Verifying Fixed-Point Digital Filters using SMT-Based Bounded Model Checking

Renato B. Abreu, Lucas Cordeiro and Eddie B. L. Filho
renato.abreu@indt.org.br, lucascordeiro@ufam.edu.br, eddie@ctpim.org.br

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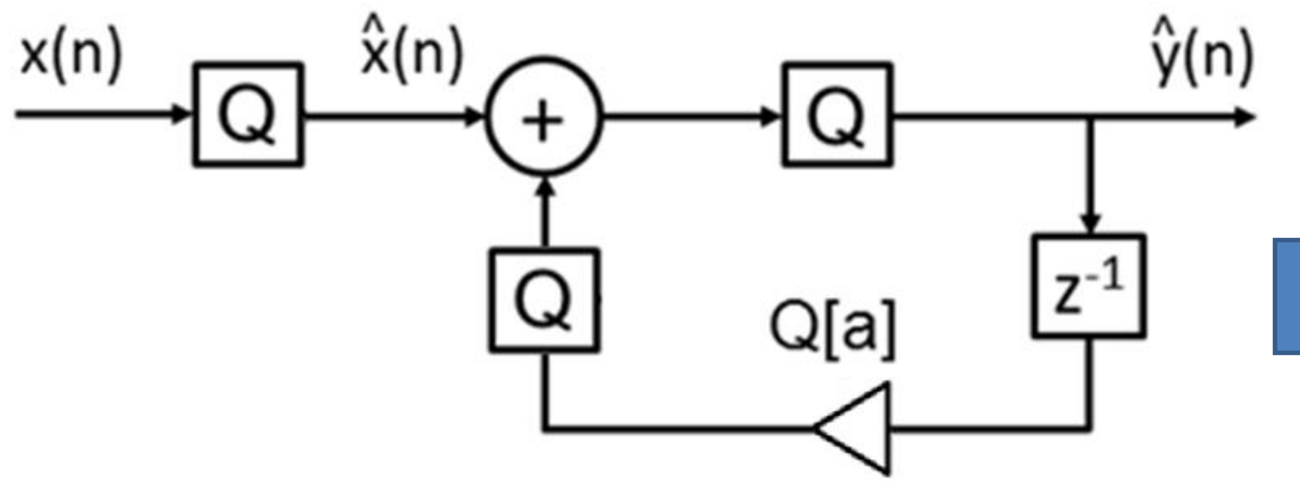
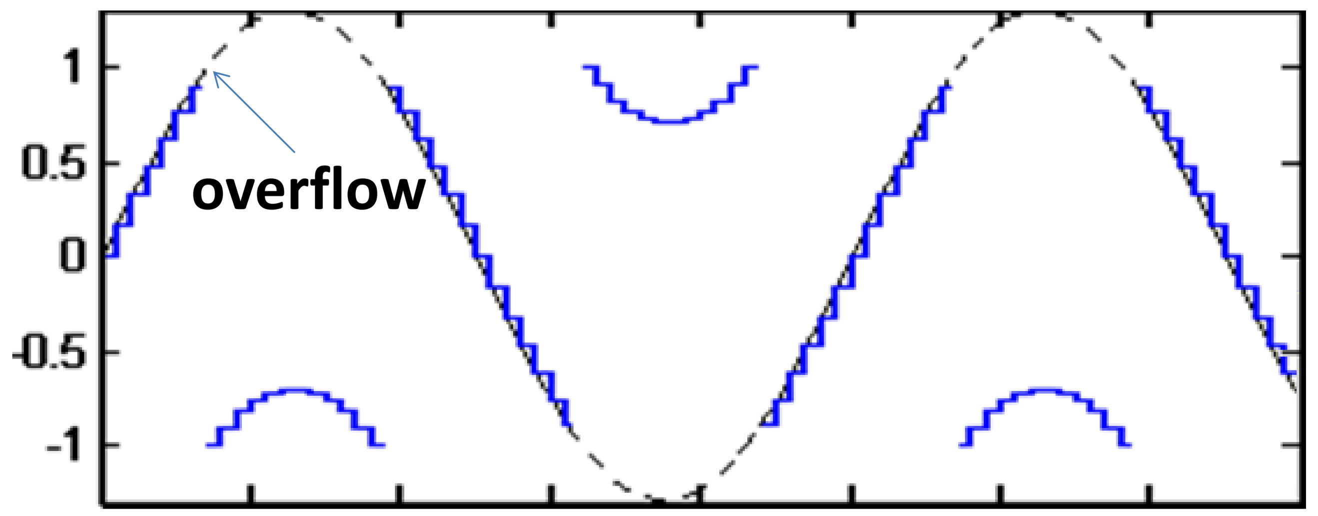
- Fixed-point implementation leads to quantization nonlinearities, round off errors, and overflows due to operations with finite word-length
- Testing and simulation can lead to a limited number of scenarios, which do not exploit all possible behaviors of the system

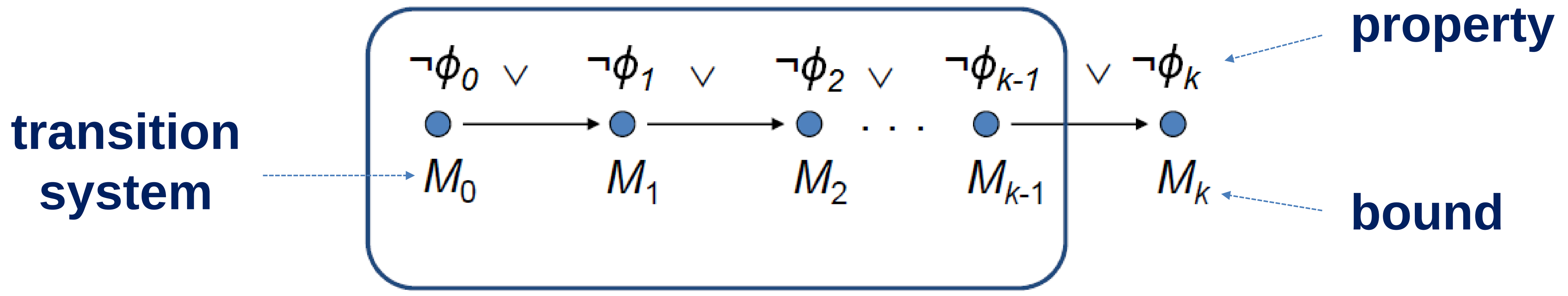




$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

2. SMT-based BMC of Digital Filter

- Check the negation of a given property at a given depth



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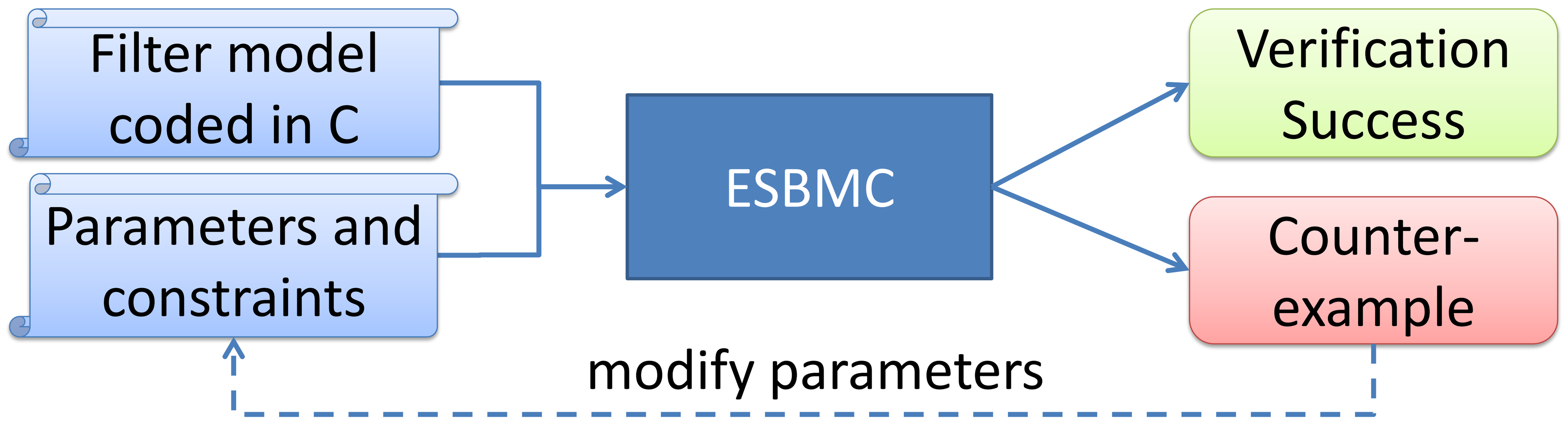

4. Experimental Setup

- Environment*: Fedora 64 bits, ESBMC v1.21, SMT Z3 v3.2
- Filters tested (from Matlab design toolbox and from literature)
  - Low Pass, High Pass, Band Pass, Band Stop
  - Up to 6th order IIR and up to 30th order FIR
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5. Conclusions

- The method can detect overflow, limit cycle and time constraint failures in filters of different types and orders
- Can find problems that are hard to detect using tests and simulations
- Verification time tends to be higher for high order filters and for longest word-length formats since these lead to harder verification conditions
- Exploits the advantages of an state of art model checker ESBMC over a model coded in C

* Tools and benchmarks available in www.esbmc.org

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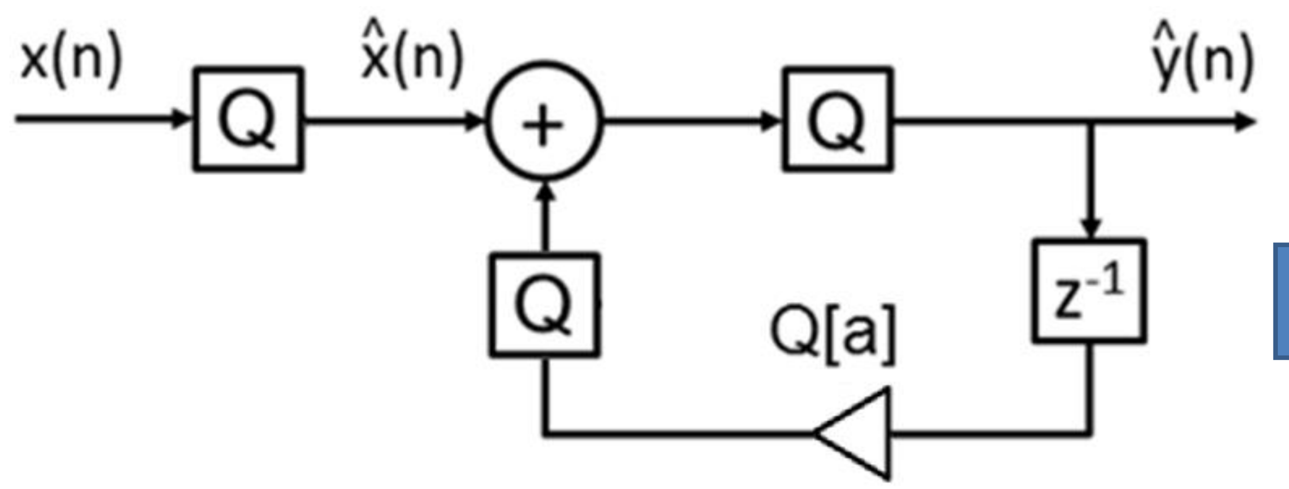

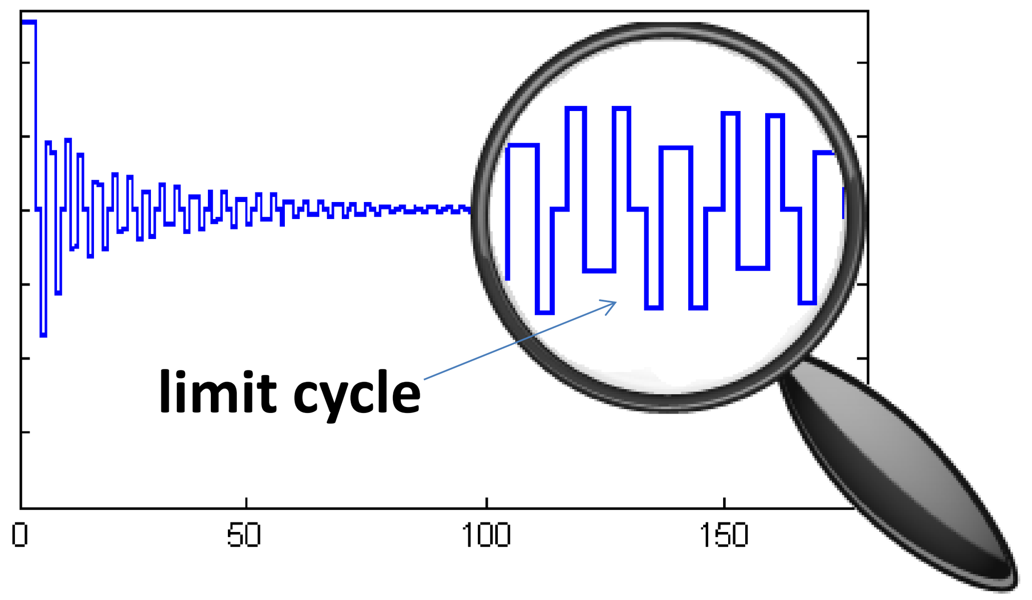

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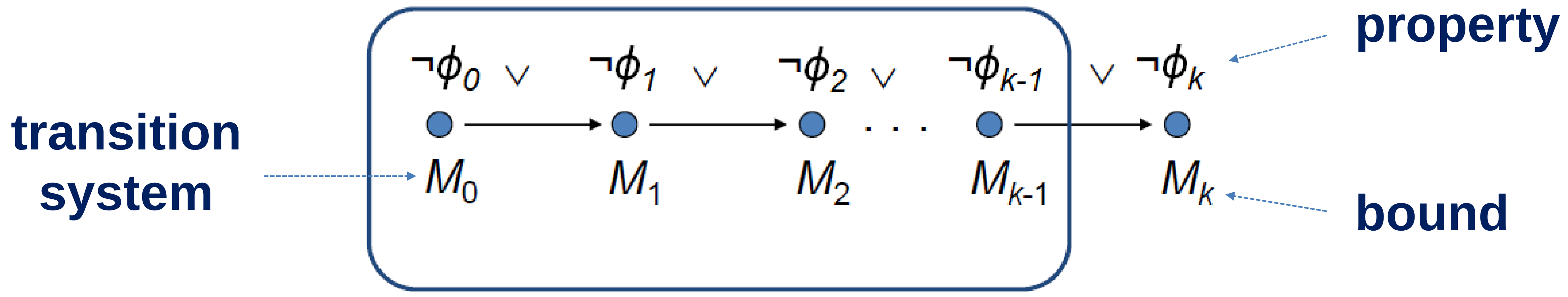

