Chapter 7

Trigonometric Functions

7.R Chapter Review

1. $135° = 135 \cdot \frac{\pi}{180}$ radian $= \frac{3\pi}{4}$ radians

2. $210° = 210 \cdot \frac{\pi}{180}$ radian $= \frac{7\pi}{6}$ radians

3. $18° = 18 \cdot \frac{\pi}{180}$ radian $= \frac{\pi}{10}$ radians

4. $15° = 15 \cdot \frac{\pi}{180}$ radian $= \frac{\pi}{12}$ radians

5. $\frac{3\pi}{4} = \frac{3\pi}{4} \cdot \frac{180}{\pi}$ degrees $= 135°$

6. $\frac{2\pi}{3} = \frac{2\pi}{3} \cdot \frac{180}{\pi}$ degrees $= 120°$

7. $-\frac{5\pi}{2} = -\frac{5\pi}{2} \cdot \frac{180}{\pi}$ degrees $= -450°$

8. $-\frac{3\pi}{2} = -\frac{3\pi}{2} \cdot \frac{180}{\pi}$ degrees $= -270°$

9. $\tan\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right) = 1 - \frac{1}{2} = \frac{1}{2}$

10. $\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{2}\right) = \frac{1}{2} + 1 = \frac{3}{2}$

11. $3\sin(45°) - 4\tan\left(\frac{\pi}{6}\right) = 3\left(\frac{\sqrt{2}}{2}\right) - 4\left(\frac{\sqrt{3}}{3}\right) = \frac{3\sqrt{2}}{2} - \frac{4\sqrt{3}}{3}$

12. $4\cos(60°) + 3\tan\left(\frac{\pi}{3}\right) = 4\left(\frac{1}{2}\right) + 3\left(\sqrt{3}\right) = 2 + 3\sqrt{3}$

13. $6\cos\left(\frac{3\pi}{4}\right) + 2\tan\left(-\frac{\pi}{3}\right) = 6\left(-\frac{\sqrt{2}}{2}\right) + 2\left(-\sqrt{3}\right) = -3\sqrt{2} - 2\sqrt{3}$

14. $3\sin\left(\frac{2\pi}{3}\right) - 4\cos\left(\frac{5\pi}{2}\right) = 3\left(\frac{\sqrt{3}}{2}\right) - 4(0) = \frac{3\sqrt{3}}{2}$

15. $\sec\left(-\frac{\pi}{3}\right) - \cot\left(-\frac{5\pi}{4}\right) = \sec\left(\frac{\pi}{3}\right) + \cot\left(\frac{5\pi}{4}\right) = 2 + 1 = 3$

16. $4\csc\left(\frac{3\pi}{4}\right) - \cot\left(-\frac{\pi}{4}\right) = 4\csc\left(\frac{3\pi}{4}\right) + \cot\left(\frac{\pi}{4}\right) = 4\sqrt{2} + 1$

17. $\tan(\pi) + \sin(\pi) = 0 + 0 = 0$

18. $\cos\left(\frac{\pi}{2}\right) - \csc\left(-\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) + \csc\left(\frac{\pi}{2}\right) = 0 + 1 = 1$

19. $\cos(180^\circ) - \tan(-45^\circ) = -1 - (-1) = -1 + 1 = 0$

20. $\sin(270^\circ) + \cos(-180^\circ) = -1 + (-1) = -2$

21. $\sin^2(20^\circ) + \frac{1}{\sec^2(20^\circ)} = \sin^2(20^\circ) + \cos^2(20^\circ) = 1$

22. $\frac{1}{\cos^2(40^\circ)} - \frac{1}{\cot^2(20^\circ)} = \sec^2(40^\circ) - \tan^2(40^\circ) = 1$

23. $\sec(50^\circ)\cos(50^\circ) = \frac{1}{\cos(50^\circ)} \cos(50^\circ) = 1$

24. $\tan(10^\circ)\cot(10^\circ) = \tan(10^\circ) \frac{1}{\tan(10^\circ)} = 1$

25. $\frac{\sin(50^\circ)}{\cos(40^\circ)} = \frac{\cos(40^\circ)}{\cos(40^\circ)} = 1$

26. $\frac{\tan(20^\circ)}{\cot(70^\circ)} = \frac{\tan(20^\circ)}{\tan(20^\circ)} = 1$

27. $\frac{\sin(-40^\circ)}{\cos(50^\circ)} = \frac{-\sin(40^\circ)}{\cos(50^\circ)} = \frac{-\cos(50^\circ)}{\cos(50^\circ)} = -1$

28. $\tan(-20^\circ)\cot(20^\circ) = -\tan(20^\circ)\cot(20^\circ) = -\tan(20^\circ) \frac{1}{\cot(20^\circ)} = -1$

29. $\sin(400^\circ)\sec(-50^\circ) = \sin(40^\circ+360^\circ)\sec(50^\circ) = \sin(40^\circ)\csc(40^\circ) = \sin(40^\circ)\left(\frac{1}{\sin(40^\circ)}\right) = 1$

30. $\cot(200^\circ)\cot(-70^\circ) = \cot(20^\circ+180^\circ)(-\cot 70^\circ) = \cot(20^\circ)(-\tan 20^\circ) = \cot(20^\circ)\left(\frac{-1}{\cot(20^\circ)}\right) = -1$

31. $\sin\theta = -\frac{4}{5}, \quad \cos\theta > 0, \qquad \theta$ in quadrant IV

Solve for $\cos\theta$:

$$\sin^2\theta + \cos^2\theta = 1 \rightarrow \cos^2\theta = 1-\sin^2\theta \rightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Since θ is in quadrant IV, $\cos\theta > 0$.

