Chapter 9

Applications of Trigonometric Functions

9.4 The Area of a Triangle

1. $a = 2,\ c = 4,\ \beta = 45^\circ$

   $A = \frac{1}{2}ac\sin\beta = \frac{1}{2}(2)(4)\sin(45^\circ) \approx 2.83$

2. $b = 3,\ c = 4,\ \alpha = 30^\circ$

   $A = \frac{1}{2}bc\sin\alpha = \frac{1}{2}(3)(4)\sin(30^\circ) = 3$

3. $a = 2,\ b = 3,\ \gamma = 95^\circ$

   $A = \frac{1}{2}ab\sin\gamma = \frac{1}{2}(2)(3)\sin(95^\circ) \approx 2.99$

4. $a = 2,\ c = 5,\ \beta = 20^\circ$

   $A = \frac{1}{2}ac\sin\beta = \frac{1}{2}(2)(5)\sin(20^\circ) \approx 1.71$

5. $a = 6,\ b = 5,\ c = 8$

   $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(6+5+8) = \frac{19}{2}$

   $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{19}{2}\left(\frac{7}{2}\right)\left(\frac{9}{2}\right)\left(\frac{3}{2}\right)} = \sqrt{\frac{3591}{16}} \approx 14.98$

6. $a = 8,\ b = 5,\ c = 4$

   $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(8+5+4) = \frac{17}{2}$

   $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{17}{2}\left(\frac{1}{2}\right)\left(\frac{7}{2}\right)\left(\frac{9}{2}\right)} = \sqrt{\frac{1071}{16}} \approx 8.18$

7. $a = 9,\ b = 6,\ c = 4$

   $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(9+6+4) = \frac{19}{2}$

   $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{19}{2}\left(\frac{1}{2}\right)\left(\frac{7}{2}\right)\left(\frac{11}{2}\right)} = \sqrt{\frac{1463}{16}} \approx 9.56$

8. $a = 4,\ b = 3,\ c = 4$

$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(4+3+4) = \frac{11}{2}$

$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{11}{2}\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{3}{2}\right)} = \sqrt{\frac{495}{16}} \approx 5.56$

9. $a = 3,\ b = 4,\ \gamma = 40°$

$A = \frac{1}{2}ab\sin\gamma = \frac{1}{2}(3)(4)\sin(40°) \approx 3.86$

10. $a = 2,\ c = 1,\ \beta = 10°$

$A = \frac{1}{2}ac\sin\beta = \frac{1}{2}(2)(1)\sin(10°) \approx 0.17$

11. $b = 1,\ c = 3,\ \alpha = 80°$

$A = \frac{1}{2}bc\sin\alpha = \frac{1}{2}(1)(3)\sin(80°) \approx 1.48$

12. $a = 6,\ b = 4,\ \gamma = 60°$

$A = \frac{1}{2}ab\sin\gamma = \frac{1}{2}(6)(4)\sin(60°) \approx 10.39$

13. $a = 3,\ c = 2,\ \beta = 110°$

$A = \frac{1}{2}ac\sin\beta = \frac{1}{2}(3)(2)\sin(110°) \approx 2.82$

14. $b = 4,\ c = 1,\ \alpha = 120°$

$A = \frac{1}{2}bc\sin\alpha = \frac{1}{2}(4)(1)\sin(120°) \approx 1.73$

15. $a = 2,\ b = 2,\ \gamma = 50°$

$A = \frac{1}{2}ab\sin\gamma = \frac{1}{2}(2)(2)\sin(50°) \approx 1.53$

16. $a = 3,\ c = 2,\ \beta = 90°$

$A = \frac{1}{2}ac\sin\beta = \frac{1}{2}(3)(2)\sin(90°) = 3$

17. $a = 12,\ b = 13,\ c = 5$

$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(12+13+5) = 15$

$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15(3)(2)(10)} = \sqrt{900} \approx 30$

18. $a = 4,\ b = 5,\ c = 3$
$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(4+5+3) = 6$
$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{6(2)(1)(3)} = \sqrt{36} = 6$

19. $a = 2,\ b = 2,\ c = 2$
$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(2+2+2) = 3$
$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{3(1)(1)(1)} = \sqrt{3} \approx 1.73$

20. $a = 3,\ b = 3,\ c = 2$
$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(3+3+2) = 4$
$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{4(1)(1)(2)} = \sqrt{8} \approx 2.83$

21. $a = 5,\ b = 8,\ c = 9$
$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(5+8+9) = 11$
$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{11(6)(3)(2)} = \sqrt{396} \approx 19.90$

