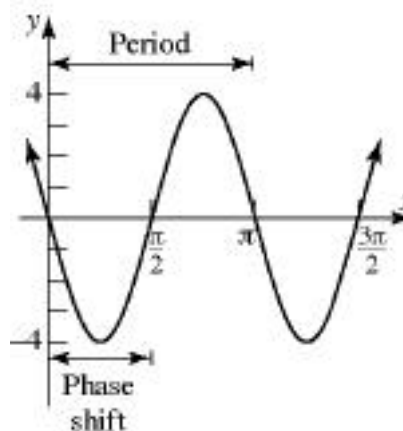


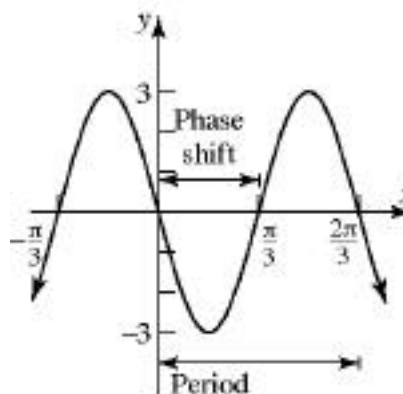
Trigonometric Functions

7.8 Phase Shift; Sinusoidal Curve Fitting

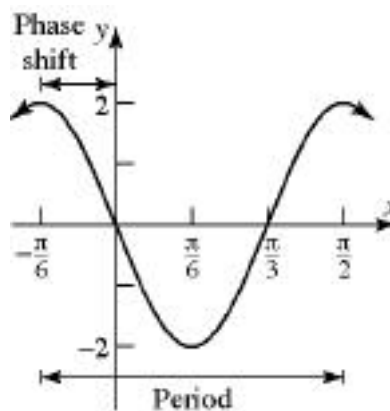
- $y = 4 \sin(2x - \frac{\pi}{2})$
 Amplitude: $|A| = |4| = 4$
 Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$
 Phase Shift: $\frac{\phi}{\omega} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{4}$



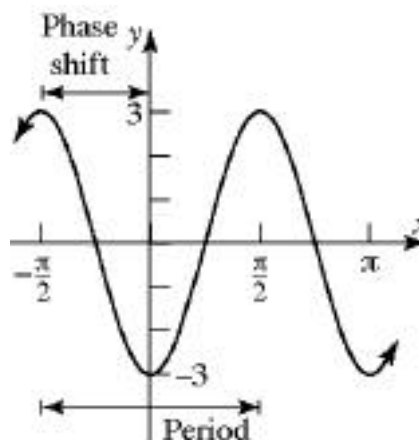
- $y = 3 \sin(3x - \frac{\pi}{3})$
 Amplitude: $|A| = |3| = 3$
 Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$
 Phase Shift: $\frac{\phi}{\omega} = \frac{-\frac{\pi}{3}}{3} = -\frac{\pi}{9}$



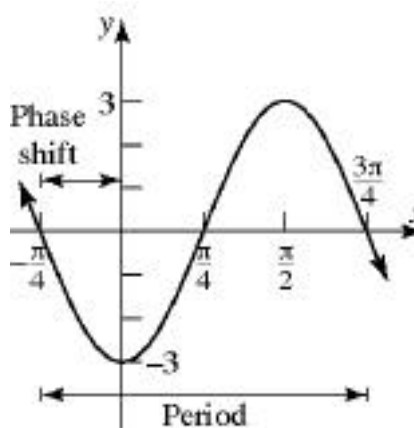
- $y = 2 \cos 3x + \frac{\pi}{2}$
 Amplitude: $|A| = |2| = 2$
 Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$
 Phase Shift: $\frac{\phi}{\omega} = \frac{\frac{\pi}{2}}{3} = \frac{\pi}{6}$



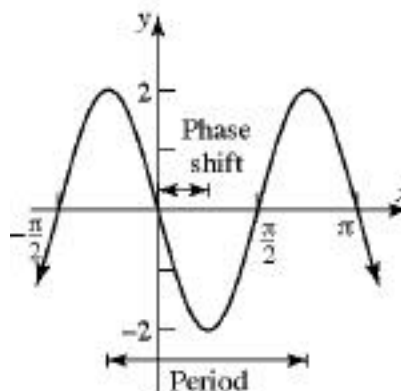
4. $y = 3\cos(2x + \frac{\pi}{2})$
 Amplitude: $|A| = |3| = 3$
 Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$
 Phase Shift: $\frac{\phi}{\omega} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{4}$



