Chapter 9

Applications of Trigonometric Functions

9.R Chapter Review

1. $c=10,\ \beta=20^\circ$

$$\sin\beta=\frac{b}{c}\quad\to\quad\sin(20^\circ)=\frac{b}{10}\quad\to\quad b=10\sin(20^\circ)\approx10(0.3420)\approx3.42$$

$$\cos\beta=\frac{a}{c}\quad\to\quad\cos(20^\circ)=\frac{a}{10}\quad\to\quad a=10\cos(20^\circ)\approx10(0.9397)\approx9.40$$

$$\alpha=90^\circ-\beta=90^\circ-20^\circ=70^\circ$$

2. $a=5,\ \alpha=35^\circ$

$$\sin\alpha=\frac{a}{c}\quad\to\quad\sin(35^\circ)=\frac{5}{c}\quad\to\quad c=\frac{5}{\sin(35^\circ)}\approx\frac{5}{0.5736}\approx8.72$$

$$\tan\alpha=\frac{a}{b}\quad\to\quad\tan(35^\circ)=\frac{5}{b}\quad\to\quad b=\frac{5}{\tan(35^\circ)}\approx\frac{5}{0.7002}\approx7.14$$

$$\beta=90^\circ-\alpha=90^\circ-35^\circ=55^\circ$$

3. $b=2,\ c=5$

$$c^2=a^2+b^2\quad\to\quad a^2=c^2-b^2=5^2-2^2=25-4=21\quad\to\quad a=\sqrt{21}\approx4.58$$

$$\sin\beta=\frac{b}{c}=\frac{2}{5}=0.4000\quad\to\quad\beta\approx23.6^\circ$$

$$\alpha=90^\circ-\beta=90^\circ-23.6^\circ=66.4^\circ$$

4. $b=1,\ a=3$

$$c^2=a^2+b^2\quad\to\quad c^2=3^2+1^2=9+1=10\quad\to\quad c=\sqrt{10}\approx3.16$$

$$\tan\beta=\frac{b}{a}=\frac{1}{3}\approx0.3333\quad\to\quad\beta\approx18.4^\circ$$

$$\alpha=90^\circ-\beta=90^\circ-18.4^\circ=71.6^\circ$$

5. $\alpha=50^\circ,\ \beta=30^\circ,\ a=1$

$$\gamma=180^\circ-\alpha-\beta=180^\circ-50^\circ-30^\circ=100^\circ$$

$$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}\quad\to\quad\frac{\sin(50^\circ)}{1}=\frac{\sin(30^\circ)}{b}\quad\to\quad b=\frac{1\sin(30^\circ)}{\sin(50^\circ)}\approx0.65$$

$$\frac{\sin\gamma}{c}=\frac{\sin\alpha}{a}\quad\to\quad\frac{\sin(100^\circ)}{c}=\frac{\sin(50^\circ)}{1}\quad\to\quad c=\frac{1\sin(100^\circ)}{\sin(50^\circ)}\approx1.29$$

6. $\alpha = 10^\circ$, $\gamma = 40^\circ$, $c = 2$

$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 10^\circ - 40^\circ = 130^\circ$

$$\frac{\sin\gamma}{c} = \frac{\sin\alpha}{a} \quad \to \quad \frac{\sin(40^\circ)}{2} = \frac{\sin(10^\circ)}{a} \quad \to \quad a = \frac{2\sin(10^\circ)}{\sin(40^\circ)} \approx 0.54$$

$$\frac{\sin\gamma}{c} = \frac{\sin\beta}{b} \quad \to \quad \frac{\sin(40^\circ)}{2} = \frac{\sin(130^\circ)}{b} \quad \to \quad b = \frac{2\sin(130^\circ)}{\sin(40^\circ)} \approx 2.38$$

7. $a = 5$, $c = 2$, $\alpha = 100^\circ$

$$\frac{\sin\gamma}{c} = \frac{\sin\alpha}{a} \quad \to \quad \frac{\sin\gamma}{2} = \frac{\sin(100^\circ)}{5} \quad \to \quad \sin\gamma = \frac{2\sin(100^\circ)}{5} \approx 0.3939$$

$$\to \quad \gamma = 23.2^\circ \ \text{ or } \gamma = 156.8^\circ$$

The second value is discarded because $\alpha + \gamma > 180^\circ$.

$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 100^\circ - 23.2^\circ = 56.8^\circ$

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a} \quad \to \quad \frac{\sin(56.8^\circ)}{b} = \frac{\sin(100^\circ)}{5} \quad \to \quad b = \frac{5\sin(56.8^\circ)}{\sin(100^\circ)} \approx 4.25$$

8. $a = 2$, $c = 5$, $\alpha = 60^\circ$

$$\frac{\sin\gamma}{c} = \frac{\sin\alpha}{a} \quad \to \quad \frac{\sin\gamma}{5} = \frac{\sin(60^\circ)}{2} \quad \to \quad \sin\gamma = \frac{5\sin(60^\circ)}{2} \approx 2.1651$$

Since $\sin\gamma$ cannot be greater than 1, this is impossible. No triangle.