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(-\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{4}{5}\right)}{\left(\frac{3}{5}\right)} = -\frac{4}{5}\ \frac{5}{3} = -\frac{4}{3} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{4}{3}\right)} = -\frac{3}{4}$$

32. $\cos\theta = -\frac{3}{5}, \quad \sin\theta < 0, \qquad \theta$ in quadrant III

Solve for $\sin\theta$:

$$\sin^2\theta + \cos^2\theta = 1 \rightarrow \sin^2\theta = 1-\cos^2\theta \rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(-\frac{3}{5}\right)^2} = -\sqrt{1-\frac{9}{25}} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{4}{5}\right)}{\left(-\frac{3}{5}\right)} = -\frac{4}{5}\ \frac{5}{3} = \frac{4}{3} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{3}{5}\right)} = -\frac{5}{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(\frac{4}{3}\right)} = \frac{3}{4}$$

33. $\tan\theta = \frac{12}{5}, \quad \sin\theta < 0, \qquad \theta$ in quadrant III

Solve for $\sec\theta$:

$$\sec^2\theta = 1+\tan^2\theta \rightarrow \sec\theta = \pm\sqrt{1+\tan^2\theta}$$

Since θ is in quadrant III, $\sec\theta < 0$.

$$\sec\theta = -\sqrt{1+\tan^2\theta} = -\sqrt{1+\left(\frac{12}{5}\right)^2} = -\sqrt{1+\frac{144}{25}} = -\sqrt{\frac{169}{25}} = -\frac{13}{5}$$

$$\cos\theta = -\frac{5}{13}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(-\frac{5}{13}\right)^2} = -\sqrt{1-\frac{25}{169}} = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(\frac{12}{5}\right)} = \frac{5}{12}$$

34. $\cot\theta = \frac{12}{5}$, $\cos\theta < 0$, θ in quadrant III

Solve for $\csc\theta$:

$$\csc^2\theta = 1+\cot^2\theta \rightarrow \csc\theta = \pm\sqrt{1+\cot^2\theta}$$

Since θ is in quadrant III, $\csc\theta < 0$.

$$\csc\theta = -\sqrt{1+\cot^2\theta} = -\sqrt{1+\left(\frac{12}{5}\right)^2} = -\sqrt{1+\frac{144}{25}} = -\sqrt{\frac{169}{25}} = -\frac{13}{5}$$

$$\sin\theta = -\frac{5}{13}$$

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(-\frac{5}{13}\right)^2} = -\sqrt{1-\frac{25}{169}} = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12} \qquad \tan\theta = \frac{1}{\cot\theta} = \frac{1}{\left(\frac{12}{5}\right)} = \frac{5}{12}$$

35. $\sec\theta = -\frac{5}{4}$, $\tan\theta < 0$, θ in quadrant II

Solve for $\cos\theta$:

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\left(-\frac{5}{4}\right)} = -\frac{4}{5}$$

Solve for $\sin\theta$:

$$\sin^2\theta + \cos^2\theta = 1 \rightarrow \sin^2\theta = 1-\cos^2\theta \rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$$

Since θ is in quadrant II, $\sin\theta > 0$.

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(-\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = \frac{3}{5} \quad \frac{5}{4} = -\frac{3}{4}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{3}{4}\right)} = -\frac{4}{3}$$

36. $\csc\theta = -\frac{5}{3}$, $\cot\theta < 0$, θ in quadrant IV

$$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

Solve for $\cos\theta$:

$$\sin^2\theta + \cos^2\theta = 1 \rightarrow \cos^2\theta = 1 - \sin^2\theta \rightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Since θ is in quadrant IV, $\cos\theta > 0$.

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(-\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{4}{5}\right)} = \frac{5}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{3}{5}\right)}{\left(\frac{4}{5}\right)} = -\frac{3}{5} \quad \frac{5}{4} = -\frac{3}{4} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{3}{4}\right)} = -\frac{4}{3}$$

37. $\sin\theta = \frac{12}{13}$, θ in quadrant II

Solve for $\cos\theta$:

$$\sin^2\theta + \cos^2\theta = 1 \rightarrow \cos^2\theta = 1 - \sin^2\theta \rightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Since θ is in quadrant II, $\cos\theta < 0$.

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(\frac{12}{13}\right)^2} = -\sqrt{1-\frac{144}{169}} = -\sqrt{\frac{25}{169}} = -\frac{5}{13}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{12}{13}\right)}{\left(-\frac{5}{13}\right)} = \frac{12}{13} \quad \frac{13}{5} = -\frac{12}{5} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{5}{13}\right)} = -\frac{13}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{12}{13}\right)} = \frac{13}{12} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

38. $\cos\theta = -\frac{3}{5}, \quad \theta$ in quadrant III

Solve for $\sin\theta$:

$$\sin^2\theta + \cos^2\theta = 1 \rightarrow \sin^2\theta = 1 - \cos^2\theta \rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$$

Since θ is in quadrant III, $\sin\theta < 0$.

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(-\frac{3}{5}\right)^2} = -\sqrt{1-\frac{9}{25}} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{4}{5}\right)}{\left(-\frac{3}{5}\right)} = -\frac{4}{5}\ \ \frac{5}{3} = \frac{4}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{3}{5}\right)} = -\frac{5}{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(\frac{4}{3}\right)} = \frac{3}{4}$$

39. $\sin\theta = -\frac{5}{13}, \quad \frac{3\pi}{2} < \theta < 2\pi, \quad \theta$ in quadrant IV

Solve for $\cos\theta$:

$$\sin^2\theta + \cos^2\theta = 1 \rightarrow \cos^2\theta = 1 - \sin^2\theta \rightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Since θ is in quadrant IV, $\cos\theta > 0$.

