta > 0$ and $\sec\theta > 0$.
If α is the reference angle for θ, then $\sec\alpha = 2$.
Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \dfrac{1}{2}$ $\sin\alpha = \dfrac{\sqrt{3}}{2}$ $\tan\alpha = \sqrt{3}$

$\csc\alpha = \dfrac{2\sqrt{3}}{3}$ $\cot\alpha = \dfrac{\sqrt{3}}{3}$

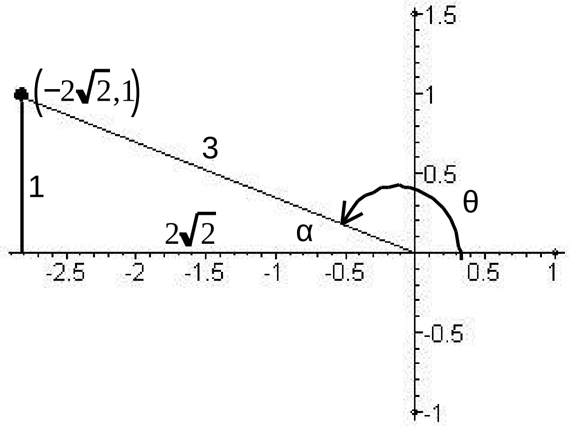
Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = \dfrac{1}{2}$ $\sin\theta = -\dfrac{\sqrt{3}}{2}$ $\tan\theta = -\sqrt{3}$

$\csc\theta = -\dfrac{2\sqrt{3}}{3}$ $\cot\theta = -\dfrac{\sqrt{3}}{3}$

90. $\csc\theta = 3$, $\cot\theta < 0$, θ in quadrant II

Since θ is in quadrant II, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$; while $\sin\theta > 0$ and $\csc\theta > 0$.
If α is the reference angle for θ, then $\csc\alpha = 3$.
Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \dfrac{2\sqrt{2}}{3}$ $\sin\alpha = \dfrac{1}{3}$ $\tan\alpha = \dfrac{\sqrt{2}}{4}$

$\sec\alpha = \dfrac{3\sqrt{2}}{4}$ $\cot\alpha = 2\sqrt{2}$

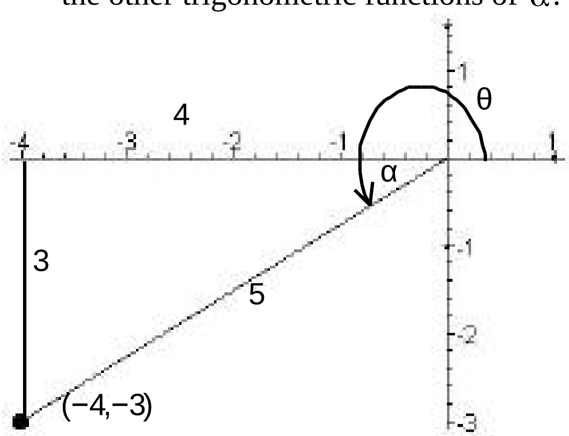
Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = -\dfrac{2\sqrt{2}}{3}$ $\sin\theta = \dfrac{1}{3}$ $\tan\theta = -\dfrac{\sqrt{2}}{4}$

$\csc\theta = \dfrac{3\sqrt{2}}{4}$ $\cot\theta = -2\sqrt{2}$

91. $\tan\theta = \dfrac{3}{4}$, $\sin\theta < 0$, θ in quadrant III

Since θ is in quadrant III, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$; while $\tan\theta > 0$ and $\cot\theta > 0$.
If α is the reference angle for θ, then $\tan\alpha = \dfrac{3}{4}$.
Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \dfrac{4}{5}$ $\sin\alpha = \dfrac{3}{5}$ $\sec\alpha = \dfrac{5}{4}$

$\csc\alpha = \dfrac{5}{3}$ $\cot\alpha = \dfrac{4}{3}$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{4}{5}$ $\sin\theta = -\frac{3}{5}$ $\sec\theta = -\frac{5}{4}$