9. $a = 3$, $c = 1$, $\gamma = 110^\circ$

$$\frac{\sin\gamma}{c} = \frac{\sin\alpha}{a} \quad \to \quad \frac{\sin(110^\circ)}{1} = \frac{\sin\alpha}{3} \quad \to \quad \sin\alpha = \frac{3\sin(110^\circ)}{1} \approx 2.8191$$

There is no angle α for which $\sin\alpha > 1$. Therefore, there is no triangle with the given measurements.

10. $a = 3$, $c = 1$, $\gamma = 20^\circ$

$$\frac{\sin\gamma}{c} = \frac{\sin\alpha}{a} \quad \to \quad \frac{\sin(20^\circ)}{1} = \frac{\sin\alpha}{3} \quad \to \quad \sin\alpha = \frac{3\sin(20^\circ)}{1} \approx 1.0260$$

There is no angle α for which $\sin\alpha > 1$. Therefore, there is no triangle with the given measurements.

11. $a = 3$, $c = 1$, $\beta = 100^\circ$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$b^2 = 3^2 + 1^2 - 2 \cdot 3 \cdot 1\cos(100^\circ) \approx 11.0419$$

$$b \approx 3.32$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad \to \quad \cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{3.32^2 + 1^2 - 3^2}{2(3.32)(1)} \approx 0.4552 \to \alpha \approx 62.9^\circ$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \quad \rightarrow \quad \cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos\gamma = \frac{3^2 + 3.32^2 - 1^2}{2(3)(3.32)} \approx 0.9549 \rightarrow \gamma = 17.3^\circ$$

12. $a = 3, \quad b = 5, \quad \beta = 80^\circ$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} \quad \rightarrow \quad \frac{\sin\alpha}{3} = \frac{\sin(80^\circ)}{5} \quad \rightarrow \quad \sin\alpha = \frac{3\sin(80^\circ)}{5} \approx 0.5909$$

$$\rightarrow \quad \alpha = 36.2^\circ \quad \text{or}\ \alpha = 143.8^\circ$$

The second value is discarded because $\alpha + \beta > 180^\circ$.

$$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 36.2^\circ - 80^\circ = 63.8^\circ$$

$$\frac{\sin\gamma}{c} = \frac{\sin\beta}{b} \quad \rightarrow \quad \frac{\sin(63.8^\circ)}{c} = \frac{\sin(80^\circ)}{5} \quad \rightarrow \quad c = \frac{5\sin(63.8^\circ)}{\sin(80^\circ)} \approx 4.56$$

13. $a = 2, \quad b = 3, \quad c = 1$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad \rightarrow \quad \cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{3^2 + 1^2 - 2^2}{2(3)(1)} = 1.000 \rightarrow \alpha \approx 0^\circ$$

No triangle exists with an angle of 0°.

14. $a = 10, \quad b = 7, \quad c = 8$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad \rightarrow \quad \cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{7^2 + 8^2 - 10^2}{2(7)(8)} \approx 0.1161 \rightarrow \alpha \approx 83.3^\circ$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta \quad \rightarrow \quad \cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos\beta = \frac{10^2 + 8^2 - 7^2}{2(10)(8)} \approx 0.7188 \rightarrow \beta = 44.0^\circ$$

$$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 83.3^\circ - 44.0^\circ = 52.7^\circ$$

15. $a = 1, \quad b = 3, \quad \gamma = 40^\circ$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 = 1^2 + 3^2 - 2\ 1\ 3\cos(40^\circ) \approx 5.4037$$

$$c \approx 2.32$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad \rightarrow \quad \cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{3^2 + 2.32^2 - 1^2}{2(3)(2.32)} \approx 0.9614 \rightarrow \alpha \approx 16.0^\circ$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta \quad \rightarrow \quad \cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos\beta = \frac{1^2 + 2.32^2 - 3^2}{2(1)(2.32)} \approx -0.5641 \rightarrow \beta = 124.3^\circ$$

16. $a = 4,\ b = 1,\ \gamma = 100^\circ$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = 4^2 + 1^2 - 2 \quad 4 \quad 1\cos(100^\circ) \approx 18.3892$

$c \approx 4.29$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad \rightarrow \quad \cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{1^2 + 4.29^2 - 4^2}{2(1)(4.29)} \approx 0.3967 \rightarrow \alpha \approx 66.6^\circ$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta \quad \rightarrow \quad \cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos\beta = \frac{4^2 + 4.29^2 - 1^2}{2(4)(4.29)} \approx 0.9733 \rightarrow \beta = 13.3^\circ$$

17. $a = 5,\ b = 3,\ \alpha = 80^\circ$

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a} \quad \rightarrow \quad \frac{\sin\beta}{3} = \frac{\sin(80^\circ)}{5} \quad \rightarrow \quad \sin\beta = \frac{3\sin(80^\circ)}{5} \approx 0.5909$$

$$\rightarrow \quad \beta = 36.2^\circ \quad \text{or} \quad \beta = 143.8^\circ$$

The second value is discarded because $\alpha + \beta > 180^\circ$.

