Chapter 9

Applications of Trigonometric Functions

9.1 Applications Involving Right Triangles

1. $b = 5, \ \beta = 20^\circ$

$$\sin\beta = \frac{b}{c} \rightarrow \sin(20^\circ) = \frac{5}{c} \rightarrow c = \frac{5}{\sin(20^\circ)} \approx \frac{5}{0.3420} \approx 14.62$$

$$\tan\beta = \frac{b}{a} \rightarrow \tan(20^\circ) = \frac{5}{a} \rightarrow a = \frac{5}{\tan(20^\circ)} \approx \frac{5}{0.3640} \approx 13.74$$

$$\alpha = 90^\circ - \beta = 90^\circ - 20^\circ = 70^\circ$$

2. $b = 4, \ \beta = 10^\circ$

$$\sin\beta = \frac{b}{c} \rightarrow \sin(10^\circ) = \frac{4}{c} \rightarrow c = \frac{4}{\sin(10^\circ)} \approx \frac{4}{0.1736} \approx 23.0$$

$$\tan\beta = \frac{b}{a} \rightarrow \tan(10^\circ) = \frac{4}{a} \rightarrow a = \frac{4}{\tan(10^\circ)} \approx \frac{4}{0.1763} \approx 22.7$$

$$\alpha = 90^\circ - \beta = 90^\circ - 10^\circ = 80^\circ$$

3. $a = 6, \ \beta = 40^\circ$

$$\cos\beta = \frac{a}{c} \rightarrow \cos(40^\circ) = \frac{6}{c} \rightarrow c = \frac{6}{\cos(40^\circ)} \approx \frac{6}{0.7660} \approx 7.83$$

$$\tan\beta = \frac{b}{a} \rightarrow \tan(40^\circ) = \frac{b}{6} \rightarrow b = 6\tan(40^\circ) \approx 6 \ (0.8391) \approx 5.03$$

$$\alpha = 90^\circ - \beta = 90^\circ - 40^\circ = 50^\circ$$

4. $a = 7, \ \beta = 50^\circ$

$$\cos\beta = \frac{a}{c} \rightarrow \cos(50^\circ) = \frac{7}{c} \rightarrow c = \frac{7}{\cos(50^\circ)} \approx \frac{7}{0.6428} \approx 10.9$$

$$\tan\beta = \frac{b}{a} \rightarrow \tan(50^\circ) = \frac{b}{7} \rightarrow b = 7\tan(50^\circ) \approx 7 \ (1.1918) \approx 8.3$$

$$\alpha = 90^\circ - \beta = 90^\circ - 50^\circ = 40^\circ$$

5. $b = 4,\ \alpha = 10^\circ$

$$\tan\alpha = \frac{a}{b} \ \rightarrow\ \tan(10^\circ) = \frac{a}{4} \ \rightarrow\ a = 4\tan(10^\circ) \approx 4\ (0.1763) \approx 0.71$$

$$\cos\alpha = \frac{b}{c} \ \rightarrow\ \cos(10^\circ) = \frac{4}{c} \ \rightarrow\ c = \frac{4}{\cos(10^\circ)} \approx \frac{4}{0.9848} \approx 4.06$$

$$\beta = 90^\circ - \alpha = 90^\circ - 10^\circ = 80^\circ$$

6. $b = 6,\ \alpha = 20^\circ$

$$\tan\alpha = \frac{a}{b} \ \rightarrow\ \tan(20^\circ) = \frac{a}{6} \ \rightarrow\ a = 6\tan(20^\circ) \approx 6\ (0.3640) \approx 2.2$$

$$\cos\alpha = \frac{b}{c} \ \rightarrow\ \cos(20^\circ) = \frac{6}{c} \ \rightarrow\ c = \frac{6}{\cos(20^\circ)} \approx \frac{6}{0.9397} \approx 6.4$$

$$\beta = 90^\circ - \alpha = 90^\circ - 20^\circ = 70^\circ$$

7. $a = 5,\ \alpha = 25^\circ$

$$\cot\alpha = \frac{a}{b} \ \rightarrow\ \cot(25^\circ) = \frac{b}{5} \ \rightarrow\ b = 5\cot(25^\circ) \approx 5\ (2.1445) \approx 10.72$$

$$\csc\alpha = \frac{c}{a} \ \rightarrow\ \csc(25^\circ) = \frac{c}{5} \ \rightarrow\ c = 5\csc(25^\circ) \approx 5\ (2.3662) \approx 11.83$$