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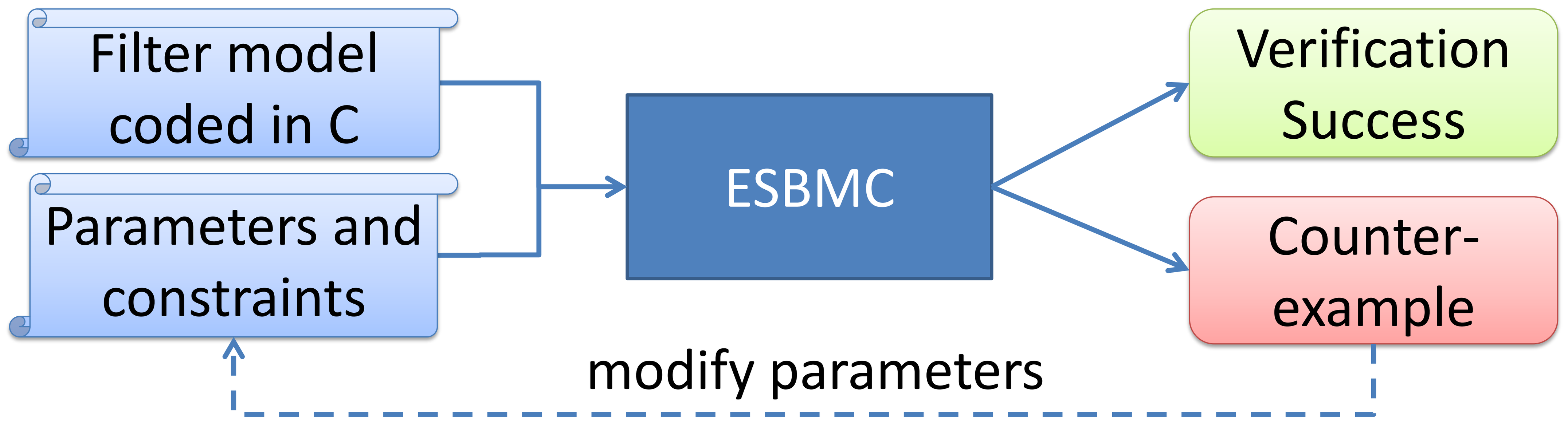

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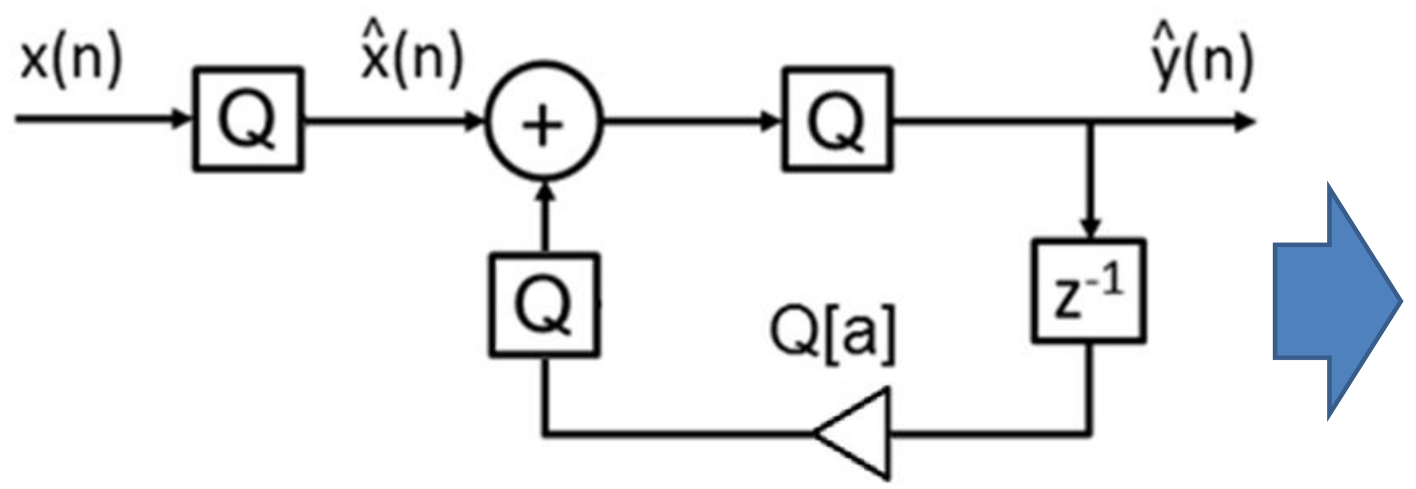
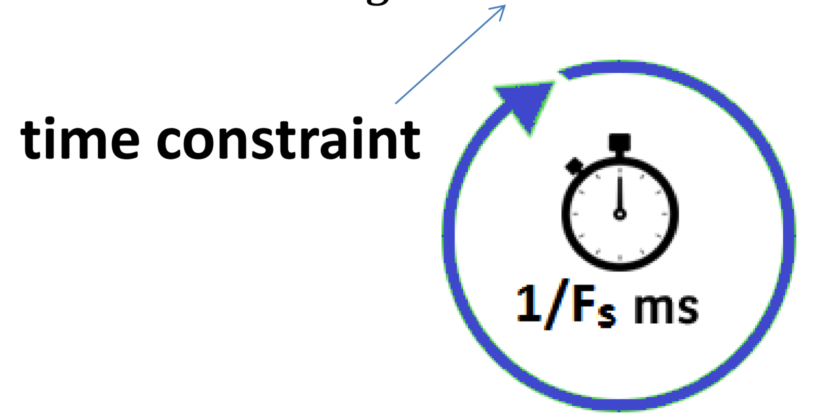


$$l_{timing} \Leftrightarrow ((N \times T) \le D)$$



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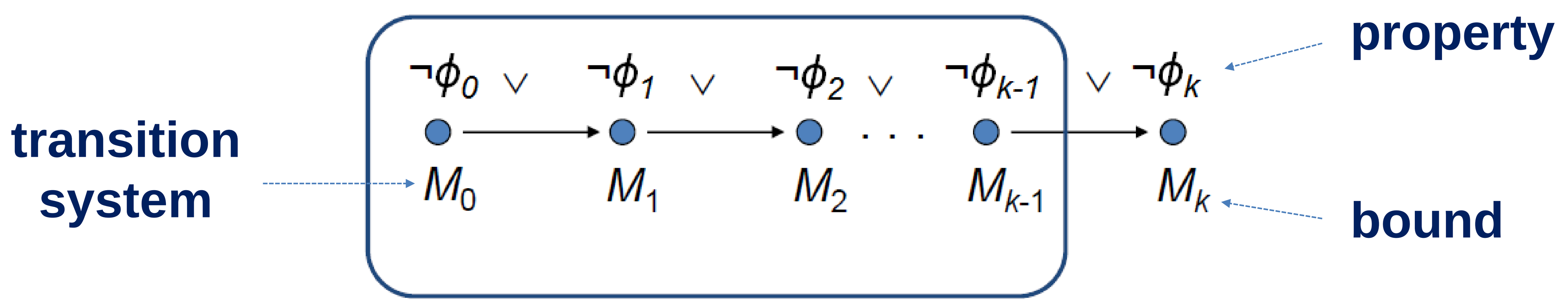

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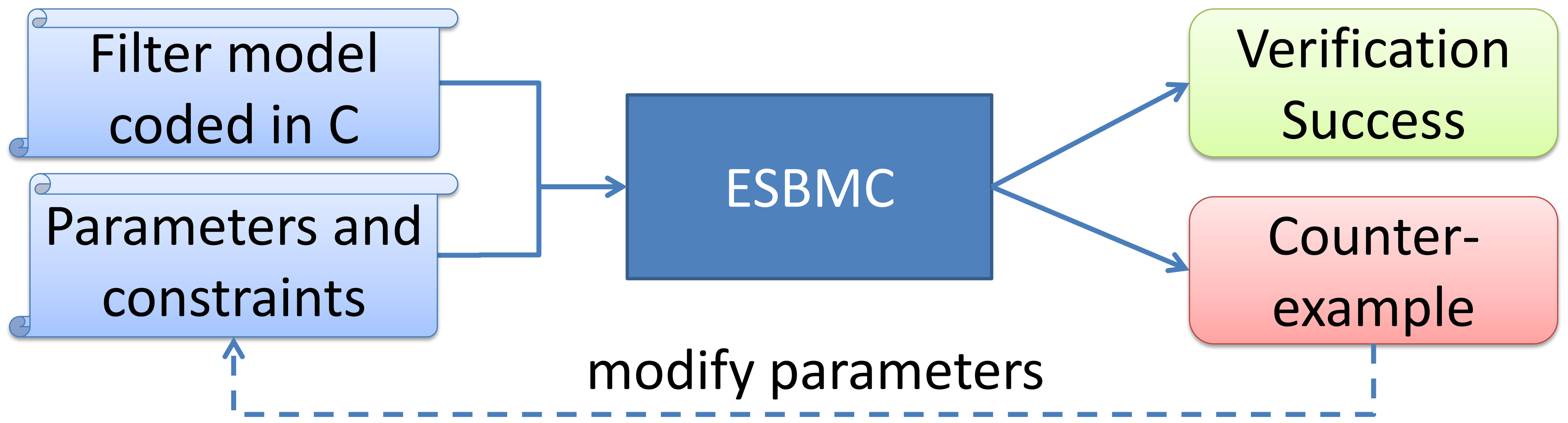

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Theory	Example
Equality	$x_1 = x_2 \land \neg(x_1 = x_3) \Rightarrow \neg(x_1 = x_3)$
Bit vector	$(b \gg i)\&1 = 1$
Linear arithmetic	$(4y_1 + 3y_2 \geq 4) \lor (y_2 - 3y_3 \leq 3)$
Arrays	$(j = k \land a[k] = 2) \Rightarrow a[j] = 2$
Combined theories	$(j \leq k \land a[j] = 2) \Rightarrow a[i] < 3$