$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 80^\circ - 36.2^\circ = 63.8^\circ$

$$\frac{\sin\gamma}{c} = \frac{\sin\alpha}{a} \quad \rightarrow \quad \frac{\sin(63.8^\circ)}{c} = \frac{\sin(80^\circ)}{5} \quad \rightarrow \quad c = \frac{5\sin(63.8^\circ)}{\sin(80^\circ)} \approx 4.56$$

18. $a = 2,\ b = 3,\ \alpha = 20^\circ$

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a} \quad \rightarrow \quad \frac{\sin\beta}{3} = \frac{\sin(20^\circ)}{2} \quad \rightarrow \quad \sin\beta = \frac{3\sin(20^\circ)}{2} \approx 0.5130$$

$$\rightarrow \quad \beta_1 = 30.9^\circ \quad \text{or}\ \beta_2 = 149.1^\circ$$

For both values, $\alpha + \beta < 180^\circ$. Therefore, there are two triangles.

$\gamma_1 = 180^\circ - \alpha - \beta_1 = 180^\circ - 20^\circ - 30.9^\circ \approx 129.1^\circ$

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma_1}{c_1} \quad \rightarrow \quad \frac{\sin(20^\circ)}{2} = \frac{\sin(129.1^\circ)}{c_1} \quad \rightarrow \quad c_1 = \frac{2\sin(129.1^\circ)}{\sin(20^\circ)} \approx 4.54$$

$\gamma_2 = 180^\circ - \alpha - \beta_2 = 180^\circ - 20^\circ - 149.1^\circ \approx 10.9^\circ$

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma_2}{c_2} \quad \rightarrow \quad \frac{\sin(20^\circ)}{2} = \frac{\sin(10.9^\circ)}{c_2} \quad \rightarrow \quad c_2 = \frac{2\sin(10.9^\circ)}{\sin(20^\circ)} \approx 1.11$$

Two triangles: $\beta_1 \approx 30.9^\circ, \gamma_1 \approx 129.1^\circ, \; c_1 \approx 4.54$
or $\beta_2 \approx 149.1^\circ, \; \gamma_2 \approx 10.9^\circ, c_2 \approx 1.11$

19. $a = 1, \; b = \frac{1}{2}, \; c = \frac{4}{3}$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad \rightarrow \quad \cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{\left(\left(\frac{1}{2}\right)^2 + \left(\frac{4}{3}\right)^2 - 1^2\right)}{2\left(\frac{1}{2}\right)\left(\frac{4}{3}\right)} \approx 0.7708 \rightarrow \alpha \approx 39.6^\circ$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta \quad \rightarrow \quad \cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos\beta = \frac{\left(1^2 + \left(\frac{4}{3}\right)^2 - \left(\frac{1}{2}\right)^2\right)}{2(1)\left(\frac{4}{3}\right)} \approx 0.9479 \rightarrow \beta \approx 18.6^\circ$$

$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 39.6^\circ - 18.6^\circ = 121.8^\circ$

20. $a = 3, \; b = 2, \; c = 2$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad \rightarrow \quad \cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{2^2 + 2^2 - 3^2}{2(2)(2)} \approx -0.1250 \rightarrow \alpha \approx 97.2^\circ$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta \quad \rightarrow \quad \cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos\beta = \frac{3^2 + 2^2 - 2^2}{2(3)(2)} \approx 0.7500 \rightarrow \beta = 41.4^\circ$$

$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 97.2^\circ - 41.4^\circ = 41.4^\circ$

21. $a = 3, \; b = 4, \; \alpha = 10^\circ$

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a} \quad \rightarrow \quad \frac{\sin\beta}{4} = \frac{\sin(10^\circ)}{3} \quad \rightarrow \quad \sin\beta = \frac{4\sin(10^\circ)}{3} \approx 0.2315$$

$\rightarrow \quad \beta_1 \approx 13.4^\circ \;$ or $\beta_2 \approx 166.6^\circ$

For both values, $\alpha + \beta < 180^\circ$. Therefore, there are two triangles.