$$\beta = 90^\circ - \alpha = 90^\circ - 25^\circ = 65^\circ$$

8. $a = 6,\ \alpha = 40^\circ$

$$\cot\alpha = \frac{a}{b} \ \rightarrow\ \cot(40^\circ) = \frac{b}{6} \ \rightarrow\ b = 6\cot(40^\circ) \approx 6\ (1.1918) \approx 7.15$$

$$\csc\alpha = \frac{c}{a} \ \rightarrow\ \csc(45^\circ) = \frac{c}{6} \ \rightarrow\ c = 6\csc(40^\circ) \approx 6\ (1.5557) \approx 9.33$$

$$\beta = 90^\circ - \alpha = 90^\circ - 40^\circ = 50^\circ$$

9. $c = 9,\ \beta = 20^\circ$

$$\sin\beta = \frac{b}{c} \ \rightarrow\ \sin(20^\circ) = \frac{b}{9} \ \rightarrow\ b = 9\sin(20^\circ) \approx 9\ (0.3420) \approx 3.08$$

$$\cos\alpha = \frac{a}{c} \ \rightarrow\ \cos(20^\circ) = \frac{a}{9} \ \rightarrow\ a = 9\cos(20^\circ) \approx 9\ (0.9397) \approx 8.46$$

$$\alpha = 90^\circ - \alpha = 90^\circ - 20^\circ = 70^\circ$$

10. $c = 10,\ \alpha = 40^\circ$

$$\sin\alpha = \frac{a}{c} \ \rightarrow\ \sin(40^\circ) = \frac{a}{10} \ \rightarrow\ a = 10\sin(40^\circ) \approx 10\ (0.6428) \approx 6.43$$

$$\cos\alpha = \frac{b}{c} \ \rightarrow\ \cos(40^\circ) = \frac{b}{10} \ \rightarrow\ b = 10\cos(40^\circ) \approx 10\ (0.7660) \approx 7.66$$

$$\beta = 90^\circ - \alpha = 90^\circ - 40^\circ = 50^\circ$$

11. $a = 5, \quad b = 3$

$c^2 = a^2 + b^2 = 5^2 + 3^2 = 25 + 9 = 34 \quad \rightarrow \quad c = \sqrt{34} \approx 5.83$

$\tan\alpha = \dfrac{a}{b} = \dfrac{5}{3} \approx 1.6667 \quad \rightarrow \quad \alpha \approx 59.0^\circ$

$\beta = 90^\circ - \alpha = 90^\circ - 59.0^\circ = 31.0^\circ$

12. $a = 2, \quad b = 8$

$c^2 = a^2 + b^2 = 2^2 + 8^2 = 4 + 64 = 68 \quad \rightarrow \quad c = \sqrt{68} \approx 8.25$

$\tan\alpha = \dfrac{a}{b} = \dfrac{2}{8} = 0.2500 \quad \rightarrow \quad \alpha \approx 14.0^\circ$

$\beta = 90^\circ - \alpha = 90^\circ - 14.0^\circ = 76.0^\circ$

13. $a = 2, \quad c = 5$

$c^2 = a^2 + b^2 \quad \rightarrow \quad b^2 = c^2 - a^2 = 5^2 - 2^2 = 25 - 4 = 21 \quad \rightarrow \quad b = \sqrt{21} \approx 4.58$

$\sin\alpha = \dfrac{a}{c} = \dfrac{2}{5} = 0.4000 \quad \rightarrow \quad \alpha \approx 23.6^\circ$

$\beta = 90^\circ - \alpha = 90^\circ - 23.6^\circ = 66.4^\circ$

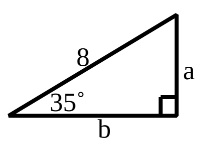
14. $b = 4, \quad c = 6$

$c^2 = a^2 + b^2 \quad \rightarrow \quad a^2 = c^2 - b^2 = 6^2 - 4^2 = 36 - 16 = 20 \quad \rightarrow \quad b = \sqrt{20} \approx 4.47$

$\cos\alpha = \dfrac{b}{c} = \dfrac{4}{6} = 0.6667 \quad \rightarrow \quad \alpha \approx 48.2^\circ$

$\beta = 90^\circ - \alpha = 90^\circ - 48.2^\circ = 41.8^\circ$

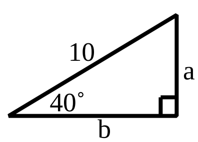
15. $c = 8, \quad \alpha = 35^\circ$



$\sin(35^\circ) = \dfrac{a}{8} \quad \rightarrow \quad a = 8\sin(35^\circ) \approx 8(0.5736) \approx 4.59 \text{ in.}$