5. $y = -3\sin 2x + \frac{\pi}{2}$
 Amplitude: $|A| = |-3| = 3$
 Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$
 Phase Shift: $\frac{\phi}{\omega} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{4}$



6. $y = -2\cos 2x - \frac{\pi}{2}$
 Amplitude: $|A| = |-2| = 2$
 Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$
 Phase Shift: $\frac{\phi}{\omega} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{4}$



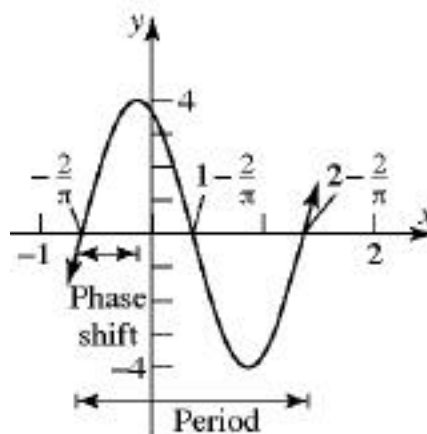
Chapter 7 Trigonometric Functions

7. $y = 4\sin(x + 2)$

Amplitude: $|A| = |4| = 4$

Period: $T = \frac{2}{\omega} = \frac{2}{1} = 2$

Phase Shift: $\frac{\phi}{\omega} = -\frac{2}{1}$

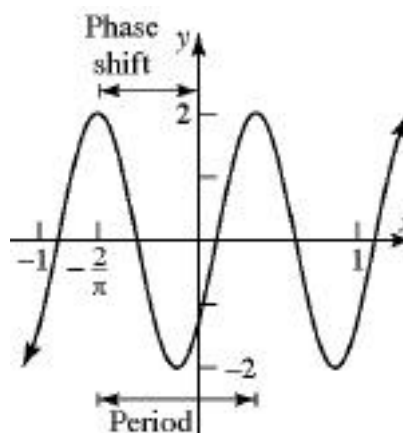


8. $y = 2\cos(2x + 4)$

Amplitude: $|A| = |2| = 2$

Period: $T = \frac{2}{\omega} = \frac{2}{2} = 1$

Phase Shift: $\frac{\phi}{\omega} = \frac{-4}{2} = -\frac{2}{1}$

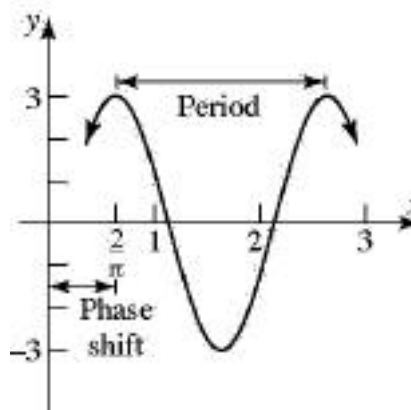


9. $y = 3\cos(x - 2)$

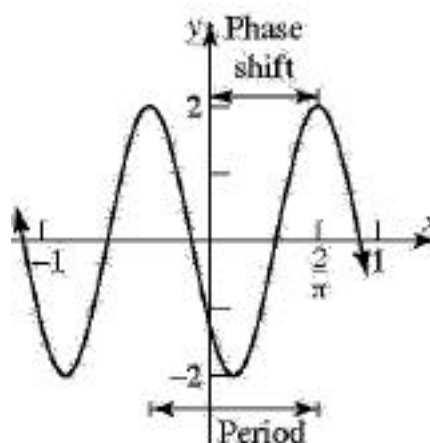
Amplitude: $|A| = |3| = 3$

Period: $T = \frac{2}{\omega} = \frac{2}{1} = 2$

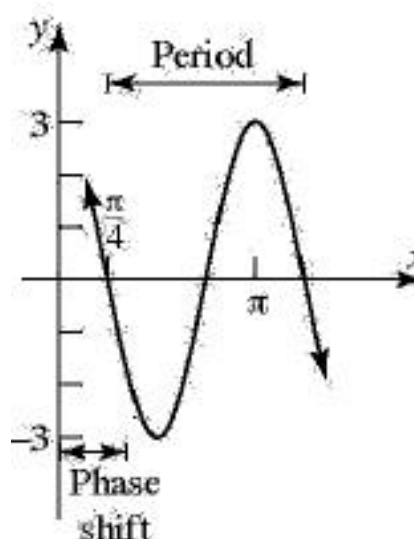
Phase Shift: $\frac{\phi}{\omega} = \frac{2}{1}$