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(-\frac{5}{13}\right)^2} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{5}{13}\right)}{\left(\frac{12}{13}\right)} = -\frac{5}{13}\ \ \frac{13}{12} = -\frac{5}{12}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{12}{13}\right)} = \frac{13}{12}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{5}{13}\right)} = -\frac{13}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

40. $\cos\theta = \frac{12}{13}, \quad \frac{3\pi}{2} < \theta < 2\pi, \quad \theta$ in quadrant IV

Solve for $\sin\theta$:

$$\sin^2\theta + \cos^2\theta = 1 \rightarrow \sin^2\theta = 1 - \cos^2\theta \rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$$

Since θ is in quadrant IV, $\sin\theta < 0$.

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(\frac{12}{13}\right)^2} = -\sqrt{1-\frac{144}{169}} = -\sqrt{\frac{25}{169}} = -\frac{5}{13}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{5}{13}\right)}{\left(\frac{12}{13}\right)} = -\frac{5}{13}\ \frac{13}{12} = -\frac{5}{12} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{12}{13}\right)} = \frac{13}{12}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{5}{13}\right)} = -\frac{13}{5} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

41. $\tan\theta = \frac{1}{3},\ \ 180^{\circ} < \theta < 270^{\circ}, \qquad \theta$ in quadrant III

Solve for $\sec\theta$:

$$\sec^2\theta = 1+\tan^2\theta \rightarrow \sec\theta = \pm\sqrt{1+\tan^2\theta}$$

Since θ is in quadrant III, $\sec\theta < 0$.

$$\sec\theta = -\sqrt{1+\tan^2\theta} = -\sqrt{1+\left(\frac{1}{3}\right)^2} = -\sqrt{1+\frac{1}{9}} = -\sqrt{\frac{10}{9}} = -\frac{\sqrt{10}}{3}$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\left(-\frac{\sqrt{10}}{3}\right)} = -\frac{3}{\sqrt{10}}\ \frac{\sqrt{10}}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(-\frac{3\sqrt{10}}{3}\right)^2} = -\sqrt{1-\frac{90}{100}} = -\sqrt{\frac{10}{100}} = -\frac{\sqrt{10}}{10}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{\sqrt{10}}{10}\right)} = -\frac{10}{\sqrt{10}} = -\sqrt{10} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(\frac{1}{3}\right)} = 3$$

42. $\tan\theta = -\frac{2}{3},\ \ 90^{\circ} < \theta < 180^{\circ}, \qquad \theta$ in quadrant II

Solve for $\sec\theta$:

$$\sec^2\theta = 1+\tan^2\theta \quad \rightarrow \quad \sec\theta = \pm\sqrt{1+\tan^2\theta}$$

Since θ is in quadrant II, $\sec\theta < 0$.

$$\sec\theta = -\sqrt{1+\tan^2\theta} = -\sqrt{1+\left(-\frac{2}{3}\right)^2} = -\sqrt{1+\frac{4}{9}} = -\sqrt{\frac{13}{9}} = -\frac{\sqrt{13}}{3}$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\left(-\frac{\sqrt{13}}{3}\right)} = -\frac{3}{\sqrt{13}}\ \frac{\sqrt{13}}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(-\frac{3\sqrt{13}}{13}\right)^2} = \sqrt{1-\frac{117}{169}} = \sqrt{\frac{52}{169}} = \frac{2\sqrt{13}}{13}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{2\sqrt{13}}{13}\right)} = \frac{13}{2\sqrt{13}} = \frac{\sqrt{13}}{2} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{2}{3}\right)} = -\frac{3}{2}$$

43. $\sec\theta = 3,\ \frac{3\pi}{2} < \theta < 2\pi,\qquad \theta$ in quadrant IV

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{3}$

Solve for $\sin\theta$:

$$\sin^2\theta + \cos^2\theta = 1 \quad \rightarrow \quad \sin^2\theta = 1 - \cos^2\theta \quad \rightarrow \quad \sin\theta = \pm\sqrt{1-\cos^2\theta}$$

Since θ is in quadrant IV, $\sin\theta < 0$.

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(\frac{1}{3}\right)^2} = -\sqrt{1-\frac{1}{9}} = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{2\sqrt{2}}{3}\right)}{\left(\frac{1}{3}\right)} = -\frac{2\sqrt{2}}{3}\ \ \frac{3}{1} = -2\sqrt{2}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{2\sqrt{2}}{3}\right)} = -\frac{3}{2\sqrt{2}} = -\frac{3\sqrt{2}}{4} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{-2\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

44. $\csc\theta = -4,\ \pi < \theta < \frac{3\pi}{2},\qquad \theta$ in quadrant III

$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-4} = -\frac{1}{4}$

Solve for $\cos\theta$:

$$\sin^2\theta + \cos^2\theta = 1 \quad \rightarrow \quad \cos^2\theta = 1 - \sin^2\theta \quad \rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Since θ is in quadrant III, $\cos\theta < 0$.