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Overflow verification

BMC using the ESBMC

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Code
C/C++

```
int main() {
  int a[2], i, x;
  if (x==0)
    a[i]=0;
  else
    a[i+2]=1;
  assert(a[i+1]==1);
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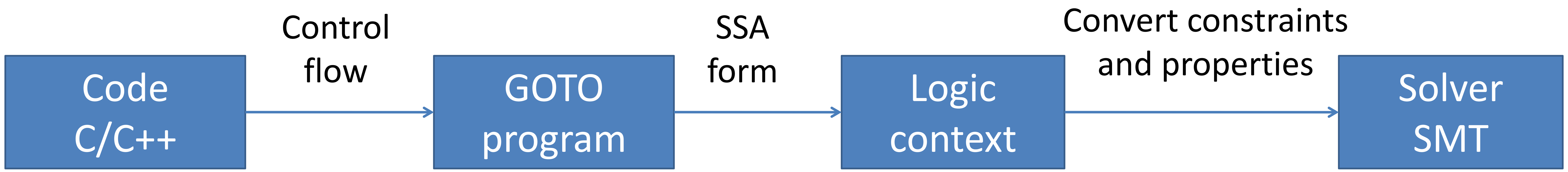
$$
\begin{aligned}
g_1 &= x_1 == 0 \\
a_1 &= a_0 \text{ WITH } [i_0{:=}0] \\
a_2 &= a_0 \\
a_3 &= a_2 \text{ WITH } [2+i_0{:=}1] \\
a_4 &= g_1 \; ? \; a_1 : a_3 \\
t_1 &= a_4\,[1+i_0] == 1
\end{aligned}
$$

BMC using the ESBMC

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$$C := \left[\begin{array}{l} g_1 := (x_1 = 0) \\ \wedge\, a_1 := store(a_0, i_0, 0) \\ \wedge\, a_2 := a_0 \\ \wedge\, a_3 := store(a_2, 2+i_0, 1) \\ \wedge\, a_4 := ite(g_1, a_1, a_3) \end{array}\right]$$

$$P := \left[\begin{array}{l} i_0 \geq 0 \wedge i_0 < 2 \\ \wedge\, 2+i_0 \geq 0 \wedge 2+i_0 < 2 \\ \wedge\, 1+i_0 \geq 0 \wedge 1+i_0 < 2 \\ \wedge\, select(a_4, i_0+1) = 1 \end{array}\right]$$

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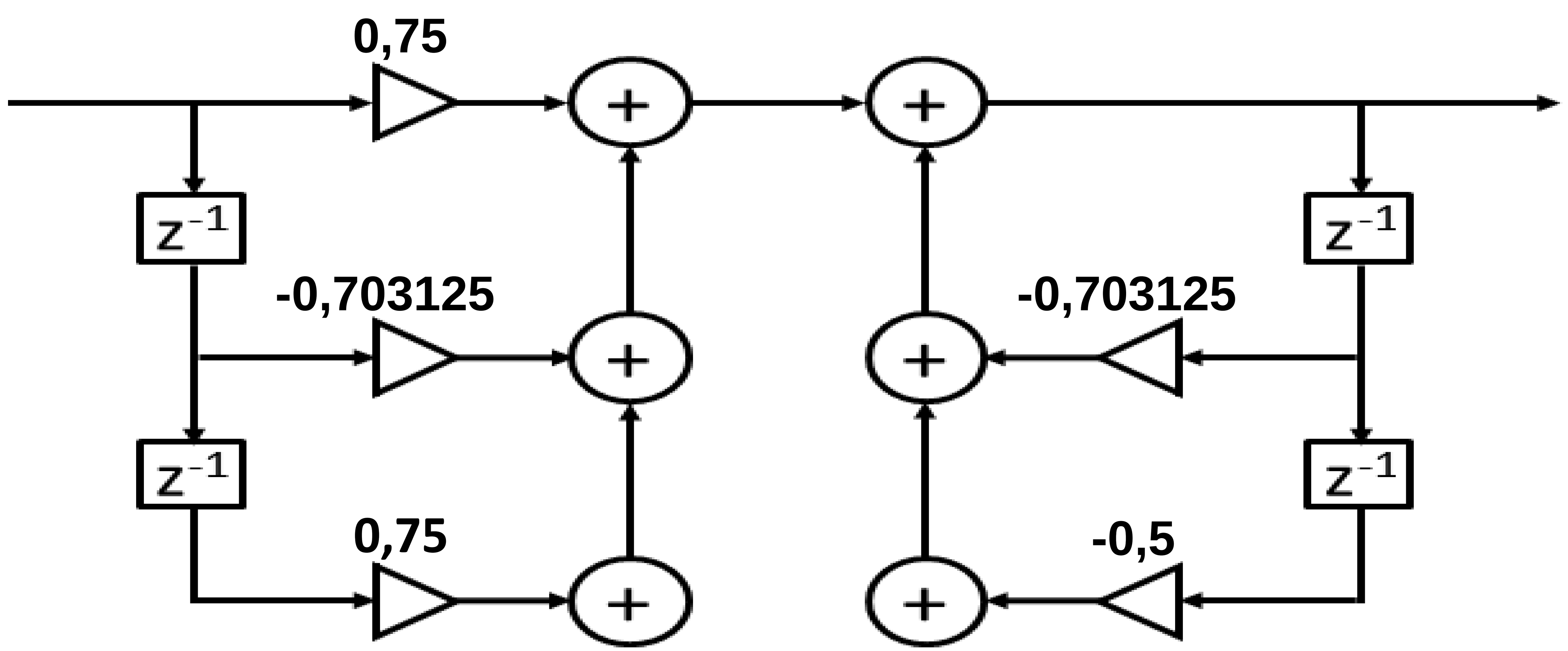

- $\sum_{k=0}^{100}|h_k|$ = 1,8178
- For x between [-1,1], $|y(n)| \leq 1{,}82$
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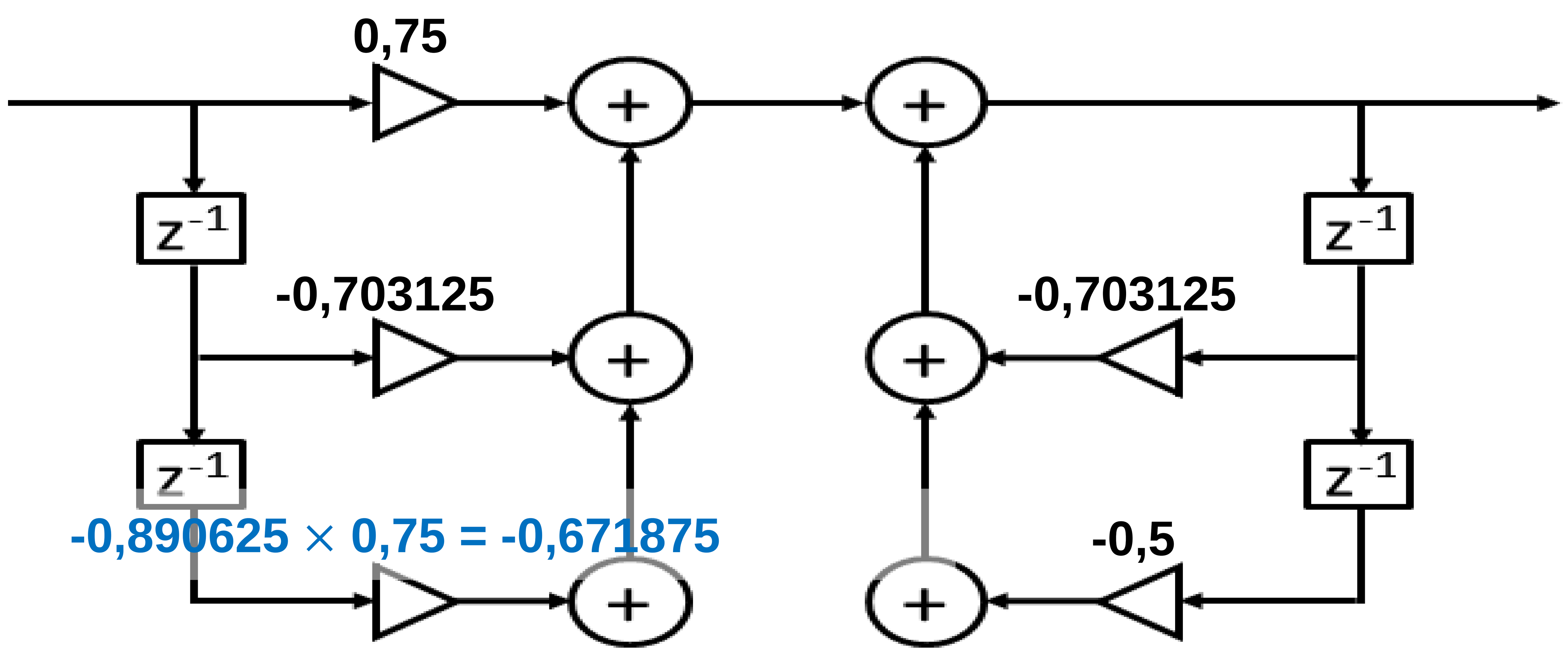

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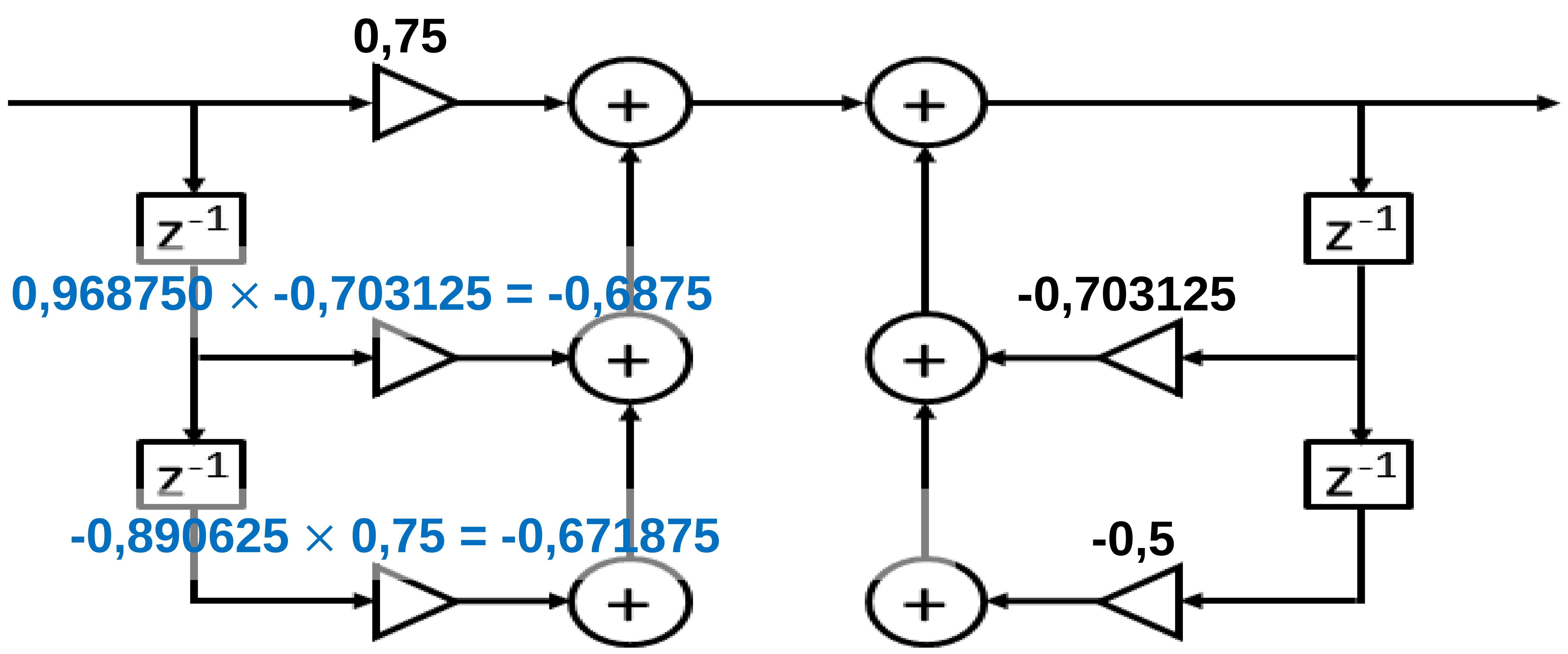

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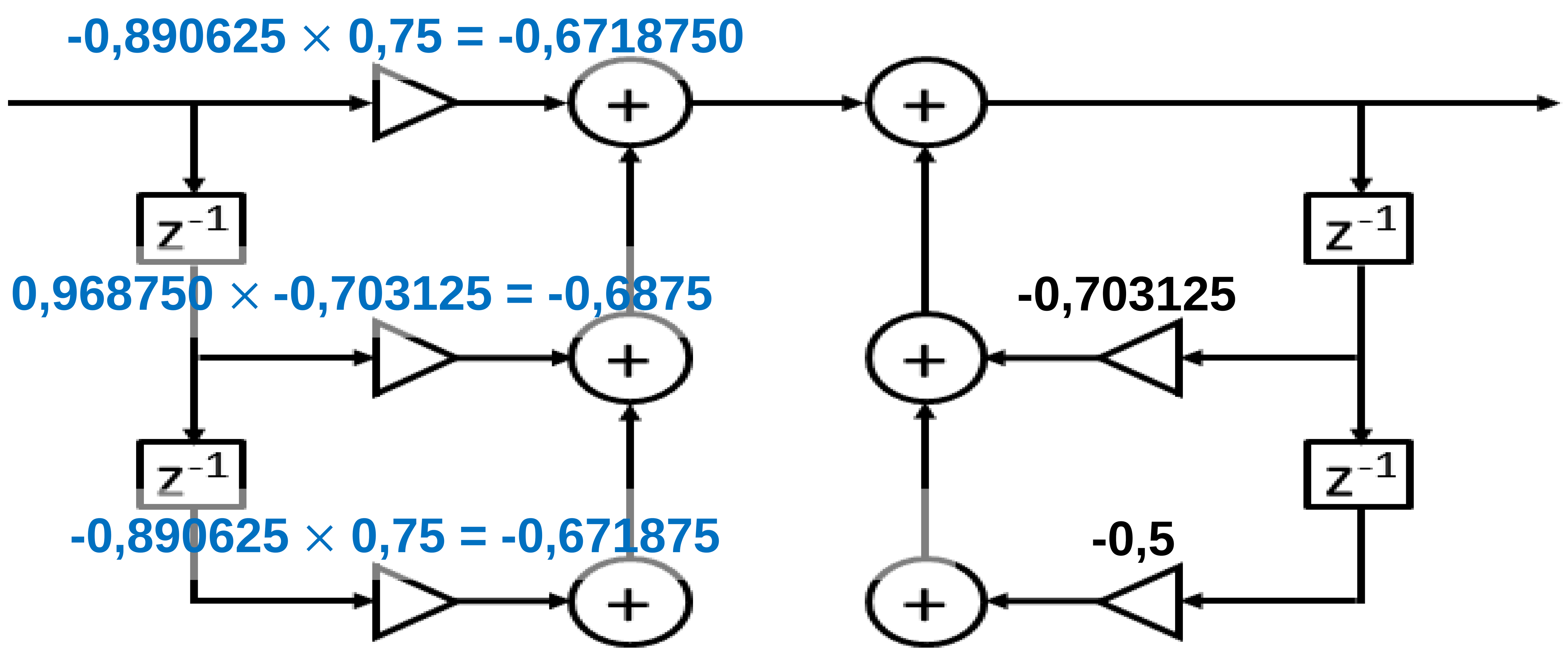

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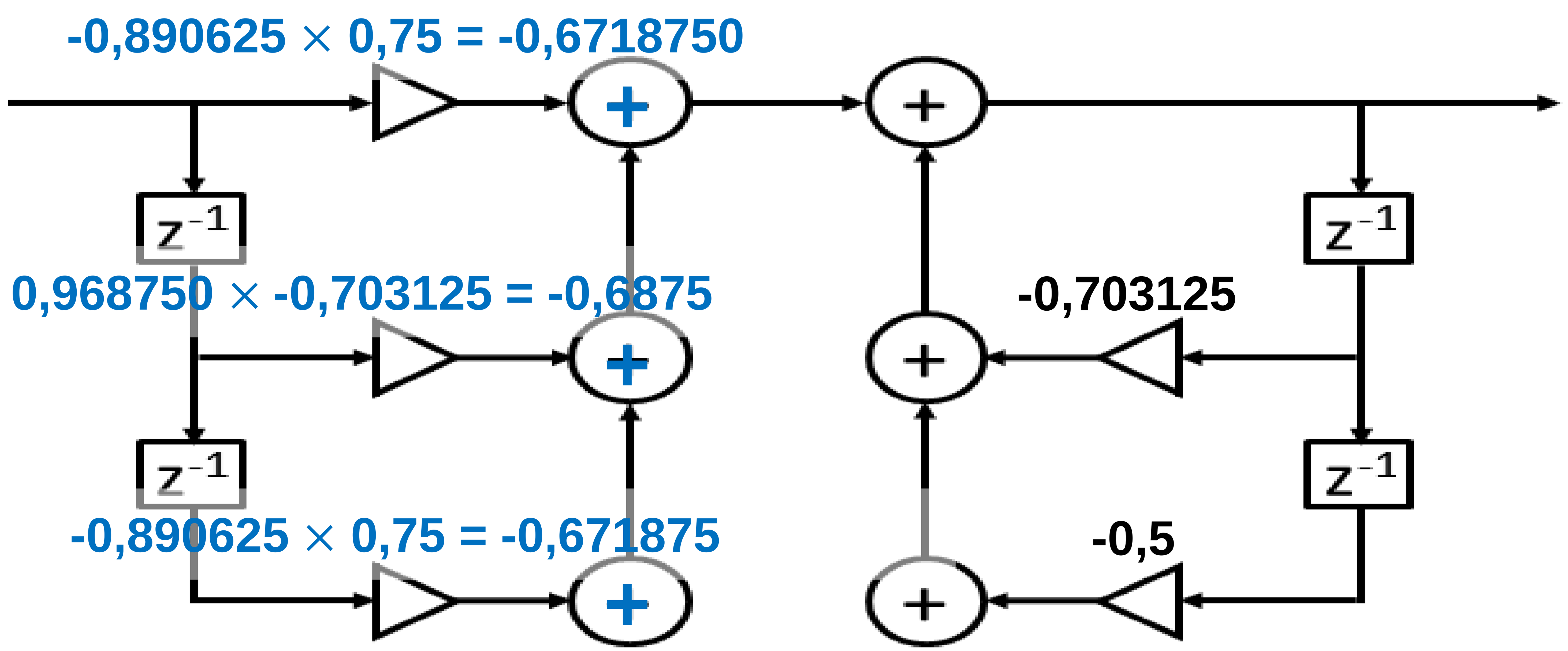

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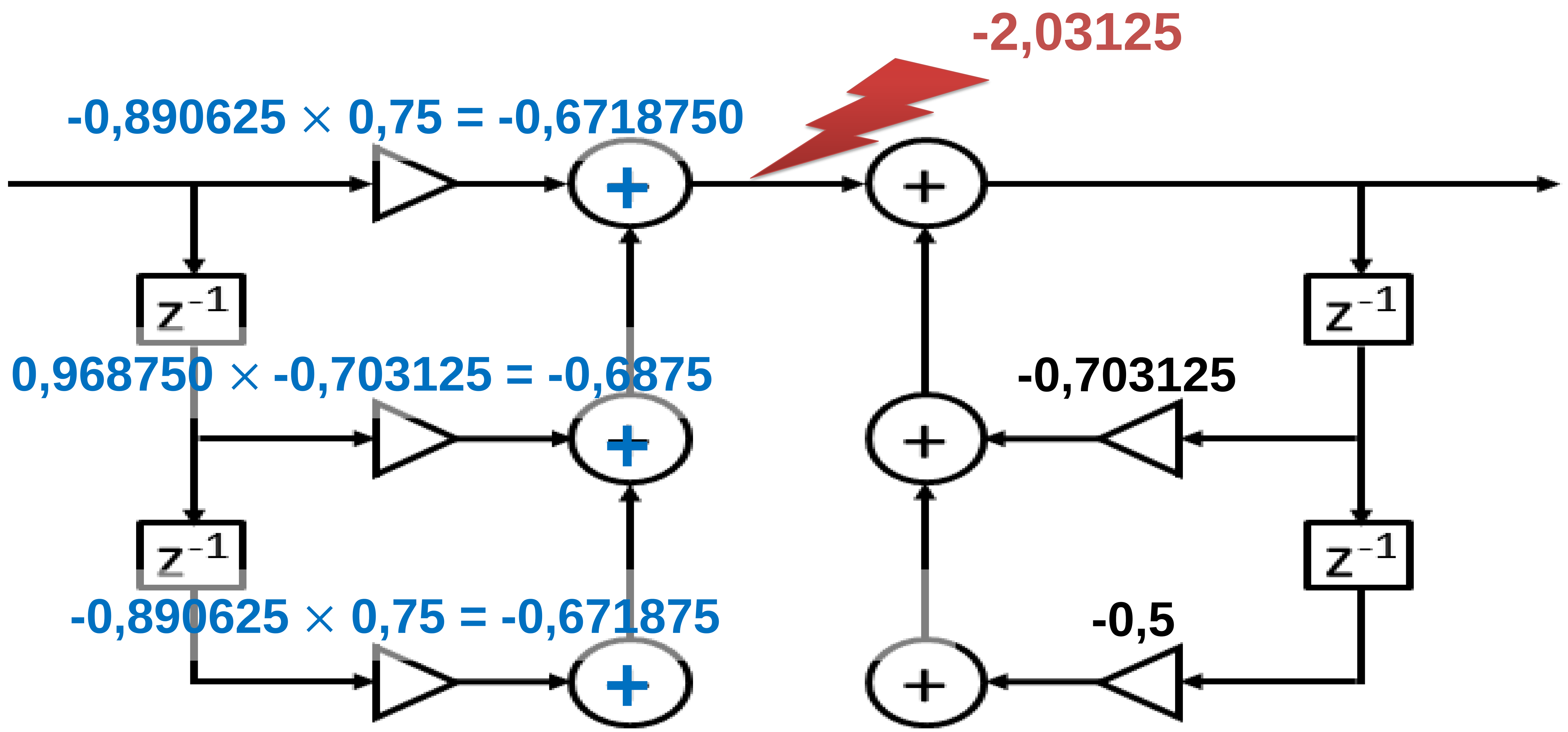

