Chapter 8

Analytic Trigonometry

8.1 The Inverse Sine, Cosine and Tangent Functions

1. $\sin^{-1}(0)$

   We are finding the angle θ, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, whose sine equals 0.

   $$\sin\theta = 0 \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

   $$\theta = 0 \to \sin^{-1}(0) = 0$$

2. $\cos^{-1}(1)$

   We are finding the angle θ, $0 \le \theta \le \pi$, whose cosine equals 1.

   $$\cos\theta = 1 \qquad 0 \le \theta \le \pi$$

   $$\theta = 0 \to \cos^{-1}(1) = 0$$

3. $\sin^{-1}(-1)$

   We are finding the angle θ, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, whose sine equals –1.

   $$\sin\theta = -1 \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

   $$\theta = -\frac{\pi}{2} \to \sin^{-1}(-1) = -\frac{\pi}{2}$$

4. $\cos^{-1}(-1)$

   We are finding the angle θ, $0 \le \theta \le \pi$, whose cosine equals –1.

   $$\cos\theta = -1 \qquad 0 \le \theta \le \pi$$

   $$\theta = \pi \to \cos^{-1}(-1) = \pi$$

5. $\tan^{-1}(0)$

   We are finding the angle θ, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, whose tangent equals 0.

   $$\tan\theta = 0 \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

   $$\theta = 0 \to \tan^{-1}(0) = 0$$

6. $\tan^{-1}(-1)$

We are finding the angle $\theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$, whose tangent equals –1.

$$\tan\theta = -1 \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\theta = -\frac{\pi}{4} \rightarrow \tan^{-1}(-1) = -\frac{\pi}{4}$$

7. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

We are finding the angle $\theta, -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, whose sine equals $\frac{\sqrt{2}}{2}$.

$$\sin\theta = \frac{\sqrt{2}}{2} \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4} \rightarrow \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

8. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

We are finding the angle $\theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$, whose tangent equals $\frac{\sqrt{3}}{3}$.

$$\tan\theta = \frac{\sqrt{3}}{3} \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\theta = \frac{\pi}{6} \rightarrow \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

9. $\tan^{-1}\left(\sqrt{3}\right)$

We are finding the angle $\theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$, whose tangent equals $\sqrt{3}$.

$$\tan\theta = \sqrt{3} \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\theta = \frac{\pi}{3} \rightarrow \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$$

10. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

We are finding the angle $\theta, -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, whose sine equals $-\frac{\sqrt{3}}{2}$.

$$\sin\theta = -\frac{\sqrt{3}}{2} \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$\theta = -\frac{\pi}{3} \rightarrow \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

11. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

We are finding the angle θ, $0 \le \theta \le \pi$, whose cosine equals $-\frac{\sqrt{3}}{2}$.

$$\cos\theta = -\frac{\sqrt{3}}{2} \qquad 0 \le \theta \le \pi$$

$$\theta = \frac{5\pi}{6} \rightarrow \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

12. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

We are finding the angle θ, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, whose sine equals $-\frac{\sqrt{2}}{2}$.

$$\sin\theta = -\frac{\sqrt{2}}{2} \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$\theta = -\frac{\pi}{4} \rightarrow \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