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(-\frac{1}{4}\right)^2} = -\sqrt{1-\frac{1}{16}} = -\sqrt{\frac{15}{16}} = -\frac{\sqrt{15}}{4}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{\sqrt{15}}{4}\right)} = -\frac{4}{\sqrt{15}}\ \ \frac{\sqrt{15}}{\sqrt{15}} = -\frac{4\sqrt{15}}{15}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{1}{4}\right)}{\left(-\frac{\sqrt{15}}{4}\right)} = -\frac{1}{4}\ \ \frac{4}{\sqrt{15}}\ \ \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{15}}{15}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(\frac{\sqrt{15}}{5}\right)} = \frac{15}{\sqrt{15}} = \sqrt{15}$$

45. $\cot\theta = -2,\ \frac{\pi}{2} < \theta < \pi,\quad \theta$ in quadrant II

$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{-2} = -\frac{1}{2}$

Solve for $\sec\theta$:

$$\sec^2\theta = 1 + \tan^2\theta \rightarrow \sec\theta = \pm\sqrt{1+\tan^2\theta}$$

Since θ is in quadrant II, $\sec\theta < 0$.

$$\sec\theta = -\sqrt{1+\tan^2\theta} = -\sqrt{1+\left(-\frac{1}{2}\right)^2} = -\sqrt{1+\frac{1}{4}} = -\sqrt{\frac{5}{4}} = -\frac{\sqrt{5}}{2}$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\left(-\frac{\sqrt{5}}{2}\right)} = -\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(-\frac{2\sqrt{5}}{2}\right)^2} = \sqrt{1-\frac{20}{25}} = \sqrt{\frac{5}{25}} = \frac{\sqrt{5}}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{\sqrt{5}}{5}\right)} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

46. $\tan\theta = -2,\ \frac{3\pi}{2} < \theta < 2\pi,\quad \theta$ in quadrant IV

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-2} = -\frac{1}{2}$

Solve for $\sec\theta$:

$$\sec^2\theta = 1 + \tan^2\theta \rightarrow \sec\theta = \pm\sqrt{1+\tan^2\theta}$$

Since θ is in quadrant IV, $\sec\theta > 0$.

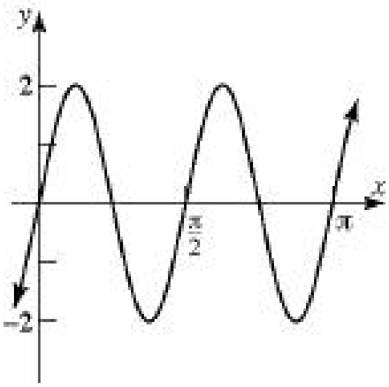
$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+(-2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

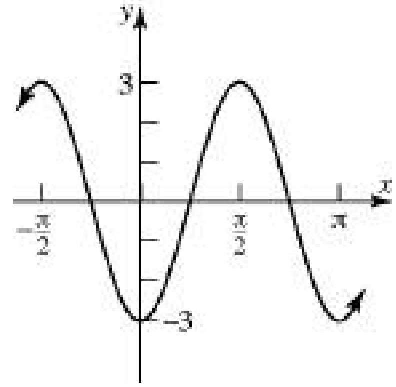
$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(\frac{\sqrt{5}}{5}\right)^2} = -\sqrt{1-\frac{5}{25}} = -\sqrt{\frac{20}{25}} = -\frac{2\sqrt{5}}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{2\sqrt{5}}{5}\right)} = -\frac{5}{2\sqrt{5}} = -\frac{\sqrt{5}}{2}$$

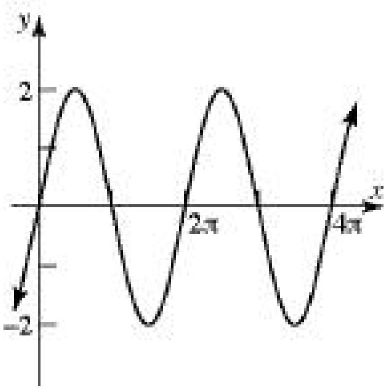
47. $y = 2\sin(4x)$



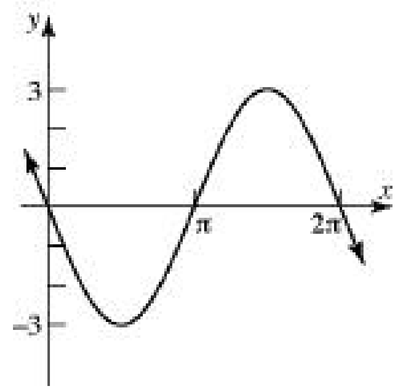
48. $y = -3\cos(2x)$



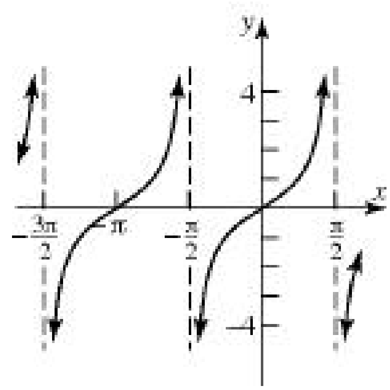
49. $y = -2\cos\left(x + \frac{\pi}{2}\right)$