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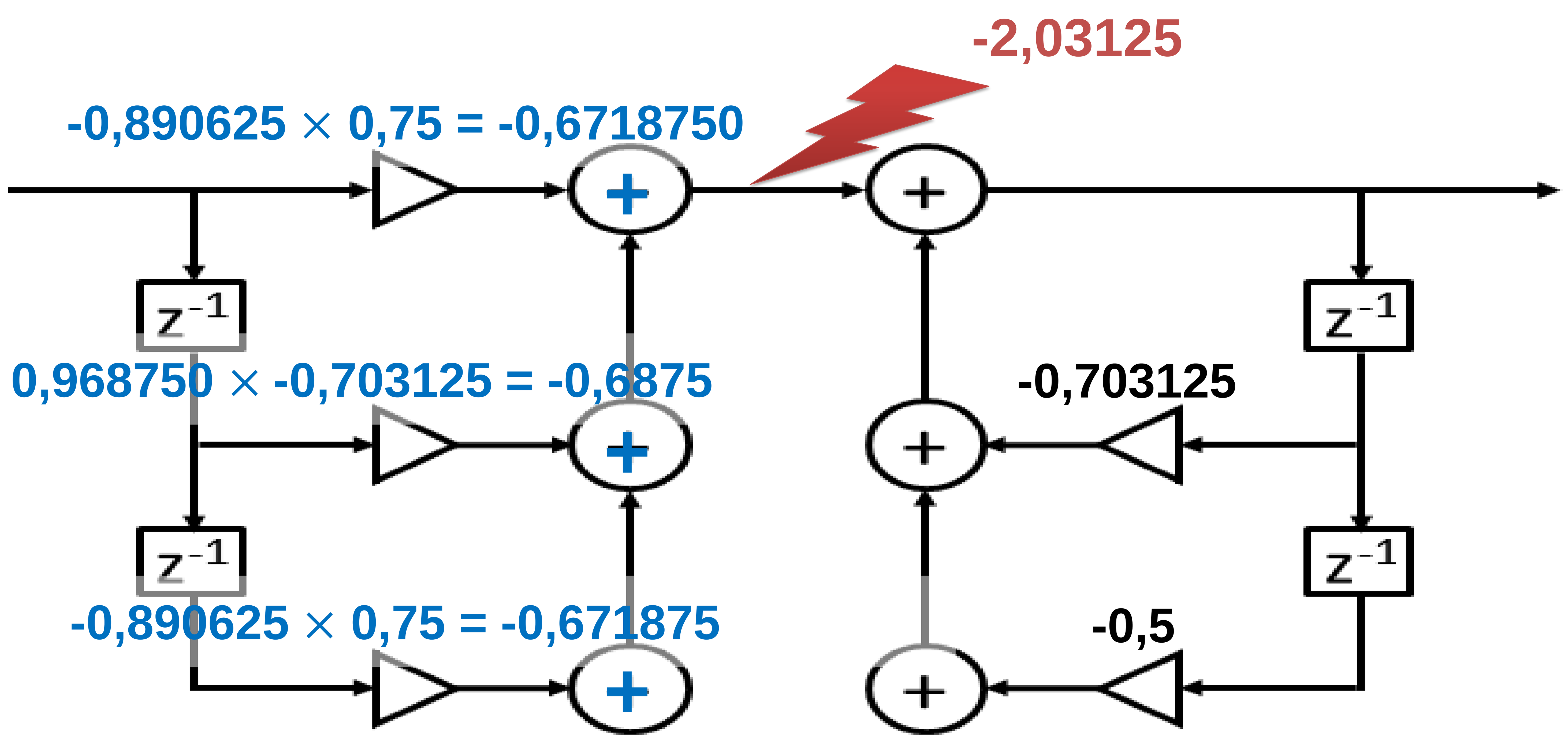

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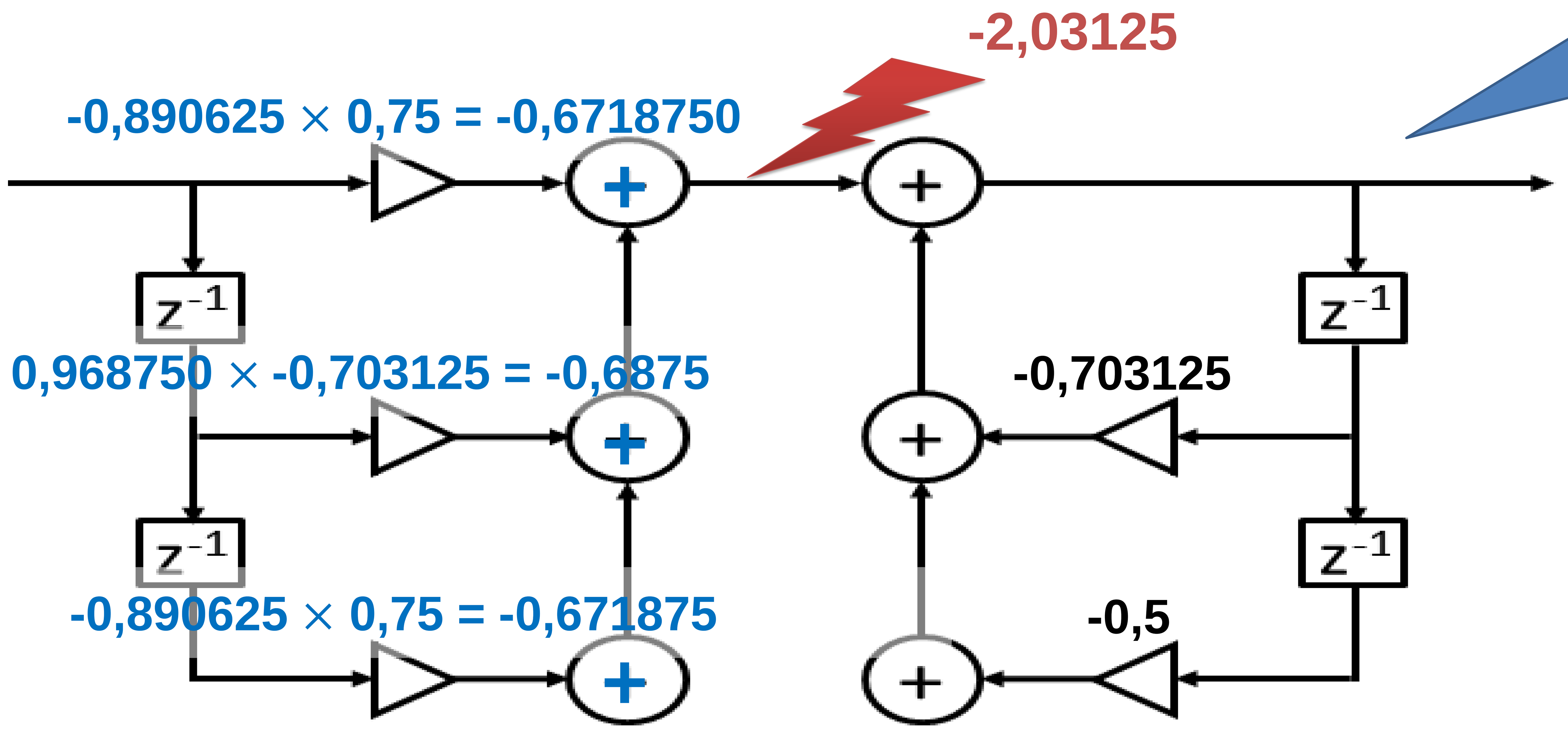
The filter also fails in Direct Form II, but does not fail for the Transposed Form II due to the order of operations



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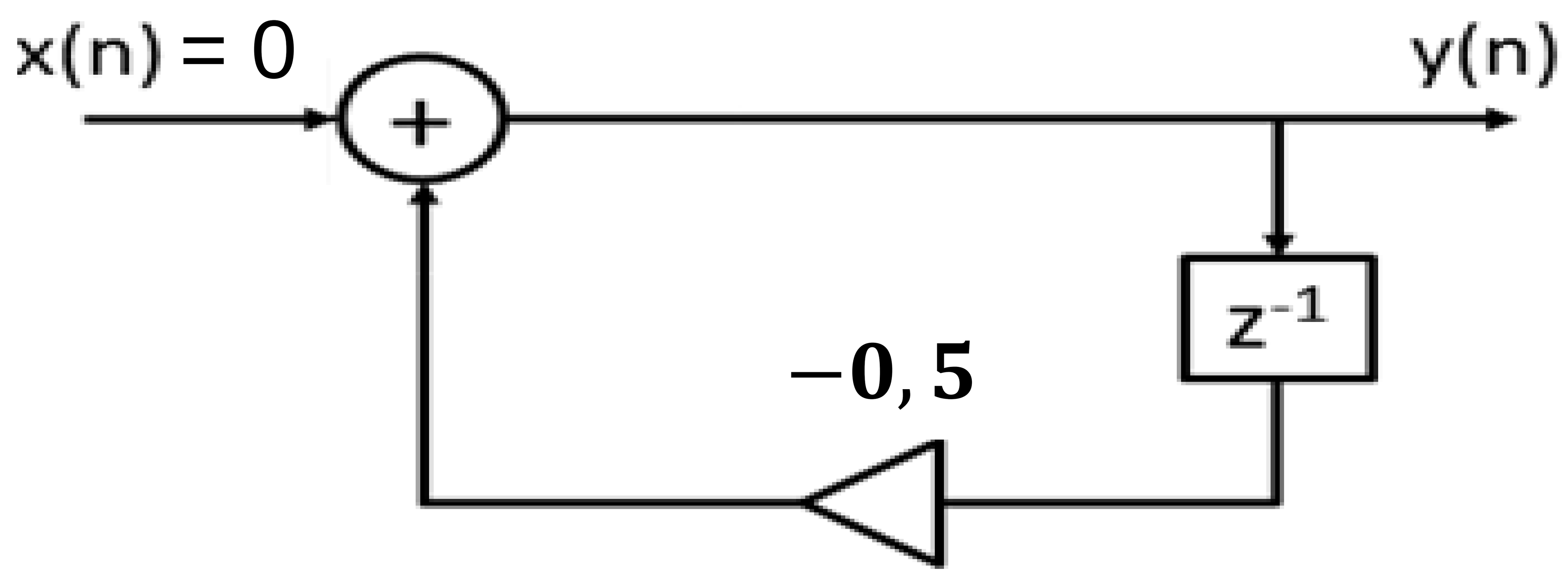

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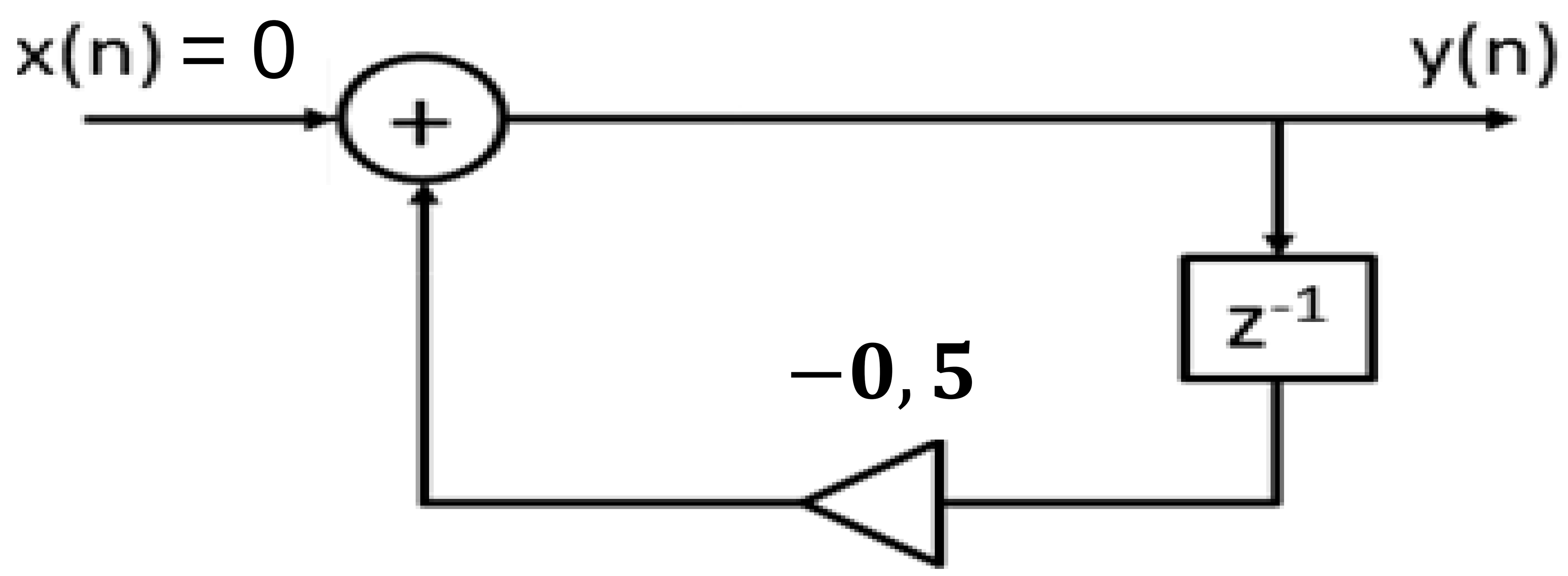

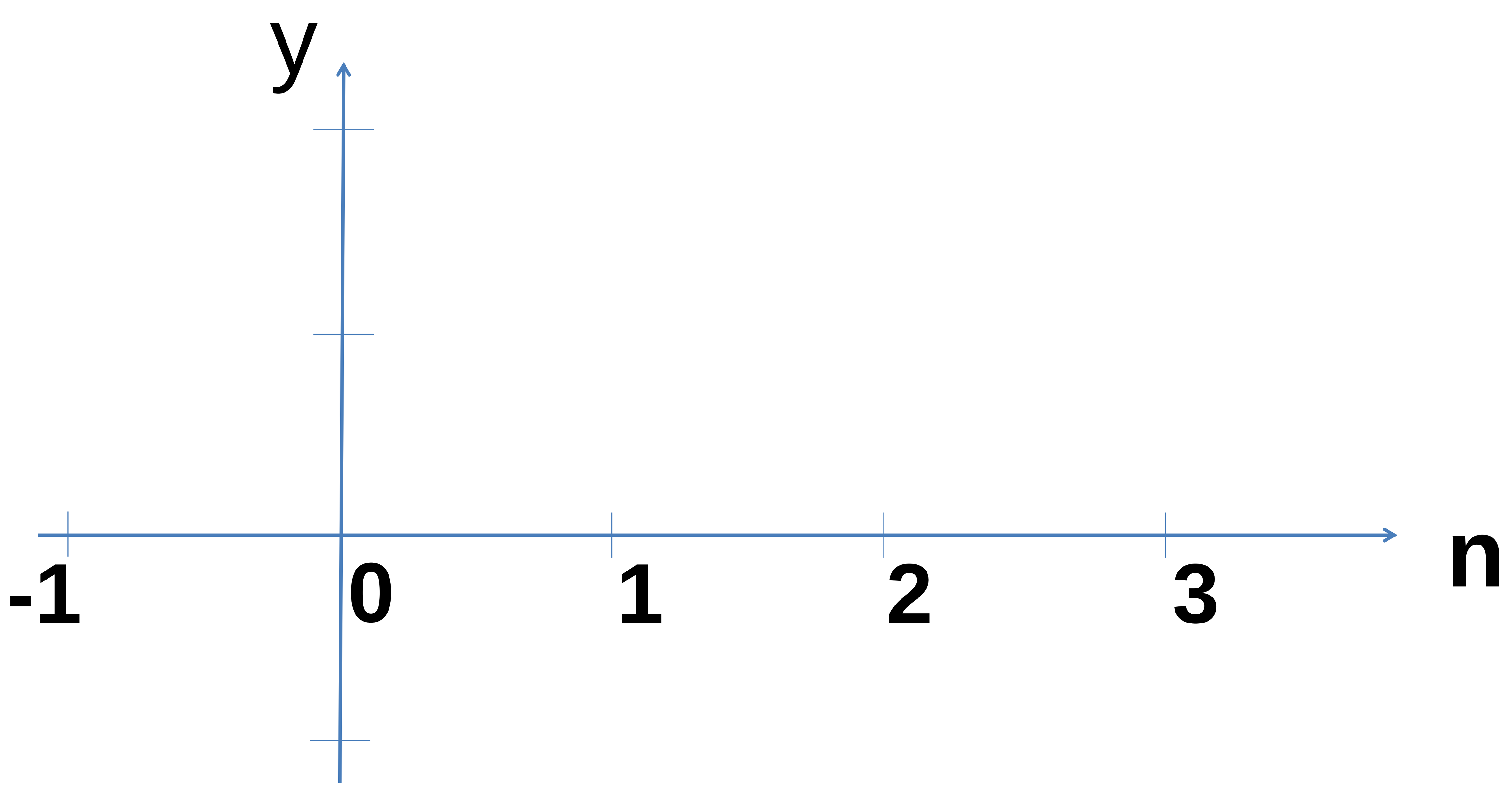

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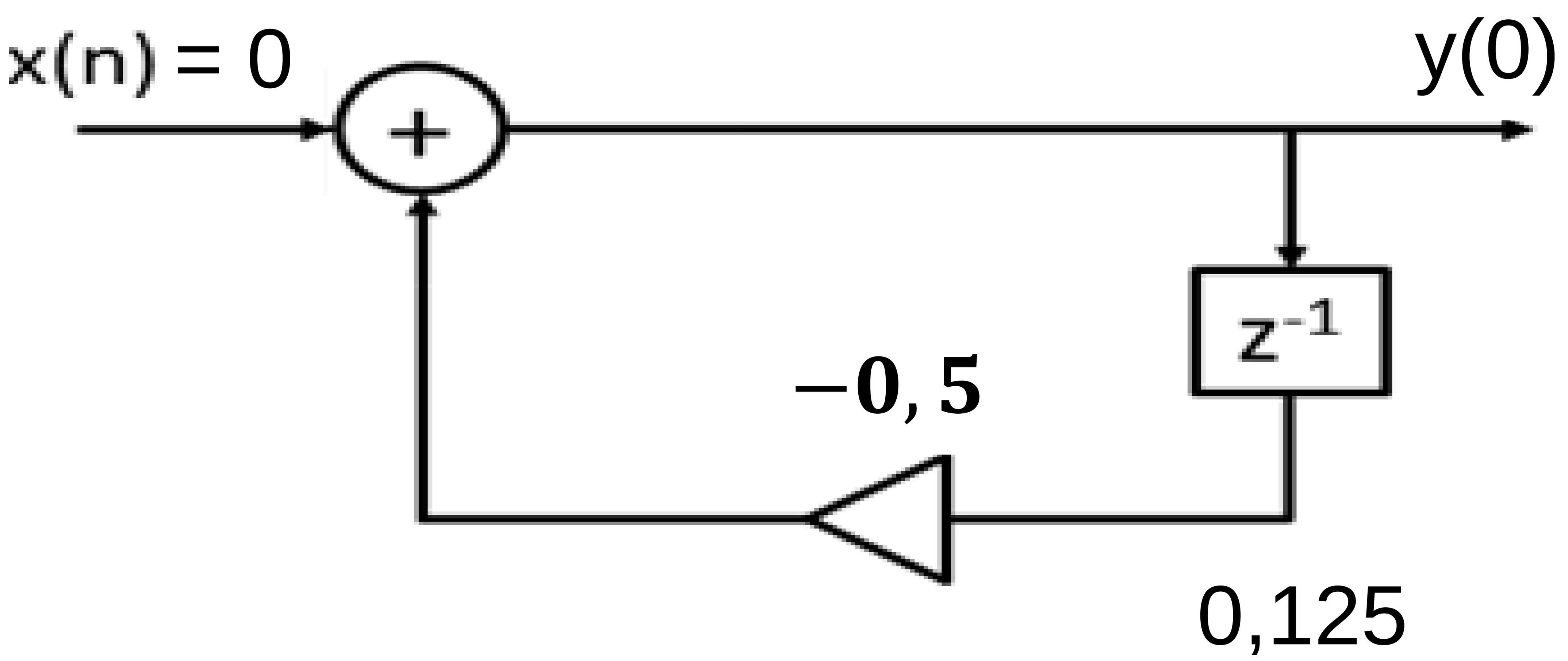

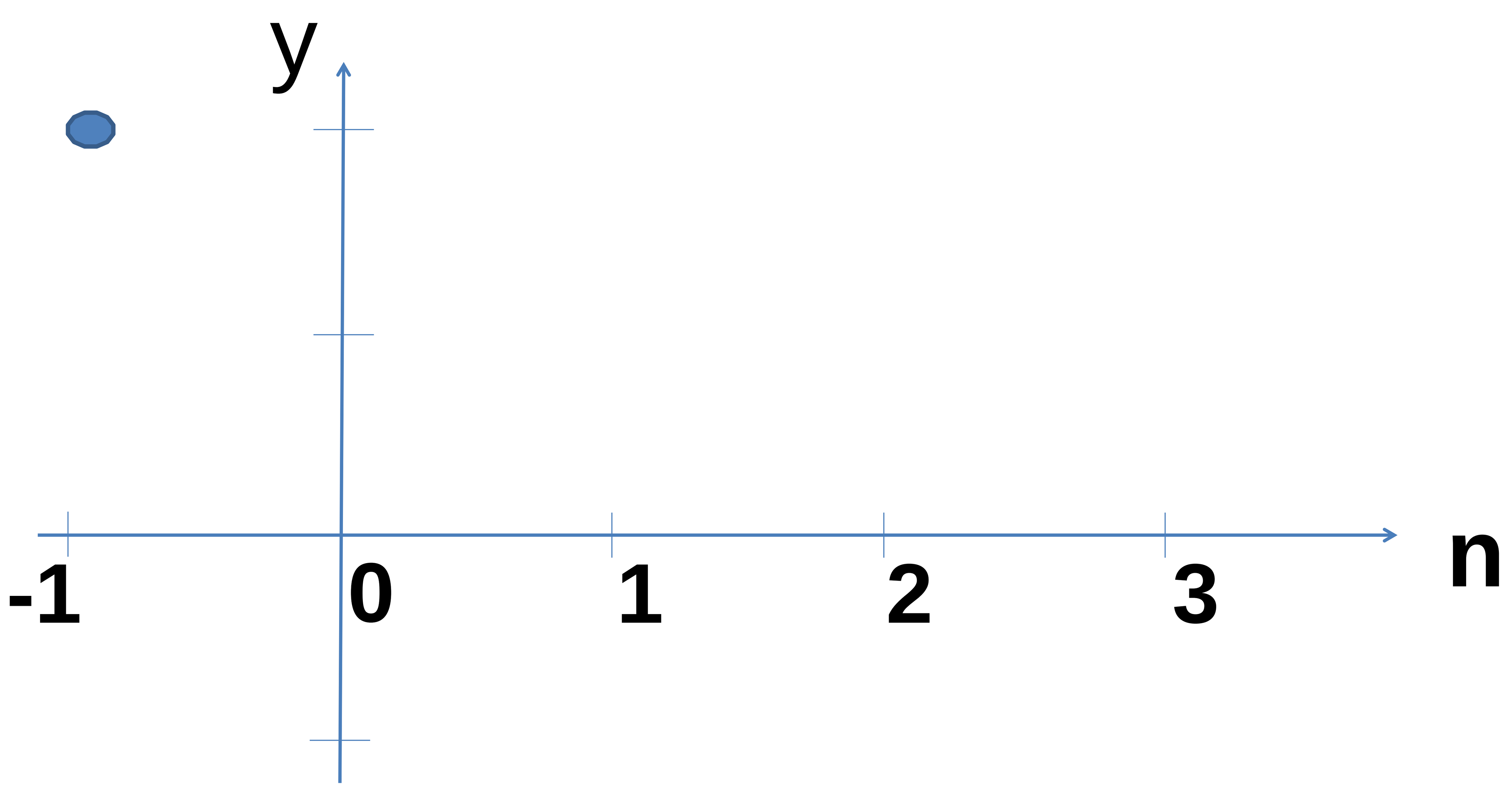

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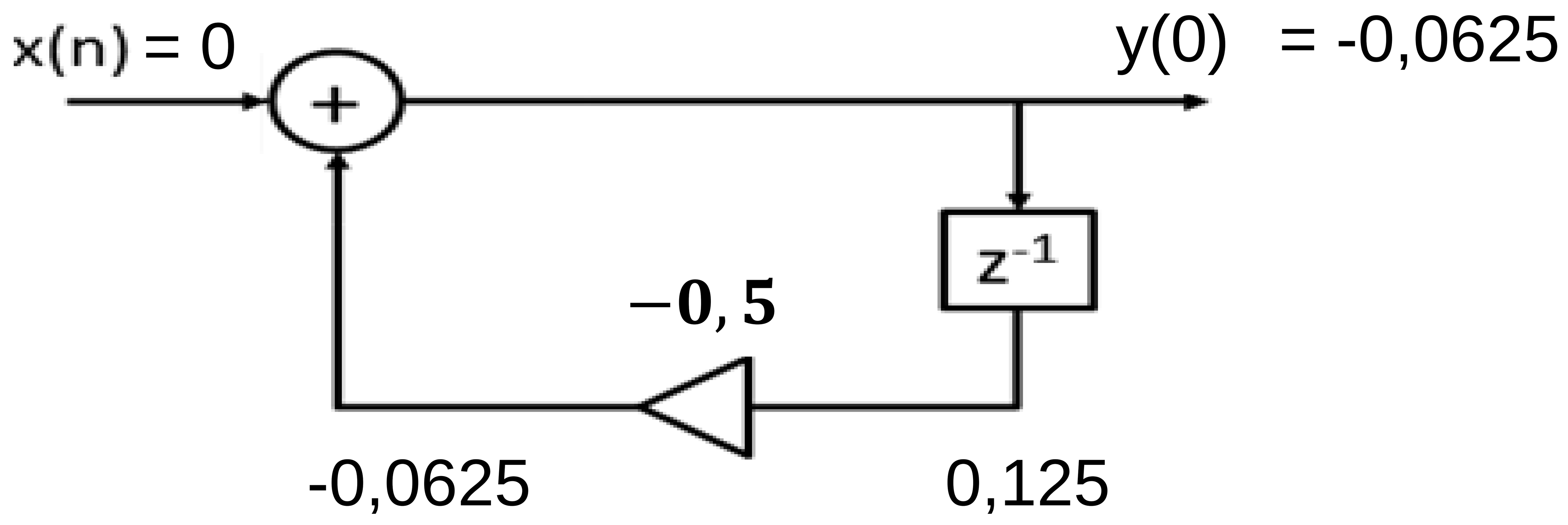

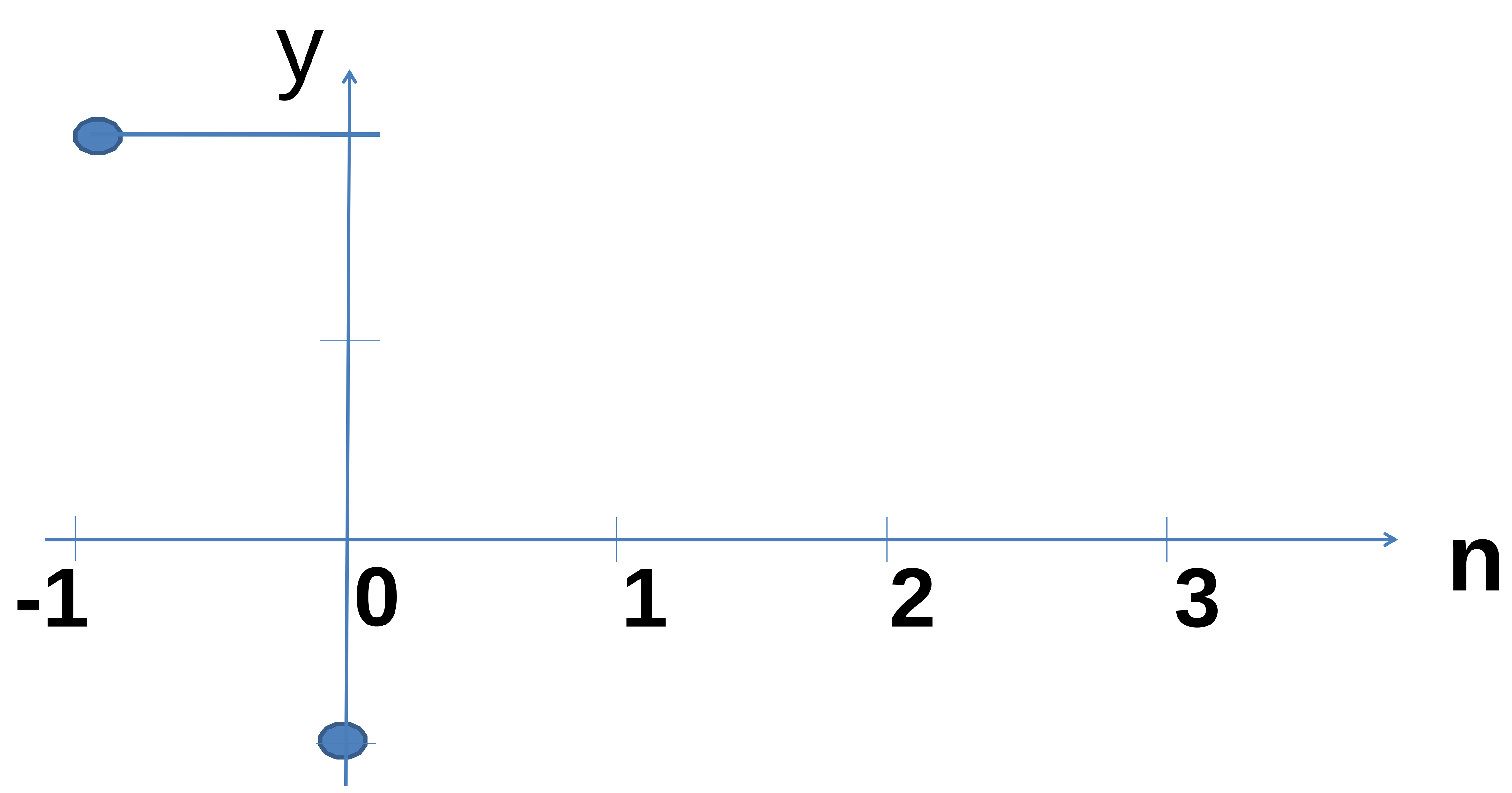

Example 2: Limit Cycle Verification

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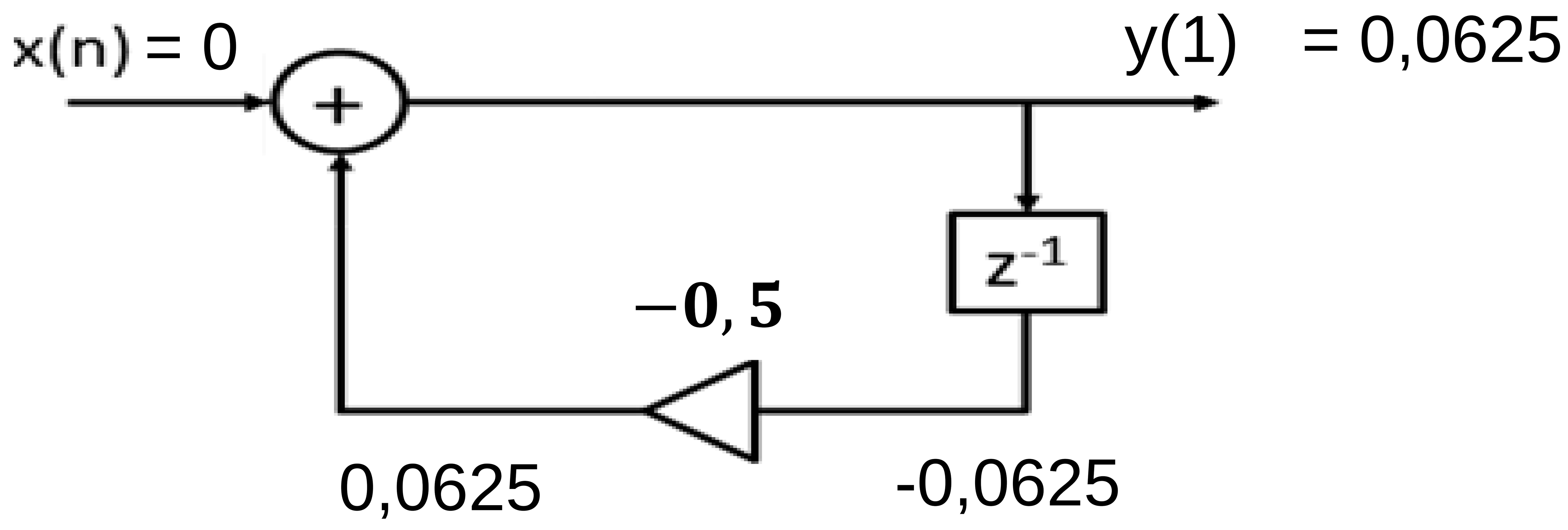

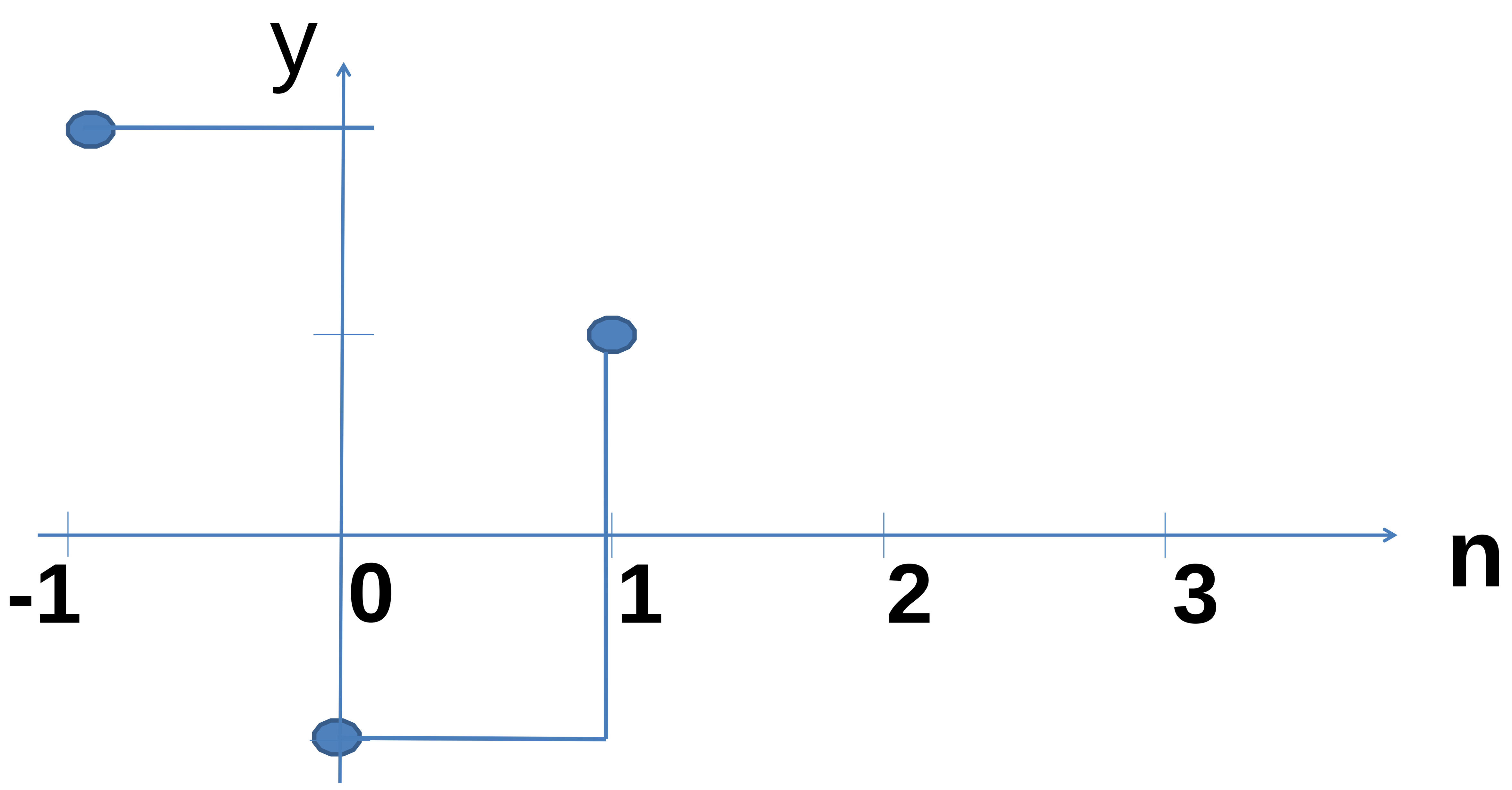

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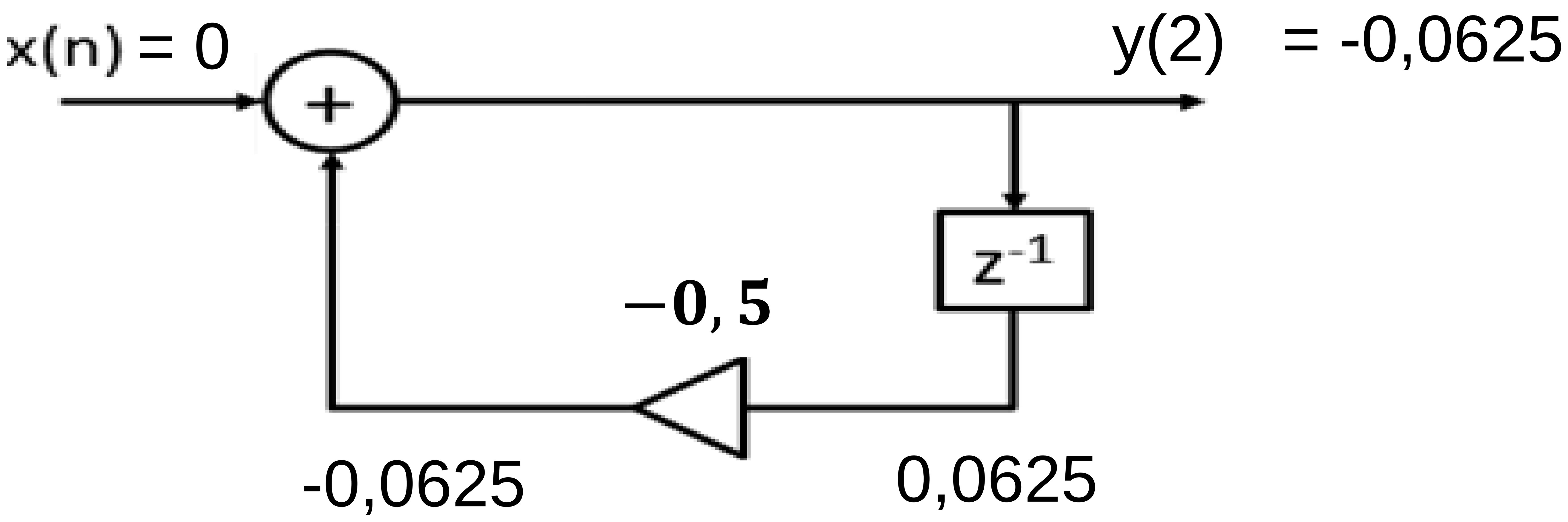

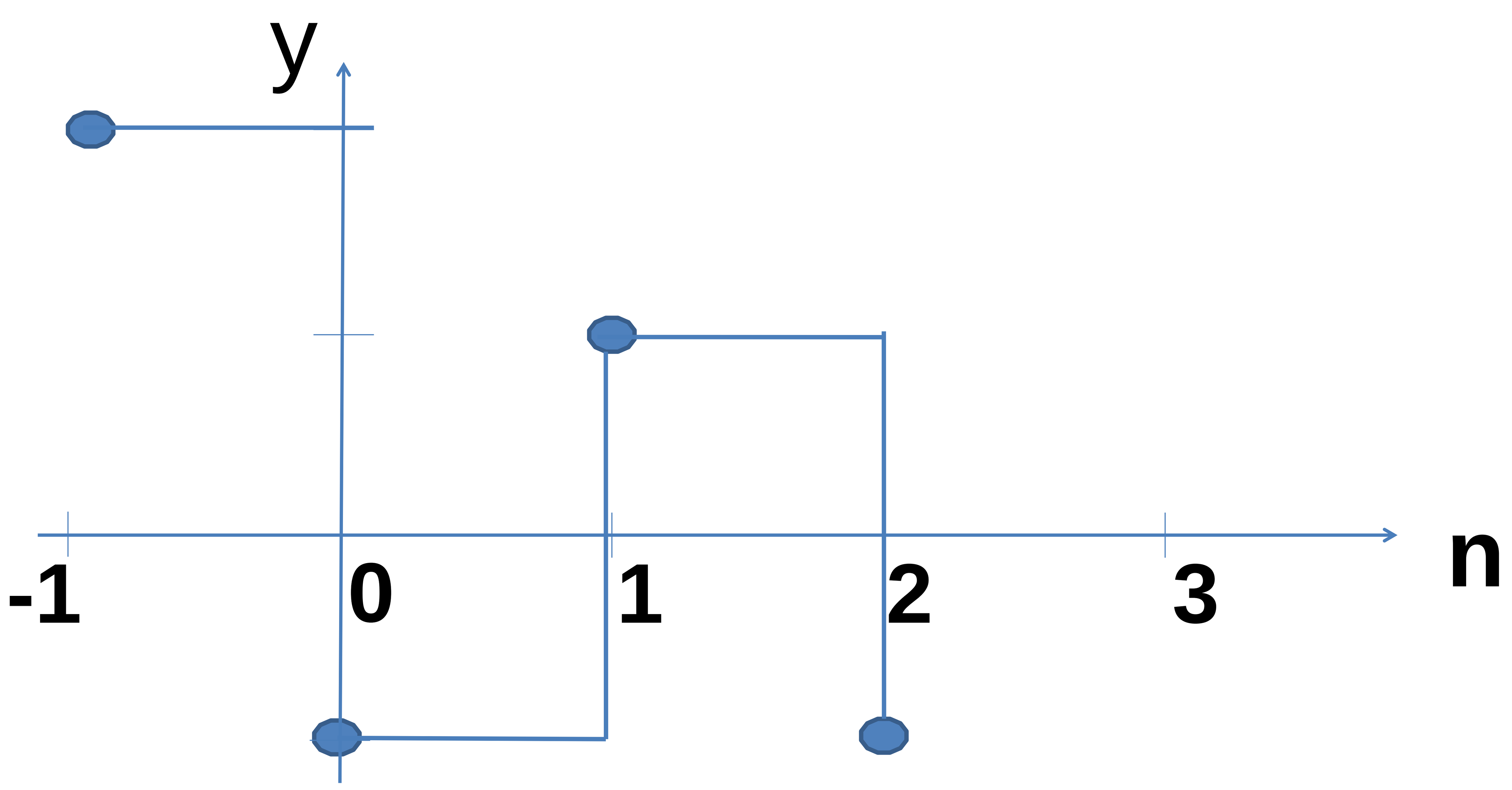

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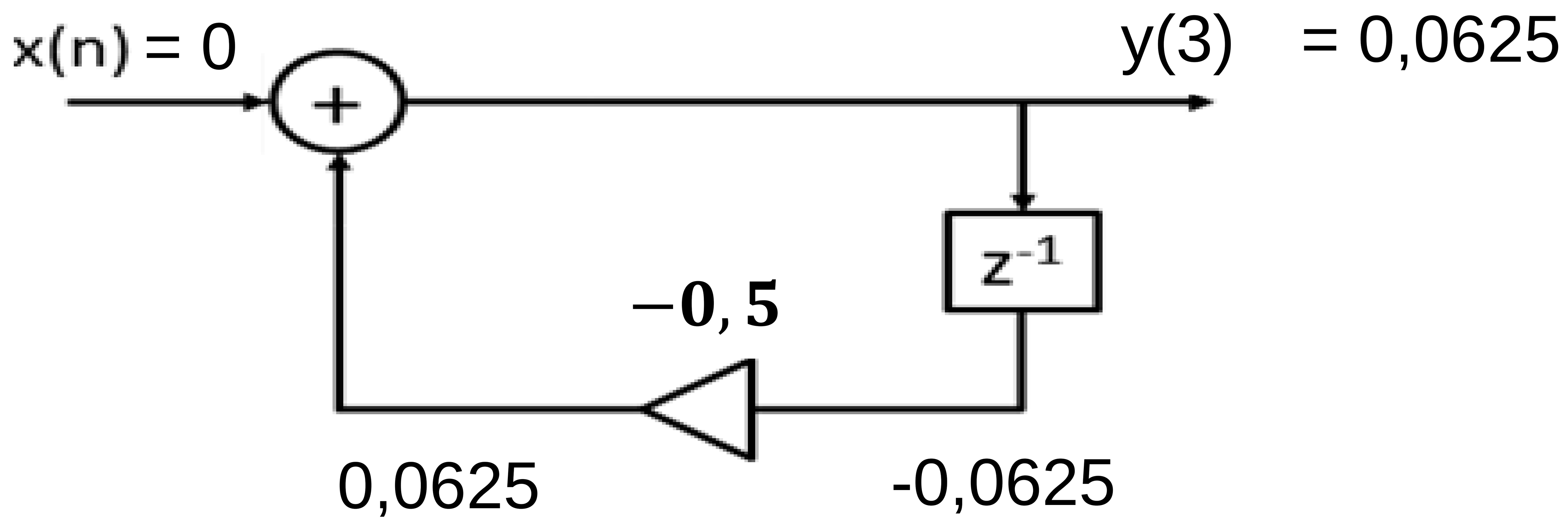

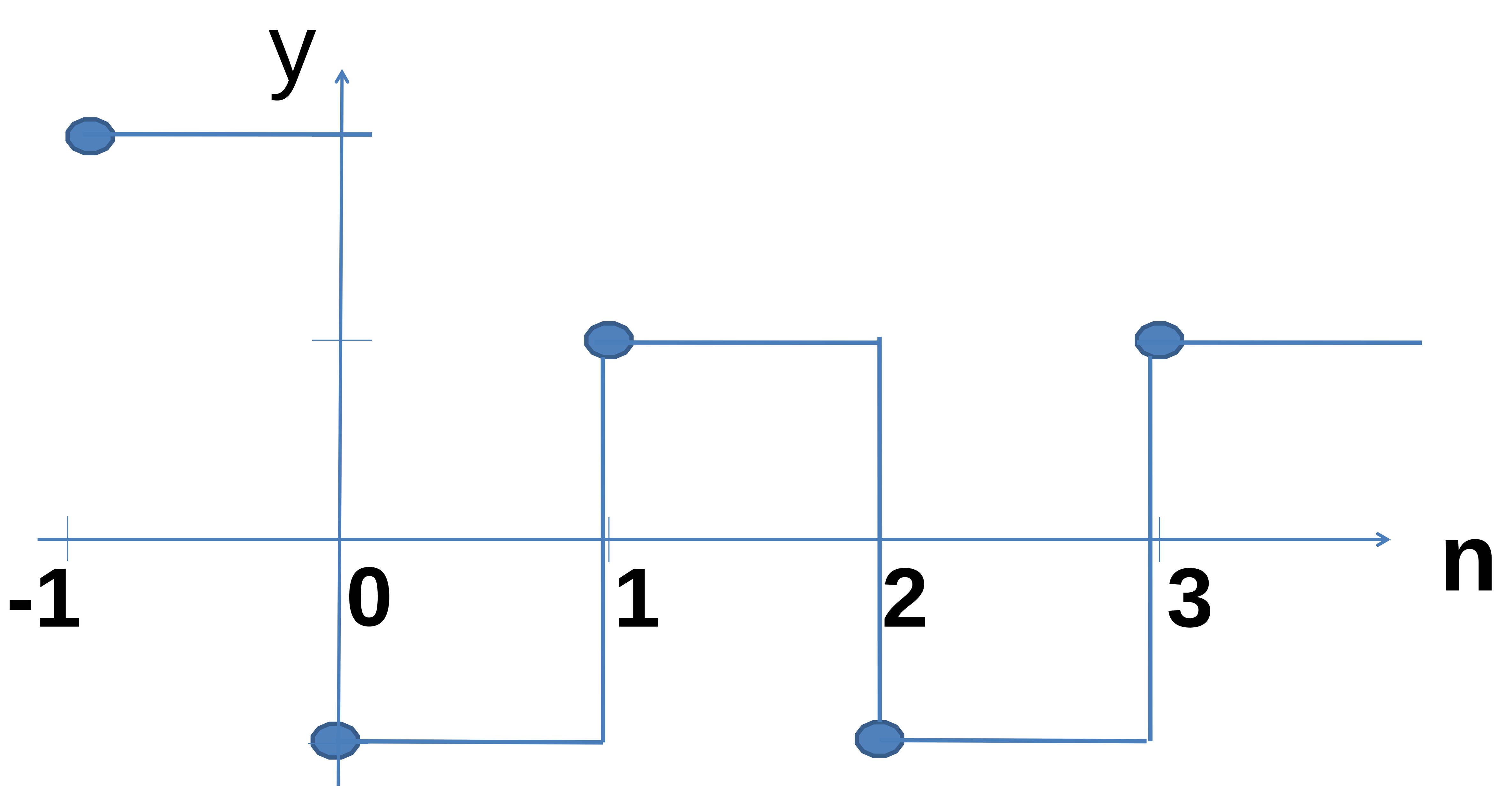

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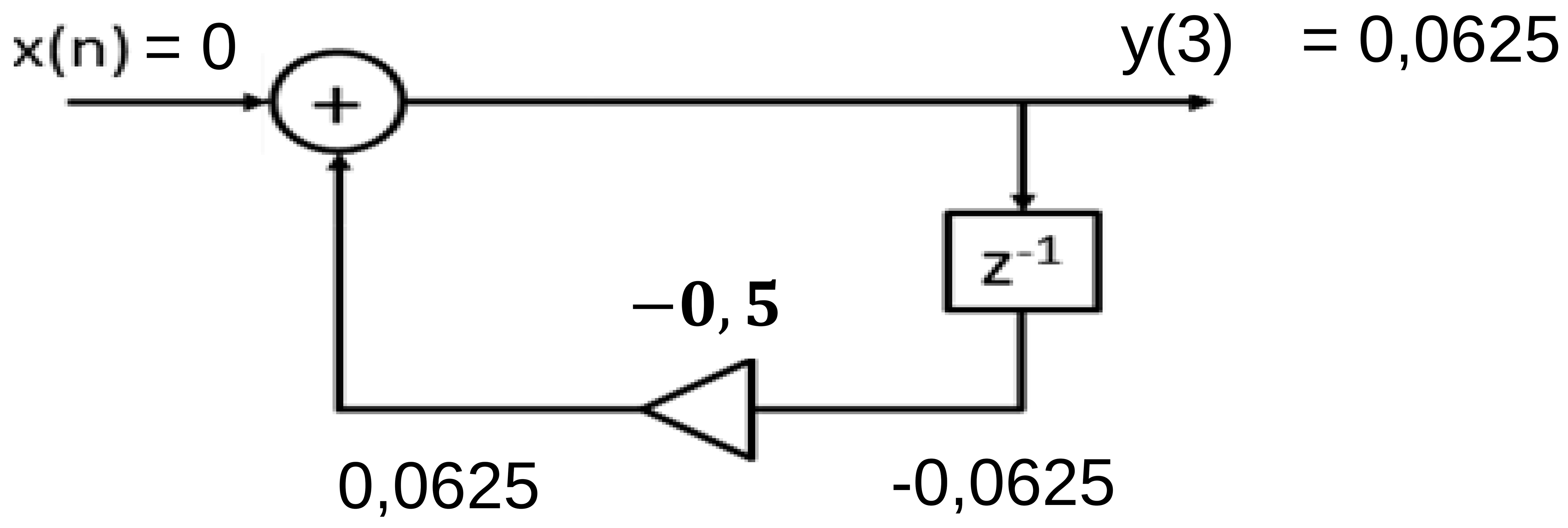

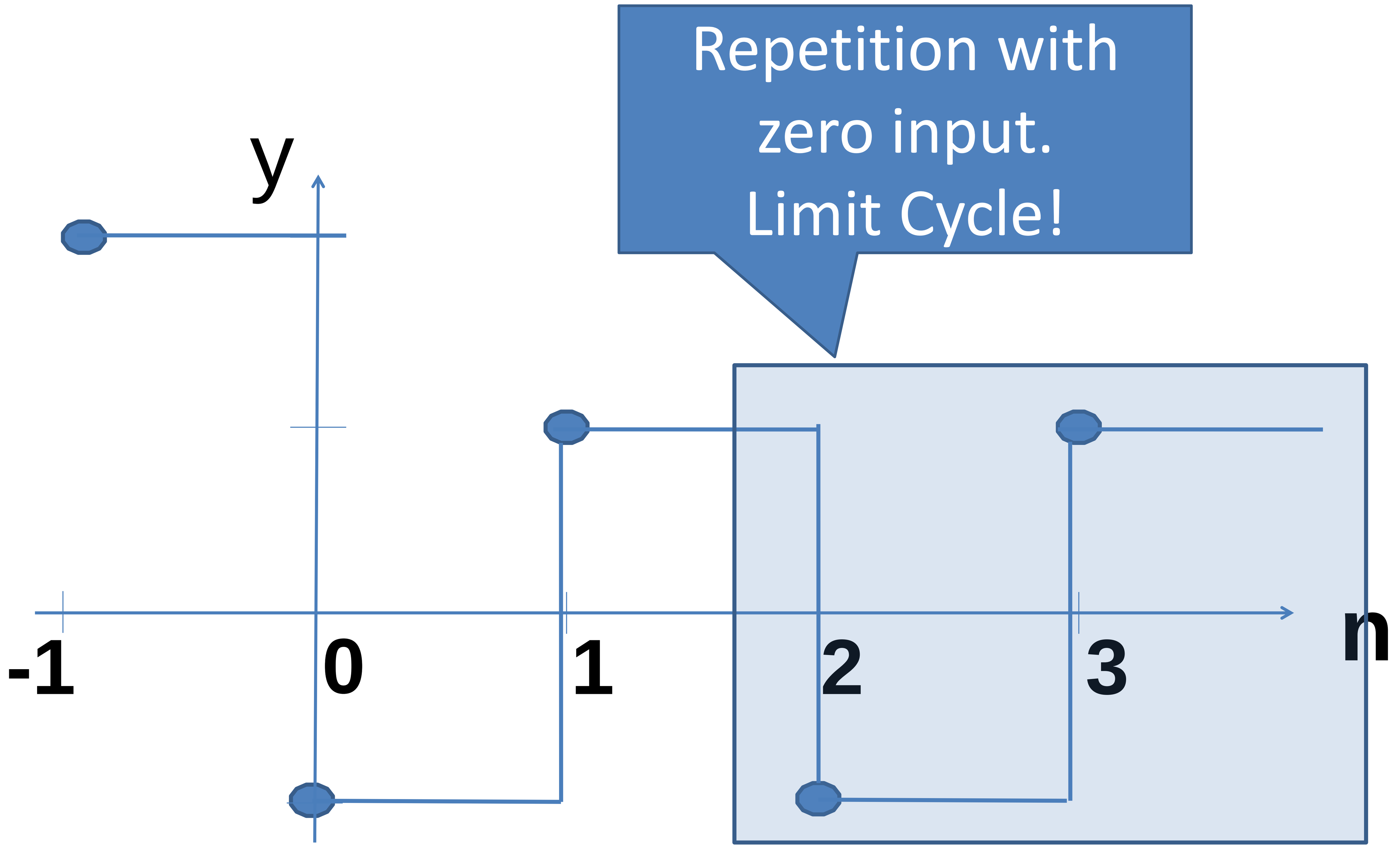

Example 3: Time Constraints Verification

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MOV.W	@r9+,r12	5 cycles
MOV.W	@r9+,r13	5 cycles
SUB.W	#4,r10	5 cycles
MOV.W	4(r10),r14	3 cycles
MOV.W	6(r10),r15	3 cycles
CALL	#__fs_mpy	5 cycles
MOV.W	r7,r14	1 cycle