$\gamma_1 = 180^\circ - \alpha - \beta_1 = 180^\circ - 10^\circ - 13.4^\circ \approx 156.6^\circ$

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma_1}{c_1} \quad \to \quad \frac{\sin(10^\circ)}{3} = \frac{\sin(156.6^\circ)}{c_1} \quad \to \quad c_1 = \frac{3\sin(156.6^\circ)}{\sin(10^\circ)} \approx 6.86$$

$\gamma_2 = 180^\circ - \alpha - \beta_2 = 180^\circ - 10^\circ - 166.6^\circ \approx 3.4^\circ$

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma_2}{c_2} \quad \to \quad \frac{\sin(10^\circ)}{3} = \frac{\sin(3.4^\circ)}{c_2} \quad \to \quad c_2 = \frac{3\sin(3.4^\circ)}{\sin(10^\circ)} \approx 1.02$$

Two triangles: $\beta_1 \approx 13.4^\circ, \gamma_1 \approx 156.6^\circ, c_1 \approx 6.86$

or $\beta_2 \approx 166.6^\circ, \gamma_2 \approx 3.4^\circ, \quad c_2 \approx 1.02$

22. $a = 4, \ \alpha = 20^\circ, \beta = 100^\circ$

$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 20^\circ - 100^\circ = 60^\circ$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} \quad \to \quad \frac{\sin(20^\circ)}{4} = \frac{\sin(100^\circ)}{b} \quad \to \quad b = \frac{4\sin(100^\circ)}{\sin(20^\circ)} \approx 11.52$$

$$\frac{\sin\gamma}{c} = \frac{\sin\alpha}{a} \quad \to \quad \frac{\sin(60^\circ)}{c} = \frac{\sin(20^\circ)}{4} \quad \to \quad c = \frac{4\sin(60^\circ)}{\sin(20^\circ)} \approx 10.13$$

23. $b = 4, \ c = 5, \ \alpha = 70^\circ$

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$a^2 = 4^2 + 5^2 - 2 \;\; 4 \;\; 5\cos(70^\circ) \approx 27.3192$

$a \approx 5.23$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \quad \to \quad \cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos\gamma = \frac{5.23^2 + 4^2 - 5^2}{2(5.23)(4)} \approx 0.4386 \to \gamma \approx 64.0^\circ$$

$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 70^\circ - 64^\circ \approx 46.0^\circ$

24. $a = 1, \ b = 2, \ \gamma = 60^\circ$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = 1^2 + 2^2 - 2 \;\; 1 \;\; 2\cos(60^\circ) = 3$

$c \approx 1.73$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad \to \quad \cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\alpha = \frac{2^2 + 1.73^2 - 1^2}{2(2)(1.73)} \approx 0.8660 \to \alpha \approx 30^\circ$$

$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 30^\circ - 60^\circ = 90^\circ$

25. $a = 2, \ b = 3, \ \gamma = 40^\circ$

$$A = \frac{1}{2}ab\sin\gamma = \frac{1}{2}(2)(3)\sin(40^\circ) \approx 1.93$$

26. $b = 5, \ c = 5, \ \alpha = 20^\circ$

$$A=\frac{1}{2}bc\sin\alpha=\frac{1}{2}(5)(5)\sin\left(20^\circ\right)\approx 4.28$$

27. $b=4,\ c=10,\ \alpha=70^\circ$
$$A=\frac{1}{2}bc\sin\alpha=\frac{1}{2}(4)(10)\sin\left(70^\circ\right)\approx 18.79$$

28. $a=2,\ b=1,\ \gamma=100^\circ$
$$A=\frac{1}{2}ab\sin\gamma=\frac{1}{2}(2)(1)\sin\left(100^\circ\right)\approx 0.98$$

29. $a=4,\ b=3,\ c=5$
$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(4+3+5)=6$$
$$A=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{6(2)(3)(1)}=\sqrt{36}=6$$

30. $a=10,\ b=7,\ c=8$
$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(10+7+8)=\frac{25}{2}$$
$$A=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{\frac{25}{2}\left(\frac{5}{2}\right)\left(\frac{11}{2}\right)\left(\frac{9}{2}\right)}=\sqrt{\frac{12375}{16}}\approx 27.81$$

31. $a=4,\ b=2,\ c=5$
$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(4+2+5)=\frac{11}{2}$$
$$A=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{\frac{11}{2}\left(\frac{3}{2}\right)\left(\frac{7}{2}\right)\left(\frac{1}{2}\right)}=\sqrt{\frac{231}{16}}\approx 3.80$$

32. $a=3,\ b=2,\ c=2$
$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(3+2+2)=\frac{7}{2}$$
$$A=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{\frac{7}{2}\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)}=\sqrt{\frac{63}{16}}\approx 1.98$$

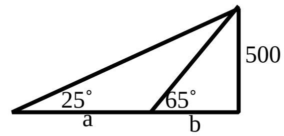
33. $\alpha=50^\circ,\ \beta=30^\circ,\ a=1$ $\quad\gamma=180^\circ-\alpha-\beta=180^\circ-50^\circ-30^\circ=100^\circ$
$$A=\frac{a^2\sin\beta\sin\gamma}{2\sin\alpha}=\frac{1^2\sin\left(30^\circ\right)\sin\left(100^\circ\right)}{2\sin\left(50^\circ\right)}\approx\frac{1(0.5000)(0.9848)}{2(0.7660)}\approx 0.32$$