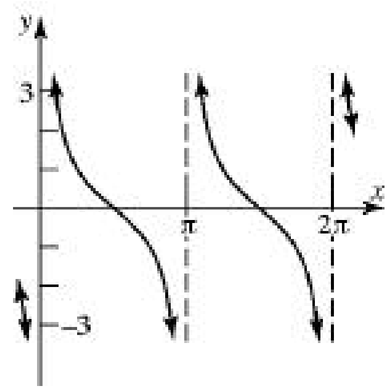
50. $y = 3\sin(x - \pi)$



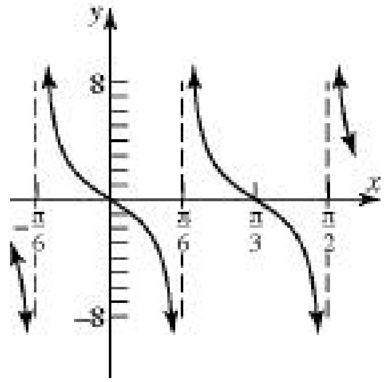
51. $y = \tan(x + \pi)$



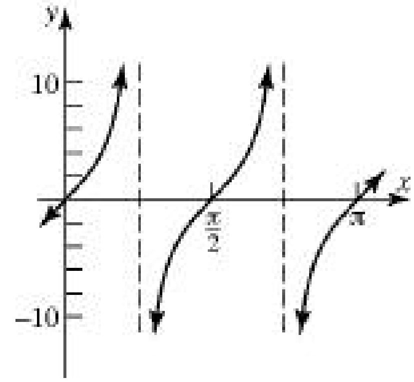
52. $y = -\tan\left(x - \frac{\pi}{2}\right)$



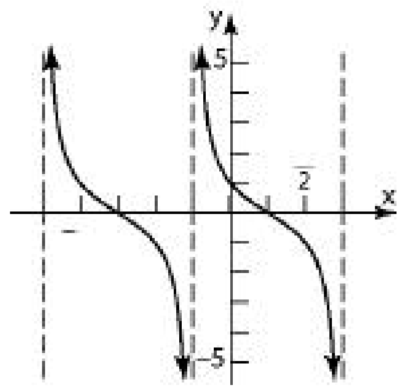
53. $y = -2\tan(3x)$



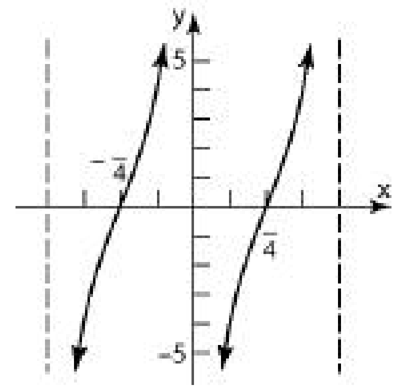
54. $y = 4\tan(2x)$



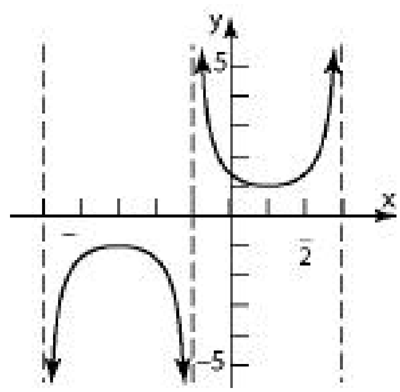
55. $y = \cot\left(x + \frac{\pi}{8}\right)$



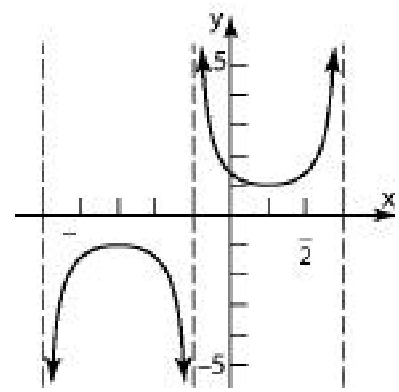
56. $y = -4\cot(2x)$



57. $y = \sec\left(x - \frac{\pi}{4}\right)$



58. $y = \csc\left(x + \frac{\pi}{4}\right)$



59. $y = 4\cos x$
Amplitude = 4
Period = 2π

60. $y = \sin(2x)$
Amplitude = 1
Period = π

61. $y = -8\sin\left(\frac{\pi}{2}x\right)$

Amplitude = 8

Period = 4

62. $y = -2\cos(3\pi x)$

Amplitude = 2

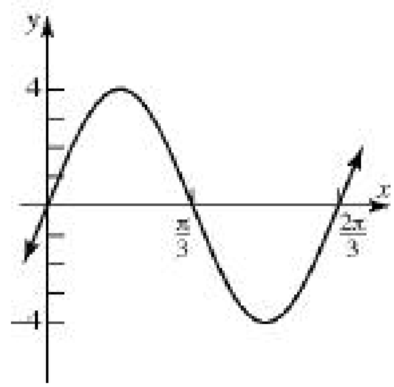
Period = $\frac{2}{3}$

63. $y = 4\sin(3x)$

Amplitude: $|A| = |4| = 4$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$

Phase Shift: $\frac{\phi}{\omega} = \frac{0}{3} = 0$

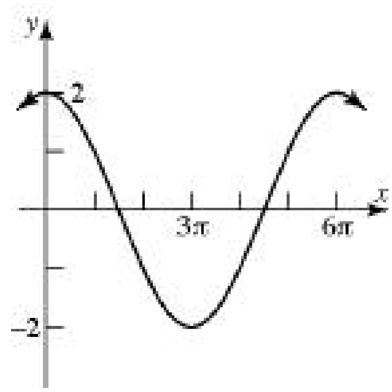


64. $y = 2\cos\left(\frac{1}{3}x\right)$

Amplitude: $|A| = |2| = 2$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(\frac{1}{3}\right)} = 6\pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{0}{\left(\frac{1}{3}\right)} = 0$