22. $a = 4,\ b = 3,\ c = 6$
$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(4+3+6) = \frac{13}{2}$
$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{13}{2}\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)\left(\frac{1}{2}\right)} = \sqrt{\frac{455}{16}} \approx 5.33$

23. $a = 10,\ b = 8,\ c = 5$
$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(10+8+5) = \frac{23}{2}$
$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{23}{2}\left(\frac{3}{2}\right)\left(\frac{7}{2}\right)\left(\frac{13}{2}\right)} = \sqrt{\frac{6279}{16}} \approx 19.81$

24. $a = 9,\ b = 7,\ c = 10$
$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(9+7+10) = 13$
$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{13(4)(6)(3)} = \sqrt{936} \approx 30.59$

25. Area of a sector $= \frac{1}{2}r^2\theta$ where θ is in radians.
$\theta = 70^\circ \ \frac{\pi}{180} = \frac{7\pi}{18}$
Area of the sector $= \frac{1}{2}\ 8^2\ \frac{7\pi}{18} = \frac{112\pi}{9} \approx 39.10$ square feet
Area of the triangle $= \frac{1}{2}\ 8\ 8\sin(70^\circ) = 32\sin(70^\circ) \approx 30.07$ square feet
Area of the segment $= 39.10 - 30.07 = 9.03$ square feet

26. Area of a sector $= \frac{1}{2}r^2\theta$ where θ is in radians.

$$\theta = 40^\circ \cdot \frac{\pi}{180} = \frac{2\pi}{9}$$

$$\text{Area of the sector} = \frac{1}{2} \cdot 5^2 \cdot \frac{2\pi}{9} = \frac{25\pi}{9} \approx 8.73 \text{ square inches}$$

$$\text{Area of the triangle} = \frac{1}{2} \cdot 5 \cdot 5\sin\left(40^\circ\right) = \frac{25}{2}\sin\left(40^\circ\right) \approx 8.03 \text{ square inches}$$

$$\text{Area of the segment} = 8.73 - 8.03 = 0.70 \text{ square inches}$$

27. Find the area of the lot using Heron's Formula:

$$a = 100, \quad b = 50, \quad c = 75$$

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(100+50+75) = \frac{225}{2}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{225}{2}\left(\frac{25}{2}\right)\left(\frac{125}{2}\right)\left(\frac{75}{2}\right)} = \sqrt{\frac{52{,}734{,}375}{16}} \approx 1815.46$$

The cost is \$3 times the area:

$$\text{Cost} = \$3(1815.46) = \$5446.38$$

28. Diameter of canvas is 24 feet; radius of canvas is 12 feet; angle is 260˚.

Area of a sector $= \frac{1}{2}r^2\theta$ where θ is in radians.

$$\theta = 260^\circ \cdot \frac{\pi}{180} = \frac{13\pi}{9}$$

$$\text{Area of the sector} = \frac{1}{2} \cdot 12^2 \cdot \frac{13\pi}{9} = \frac{936\pi}{9} = 104\pi \approx 326.73 \text{ square feet}$$

29. The area of the shaded region = the area of the semicircle – the area of the triangle.

$$\text{Area of the semicircle} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(4)^2 = 8\pi \text{ square centimeters}$$

The triangle is a right triangle. Find the other leg:

$$6^2 + b^2 = 8^2 \quad \rightarrow \quad b^2 = 64 - 36 = 28 \quad \rightarrow \quad b = \sqrt{28} = 2\sqrt{7}$$

$$\text{Area of the triangle} = \frac{1}{2} \cdot 6 \cdot 2\sqrt{7} = 6\sqrt{7} \text{ square centimeters}$$

$$\text{Area of the shaded region} = 8\pi - 6\sqrt{7} \approx 9.26 \text{ square centimeters}$$

30. The area of the shaded region = the area of the semicircle – the area of the triangle.

$$\text{Area of the semicircle} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(5)^2 = \frac{25}{2}\pi \text{ square inches}$$

The triangle is a right triangle. Find the other leg:

$$8^2 + b^2 = 10^2 \quad \rightarrow \quad b^2 = 100 - 64 = 36 \quad \rightarrow \quad b = \sqrt{36} = 6$$

$$\text{Area of the triangle} = \frac{1}{2} \cdot 8 \cdot 6 = 24 \text{ square inches}$$

$$\text{Area of the shaded region} = 12.5\pi - 24 \approx 15.27 \text{ square inches}$$

31. Use the Law of Sines in the area of the triangle formula:

$$A = \frac{1}{2}ab\sin\gamma = \frac{1}{2}a\sin\gamma\left(\frac{a\sin\beta}{\sin\alpha}\right) = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$$