13. $\sin^{-1}(0.1) \approx 0.10$

14. $\cos^{-1}(0.6) \approx 0.93$

15. $\tan^{-1}(5) \approx 1.37$

16. $\tan^{-1}(0.2) \approx 0.20$

17. $\cos^{-1}\left(\frac{7}{8}\right) \approx 0.51$

18. $\sin^{-1}\left(\frac{1}{8}\right) \approx 0.13$

19. $\tan^{-1}(-0.4) \approx -0.38$

20. $\tan^{-1}(-3) \approx -1.25$

21. $\sin^{-1}(-0.12) \approx -0.12$

22. $\cos^{-1}(-0.44) \approx 2.03$

23. $\cos^{-1}\left(\frac{\sqrt{2}}{3}\right) \approx 1.08$

24. $\sin^{-1}\left(\frac{\sqrt{3}}{5}\right) \approx 0.35$

25. $\sin\left[\sin^{-1}(0.54)\right] = 0.54$

26. $\tan\left[\tan^{-1}(7.4)\right] = 7.4$

27. $\cos^{-1}\left[\cos\left(\frac{4\pi}{5}\right)\right] = \frac{4\pi}{5}$

28. $\sin^{-1}\left[\sin\left(-\frac{\pi}{10}\right)\right] = -\frac{\pi}{10}$

29. $\tan\left[\tan^{-1}(-3.5)\right] = -3.5$

30. $\cos\left[\cos^{-1}(-0.05)\right] = -0.05$

31. $\sin^{-1}\left[\sin\left(-\frac{3\pi}{7}\right)\right] = -\frac{3\pi}{7}$

32. $\tan^{-1}\left[\tan\left(\frac{2\pi}{5}\right)\right] = \frac{2\pi}{5}$

33. yes, $\sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right] = -\frac{\pi}{6}$ since $\sin^{-1}\left[\sin(x)\right] = x$ where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ and $-\frac{\pi}{6}$ is in the restricted domain of $f(x) = \sin(x)$.

34. no, $\sin^{-1}\left[\sin\left(\frac{2\pi}{3}\right)\right] \ne \frac{2\pi}{3}$ since $\sin^{-1}\left[\sin(x)\right] = x$ where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ and $\frac{2\pi}{3}$ is not in the restricted domain of $f(x) = \sin(x)$.

35. no, $\sin\left[\sin^{-1}(2)\right] \ne 2$ since $\sin\left[\sin^{-1}(x)\right] = x$ where $-1 \le x \le 1$ and 2 is not in the domain of $f(x) = \sin^{-1}(x)$.

36. yes, $\sin\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = -\frac{1}{2}$ since $\sin\left[\sin^{-1}(x)\right] = x$ where $-1 \le x \le 1$ and $-\frac{1}{2}$ is in the domain of $f(x) = \sin^{-1}(x)$.

37. no, $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right] \ne -\frac{\pi}{6}$ since $\cos^{-1}\left[\cos(x)\right] = x$ where $0 \le x \le \pi$ and $-\frac{\pi}{6}$ is not in the restricted domain of $f(x) = \cos(x)$.

38. yes, $\cos^{-1}\left[\cos\left(\frac{2\pi}{3}\right)\right] = \frac{2\pi}{3}$ since $\cos^{-1}\left[\cos(x)\right] = x$ where $0 \le x \le \pi$ and $\frac{2\pi}{3}$ is in the restricted domain of $f(x) = \cos(x)$.

39. yes, $\cos\left[\cos^{-1}\left(-\frac{1}{2}\right)\right] = -\frac{1}{2}$ since $\cos\left[\cos^{-1}(x)\right] = x$ where $-1 \le x \le 1$

and $-\frac{1}{2}$ is in the domain of $f(x)=\cos^{-1}(x)$.

40. no, $\cos\left[\cos^{-1}(2)\right]\neq 2$ since $\cos\left[\cos^{-1}(x)\right]=x$ where $-1\leq x\leq 1$

and 2 is not in the domain of $f(x)=\cos^{-1}(x)$.

41. yes, $\tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right]=-\frac{\pi}{3}$ since $\tan^{-1}\left[\tan(x)\right]=x$ where $-\frac{\pi}{2}<x<\frac{\pi}{2}$

and $-\frac{\pi}{3}$ is in the restricted domain of $f(x)=\tan(x)$.

42. no, $\tan^{-1}\left[\tan\left(\frac{2\pi}{3}\right)\right]\neq\frac{2\pi}{3}$ since $\tan^{-1}\left[\tan(x)\right]=x$ where $-\frac{\pi}{2}<x<\frac{\pi}{2}$

and $\frac{2\pi}{3}$ is not in the restricted domain of $f(x)=\tan(x)$.