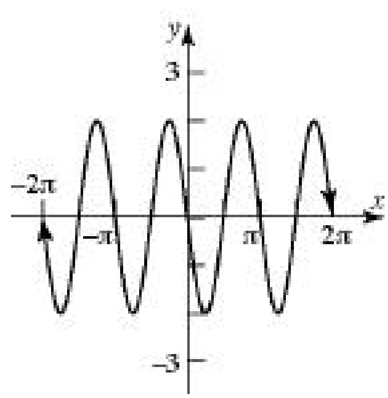


65. $y = 2\sin(2x - \pi)$

Amplitude: $|A| = |2| = 2$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{-\pi}{2} = -\frac{\pi}{2}$

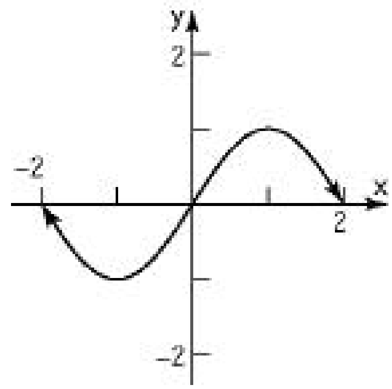


66. $y = -\cos\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

Amplitude: $|A| = |-1| = 1$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(\frac{1}{2}\right)} = 4\pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\right)} = \pi$

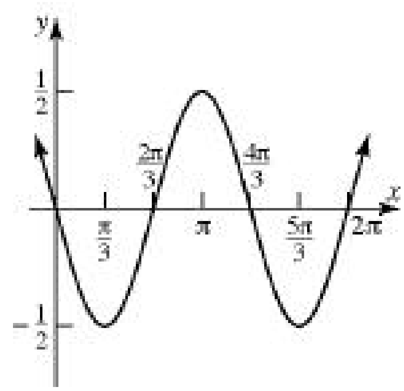


67. $y = \frac{1}{2}\sin\left(\frac{3}{2}x - \pi\right)$

Amplitude: $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(\frac{3}{2}\right)} = \frac{4\pi}{3}$

Phase Shift : $\frac{\phi}{\omega} = \frac{\pi}{\left(\frac{3}{2}\right)} = \frac{2\pi}{3}$

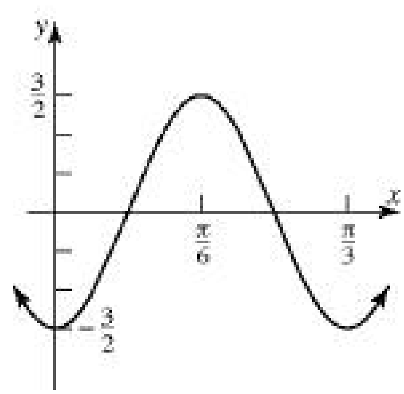


68. $y = \frac{3}{2}\cos(6x + 3\pi)$

Amplitude: $|A| = \left|\frac{3}{2}\right| = \frac{3}{2}$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{6} = \frac{\pi}{3}$

Phase Shift : $\frac{\phi}{\omega} = \frac{-3\pi}{6} = -\frac{\pi}{2}$

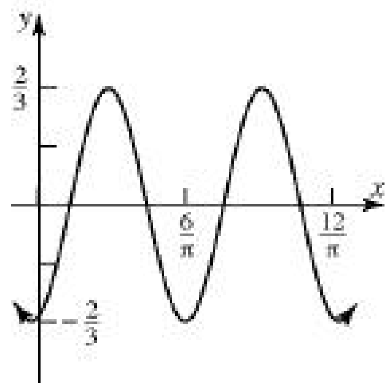


69. $y = -\frac{2}{3}\cos(\pi x - 6)$

Amplitude: $|A| = \left|-\frac{2}{3}\right| = \frac{2}{3}$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

Phase Shift : $\frac{\phi}{\omega} = \frac{6}{\pi}$

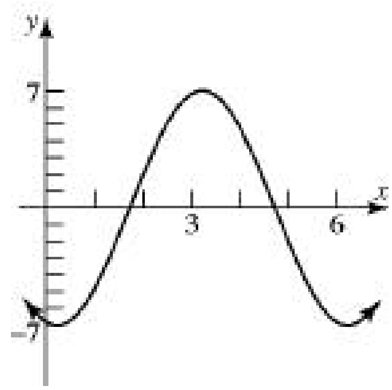


70. $y = -7\sin\left(\frac{\pi}{3}x + \frac{4}{3}\right)$

Amplitude: $|A| = |-7| = 7$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(\frac{\pi}{3}\right)} = 6$

Phase Shift : $\frac{\phi}{\omega} = \frac{\left(-\frac{4}{3}\right)}{\left(\frac{\pi}{3}\right)} = -\frac{4}{\pi}$



71. The graph is a cosine graph with an amplitude of 5 and a period of 8π. Find ω :

$$8\pi = \frac{2\pi}{\omega} \rightarrow 8\pi\omega = 2\pi \rightarrow \omega = \frac{2\pi}{8\pi} = \frac{1}{4}$$

The equation is: $y = 5\cos\left(\frac{1}{4}x\right)$.

72. The graph is a sine graph with an amplitude of 4 and a period of 8π. Find ω :

$$8\pi = \frac{2\pi}{\omega} \rightarrow 8\pi\omega = 2\pi \rightarrow \omega = \frac{2\pi}{8\pi} = \frac{1}{4}$$

The equation is: $y = 4\sin\left(\frac{1}{4}x\right)$.