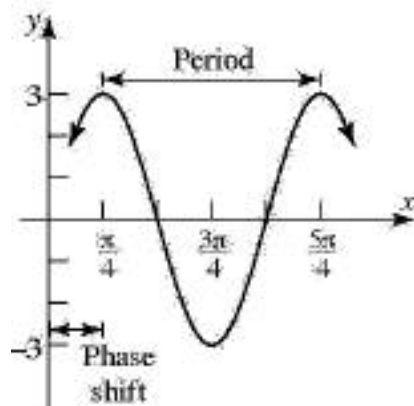
10. $y = 2\cos(2x - 4)$
 Amplitude: $|A| = |2| = 2$
 Period: $T = \frac{2}{\omega} = \frac{2}{2} = 1$
 Phase Shift: $\frac{\phi}{\omega} = \frac{4}{2} = 2$



11. $y = 3\sin -2x + \frac{\pi}{2} = -3\sin 2x - \frac{\pi}{2}$
 Amplitude: $|A| = |-3| = 3$
 Period: $T = \frac{2}{\omega} = \frac{2}{2} = 1$
 Phase Shift: $\frac{\phi}{\omega} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{4}$



12. $y = 3\cos -2x + \frac{\pi}{2}$
 Amplitude: $|A| = |3| = 3$
 Period: $T = \frac{2}{\omega} = \frac{2}{2} = 1$
 Phase Shift: $\frac{\phi}{\omega} = \frac{-\frac{\pi}{2}}{-2} = \frac{\pi}{4}$



13. $|A| = 2$; $T = \frac{1}{2}$; $\frac{\phi}{\omega} = \frac{1}{2}$; $\omega = \frac{2}{T} = \frac{2}{1/2} = 4$; $\frac{\phi}{\omega} = \frac{1}{4} = \frac{1}{4}$ $\phi = 1$
 $y = 2\sin(4x - 1) = 2\sin 4x - \frac{1}{2}$

$$14. \quad |A| = 3; \quad T = \frac{2}{2}; \quad \frac{\phi}{\omega} = 2; \quad \omega = \frac{2}{T} = \frac{2}{\frac{2}{2}} = 4; \quad \frac{\phi}{\omega} = \frac{\phi}{4} = 2 \quad \phi = 8$$

$$y = 3\sin(4x - 8) = 3\sin[4(x - 2)]$$

$$15. \quad |A| = 3; \quad T = 3; \quad \frac{\phi}{\omega} = -\frac{1}{3}; \quad \omega = \frac{2}{T} = \frac{2}{3} = \frac{2}{3};$$

$$\frac{\phi}{\omega} = \frac{\phi}{\frac{2}{3}} = -\frac{1}{3} \quad \phi = -\frac{1}{3} \cdot \frac{2}{3} = -\frac{2}{9} \quad y = 3\sin \frac{2}{3}x + \frac{2}{9} = 3\sin \frac{2}{3}x + \frac{1}{3}$$

$$16. \quad |A| = 2; \quad T = \frac{2}{2}; \quad \frac{\phi}{\omega} = -2; \quad \omega = \frac{2}{T} = \frac{2}{1} = 2; \quad \frac{\phi}{\omega} = \frac{\phi}{2} = -2 \quad \phi = -4$$