$\cos(35^\circ) = \dfrac{b}{8} \quad \rightarrow \quad b = 8\cos(35^\circ) \approx 8(0.8192) \approx 6.55 \text{ in.}$

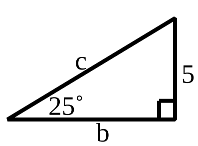
16. $c = 10, \quad \alpha = 40^\circ$



$\sin(40^\circ) = \dfrac{a}{10} \quad \rightarrow \quad a = 10\sin(40^\circ) \approx 10(0.6428) \approx 6.4 \text{ cm.}$

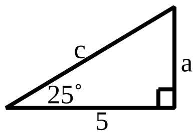
$\cos(40^\circ) = \dfrac{b}{10} \quad \rightarrow \quad b = 10\cos(40^\circ) \approx 10(0.7660) \approx 7.7 \text{ cm}$

17. $\alpha = 25^\circ, \quad a = 5$



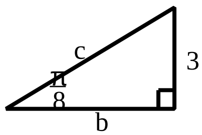
$\sin(25^\circ) = \dfrac{5}{c} \quad \rightarrow \quad c = \dfrac{5}{\sin(25^\circ)} \approx \dfrac{5}{0.4226} \approx 11.83 \text{in}$

$\alpha = 25^\circ, \quad b = 5$



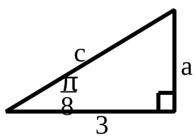
$$\cos(25^\circ)=\frac{5}{c}\quad\rightarrow\quad c=\frac{5}{\cos(25^\circ)}\approx\frac{5}{0.9063}\approx 5.52\text{ in.}$$

18. $\alpha=\dfrac{\pi}{8},\ a=3$



$$\sin\left(\frac{\pi}{8}\right)=\frac{3}{c}\quad\rightarrow\quad c=\frac{3}{\sin\left(\frac{\pi}{8}\right)}\approx\frac{3}{0.3827}\approx 7.8\text{ m.}$$

$\alpha=\dfrac{\pi}{8},\ b=3$



$$\cos\left(\frac{\pi}{8}\right)=\frac{3}{c}\quad\rightarrow\quad c=\frac{3}{\cos\left(\frac{\pi}{8}\right)}\approx\frac{3}{0.9239}\approx 3.2\text{ m.}$$

19. $c=5,\ a=2$

$$\sin\alpha=\frac{2}{5}=0.4000\quad\rightarrow\quad\alpha\approx 23.6\quad\rightarrow\quad\beta=90^\circ-\alpha=90^\circ-23.6^\circ\approx 66.4^\circ$$

20. $c=3,\ a=1$

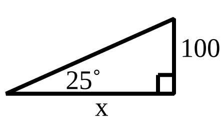
$$\sin\alpha=\frac{1}{3}\approx 0.3333\quad\rightarrow\quad\alpha\approx 19.5\quad\rightarrow\quad\beta=90^\circ-\alpha=90^\circ-19.5^\circ\approx 70.5^\circ$$

21. $\tan(35^\circ)=\dfrac{b}{100}\quad\rightarrow\quad b=100\tan(35^\circ)\approx 100(0.7002)\approx 70\text{ feet}$

22. $\tan(40^\circ)=\dfrac{b}{100}\quad\rightarrow\quad b=100\tan(40^\circ)\approx 100(0.8391)\approx 83.9\text{ feet}$

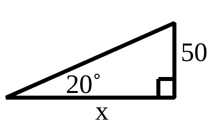
23. $\tan(85.361^\circ)=\dfrac{a}{80}\quad\rightarrow\quad a=80\tan(85.361^\circ)\approx 80(12.3239)\approx 985.9\text{ feet}$

24.



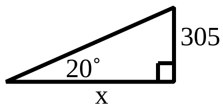
$$\tan(25^\circ)=\frac{100}{x}\quad\rightarrow\quad x=\frac{100}{\tan(25^\circ)}\approx\frac{100}{0.4663}\approx 214.5\text{ feet}$$

25.



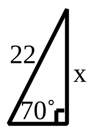
$$\tan(20^\circ)=\frac{50}{x}\quad\rightarrow\quad x=\frac{50}{\tan(20^\circ)}\approx\frac{50}{0.3640}\approx 137\text{ meters}$$

26.