73. The graph is a reflected cosine graph with an amplitude of 6 and a period of 8.
Find ω :

$$8 = \frac{2\pi}{\omega} \rightarrow 8\omega = 2\pi \rightarrow \omega = \frac{2\pi}{8} = \frac{\pi}{4}$$

The equation is: $y = -6\cos\left(\frac{\pi}{4}x\right)$.

74. The graph is a reflected sine graph with an amplitude of 7 and a period of 8. Find ω :

$$8 = \frac{2\pi}{\omega} \rightarrow 8\omega = 2\pi \rightarrow \omega = \frac{2\pi}{8} = \frac{\pi}{4}$$

The equation is: $y = -7\sin\left(\frac{\pi}{4}x\right)$.

75. $r = 2$ feet, $\theta = 30°$ or $\theta = \frac{\pi}{6}$

$$s = r\theta = 2\,\frac{\pi}{6} = \frac{\pi}{3} \text{ feet}$$

76. $r = 8$ inches, $\theta = 180°$ or $\theta = \pi$

$$s = r\theta = 8\,\pi = 8\pi \text{ inches in 30 minutes}$$

$r = 8$ inches, $\theta = 120°$ or $\theta = \frac{2\pi}{3}$

$$s = r\theta = 8\,\frac{2\pi}{3} = \frac{16\pi}{3} \text{ inches in 20 minutes}$$

77. $v = 180$ mi/hr, $d = \frac{1}{2}$ mile $\rightarrow$ $r = \frac{1}{4} = 0.25$ mile

$$\omega = \frac{v}{r} = \frac{180 \text{ mi/hr}}{0.25 \text{ mi}} = 720 \text{ rad/hr} = \frac{720 \text{ rad}}{\text{hr}}\;\frac{1 \text{ rev}}{2\pi \text{ rad}} = \frac{360 \text{ rev}}{\pi \text{ hr}} \approx 114.6 \text{ rev/hr}$$

78. $r = 25$ feet; $\omega = \frac{1 \text{ rev}}{30 \text{ sec}} = \frac{1 \text{ rev}}{30 \text{ sec}}\;\frac{2\pi \text{ radians}}{1 \text{ rev}} = \frac{\pi}{15}$ rad / sec

$v = r\omega = 25\,\frac{\pi}{15} = \frac{5\pi}{3}$ ft / sec ≈ 5.2 ft / sec.

79. Since there are two lights on opposite sides and the light is seen every 5 seconds, the beacon makes 1 revolution every 10 seconds.

$$\omega = \frac{1 \text{ rev}}{10 \text{ sec}}\;\frac{2\pi}{1 \text{ rev}} = \frac{\pi}{5} \text{ radians / second}$$

80. $r = 16$ inches; $v = 90$ mi / hr

$$\omega = \frac{v}{r} = \frac{90 \text{ mi / hr}}{16 \text{ in}}\;\frac{12 \text{ in}}{1 \text{ ft}}\;\frac{5280 \text{ ft}}{1 \text{ mi}}\;\frac{1 \text{ hr}}{60 \text{ min}}\;\frac{1 \text{ rev}}{2\pi \text{ rad}} \approx 945.38 \text{ rev / min}$$

$r = 14$ inches; $v = 90$ mi / hr

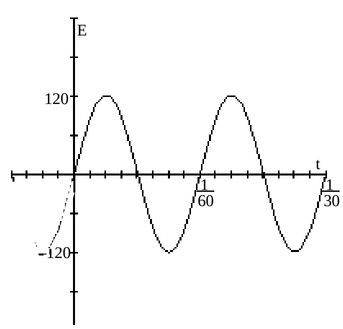
$$\omega = \frac{v}{r} = \frac{90 \text{ mi / hr}}{14 \text{ in}}\;\frac{12 \text{ in}}{1 \text{ ft}}\;\frac{5280 \text{ ft}}{1 \text{ mi}}\;\frac{1 \text{ hr}}{60 \text{ min}}\;\frac{1 \text{ rev}}{2\pi \text{ rad}} \approx 1080.43 \text{ rev / min}$$

81. $E(t) = 120\sin(120\pi t)$, $t \geq 0$

(a) The maximum value of E is the amplitude which is 120.

(b) Period $= \frac{2\pi}{120\pi} = \frac{1}{60}$

(c) Graphing:



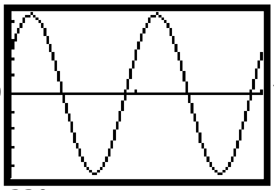
82. $I(t) = 220\sin\left(30\pi t + \frac{\pi}{6}\right), \quad t \ge 0$

(a) Period $= \frac{2\pi}{30\pi} = \frac{1}{15}$

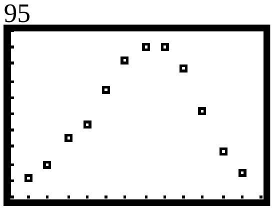
(b) The amplitude is 220.