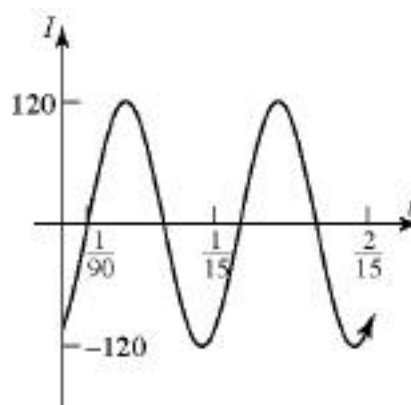
$$y = 2\sin(2x + 4) = 2\sin[2(x + 2)]$$

$$17. \quad I = 120\sin 30t - \frac{3}{3}, \quad t \geq 0$$

$$\text{Period:} \quad T = \frac{2}{\omega} = \frac{2}{30} = \frac{1}{15}$$

$$\text{Amplitude: } |A| = |120| = 120$$

$$\text{Phase Shift: } \frac{\phi}{\omega} = \frac{-\frac{3}{3}}{30} = \frac{1}{90}$$

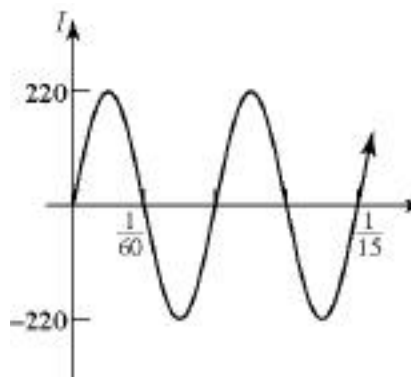


$$18. \quad I = 220\sin 60t - \frac{6}{6}, \quad t \geq 0$$

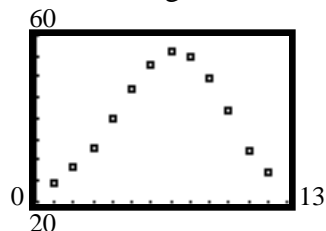
$$\text{Period:} \quad T = \frac{2}{\omega} = \frac{2}{60} = \frac{1}{30}$$

$$\text{Amplitude: } |A| = |220| = 220$$

$$\text{Phase Shift: } \frac{\phi}{\omega} = \frac{-\frac{6}{6}}{60} = \frac{1}{360}$$



19. (a) Draw a scatter diagram:



(b) Amplitude: $A = \frac{56.0 - 24.2}{2} = \frac{31.8}{2} = 15.9$

Section 7.8 Phase Shift: Sinusoidal Curve Fitting

$$\text{Vertical Shift: } \frac{56.0 + 24.2}{2} = \frac{80.2}{2} = 40.1$$

$$\omega = \frac{2}{12} = \frac{1}{6}$$

Phase shift (use $y = 24.2$, $x = 1$):

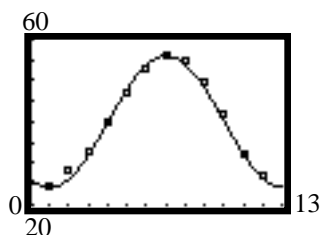
$$24.2 = 15.9 \sin \frac{1}{6} - \phi + 40.1$$

$$-15.9 = 15.9 \sin \frac{1}{6} - \phi \quad -1 = \sin \frac{1}{6} - \phi \quad -\frac{1}{2} = \frac{1}{6} - \phi$$

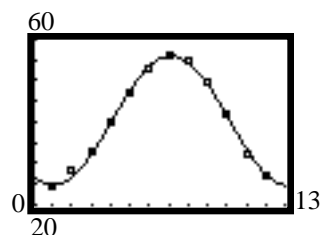
$$\phi = \frac{2}{3}$$

$$\text{Thus, } y = 15.9 \sin \frac{1}{6} x - \frac{2}{3} + 40.1$$

(c)

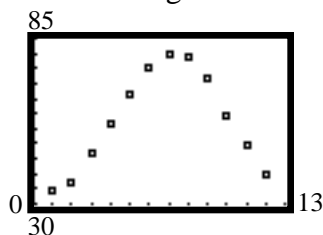


(e)



(d) $y = 15.62 \sin(0.517x - 2.096) + 40.377$

20. (a) Draw a scatter diagram:



(b) Amplitude: $A = \frac{80.0 - 34.6}{2} = \frac{45.4}{2} = 22.7$

$$\text{Vertical Shift: } \frac{80.0 + 34.6}{2} = \frac{114.6}{2} = 57.3$$

$$\omega = \frac{2}{12} = \frac{1}{6}$$

Phase shift (use $y = 34.6$, $x = 1$):