34. $\alpha=10^\circ,\ \gamma=40^\circ,\ c=3$ $\quad\beta=180^\circ-\alpha-\gamma=180^\circ-10^\circ-40^\circ=130^\circ$
$$A=\frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}=\frac{3^2\sin\left(10^\circ\right)\sin\left(130^\circ\right)}{2\sin\left(40^\circ\right)}\approx\frac{9(0.1736)(0.7660)}{2(0.6428)}\approx 0.93$$

35. Use right triangle methods:

$$\tan\left(65^\circ\right)=\frac{500}{b} \quad \rightarrow \quad b=\frac{500}{\tan\left(65^\circ\right)}\approx 233.15$$

$$\tan\left(25^\circ\right)=\frac{500}{a+b} \quad \rightarrow \quad a+b=\frac{500}{\tan\left(25^\circ\right)}\approx 1072.25$$



$$a = 1072.25-233.15=839.1 \text{ feet}$$

The lake is approximately 839 feet long.

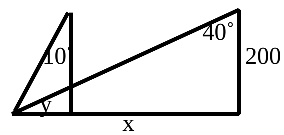
36. Use right triangle methods:

$$\tan\left(40^\circ\right)=\frac{x}{200} \quad \rightarrow \quad x=200\tan\left(40^\circ\right)\approx 167.82$$

$$\tan\left(10^\circ\right)=\frac{y}{200} \quad \rightarrow \quad y=200\tan\left(10^\circ\right)\approx 35.27$$

$$d=167.82-35.27=132.55 \text{ feet}$$

$$\text{speed}\approx\frac{132.55 \text{ feet}}{1 \text{ minute}}\cdot\frac{1 \text{ mile}}{5280 \text{ feet}}\cdot\frac{60 \text{ minutes}}{1 \text{ hour}}\approx 1.5 \text{ mi/hr}$$



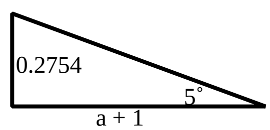
37. $\tan\left(25^\circ\right)=\dfrac{b}{50} \quad \rightarrow \quad b=50\tan\left(25^\circ\right)\approx 50(0.4663)\approx 23.3 \text{ feet}$

38. $\tan\left(25^\circ\right)=\dfrac{h}{80} \quad \rightarrow \quad h=80\tan\left(25^\circ\right)\approx 80(0.4663)\approx 37.3 \text{ feet}$

39. 1454 ft ≈ 0.2754 miles

$$\tan\left(5^\circ\right)\approx\frac{0.2754}{a+1}$$

$$a+1=\frac{0.2754}{\tan\left(5^\circ\right)}\approx 3.15 \quad \rightarrow \quad a\approx 2.15 \text{ miles}$$



The boat is about 2.15 miles offshore.

40. $\sin\theta = \dfrac{900}{4100}\approx 0.2195 \quad \rightarrow \quad \theta\approx 12.7^\circ$

41. $ABC = 180^\circ-20^\circ=160^\circ$

Find the angle at city C:

$$\frac{\sin C}{100}=\frac{\sin\left(160^\circ\right)}{300} \quad \rightarrow \quad \sin C=\frac{100\sin\left(160^\circ\right)}{300}\approx 0.1140 \quad \rightarrow \quad C\approx 6.55^\circ$$

Find the angle at city A:

$$A=180^\circ-160^\circ-6.55^\circ=13.45^\circ$$

$$\frac{\sin\left(13.45^\circ\right)}{y}=\frac{\sin\left(160^\circ\right)}{300} \quad \rightarrow \quad y=\frac{300\sin\left(13.45^\circ\right)}{\sin\left(160^\circ\right)}\approx 204 \text{ miles}$$

The distance from city B to city C is approximately 204 miles.

42. (a) The distance traveled is $420\left(\frac{1}{6}\right) = 70$ miles.

Find the remaining distance to city B:

$$a^2 = 300^2 + 70^2 - 2(300)(70)\cos\left(5^\circ\right) \approx 53059.82$$

$$a = 230.3 \text{ miles}$$

Find angle C:

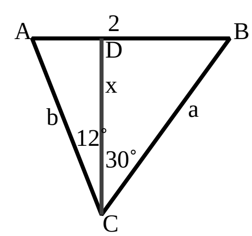
$$\frac{\sin C}{300} = \frac{\sin\left(5^\circ\right)}{230.3} \rightarrow \sin C = \frac{300\sin\left(5^\circ\right)}{230.3} \approx 0.1135 \rightarrow C \approx 173.5^\circ$$

The pilot should turn through an angle of 6.5˚ to correct the course.