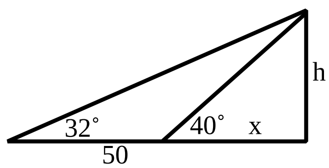
$$\tan(20^{o}) = \frac{305}{x} \quad \rightarrow \quad x = \frac{305}{\tan(20^{o})} \approx \frac{305}{0.3640} \approx 838 \text{ feet}$$

27.



$$\sin(70^{o}) = \frac{x}{22} \quad \rightarrow \quad x = 22\sin(70^{o}) \approx 22(0.9397) \approx 20.67 \text{ feet}$$

28.



$$\tan(40^{o}) = \frac{h}{x} \quad \rightarrow \quad x = \frac{h}{\tan(40^{o})}$$

$$\tan(32^{o}) = \frac{h}{x+50} \quad \rightarrow \quad h = (x+50)\tan(32^{o}) = \left(\frac{h}{\tan(40^{o})} + 50\right)\tan(32^{o})$$

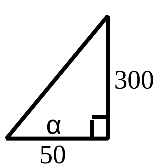
$$h = \left(\frac{h}{0.8391} + 50\right)0.6249$$

$$h = 0.7447h + 31.245 \rightarrow 0.2553h = 31.245 \rightarrow h \approx 122.4$$

29. opposite side = 10 feet, adjacent side = 35 feet

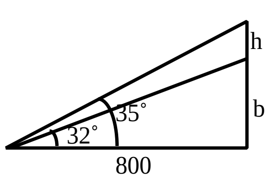
$$\tan\theta = \frac{10}{35} \approx 0.2857 \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{10}{35}\right) \approx 15.9^{o}$$

30.



$$\tan\alpha = \frac{300}{50} = 6 \quad \rightarrow \quad \alpha = 80.5^{o}$$

31. Let h represent the height of Lincoln's face.

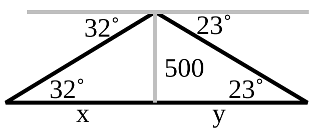


$$\tan(32^{o}) = \frac{b}{800} \quad \rightarrow \quad b = 800\tan(32^{o}) \approx 800(0.6249) \approx 499.9$$

$$\tan(35^{o}) = \frac{b+h}{800} \quad \rightarrow \quad b + h = 800\tan(35^{o}) \approx 800(0.7002) \approx 560.2$$

$$h = (b+h) - b = 560.2 - 499.9 \approx 60.3 \text{ feet}$$

32.

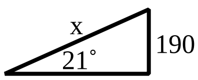


$$\tan(32^o) = \frac{500}{x} \quad \rightarrow \quad x = \frac{500}{\tan(32^o)}$$

$$\tan(23^o) = \frac{500}{y} \quad \rightarrow \quad y = \frac{500}{\tan(23^o)}$$

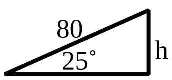
$$\text{Distance} = x + y = \frac{500}{\tan(32^o)} + \frac{500}{\tan(23^o)} \approx 800 + 1178 = 1978 \text{ feet}$$

33.



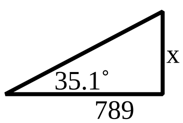
$$\sin(21^o) = \frac{190}{x} \quad \rightarrow \quad x = \frac{190}{\sin(21^o)} \approx \frac{190}{0.3584} \approx 530 \text{ ft.}$$

34.



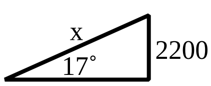
$$\sin(25^o) = \frac{h}{80} \quad \rightarrow \quad h = 80\sin(25^o) \approx 80(0.4226) \approx 33.8 \text{ ft.}$$

35.



$$\tan(35.1^o) = \frac{x}{789} \quad \rightarrow \quad x = 789\tan(35.1^o) \approx 789(0.7028) \approx 555 \text{ ft}$$

36.



$$\sin 17^o = \frac{2200}{x} \quad \rightarrow \quad x = \frac{2200}{\sin(17^o)} \approx \frac{2200}{0.2924} \approx 7525 \text{ ft.}$$

37. (a) $\tan(15^o) = \dfrac{30}{x} \quad \rightarrow \quad x = \dfrac{30}{\tan(15^o)} \approx \dfrac{30}{0.2679} \approx 111.96 \text{ feet}$

The truck is traveling at 111.96 ft/sec.