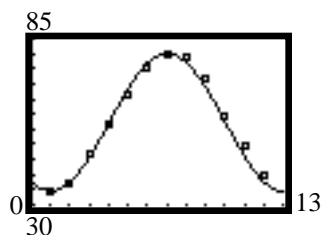
$$34.6 = 22.7 \sin \frac{1}{6} - \phi + 57.3$$

$$-22.7 = 22.7 \sin \frac{1}{6} - \phi \quad -1 = \sin \frac{1}{6} - \phi \quad -\frac{1}{2} = \frac{1}{6} - \phi$$

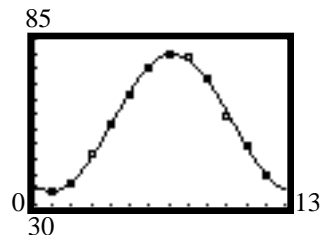
$$\phi = \frac{2}{3}$$

$$\text{Thus, } y = 22.7 \sin \frac{1}{6} x - \frac{2}{3} + 57.3$$

(c)

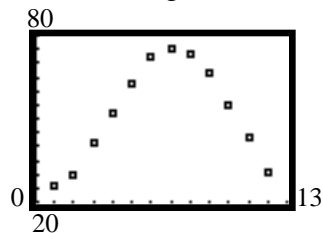


(e)



(d) $y = 22.6128\sin(0.5032x - 2.0384) + 57.1686$

21. (a) Draw a scatter diagram:



(b) Amplitude: $A = \frac{75.4 - 25.5}{2} = \frac{49.9}{2} = 24.95$

Vertical Shift: $\frac{75.4 + 25.5}{2} = \frac{100.9}{2} = 50.45$

$\omega = \frac{2}{12} = \frac{1}{6}$

Phase shift (use $y = 25.5$, $x = 1$):

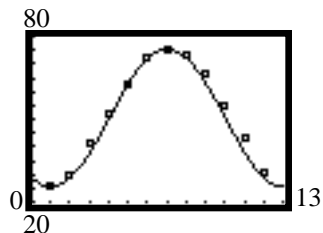
$$25.5 = 24.95\sin\left(\frac{1}{6} - \phi\right) + 50.45$$

$$-24.95 = 24.95\sin\left(\frac{1}{6} - \phi\right) \quad -1 = \sin\left(\frac{1}{6} - \phi\right) \quad -\frac{1}{2} = \frac{1}{6} - \phi$$

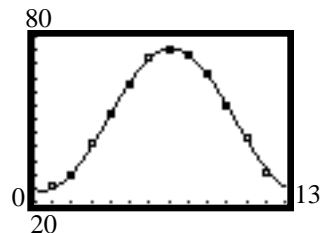
$$\phi = \frac{2}{3}$$

Thus, $y = 24.95\sin\left(\frac{1}{6}x - \frac{2}{3}\right) + 50.45$

(c)

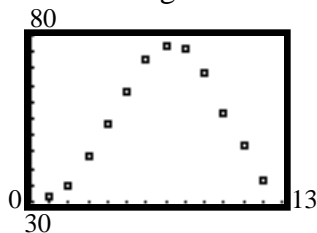


(e)



(d) $y = 25.693\sin(0.476x - 1.814) + 49.854$

22. (a) Draw a scatter diagram:



(b) Amplitude: $A = \frac{77.0 - 31.8}{2} = \frac{45.2}{2} = 22.6$

Vertical Shift: $\frac{77.0 + 31.8}{2} = \frac{108.8}{2} = 54.4$

$$\omega = \frac{2}{12} = \frac{1}{6}$$

 Phase shift (use $y = 31.8$, $x = 1$):

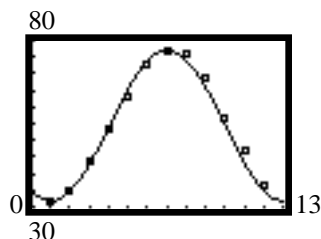
$$31.8 = 22.6 \sin \frac{1}{6} - \phi + 54.4$$

$$-22.6 = 22.6 \sin \frac{1}{6} - \phi \quad -1 = \sin \frac{1}{6} - \phi \quad -\frac{1}{2} = \frac{1}{6} - \phi$$

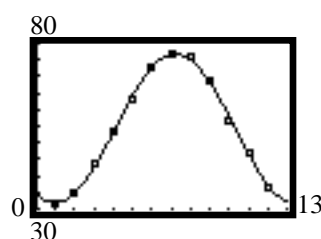
$$\phi = \frac{2}{3}$$

Thus, $y = 22.6 \sin \frac{1}{6} x - \frac{2}{3} + 54.4$

(c)



(e)



(d) $y = 22.4587 \sin(0.5058 x - 2.0602) + 54.3482$

23. (a)
- $3.6333 + 12.5 = 16.1333$
- hours which is at 4:08 p.m.