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- Number of instructions depends of filter order
- Execution time will depends of plataform and word length

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Deadline

Number of cycles

Cycle time

Experimental Results

#	Filter	N	M	$\sum\lvert h_k\rvert$	Input range	Word-length	Input size	Failures	Verification time (s)		
									OF	LC	TC
1	LP-IIR	2	1	2	[-1,1]	<2,4>	6	OF, LC	39	4	<1
2	LP-Butterworth-IIR	3	3	1.2	[-1.6,1.6]	<2,5>	6	OF	579	634	<1
3	LP-IIR	3	1	4	[-1,1]	<3,4>	6	OF, LC	210	29	<1
4	LP-IIR	3	1	1.56	[-1,1]	<2,4>	6	-	110	51	<1
5	LP-FIR	1	31	1.93	[-1,1]	<2,6>	31	TC	TO	98	1
6	HP-ChebyshevI-IIR	3	3	1.33	[-1,1]	<2,10>	6	-	853	2058	<1
7	BP-Elliptic-IIR	3	3	1.24	[-1.0,1.0]	<2,10>	6	LC	546	474	<1
8	BS-Butterworth-IIR	3	3	1.81	[-1.0,1.0]	<2,8>	6	OF	106	1299	<1
9	BP-Elliptic-IIR	5	5	0.91	[-1.1,1.1]	<1,8>	10	OF, LC	7	20	<1
10	HP-Butterworth-IIR	5	5	1.58	[-1.27, 1.27]	<2,6>	10	LC	2468	1508	<1
11	BP-ChebyshevI-IIR	5	5	1.51	[-1.33, 1.33]	<2,6>	10	-	TO	TO	<1
12	HP-Elliptic-IIR	7	7	5.39	[-1,1]	<3,13>	14	TC	73	TO	<1

OF – Overflow, LC – Limit Cycle, TC – Time constraint, TO - Time out

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Experimental Results

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Different types of filters with different size were verified

OF – Overflow, LC – Limit Cycle, TC – Time constraint, TO - Time out

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Faster verification of low order filters and with smaller word-length

OF – Overflow, LC – Limit Cycle, TC – Time constraint, TO - Time out

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Fast verification of time constraints (sequential code)