(b) The time from city A to city B is $\frac{300}{420} = \frac{5}{7}$ hour. The time from city A to the point of the error is $\frac{1}{6}$ hour. The remaining part of the trip must be completed in $\frac{5}{7} - \frac{1}{6} = \frac{23}{42}$ hour. Calculate the rate:

$$r = \frac{230.3}{\left(\frac{23}{42}\right)} \approx 420.55 \text{ mi/hr.}$$

43. Draw a line perpendicular to the shore:



(a) $ACB = 12^\circ + 30^\circ = 42^\circ$

$ABC = 90^\circ - 30^\circ = 60^\circ$

$CAB = 90^\circ - 12^\circ = 78^\circ$

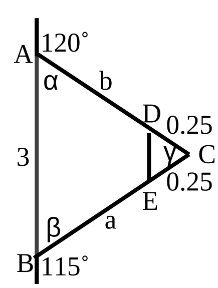
$$\frac{\sin\left(60^\circ\right)}{b} = \frac{\sin\left(42^\circ\right)}{2} \rightarrow b = \frac{2\sin\left(60^\circ\right)}{\sin\left(42^\circ\right)} \approx 2.59 \text{ miles}$$

(b) $$\frac{\sin\left(78^\circ\right)}{a} = \frac{\sin\left(42^\circ\right)}{2} \rightarrow a = \frac{2\sin\left(78^\circ\right)}{\sin\left(42^\circ\right)} \approx 2.92 \text{ mile}$$

(c) $$\cos 12^\circ = \frac{x}{2.59} \rightarrow x = 2.59\cos 12^\circ \approx 2.53 \text{ miles}$$

44. $\alpha = 180^\circ - 120^\circ = 60^\circ \quad \beta = 180^\circ - 115^\circ = 65^\circ$

$\gamma = 180^\circ - 60^\circ - 65^\circ = 55^\circ$



$$\frac{\sin\left(60^\circ\right)}{a} = \frac{\sin\left(55^\circ\right)}{3} \rightarrow a = \frac{3\sin\left(60^\circ\right)}{\sin\left(55^\circ\right)} \approx 3.17 \text{ mi}$$

$$\frac{\sin\left(65^\circ\right)}{b} = \frac{\sin\left(55^\circ\right)}{3} \rightarrow b = \frac{3\sin\left(65^\circ\right)}{\sin\left(55^\circ\right)} \approx 3.32 \text{ mi}$$

$$\overline{BE} = 3.17 - 0.25 = 2.92 \text{ mi}$$

$$\overline{AD} = 3.32 - 0.25 = 3.07 \text{ mi}$$

For the isosceles triangle,

$$CDE = CED = \frac{180^\circ - 55^\circ}{2} = 62.5^\circ$$

$$\frac{\sin\left(55^\circ\right)}{DE} = \frac{\sin\left(62.5^\circ\right)}{0.25} \rightarrow DE = \frac{0.25\sin\left(55^\circ\right)}{\sin\left(62.5^\circ\right)} \approx 0.23 \text{ miles}$$

The length of the highway is 2.92 + 3.07 + 0.23 = 6.22 miles.

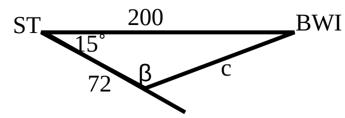
45. (a) After 4 hours, the yacht would have sailed 18(4) = 72 miles.
Find the third side of the triangle determines the distance from the island:

$a = 72,\ b = 200,\ \gamma = 15°$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = 72^2 + 200^2 - 2 \cdot 72 \cdot 200\cos(15°) \approx 17365.34$

$c \approx 131.8$ miles



The yacht is about 131.8 miles from the island.

(b) Find the measure of the angle opposite the 200 side:

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac} \rightarrow \cos\beta = \frac{72^2 + 131.8^2 - 200^2}{2(72)(131.8)} \approx -0.9192 \rightarrow \beta \approx 156.8°$$

The yacht should turn through an angle of 180° – 156.8° = 23.2° to correct its course.

(c) The original trip would have taken: $t = \frac{200}{18} \approx 11.1$ hours.

The actual trip takes: $t = 4 + \frac{131.8}{18} \approx 4 + 7.3 \approx 11.3$ hours.

The trip take about 0.2 hour or 12 minutes longer.

46. Find the third side of the triangle using the Law of Cosines:

$a = 50,\ b = 60,\ \gamma = 80°$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = 50^2 + 60^2 - 2 \cdot 50 \cdot 60\cos(80°) \approx 5058.11$

$c \approx 71.1$

The houses are approximately 71.1 feet apart.

47. Find the lengths of the two unknown sides of the middle triangle:

$x^2 = 100^2 + 125^2 - 2(100)(125)\cos(50°) \approx 9555.31 \rightarrow x \approx 97.75$ feet

$y^2 = 70^2 + 50^2 - 2(70)(50)\cos(100°) \approx 8615.54 \rightarrow y \approx 92.82$ feet

Find the areas of the three triangles:

$A_1 = \frac{1}{2}(100)(125)\sin(50°) \approx 4787.78\ \text{ft}^2$

$A_2 = \frac{1}{2}(50)(70)\sin(100°) \approx 1723.41\ \text{ft}^2$

$s = \frac{1}{2}(50 + 97.75 + 92.82) = 120.285$

$A_3 = \sqrt{120.285(70.285)(22.535)(27.465)} \approx 2287.47$

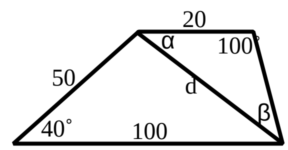
The approximate area of the lake is 4787.78 + 1723.41 + 2287.47 = 8798.66 sq.ft.