(b) Amplitude: $A = \frac{8.2 - (-0.6)}{2} = \frac{8.8}{2} = 4.4$

Vertical Shift: $\frac{8.2 + (-0.6)}{2} = \frac{7.6}{2} = 3.8$

$$\omega = \frac{2}{12.5} = \frac{1}{6.25}$$

 Phase shift (use $y = -0.6$, $x = 10.1333$):

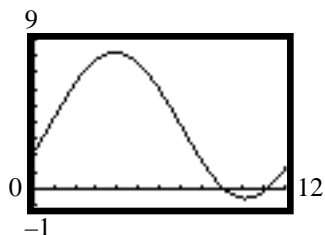
$$-0.6 = 4.4 \sin \frac{1}{6.25} 10.1333 - \phi + 3.8$$

$$-4.4 = 4.4 \sin \frac{1}{6.25} 10.1333 - \phi \quad -1 = \sin \frac{10.1333}{6.25} - \phi$$

$$-\frac{1}{2} = \frac{10.1333}{6.25} - \phi \quad \phi = 6.6643$$

$$\text{Thus, } y = 4.4 \sin \frac{x}{6.25} - 6.6643 + 3.8$$

(c)



$$(d) \quad y = 4.4 \sin \frac{16.1333}{6.25} - 6.6643 + 3.8 = 8.2 \text{ feet}$$

$$24. (a) \quad 8.1833 + 12.5 = 20.6833 \text{ hours which is at 8:41 p.m.}$$

$$(b) \quad \text{Amplitude: } A = \frac{13.2 - 2.2}{2} = \frac{11}{2} = 5.5$$

$$\text{Vertical Shift: } \frac{13.2 + 2.2}{2} = \frac{15.4}{2} = 7.7$$

$$\omega = \frac{2}{12.5} = \frac{1}{6.25}$$

Phase shift (use $y = 2.2$, $x = 14.2333$):

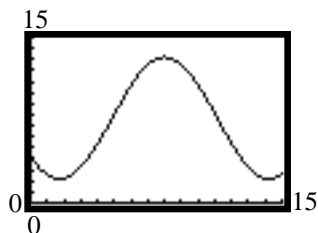
$$2.2 = 5.5 \sin \frac{14.2333}{6.25} - \phi + 7.7$$

$$-5.5 = 5.5 \sin \frac{14.2333}{6.25} - \phi \quad -1 = \sin \frac{14.2333}{6.25} - \phi$$

$$-\frac{1}{2} = \frac{14.2333}{6.25} - \phi \quad \phi = 8.7252$$

$$\text{Thus, } y = 5.5 \sin \frac{x}{6.25} - 8.7252 + 7.7$$

(c)



$$(d) \quad y = 5.5 \sin \frac{20.6833}{6.25} - 8.7252 + 7.7 = 13.2 \text{ feet}$$

Section 7.8 Phase Shift: Sinusoidal Curve Fitting

25. (a) Amplitude: $A = \frac{12.75 - 10.583}{2} = \frac{2.167}{2} = 1.0835$

Vertical Shift: $\frac{12.75 + 10.583}{2} = \frac{23.333}{2} = 11.6665$

$\omega = \frac{2}{365}$

Phase shift (use $y = 10.583$, $x = 356$):

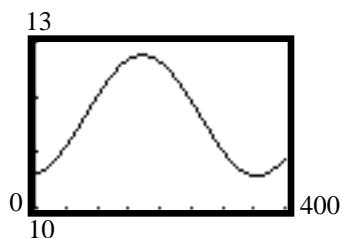
$$10.583 = 1.0835 \sin \frac{2}{365} (356 - \phi) + 11.6665$$

$$-1.0835 = 1.0835 \sin \frac{2}{365} (356 - \phi) \quad -1 = \sin \frac{712}{365} - \phi$$

$$-\frac{1}{2} = \frac{712}{365} - \phi \quad \phi = 7.6991$$

Thus, $y = 1.0835 \sin \frac{2}{365} x - 7.6991 + 11.6665$

(b)