$$\frac{111.96 \text{ ft}}{\text{sec}} \; \frac{1 \text{ mile}}{5280 \text{ ft}} \; \frac{3600 \text{ sec}}{\text{hr}} \approx 76.34 \text{ mi / hr}$$

(b) $\tan(20^o) = \dfrac{30}{x} \quad \rightarrow \quad x = \dfrac{30}{\tan(20^o)} \approx \dfrac{30}{0.3640} \approx 82.42 \text{ feet}$

The truck is traveling at 82.42 ft/sec.

$$\frac{82.42 \text{ ft}}{\text{sec}} \; \frac{1 \text{ mile}}{5280 \text{ ft}} \; \frac{3600 \text{ sec}}{\text{hr}} \approx 56.20 \text{ mi / hr}$$

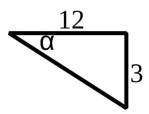
(c) A ticket is issued for traveling at a speed of 60 mi/hr or more.

$$\frac{60 \text{ mi}}{\text{hr}} \; \frac{5280 \text{ ft}}{\text{mi}} \; \frac{1\text{hr}}{3600 \text{ sec}} = 88 \text{ ft / sec.}$$

If $\tan\theta < \dfrac{30}{88}$, the trooper should issue a ticket.

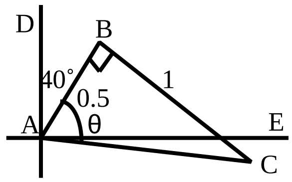
A ticket is issued if $\theta < 18.8^o$.

38.



$$\tan\alpha = \frac{3}{12} = 0.25 \quad \rightarrow \quad \alpha \approx 14.0^\circ$$

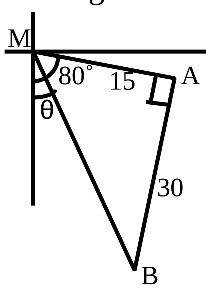
39. Find angle θ: (see the figure)



$$\tan\theta = \frac{1}{0.5} = 2 \quad \rightarrow \quad \theta = 63.4^\circ$$
$$DAC = 40^\circ + 63.4^\circ = 103.4^\circ$$
$$EAC = 103.4^\circ - 90^\circ = 13.4^\circ$$

The bearing the control tower should use is S76.6˚E.

40. Find angle AMB and subtract from 80˚ to obtain θ.



$$\tan \quad AMB = \frac{30}{15} = 2 \quad \rightarrow \quad AMB = 63.4^\circ$$
$$\theta = 80^\circ - 63.4^\circ = 16.6^\circ$$

The bearing is S16.6ºE.

41. $\tan\alpha = \dfrac{10-6}{15} = \dfrac{4}{15} \quad \rightarrow \quad \alpha = 14.9^\circ$

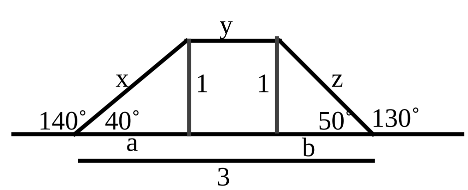
42. The height of the beam above the wall is 46 – 20 = 26 feet.

$$\tan\theta = \frac{26}{10} = 2.6 \quad \rightarrow \quad \theta \approx 69^\circ$$

43. The length of the highway $= x + y + z$

$$\sin(40^\circ) = \frac{1}{x} \quad \rightarrow \quad x = \frac{1}{\sin(40^\circ)} \approx 1.56 \text{ mi}$$

$$\sin(50^\circ) = \frac{1}{z} \quad \rightarrow \quad z = \frac{1}{\sin(50^\circ)} \approx 1.31 \text{ mi}$$



$$\tan(40^\circ) = \frac{1}{a} \quad \rightarrow \quad a = \frac{1}{\tan(40^\circ)} \approx 1.19 \text{ mi}$$

$$\tan(50^\circ) = \frac{1}{b} \quad \rightarrow \quad b = \frac{1}{\tan(50^\circ)} \approx 0.84 \text{ mi}$$

$$a + y + b = 3 \quad \rightarrow \quad y = 3 - a - b = 3 - 1.19 - 0.84 = 0.97 \text{ mi}$$

The length of the highway is: 1.56 + 0.97 + 1.31 = 3.84 miles.