OF – Overflow, LC – Limit Cycle, TC – Time constraint, TO - Time out

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Experimental Results

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OF – Overflow, LC – Limit Cycle, TC – Time constraint, TO - Time out

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Experimental Results

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Time constraint failure for high order filters

OF – Overflow, LC – Limit Cycle, TC – Time constraint, TO - Time out

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Experimental Results

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1	LP-IIR	2	1	2	[-1,1]	<2,4>	6	OF, LC	39	4	<1
2	LP-Butterworth-IIR	3	3	1.2	[-1.6,1.6]	<2,5>	6	OF	579	634	<1
3	LP-IIR	3	1	4	[-1,1]	<3,4>	6	OF, LC	210	29	<1
4	LP-IIR	3	1	1.56	[-1,1]	<2,4>	6	-	110	51	<1
5	LP-FIR	1	31	1.93	[-1,1]	<2,6>	31	TC	TO	98	1
6	HP-ChebyshevI-IIR	3	3	1.33	[-1,1]	<2,10>	6	-	853	2058	<1
7	BP-Elliptic-IIR	3	3	1.24	[-1.0,1.0]	<2,10>	6	LC	546	474	<1
8	BS-Butterworth-IIR	3	3	1.81	[-1.0,1.0]	<2,8>	6	OF	106	1299	<1
9	BP-Elliptic-IIR	5	5	0.91	[-1.1,1.1]	<1,8>	10	OF, LC	7	20	<1
10	HP-Butterworth-IIR	5	5	1.58	[-1.27, 1.27]	<2,6>	10	LC	2468	1508	<1
11	BP-ChebyshevI-IIR	5	5	1.51	[-1.33, 1.33]	<2,6>	10	-	TO	TO	<1
12	HP-Elliptic-IIR	7	7	5.39	[-1,1]	<3,13>	14	TC	73	TO	<1

OF – Overflow, LC – Limit Cycle, TC – Time constraint, TO - Time out

Experimental Results

#	Filter	N	M	$\sum\|h_k\|$	Input range	Word-length	Input size	Failures	Verification time (s)		
									OF	LC	TC
1	LP-IIR	2	1	2	[-1,1]	<2,4>	6	OF, LC	39	4	<1
2	LP-Butterworth-IIR	3	3	1.2	[-1.6,1.6]	<2,5>	6	OF	579	634	<1
3	LP-IIR	3	1	4	[-1,1]	<3,4>	6	OF, LC	210	29	<1