(c) $y = 1.0835 \sin \frac{2}{365} (92) - 7.6991 + 11.6665 = 11.85$ hours

26. (a) Amplitude: $A = \frac{13.65 - 9.067}{2} = \frac{4.583}{2} = 2.2915$

Vertical Shift: $\frac{13.65 + 9.067}{2} = \frac{22.717}{2} = 11.3585$

$\omega = \frac{2}{365}$

Phase shift (use $y = 9.067$, $x = 356$):

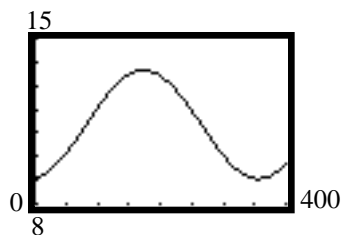
$$9.067 = 2.2915 \sin \frac{2}{365} (356 - \phi) + 11.3585$$

$$-2.2915 = 2.2915 \sin \frac{2}{365} (356 - \phi) \quad -1 = \sin \frac{712}{365} - \phi$$

$$-\frac{1}{2} = \frac{712}{365} - \phi \quad \phi = 7.6991$$

Thus, $y = 2.2915 \sin \frac{2}{365} x - 7.6991 + 11.3585$

(b)



(c) $y = 2.2915 \sin \frac{2}{365}(92) - 7.6991 + 11.3585 = 11.7$ hours

27. (a) Amplitude: $A = \frac{16.233 - 5.45}{2} = \frac{10.783}{2} = 5.3915$

Vertical Shift: $\frac{16.233 + 5.45}{2} = \frac{21.683}{2} = 10.8415$

$$\omega = \frac{2}{365}$$

Phase shift (use $y = 5.45$, $x = 356$):

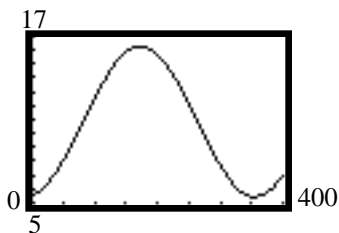
$$5.45 = 5.3915 \sin \frac{2}{365} (356 - \phi) + 10.8415$$

$$-5.3915 = 5.3915 \sin \frac{2}{365} (356 - \phi) \quad -1 = \sin \frac{712}{365} - \phi$$

$$-\frac{1}{2} = \frac{712}{365} - \phi \quad \phi = 7.6991$$

Thus, $y = 5.3915 \sin \frac{2}{365} x - 7.6991 + 10.8415$

(b)



(c) $y = 5.3915 \sin \frac{2}{365}(92) - 7.6991 + 10.8415 = 11.74$ hours

28. (a) Amplitude: $A = \frac{12.767 - 10.783}{2} = \frac{1.984}{2} = 0.992$

Vertical Shift: $\frac{12.767 + 10.783}{2} = \frac{23.55}{2} = 11.775$

$$\omega = \frac{2}{365}$$

Phase shift (use $y = 10.783$, $x = 356$):

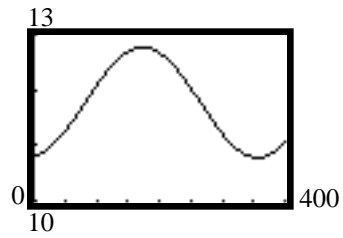
$$10.783 = 0.992 \sin \frac{2}{365} (356 - \phi) + 11.775$$

$$-0.992 = 0.992 \sin \frac{2}{365} (356 - \phi)$$

$$-1 = \sin \frac{712}{365} - \phi \quad -\frac{1}{2} = \frac{712}{365} - \phi \quad \phi = 7.6991$$

Thus, $y = 0.992 \sin \frac{2}{365}x - 7.6991 + 11.775$

(b)



(c) $y = 0.992 \sin \frac{2}{365}(92) - 7.6991 + 11.775 = 11.9$ hours