44. (a) $\cos\left(\dfrac{\theta}{2}\right) = \dfrac{3960}{3960 + h}$

(b) $d = 3960\,\theta$

(c) $\cos\left(\dfrac{d}{7920}\right) = \dfrac{3960}{3960 + h}$

(d) $\cos\left(\dfrac{2500}{7920}\right) = \dfrac{3960}{3960 + h}$

$$0.9506 = \frac{3960}{3960 + h}$$

$$0.9506(3960 + h) = 3960 \rightarrow 3764 + 0.9506h = 3960 \rightarrow 0.9506h = 196$$

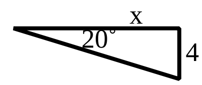
$$h \approx 206 \text{ miles}$$

(e) $\cos\left(\dfrac{d}{7920}\right) = \dfrac{3960}{3960 + 300} = \dfrac{3960}{4260} \approx 0.9296$

$$\frac{d}{7920} \approx 0.3775 \rightarrow d \approx 2990 \text{ miles}$$

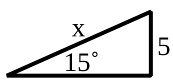
45. In order to see George's head and feet the camera must be x feet from George. Solve:

$$\tan(20^\circ) = \frac{4}{x} \quad \rightarrow \quad x = \frac{4}{\tan(20^\circ)} \approx 10.99 \text{ feet}$$



The camera will need to be moved back 1 foot to see George's feet.

46.



$$\sin(15^\circ) = \frac{5}{x} \quad \rightarrow \quad x = \frac{5}{\sin(15^\circ)} \approx \frac{5}{0.2588} \approx 19.3 \text{ ft.}$$

47. $\sin\theta = \dfrac{y}{1} = y; \qquad \cos\theta = \dfrac{x}{1} = x$

(a) $A = 2xy = 2\cos\theta\sin\theta$

(b) $2\cos\theta\sin\theta = 2\sin\theta\cos\theta = \sin(2\theta)$

(c) The largest value of the sine function is 1. Solve:

$$\sin(2\theta) = 1 \rightarrow 2\theta = \frac{\pi}{2} \rightarrow \theta = \frac{\pi}{4}$$

(d) $x = \cos\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2} \qquad y = \sin\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

The dimensions are $\sqrt{2}$ by $\dfrac{\sqrt{2}}{2}$.

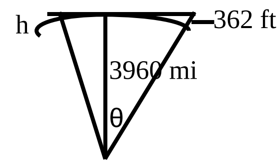
48. Let b represent the base of the triangle.

$$\cos\left(\frac{\theta}{2}\right) = \frac{h}{s} \quad \rightarrow \quad h = s\cos\left(\frac{\theta}{2}\right) \qquad \sin\left(\frac{\theta}{2}\right) = \frac{\left(\frac{1}{2}b\right)}{s} \quad \rightarrow \quad b = 2s\sin\left(\frac{\theta}{2}\right)$$

$$A = \frac{1}{2}b\,h = \frac{1}{2}\left(2s\sin\left(\frac{\theta}{2}\right)\right)\left(s\cos\left(\frac{\theta}{2}\right)\right) = s^2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = \frac{1}{2}s^2\sin\theta$$

49. Find θ: (see figure)

$$\cos\theta = \frac{3960}{\left(3960 + \frac{362}{5280}\right)} \approx 0.99998269$$



$$\theta \approx 0.00588439 \text{ radians}$$

Find the arc length from the base of the lighthouse to the horizon:

$$s = r\theta = 3960(0.00588439) \approx 23.3 \text{ miles}$$

The distance from the ship to the horizon point is 40 – 23.3 = 16.7 miles.

$$\theta = \frac{s}{r} = \frac{16.7}{3960} \approx 0.00421717$$

If h is the height of the ship, $\cos(0.00421717) = \dfrac{3960}{3960 + h}$.

Solve for h:

$$(3960 + h)\cos(0.00421717) = 3960$$

$$h = \frac{3960}{\cos(0.00421717)} - 3960 \approx 0.0352 \text{ miles or } \approx 186 \text{ feet}$$

The ship would have to be 186 feet tall to see the lighthouse from 40 miles away.

The distance from the plane to the horizon point is 120 – 23.3 = 96.7 miles.

$$\theta = \frac{s}{r} = \frac{96.7}{3960} \approx 0.02441919$$

If h is the height of the plane, $\cos(0.02441919) = \dfrac{3960}{3960 + h}$.

Solve for h:

$$(3960 + h)\cos(0.02441919) = 3960$$

$$h = \frac{3960}{\cos(0.02441919)} - 3960 \approx 1.18 \text{ miles or } \approx 6230 \text{ feet}$$

A plane at an altitude of 6230 feet could see the lighthouse from 120 miles away.
The brochure understates the distance from which the lighthouse can be seen.