4	LP-IIR	3	1	1.56	[-1,1]	<2,4>	6	-	110	51	<1
5	LP-FIR	1	31	1.93	[-1,			C	TO	98	1
6	HP-ChebyshevI-IIR	3	3	1.33	[-1,			-	853	2058	<1
7	BP-Elliptic-IIR	3	3	1.24	[-1.0,			C	546	474	<1
8	BS-Butterworth-IIR	3	3	1.81	[-1.0,1.0]	<2,8>	6	OF	106	1299	<1
9	BP-Elliptic-IIR	5	5	0.91	[-1.1,1.1]	<1,8>	10	OF, LC	7	20	<1
10	HP-Butterworth-IIR	5	5	1.58	[-1.27, 1.27]	<2,6>	10	LC	2468	1508	<1
11	BP-ChebyshevI-IIR	5	5	1.51	[-1.33, 1.33]	<2,6>	10	-	TO	TO	<1
12	HP-Elliptic-IIR	7	7	5.39	[-1,1]	<3,13>	14	TC	73	TO	<1

Overflow detected even for conservative cases

OF – Overflow, LC – Limit Cycle, TC – Time constraint, TO - Time out

Experimental Results

#	Filter	N	M	$\sum\|h_k\|$	Input range	Word-length	Input size	Failures	Verification time (s)		
									OF	LC	TC
1	LP-IIR	2	1	2	[-1,1]	<2,4>	6	OF, LC	39	4	<1
2	LP-Butterworth-IIR	3	3	1.2	[-1.6,1.6]	<2,5>	6	OF	579	634	<1
3	LP-IIR	3	1	4	[-1,1]	<3,4>	6	OF, LC	210	29	<1
4	LP-IIR	3	1	1.56	[-1,1]	<2,4>	6	-	110	51	<1
5	LP-FIR	1	31	1.93	[-1,1]	<2,6>	31	TC	TO	98	1
6	HP-ChebyshevI-IIR	3	3	1.33	[-1,1]	<2,10>	6	-	853	2058	<1
7	BP-Elliptic-IIR	3	3	1.24	[-1.0,1.0]	<2,10>	6	LC	546	474	<1
8	BS-Butterworth-IIR	3	3	1.81	[-1.0,1.0]	<2,8>	6	OF	106	1299	<1
9	BP-Elliptic-IIR	5	5	0.91	[-1.1,1.1]	<1,8>	10	OF, LC	7	20	<1
10	HP-Butterworth-IIR	5	5	1.58	[-1.27, 1.27]	<2,6>	10	LC	2468	1508	<1
11	BP-ChebyshevI-IIR	5	5	1.51	[-1.33, 1.33]	<2,6>	10	-	TO	TO	<1
12	HP-Elliptic-IIR	7	7	5.39	[-1,1]	<3,13>	14	TC	73	TO	<1

OF – Overflow, LC – Limit Cycle, TC – Time constraint, TO - Time out

Experimental Results

#	Filter	N	M	∑|h$_k$|	Input range	Word-length	Input size	Failures	Verification time (s)		
									OF	LC	TC
1	LP-IIR	2	1	2	[-1,1]	<2,4>	6	OF, LC	39	4	<1
2	LP-Butterworth-IIR	3	3	1.2	[-1.6,1.6]	<2,5>	6	OF	579	634	<1