32. $A = \frac{1}{2}bc\sin\alpha = \frac{1}{2}b\sin\alpha\left(\frac{b\sin\gamma}{\sin\beta}\right) = \frac{b^2\sin\alpha\sin\gamma}{2\sin\beta}$

$A = \frac{1}{2}ac\sin\beta = \frac{1}{2}c\sin\beta\left(\frac{c\sin\alpha}{\sin\gamma}\right) = \frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$

33. $\alpha = 40^\circ,\ \beta = 20^\circ,\ a = 2 \qquad \gamma = 180^\circ - \alpha - \beta = 180^\circ - 40^\circ - 20^\circ = 120^\circ$

$$A = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha} = \frac{2^2\sin(20^\circ)\sin(120^\circ)}{2\sin(40^\circ)} \approx \frac{4(0.3420)(0.8660)}{2(0.6428)} \approx 0.92$$

34. $\alpha = 50^\circ,\ \gamma = 20^\circ,\ a = 3 \qquad \beta = 180^\circ - \alpha - \gamma = 180^\circ - 50^\circ - 20^\circ = 110^\circ$

$$A = \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha} = \frac{3^2\sin(110^\circ)\sin(20^\circ)}{2\sin(50^\circ)} \approx \frac{9(0.9397)(0.3420)}{2(0.7660)} \approx 1.89$$

35. $\beta = 70^\circ,\ \gamma = 10^\circ,\ b = 5 \qquad \alpha = 180^\circ - \beta - \gamma = 180^\circ - 70^\circ - 10^\circ = 100^\circ$

$$A = \frac{b^2\sin\alpha\sin\gamma}{2\sin\beta} = \frac{5^2\sin(100^\circ)\sin(10^\circ)}{2\sin(70^\circ)} \approx \frac{25(0.9848)(0.1736)}{2(0.9397)} \approx 2.27$$

36. $\alpha = 70^\circ,\ \beta = 60^\circ,\ c = 4 \qquad \gamma = 180^\circ - \alpha - \beta = 180^\circ - 70^\circ - 60^\circ = 50^\circ$

$$A = \frac{c^2\sin\alpha\sin\beta}{2\sin\gamma} = \frac{4^2\sin(70^\circ)\sin(60^\circ)}{2\sin(50^\circ)} \approx \frac{16(0.9397)(0.8660)}{2(0.7660)} \approx 8.50$$

37. $\alpha = 110^\circ,\ \gamma = 30^\circ,\ c = 3 \qquad \beta = 180^\circ - \alpha - \gamma = 180^\circ - 110^\circ - 30^\circ = 40^\circ$

$$A = \frac{c^2\sin\alpha\sin\beta}{2\sin\gamma} = \frac{3^2\sin(110^\circ)\sin(40^\circ)}{2\sin(30^\circ)} \approx \frac{9(0.9397)(0.6428)}{2(0.5000)} \approx 5.44$$

38. $\beta = 10^\circ,\ \gamma = 100^\circ,\ b = 2 \qquad \alpha = 180^\circ - \beta - \gamma = 180^\circ - 10^\circ - 100^\circ = 70^\circ$

$$A = \frac{b^2\sin\alpha\sin\gamma}{2\sin\beta} = \frac{2^2\sin(70^\circ)\sin(100^\circ)}{2\sin(10^\circ)} \approx \frac{4(0.9397)(0.9848)}{2(0.1736)} \approx 10.66$$

39. The area is the sum of the area of a triangle and a sector.

Area of the triangle $= \frac{1}{2}r\ \ r\sin(\pi - \theta) = \frac{1}{2}r^2\sin(\pi - \theta)$

Area of the sector $= \frac{1}{2}r^2\theta$

$$A = \frac{1}{2}r^2\sin(\pi - \theta) + \frac{1}{2}r^2\theta = \frac{1}{2}r^2(\sin(\pi - \theta) + \theta)$$
$$= \frac{1}{2}r^2(\sin\pi\cos\theta - \cos\pi\sin\theta + \theta) = \frac{1}{2}r^2(0 + \sin\theta + \theta) = \frac{1}{2}r^2(\theta + \sin\theta)$$

40. Find the lengths of the diagonals of the polygon. Use the Law of Cosines:

$x^2 = 35^2 + 80^2 - 2\ \ 35\ \ 80\cos(15^\circ) \approx 2215.82 \rightarrow x \approx 47.1$ feet

The interior angle of the third triangle is: $180^\circ - 100^\circ = 80^\circ$.