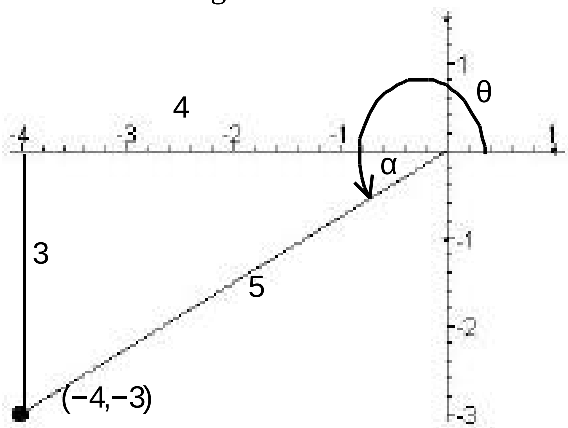
$\csc\theta = -\frac{5}{3}$ $\cot\theta = \frac{4}{3}$

92. $\cot\theta = \frac{4}{3}$, $\cos\theta < 0$, θ in quadrant III

Since θ is in quadrant III, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$; while $\tan\theta > 0$ and $\cot\theta > 0$.

If α is the reference angle for θ, then $\cot\alpha = \frac{4}{3}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{4}{5}$ $\sin\alpha = \frac{3}{5}$ $\sec\alpha = \frac{5}{4}$

$\csc\alpha = \frac{5}{3}$ $\tan\alpha = \frac{3}{4}$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{4}{5}$ $\sin\theta = -\frac{3}{5}$ $\sec\theta = -\frac{5}{4}$

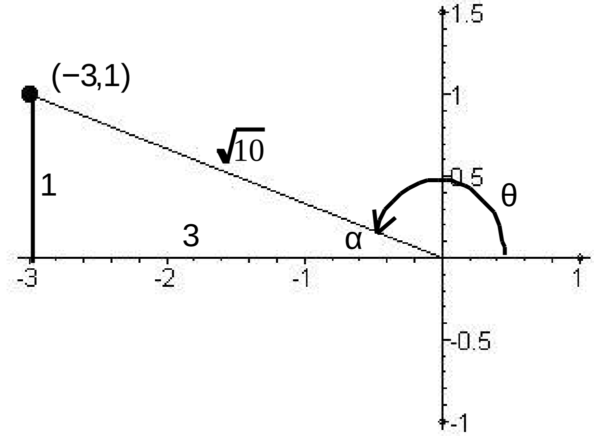
$\csc\theta = -\frac{5}{3}$ $\tan\theta = \frac{3}{4}$

93. $\tan\theta = -\frac{1}{3}$, $\sin\theta > 0$ θ in quadrant II

Since θ is in quadrant II, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$; while $\sin\theta > 0$ and $\csc\theta > 0$.

If α is the reference angle for θ, then $\tan\alpha = \frac{1}{3}$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{3\sqrt{10}}{10}$ $\sin\alpha = \frac{\sqrt{10}}{10}$ $\sec\alpha = \frac{\sqrt{10}}{3}$

$\csc\alpha = \sqrt{10}$ $\cot\alpha = 3$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{3\sqrt{10}}{10}$ $\sin\theta = \frac{\sqrt{10}}{10}$ $\sec\theta = -\frac{\sqrt{10}}{3}$

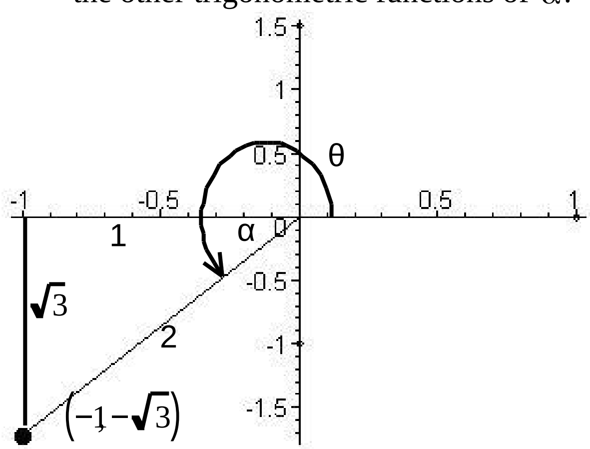
$\csc\theta = \sqrt{10}$ $\cot\theta = -3$

94. $\sec\theta = -2,\ \tan\theta > 0$ θ in quadrant III

Since θ is in quadrant III, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$; while $\tan\theta > 0$ and $\cot\theta > 0$.

If α is the reference angle for θ, then $\sec\alpha = 2$.

Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \frac{1}{2}$ $\sin\alpha = \frac{\sqrt{3}}{2}$ $\cot\alpha = \frac{\sqrt{3}}{3}$

$\csc\alpha = \frac{2\sqrt{3}}{3}$ $\tan\alpha = \sqrt{3}$

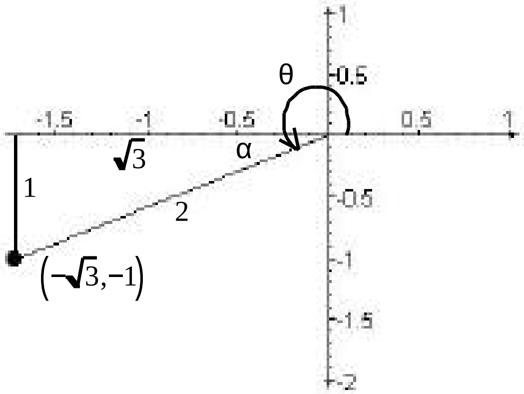
Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = -\frac{1}{2}$ $\sin\theta = -\frac{\sqrt{3}}{2}$ $\cot\theta = -\frac{\sqrt{3}}{3}$

$\csc\theta = -\frac{2\sqrt{3}}{3}$ $\tan\theta = \sqrt{3}$

95. $\csc\theta = -2, \; \tan\theta > 0 \qquad \theta$ in quadrant III

Since θ is in quadrant III, we know that $\cos\theta < 0$, $\sec\theta < 0$, $\sin\theta < 0$ and $\csc\theta < 0$; while $\tan\theta > 0$ and $\cot\theta > 0$.
If α is the reference angle for θ, then $\csc\alpha = 2$.
Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\cos\alpha = \dfrac{\sqrt{3}}{2} \qquad \sin\alpha = \dfrac{1}{2} \qquad \tan\alpha = \dfrac{\sqrt{3}}{3}$

$\sec\alpha = \dfrac{2\sqrt{3}}{3} \qquad \cot\alpha = \sqrt{3}$

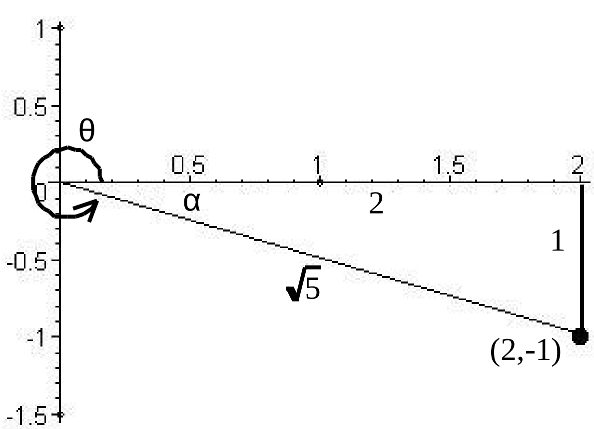
Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\cos\theta = -\dfrac{\sqrt{3}}{2} \qquad \sin\theta = -\dfrac{1}{2} \qquad \cot\theta = \sqrt{3}$

$\sec\theta = -\dfrac{2\sqrt{3}}{3} \qquad \tan\theta = \dfrac{\sqrt{3}}{3}$

96. $\cot\theta = -2, \; \sec\theta > 0 \qquad \theta$ in quadrant IV

Since θ is in quadrant IV, we know that $\cos\theta > 0$ and $\sec\theta > 0$; while $\sin\theta < 0$, $\csc\theta < 0$, $\tan\theta < 0$ and $\cot\theta < 0$.
If α is the reference angle for θ, then $\cot\alpha = 2$.
Now draw the appropriate triangle and use the Pythagorean Theorem to find the values of the other trigonometric functions of α.



$\sin\alpha = \dfrac{\sqrt{5}}{5} \qquad \cos\alpha = \dfrac{2\sqrt{5}}{5} \qquad \tan\alpha = \dfrac{1}{2}$

$\csc\alpha = \sqrt{5} \qquad \sec\alpha = \dfrac{\sqrt{5}}{2}$

Finally, we assign the appropriate sign to find the values of the other trigonometric functions of θ.