43. yes, $\tan\left[\tan^{-1}(2)\right]=2$ since $\tan\left[\tan^{-1}(x)\right]=x$ where $-\infty<x<\infty$

44. yes, $\tan\left[\tan^{-1}\left(-\frac{1}{2}\right)\right]=-\frac{1}{2}$ since $\tan\left[\tan^{-1}(x)\right]=x$ where $-\infty<x<\infty$

45. (a) $$D=24\left[1-\frac{\cos^{-1}\left(\tan\left(23.5\frac{\pi}{180}\right)\tan\left(29.75\frac{\pi}{180}\right)\right)}{\pi}\right]\approx 13.92\text{ hours}$$

(b) $$D=24\left[1-\frac{\cos^{-1}\left(\tan\left(0\frac{\pi}{180}\right)\tan\left(29.75\frac{\pi}{180}\right)\right)}{\pi}\right]\approx 12\text{ hours}$$

(c) $$D=24\left[1-\frac{\cos^{-1}\left(\tan\left(22.8\frac{\pi}{180}\right)\tan\left(29.75\frac{\pi}{180}\right)\right)}{\pi}\right]\approx 13.85\text{ hours}$$

46. (a) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(23.5\cdot\frac{\pi}{180}\right)\tan\left(40.75\cdot\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 14.93 \text{ hours}$$

(b) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(0\cdot\frac{\pi}{180}\right)\tan\left(40.75\cdot\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 12 \text{ hours}$$

(c) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(22.8\cdot\frac{\pi}{180}\right)\tan\left(40.75\cdot\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 14.83 \text{ hours}$$

47. (a) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(23.5\cdot\frac{\pi}{180}\right)\tan\left(21.3\cdot\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 13.30 \text{ hours}$$

(b) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(0\cdot\frac{\pi}{180}\right)\tan\left(21.3\cdot\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 12 \text{ hours}$$

(c) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(22.8\cdot\frac{\pi}{180}\right)\tan\left(21.3\cdot\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 13.26 \text{ hours}$$

48. (a) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(23.5\cdot\frac{\pi}{180}\right)\tan\left(61.167\cdot\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 18.96 \text{ hours}$$

(b) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(0\cdot\frac{\pi}{180}\right)\tan\left(61.167\cdot\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 12 \text{ hours}$$

(c) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(22.8\frac{\pi}{180}\right)\tan\left(61.167\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 18.64 \text{ hours}$$

49. (a) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(23.5\frac{\pi}{180}\right)\tan\left(0\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 12 \text{ hours}$$

(b) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(0\frac{\pi}{180}\right)\tan\left(0\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 12 \text{ hours}$$

(c) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(22.8\frac{\pi}{180}\right)\tan\left(0\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 12 \text{ hours}$$

50. (a) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(23.5\frac{\pi}{180}\right)\tan\left(66.5\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 24 \text{ hours}$$

(b) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(0\frac{\pi}{180}\right)\tan\left(66.5\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 12 \text{ hours}$$

(c) $$D = 24\left(1 - \frac{\cos^{-1}\left(\tan\left(22.8\frac{\pi}{180}\right)\tan\left(66.5\frac{\pi}{180}\right)\right)}{\pi}\right) \approx 22.02 \text{ hours}$$

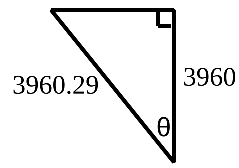
(d) The number of hours of daylight at this location on the winter solstice is: $24 - 24 = 0$. On the winter solstice, there is no daylight.

51. $1530 \text{ ft} \; \dfrac{1 \text{ mile}}{5280 \text{ feet}} = 0.29 \text{ mile}$

$$\cos\theta = \frac{3960}{3960.29}$$

$$\theta \approx 0.0121 \text{ radians}$$

$$s = r\theta = 3960(0.0121) = 47.92 \text{ miles}$$



$$\frac{2\pi(2710)}{24} = \frac{47.92}{t}$$

$$t \approx 0.0675 \text{ hour} = 4.05 \text{ minutes}$$