(c) The phase shift is: $\frac{\phi}{\omega} = \frac{\left(-\frac{\pi}{6}\right)}{30\pi} = -\frac{\pi}{6} \cdot \frac{1}{30\pi} = -\frac{1}{180}$

(d) Graphing:



83. (a) Draw a scatter diagram:



(b) Amplitude: $A = \frac{90 - 51}{2} = \frac{39}{2} = 19.5$

Vertical Shift: $\frac{90 + 51}{2} = \frac{141}{2} = 70.5$

$\omega = \frac{2\pi}{12} = \frac{\pi}{6}$

Phase shift (use $y = 51$, $x = 1$):

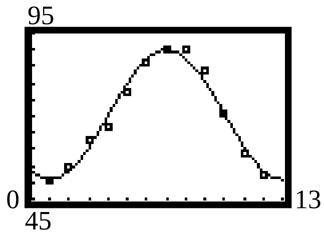
$$51 = 19.5\sin\left(\frac{\pi}{6} \cdot 1 - \phi\right) + 70.5$$

$$-19.5 = 19.5\sin\left(\frac{\pi}{6} - \phi\right) \to -1 = \sin\left(\frac{\pi}{6} - \phi\right)$$

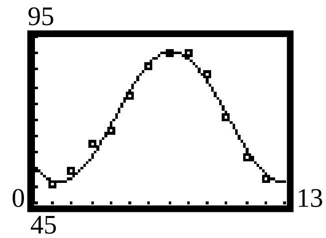
$$-\frac{\pi}{2} = \frac{\pi}{6} - \phi \to \phi = \frac{2\pi}{3}$$

Thus, $y = 19.5\sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 70.5$

(c)

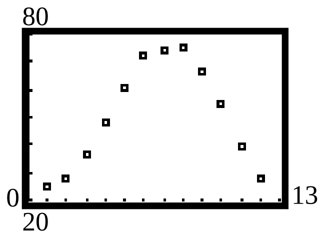


(e)



(d) $y = 19.518\sin(0.541x - 2.283) + 71.01$

84. (a) Draw a scatter diagram:



(b) Amplitude: $A = \dfrac{75-25}{2} = \dfrac{50}{2} = 25$

Vertical Shift: $\dfrac{75+25}{2} = \dfrac{100}{2} = 50$

$\omega = \dfrac{2\pi}{12} = \dfrac{\pi}{6}$

Phase shift (use $y = 25$, $x = 1$):

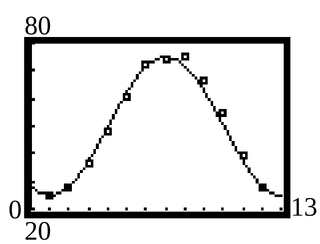
$$25 = 25\sin\left(\frac{\pi}{6}\cdot 1 - \phi\right) + 50$$

$$-25 = 25\sin\left(\frac{\pi}{6} - \phi\right) \rightarrow -1 = \sin\left(\frac{\pi}{6} - \phi\right)$$

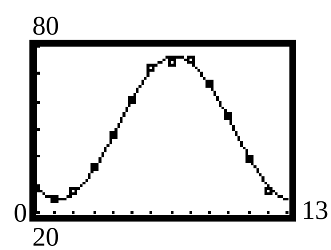
$$-\frac{\pi}{2} = \frac{\pi}{6} - \phi \rightarrow \phi = \frac{2\pi}{3}$$

Thus, $y = 25\sin\left(\dfrac{\pi}{6}x - \dfrac{2\pi}{3}\right) + 50$

(c)



(e)



(d) $y = 25.815\sin(0.521x - 2.175) + 50.46$

85. (a) Amplitude: $A = \dfrac{13.367 - 9.667}{2} = \dfrac{3.7}{2} = 1.85$

Vertical Shift: $\dfrac{13.367 + 9.667}{2} = \dfrac{23.034}{2} = 11.517$

$\omega = \dfrac{2\pi}{365}$

Phase shift (use $y = 9.667$, $x = 355$):

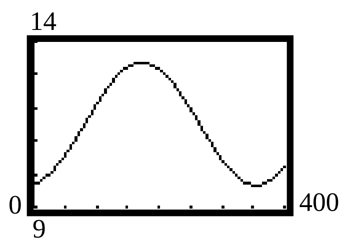
$$9.667 = 1.85\sin\left(\frac{2\pi}{365}\ 355 - \phi\right) + 11.517$$

$$-1.85 = 1.85\sin\left(\frac{2\pi}{365}\ 355 - \phi\right) \to -1 = \sin\left(\frac{710\pi}{365} - \phi\right)$$

$$-\frac{\pi}{2} = \frac{710\pi}{365} - \phi \to \phi = 7.6818$$

Thus, $y = 1.85\sin\left(\dfrac{2\pi}{365}x - 7.6818\right) + 11.517$

(b)



(c) $y = 1.85\sin\left(\dfrac{2\pi}{365}(91) - 7.6818\right) + 11.517 = 11.83$ hours

86. (a) Amplitude: $A = \dfrac{13.967 - 8.417}{2} = \dfrac{5.55}{2} = 2.775$

Vertical Shift: $\dfrac{13.967 + 8.417}{2} = \dfrac{22.384}{2} = 11.192$

$\omega = \dfrac{2\pi}{365}$

Phase shift (use $y = 8.417$, $x = 355$):

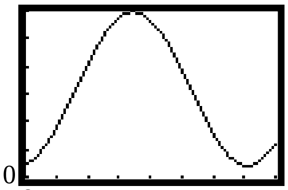
$$8.417 = 2.775\sin\left(\frac{2\pi}{365}\ 355 - \phi\right) + 11.192$$

$$-2.775 = 2.775\sin\left(\frac{2\pi}{365}\ 355 - \phi\right) \to -1 = \sin\left(\frac{710\pi}{365} - \phi\right)$$

$$-\frac{\pi}{2} = \frac{710\pi}{365} - \phi \to \phi = 7.6818$$

Thus, $y = 2.775\sin\left(\dfrac{2\pi}{365}x - 7.6818\right) + 11.192$

(b)



(c) $y = 2.775\sin\left(\frac{2\pi}{365}(91) - 7.6818\right) + 11.192 \approx 11.66$ hours