3	LP-IIR	3	1	4	[-1,1]	<3,4>	6	OF, LC	210	29	<1
4	LP-IIR	3	1	1.56	[-1,1]	<2,4>	6	-	110	51	<1
5	LP-FIR	1	31	1.93	[-1,1]	<2,6>	31	TC	TO	98	1
6	HP-ChebyshevI-IIR	3	3	1.33	[-1,1]	<2,10>	6	-	853	2058	<1
7	BP-Elliptic-IIR	3	3	1.24	[-1.0,1.0]	<2,10>	6	[illegible]	[illegible]	[illegible]	[illegible]
8	BS-Butterworth-IIR	3	3	1.81	[-1.0,1.0]	<2,8>	6	[illegible]	[illegible]	[illegible]	[illegible]
9	BP-Elliptic-IIR	5	5	0.91	[-1.1,1.1]	<1,8>	10	[illegible]	7	20	<1
10	HP-Butterworth-IIR	5	5	1.58	[-1.27, 1.27]	<2,6>	10	LC	2468	1508	<1
11	BP-ChebyshevI-IIR	5	5	1.51	[-1.33, 1.33]	<2,6>	10	-	TO	TO	<1
12	HP-Elliptic-IIR	7	7	5.39	[-1,1]	<3,13>	14	TC	73	TO	<1

Higher verification time for high order filters

OF – Overflow, LC – Limit Cycle, TC – Time constraint, TO - Time out

Experimental Results

#	Filter	N	M	$\sum\lvert h_k\rvert$	Input range	Word-length	Input size	Failures	Verification time (s)		
									OF	LC	TC
1	LP-IIR	2	1	2	[-1,1]	<2,4>	6	OF, LC	39	4	<1
2	LP-Butterworth-IIR	3	3	1.2	[-1.6,1.6]	<2,5>	6	OF	579	634	<1
3	LP-IIR	3	1	4	[-1,1]	<3,4>	6	OF, LC	210	29	<1
4	LP-IIR	3	1	1.56	[-1,1]	<2,4>	6	-	110	51	<1
5	LP-FIR	1	31	1.93	[-1,1]	<2,6>	31	TC	TO	98	1
6	HP-ChebyshevI-IIR	3	3	1.33	[-1,1]	<2,10>	6	-	853	2058	<1
7	BP-Elliptic-IIR	3	3	1.24	[-1.0,1.0]	<2,10>	6	LC	546	474	<1
8	BS-Butterworth-IIR	3	3	1.81	[-1.0,1.0]	<2,8>	6	OF	106	1299	<1
9	BP-Elliptic-IIR	5	5	0.91	[-1.1,1.1]	<1,8>	10	OF, LC	7	20	<1
10	HP-Butterworth-IIR	5	5	1.58	[-1.27, 1.27]	<2,6>	10	LC	2468	1508	<1
11	BP-ChebyshevI-IIR	5	5	1.51	[-1.33, 1.33]	<2,6>	10	-	TO	TO	<1
12	HP-Elliptic-IIR	7	7	5.39	[-1,1]	<3,13>	14	TC	73	TO	<1

OF – Overflow, LC – Limit Cycle, TC – Time constraint, TO - Time out