$y^2 = 45^2 + 20^2 - 2\ \ 45\ \ 20\cos(80^\circ) \approx 2112.43 \rightarrow y \approx 46.0$ feet

Find the area of the three triangles.

$$s_1 = \frac{1}{2}(35 + 80 + 47.1) = 81.05$$
$$A_1 = \sqrt{81.05(81.05 - 35)(81.05 - 80)(81.05 - 47.1)} \approx 364.8 \text{ ft}^2$$
$$s_2 = \frac{1}{2}(40 + 46 + 47.1) = 66.55$$
$$A_2 = \sqrt{66.55(66.55 - 40)(66.55 - 46)(66.55 - 47.1)} \approx 840.4 \text{ ft}^2$$
$$s_3 = \frac{1}{2}(45 + 20 + 46) = 55.5$$
$$A_3 = \sqrt{55.5(55.5 - 45)(55.5 - 20)(55.5 - 46)} \approx 443.3 \text{ ft}^2$$

The approximate area of the lake is $364.8 + 840.4 + 443.3 = 1648.5 \text{ ft}^2$

41. The grazing area must be considered in sections. A_1 represents $\frac{3}{4}$ of a circle:

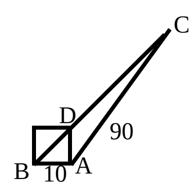
$$A_1 = \frac{3}{4}\pi(100)^2 = 7500\pi \approx 23,562 \text{ square feet}$$

Angles are needed to find A_2 and A_3: (see the figure)

In ΔABC, $CBA = 45^\circ$, $AB = 10$, $AC = 90$

Find BCA:

$$\frac{\sin CBA}{90} = \frac{\sin BCA}{10} \rightarrow \frac{\sin(45^\circ)}{90} = \frac{\sin BCA}{10}$$



$$\sin BCA = \frac{10\sin(45^\circ)}{90} \approx 0.0786$$
$$BCA \approx 4.5^\circ$$

$$m\ BAC = 180^\circ - 45^\circ - 4.5^\circ = 130.5^\circ$$
$$m\ DAC = 130.5^\circ - 90^\circ = 40.5^\circ$$

$$\text{Area of } A_3 = \frac{1}{2}(10)(90)\sin(40.5^\circ) \approx 292 \text{ square feet}$$

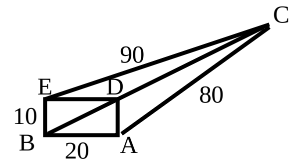
$$\text{Area of sector } A_2 = \frac{1}{2}(90)^2\left(49.5^\circ \cdot \frac{\pi}{180}\right) \approx 3499 \text{ square feet}$$

Since the cow can go in either direction around the barn, A_2 and A_3 must be doubled. Total grazing area is: $23,562 + 2(3499) + 2(292) = 31,144$ square feet

42. The grazing area must be considered in sections. A_1 represents $\frac{3}{4}$ of a circle

$$A_1 = \frac{3}{4}\pi(100)^2 = 7500\pi \approx 23,562 \text{ square feet}$$

Angles are needed to find A_2 and A_3: (see the figure)



In ΔABC, $\tan\ CBA = \dfrac{10}{20} \rightarrow\ CBA = 26.6^\circ$,

$AB = 20$, $AC = 80$

Find BCA:

$$\frac{\sin\ CBA}{80} = \frac{\sin\ BCA}{20} \rightarrow \frac{\sin(26.6^\circ)}{80} = \frac{\sin\ BCA}{20}$$

$$\sin\ BCA = \frac{20\sin(26.6^\circ)}{80} \approx 0.1119$$

$$BCA \approx 6.4^\circ$$

$$m\ BAC = 180^\circ - 26.6^\circ - 6.4^\circ = 147^\circ$$

$$m\ DAC = 147^\circ - 90^\circ = 57^\circ$$

Area of $A_3 = \dfrac{1}{2}(10)(80)\sin(57^\circ) \approx 335.5$ square feet

Area of sector $A_2 = \dfrac{1}{2}(80)^2\left(33^\circ \cdot \dfrac{\pi}{180}\right) \approx 1843.1$ square feet

There are similar areas on the opposite side of the barn. Calculate their areas.