48. Construct a diagonal. Find the area of the first triangle and the length of the diagonal:

$A_1 = \frac{1}{2} \cdot 50 \cdot 100 \cdot \sin(40°) \approx 1606.97\ \text{ft}^2$

$d^2 = 50^2 + 100^2 - 2 \cdot 50 \cdot 100\cos(40°) \approx 4839.56$

$d = 69.57$ feet



Use the Law of Sines to find β:

$$\frac{\sin\beta}{20} = \frac{\sin(100^\circ)}{69.57} \rightarrow \sin\beta = \frac{20\sin(100^\circ)}{69.57} \approx 0.2831 \rightarrow \beta \approx 16.4^\circ$$

$$\alpha = 180^\circ - 100^\circ - 16.4^\circ = 63.6^\circ$$

Find the area of the second triangle:

$$A_2 = \frac{1}{2}\ 20\ \ 69.57\ \ \sin(63.6^\circ) \approx 623.15\ \text{ft}^2$$

The cost of the parcel is: $\$100(1606.97 + 623.15) = \$223{,}012$

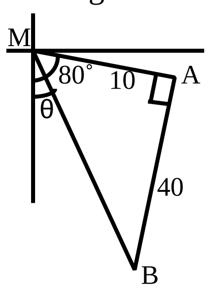
49. Area of the segment = area of the sector - area of the triangle.

$$\text{Area of sector} = \frac{1}{2}r^2\theta = \frac{1}{2}\ 6^2\left(50\ \frac{\pi}{180}\right) \approx 15.708\ \text{in}^2$$

$$\text{Area of triangle} = \frac{1}{2}ab\sin\theta = \frac{1}{2}\ 6\ \ 6\sin(50^\circ) \approx 13.789\ \text{in}^2$$

$$\text{Area of segment} = 15.708 - 13.789 = 1.919\ \text{in}^2$$

50. Find angle AMB and subtract from 80˚ to obtain θ.



$$\tan\ AMB = \frac{40}{10} = 4 \quad \rightarrow \quad AMB \approx 76.0^\circ$$

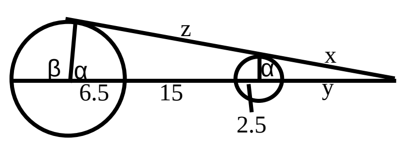
$$\theta \approx 80^\circ - 76.0^\circ \approx 4.0^\circ$$

The bearing is S4.0°E.

51. Extend the tangent line until it meets a line extended through the centers of the pulleys. Label these extensions x and y. The distance between the points of tangency is z. Two similar triangles are formed. Therefore:

$$\frac{24+y}{y} = \frac{6.5}{2.5}$$

where $24+y$ is the hypotenuse of the larger triangle and y is the hypotenuse of the smaller triangle.



Solve for y:

$$6.5y = 2.5(24+y) \rightarrow 6.5y = 60 + 2.5y \rightarrow 4y = 60 \rightarrow y = 15$$

Use the Pythagorean Theorem to find x:

$$x^2 + 2.5^2 = 15^2 \rightarrow x^2 = 225 - 6.25 = 218.75 \rightarrow x \approx 14.79$$

Use the Pythagorean Theorem to find z:

$$(z+14.79)^2 + 6.5^2 = (24+15)^2 \rightarrow (z+14.79)^2 = 1521 - 42.25 = 1478.75$$

$$z + 14.79 = 38.45 \rightarrow z = 23.66$$

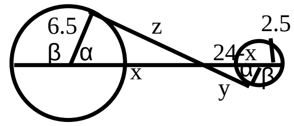
Find α:

$$\cos\alpha = \frac{2.5}{15} \approx 0.1667 \quad \rightarrow \quad \alpha \approx 1.4033\ \text{radians}$$

$$\beta = \pi - 1.4033 \approx 1.7383\ \text{radians}$$

The arc length on the larger pulley is: 6.5(1.7383) = 11.30 inches.
The arc length on the smaller pulley is 2.5(1.4033) = 3.51 inches.
The distance between the points of tangency is 23.66 inches.
The length of the belt is: 2(11.30 + 3.51 + 23.66) = 76.94 inches.