$\sin\theta = -\frac{\sqrt{5}}{5}$ $\cos\theta = \frac{2\sqrt{5}}{5}$ $\tan\theta = -\frac{1}{2}$

$\csc\theta = -\sqrt{5}$ $\sec\theta = \frac{\sqrt{5}}{2}$

97. $\sin(45^\circ) + \sin(135^\circ) + \sin(225^\circ) + \sin(315^\circ)$

$= \sin(45^\circ) + \sin(45^\circ + 90^\circ) + \sin(45^\circ + 180^\circ) + \sin(45^\circ + 270^\circ)$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right) = 0$

98. $\tan(60^\circ) + \tan(150^\circ) = \tan(60^\circ) + \tan(180^\circ - 30^\circ)$

So the reference angle for 150º is 30º, and since 150º lies in Quadrant II we have:

$\tan(60^\circ) + \tan(150^\circ) = \tan(60^\circ) - \tan(30^\circ) = \sqrt{3} - \frac{\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$

99. Given: $\sin\theta = 0.2$ θ in quadrant I

Therefore, $\theta + \pi$ is in quadrant III $\sin(\theta + \pi) = -0.2$

100. Given: $\cos\theta = 0.4$ θ in quadrant I

Therefore, $\theta + \pi$ is in quadrant III $\cos(\theta + \pi) = -0.4$

101. Given: $\tan\theta = 3$ θ in quadrant I or III

Therefore, $\theta + \pi$ is in quadrant III or I $\tan(\theta + \pi) = 3$

102. Given: $\cot\theta = -2$ θ in quadrant II or IV

Therefore, $\theta + \pi$ is in quadrant IV or II $\cot(\theta + \pi) = -2$

103. Given $\sin\theta = \frac{1}{5}$, then $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{1}{5}\right)} = 5$

104. Given $\cos\theta = \frac{2}{3}$, then $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{2}{3}\right)} = \frac{3}{2}$

105. Find the value:

$\sin(1^\circ) + \sin(2^\circ) + \sin(3^\circ) + \ldots + \sin(357^\circ) + \sin(358^\circ) + \sin(359^\circ)$

$= \sin(1^\circ) + \sin(2^\circ) + \sin(3^\circ) + \ldots + \sin(360^\circ - 3^\circ) + \sin(360^\circ - 2^\circ) + \sin(360^\circ - 1^\circ)$

$= \sin(1^\circ) + \sin(2^\circ) + \sin(3^\circ) + \ldots + \sin(-3^\circ) + \sin(-2^\circ) + \sin(-1^\circ)$

$= \sin(1^\circ) + \sin(2^\circ) + \sin(3^\circ) + \ldots - \sin(3^\circ) - \sin(2^\circ) - \sin(1^\circ)$

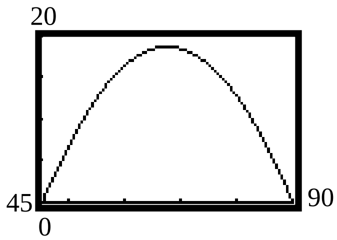
$= \sin(180^\circ) = 0$

106. Find the value:

$$\cos(1^\circ)+\cos(2^\circ)+\cos(3^\circ)+\ldots+\cos(357^\circ)+\cos(358^\circ)+\cos(359^\circ)$$
$$=\cos(1^\circ)+\cos(2^\circ)+\cos(3^\circ)+\ldots+\cos(360^\circ-3^\circ)+\cos(360^\circ-2^\circ)+\cos(360^\circ-1^\circ)$$
$$=\cos(1^\circ)+\cos(2^\circ)+\cos(3^\circ)+\ldots+\cos(-3^\circ)+\cos(-2^\circ)+\cos(-1^\circ)$$
$$=\cos(1^\circ)+\cos(2^\circ)+\cos(3^\circ)+\ldots+\cos(3^\circ)+\cos(2^\circ)+\cos(1^\circ)$$
$$=2\cos(1^\circ)+2\cos(2^\circ)+2\cos(3^\circ)+\ldots+2\cos(178^\circ)+2\cos(179^\circ)+\cos(180^\circ)$$
$$=2\cos(1^\circ)+2\cos(2^\circ)+2\cos(3^\circ)+\ldots+2\cos(180^\circ-2^\circ)+2\cos(180^\circ-1^\circ)+\cos(180^\circ)$$
$$=2\cos(1^\circ)+2\cos(2^\circ)+2\cos(3^\circ)+\ldots-2\cos(2^\circ)-2\cos(1^\circ)+\cos(180^\circ)$$
$$=\cos(180^\circ)=-1$$

107. (a) $R=\dfrac{32^2\sqrt{2}}{32}\left[\sin(2(60^\circ))-\cos(2(60^\circ))-1\right]\approx 32\sqrt{2}\left(0.866-(-0.5)-1\right)\approx 16.6\text{ f}$

(b) Graph:



(c) Using MAXIMUM, R is largest when $\theta=67.5^\circ$.