Angles are needed to find A_4 and A_5: (see the figure)

In ΔEBC, $CBE = 90^\circ - 26.6^\circ = 63.4^\circ$, $EB = 10$, $EC = 90$

Find BCE:

$$\frac{\sin\ CBE}{90} = \frac{\sin\ BCE}{10} \rightarrow \frac{\sin(63.4^\circ)}{90} = \frac{\sin\ BCE}{10}$$

$$\sin\ BCE = \frac{10\sin(63.4^\circ)}{90} \approx 0.0994$$

$$BCE \approx 5.7^\circ$$

$$m\ BEC = 180^\circ - 63.4^\circ - 5.7^\circ = 110.9^\circ$$

$$m\ DEC = 110.9^\circ - 90^\circ = 20.9^\circ$$

Area of $A_5 = \dfrac{1}{2}(20)(90)\sin 20.9^\circ \approx 321.1$ square feet

Area of sector $A_4 = \dfrac{1}{2}(90)^2\left(69.1^\circ \cdot \dfrac{\pi}{180}\right) \approx 4884.4$ square feet

Total grazing area is: $23{,}562 + 1843.1 + 335.5 + 4884.4 + 321.1 = 30{,}946.1\ \text{ft}^2$

43. $h_1 = \dfrac{2K}{a}$, $h_2 = \dfrac{2K}{b}$, $h_3 = \dfrac{2K}{c}$ where K is the area of the triangle.

$$\frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} = \frac{a}{2K} + \frac{b}{2K} + \frac{c}{2K} = \frac{a+b+c}{2K} = \frac{2s}{2K} = \frac{s}{K}$$

44. From Problem 31, we have $A = \dfrac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}$.

From Problem 43, we have $h = \dfrac{2K}{a}$ where K is the area of the triangle

$$\frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha} = K \rightarrow \frac{a\sin\beta\sin\gamma}{\sin\alpha} = \frac{2K}{a} = h$$

45. $h = \dfrac{a\sin\beta\sin\gamma}{\sin\alpha}$ where h is the altitude to side a.

In ΔOAB, c is opposite angle AOB. The two adjacent angles are $\dfrac{\alpha}{2}$ and $\dfrac{\beta}{2}$.

Then $r = \dfrac{c\sin\left(\dfrac{\alpha}{2}\right)\sin\left(\dfrac{\beta}{2}\right)}{\sin(AOB)}$

$$AOB = \pi - \left(\frac{\alpha}{2} + \frac{\beta}{2}\right)$$

$$\sin(AOB) = \sin\left(\pi - \left(\frac{\alpha}{2} + \frac{\beta}{2}\right)\right) = \sin\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\pi}{2} - \left(\frac{\alpha+\beta}{2}\right)\right)$$

$$= \cos\left(\frac{\pi - (\alpha+\beta)}{2}\right) = \cos\left(\frac{\gamma}{2}\right)$$

Thus, $r = \dfrac{c\sin\left(\dfrac{\alpha}{2}\right)\sin\left(\dfrac{\beta}{2}\right)}{\cos\left(\dfrac{\gamma}{2}\right)}$

46. $$\cot\left(\frac{\gamma}{2}\right) = \frac{\cos\left(\frac{\gamma}{2}\right)}{\sin\left(\frac{\gamma}{2}\right)} = \frac{\dfrac{c\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)}{r}}{\sqrt{\dfrac{(s-a)(s-b)}{ab}}} = \frac{c\sqrt{\dfrac{(s-b)(s-c)}{bc}}\sqrt{\dfrac{(s-a)(s-c)}{ac}}}{r\sqrt{\dfrac{(s-a)(s-b)}{ab}}}$$

$$= \frac{c\sqrt{s-b}\sqrt{s-c}\sqrt{s-a}\sqrt{s-c}\sqrt{a}\sqrt{b}}{r\sqrt{s-a}\sqrt{s-b}\sqrt{b}\sqrt{c}\sqrt{a}\sqrt{c}} = \frac{c(s-c)}{r\,c} = \frac{s-c}{r}$$

47. Use the result of Problem 46:

$$\cot\left(\frac{\alpha}{2}\right) + \cot\left(\frac{\beta}{2}\right) + \cot\left(\frac{\gamma}{2}\right) = \frac{s-a}{r} + \frac{s-b}{r} + \frac{s-c}{r} = \frac{s-a+s-b+s-c}{r}$$

$$= \frac{3s-(a+b+c)}{r} = \frac{3s-2s}{r} = \frac{s}{r}$$

48. The area of a triangle is $\dfrac{1}{2}bh$. Find the area of ΔABC by adding the areas of three triangles.

$$K = \text{Area } \Delta AOB + \text{ Area } \Delta AOC + \text{ Area } \Delta BOC$$

$$= \frac{1}{2}rc + \frac{1}{2}rb + \frac{1}{2}ra = \frac{1}{2}r(a+b+c) = rs$$

$$rs = \sqrt{s(s-a)(s-b)(s-c)}$$

$$r = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s} = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$