52. Find the lengths of x and 24 – x:



$$\frac{6.5}{2.5} = \frac{x}{24 - x}$$

$$2.5x = 6.5(24 - x)$$

$$2.5x = 156 - 6.5x$$

$$9x = 156 \rightarrow x \approx 17.3 \rightarrow 24 - x = 6.7$$

$$\cos\alpha = \frac{6.5}{17.3} \approx 0.376 \rightarrow \alpha \approx 68.0^\circ$$

$$\beta = 180^\circ - 68.0^\circ = 112^\circ$$

$$z = 6.5\tan\left(68^\circ\right) \approx 16.1 \text{ in}$$

$$y = 2.5\tan\left(68^\circ\right) \approx 6.2 \text{ in}$$

The arc length on the larger pulley is: $6.5\left(112 \cdot \frac{\pi}{180}\right) \approx 12.7$ inches.

The arc length on the smaller pulley is $2.5\left(112 \cdot \frac{\pi}{180}\right) \approx 4.9$ inches.

The distance between the points of tangency is 16.1 + 6.2 = 22.3 inches.
The length of the belt is: 2(12.7 + 4.9 + 22.3) = 79.8 inches.

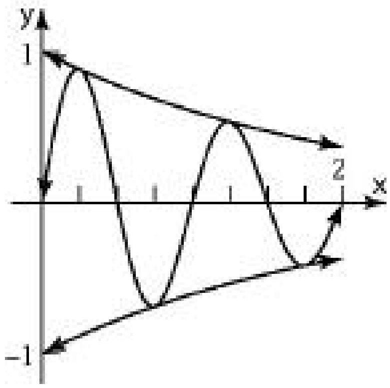
53. $d = 6\sin(2t)$
(a) Simple harmonic
(b) 6 feet
(c) π seconds
(d) $\frac{1}{\pi}$ oscillation/second

54. $d = 2\cos(4t)$
(a) Simple harmonic
(b) 2 feet
(c) $\frac{\pi}{2}$ seconds
(d) $\frac{2}{\pi}$ oscillation/second

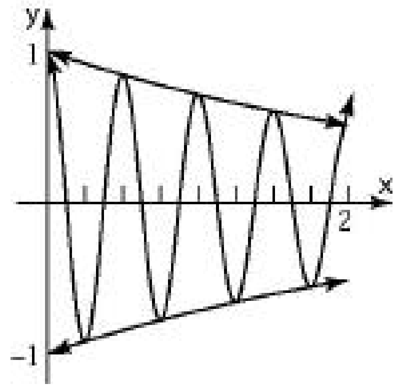
55. $d = -2\cos(\pi t)$
(a) Simple harmonic
(b) 2 feet
(c) 2 seconds
(d) $\frac{1}{2}$ oscillation/second

56. $d = -3\sin\left(\frac{\pi}{2}t\right)$
(a) Simple harmonic
(b) 3 feet
(c) 4 seconds
(d) $\frac{1}{4}$ oscillation/second

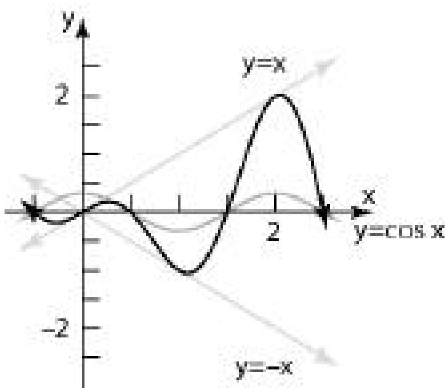
57. $y = e^{-x/2\pi}\sin(2x), \quad 0 \le x \le 2\pi$



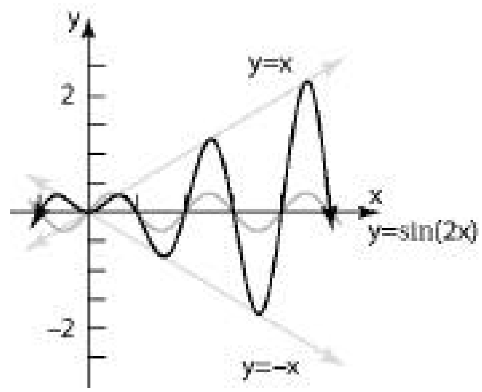
58. $y = e^{-x/3\pi}\cos(4x), \quad 0 \le x \le 2\pi$



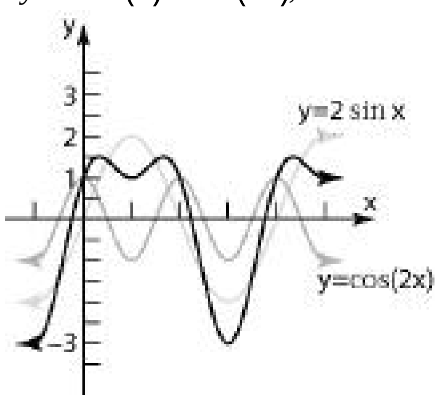
59. $y = x\cos(x), \quad 0 \le x \le 2\pi$



60. $y = x\sin(2x), \quad 0 \le x \le 2\pi$



61. $y = 2\sin(x) + \cos(2x), \quad 0 \le x \le 2\pi$



62. $y = 2\cos(2x) + \sin\left(\frac{x}{2}\right) \quad 0 \le x \le 2\pi$